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NONLOCALITY AND SUPERLUMINOSITY

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In a series of interesting papers G. C. Hegerfeldt has shown that quantum systems with positive energy initially localized in a finite region, immediately develop infinite tails. In our paper Hegerfeldt's theorem is analyzed using quantum and classical wave packets. We show that Hegerfeldt's conclusion remains valid in classical physics. No violation of Einstein's causality is ever involved. Using only positive frequencies, complex relativistic wave packets are constructed which at $t = 0$ are real and localized. We show that they are superpositions of two nonlocal wave packets. The nonlocality is initially cancelled by destructive interference. However this cancellation becomes incomplete at arbitrary times immediately afterwards. In agreement with relativity the two nonlocal wave packets move with the velocity of light, in opposite directions.

We also consider the dressing process of an atom interacting with a scalar field («photon»). The bare particle is localized while the dressed particle is delocalized. Again the Hegerfeldt's theorem applies.

1. INTRODUCTION

I am greatly honoured to receive the Bogolyubov Prize. I have always admired the scope and the depth of Professor Bogolyubov's work. He has influenced greatly the work which we did on nonequilibrium statistical mechanics in Brussels. I will tell an anecdote. Professor Bogolyubov was visiting Brussels many years ago, perhaps 20 or 30 years ago. I was astonished that he spoke very fluently French. He said to me that he worked with Krylov, and Krylov would speak about science only in French. He said that French is the language of science. This belongs probably to the past. So I shall use as the other people at this conference my broken English.

I want to give a summary of some recent work done in collaboration with Karpov, Ordonez, Petrosky, and Pronko. Of course, Karpov and Pronko are Russian. The work about which I shall speak is still in progress. We have just received the news that the first publication in this direction will appear in Physical Review. But I want also to include some more recent results which are still unpublished.

Are there deviations from Einstein's causality? G. C. Hegerfeldt has written [1]: «Positivity of the Hamiltonian alone is used to show that particles, if initially localized in a finite region, immediately develop infinite tails». This seems to imply superluminality. One of his examples is the Fermi problem [2] of two atoms coupled by a radiation field. Consider the initial condition when one of the atoms is in an excited state, the other in the ground state, and no photons are present. The probability to find the second atom in an excited state is non-vanishing immediately after the initial moment, independently of the distance between the atoms [3]. Hegerfeldt's arguments are based on the analyticity of the expectation values of the operator $N(V)$, which gives the probability to find a particle inside a finite volume V. He showed that a state in a quantum system with positive energy localized in a finite volume V at the instant $t = 0$, will develop infinite tails immediately afterwards. Positivity of energy plays an essential role in his proof. In this paper we present illustrations of Hegerfeldt's theorem, without any appeal to superluminality. We first apply Hegerfeldt's consideration to wave packets. Moreover, we show that Hegerfeldt's effect appears even for classical fields if wave packets are constructed from positive frequencies (corresponding positive energy quantum fields). We first study the positive-frequency solutions of the classical wave equation. We consider wave packets $\Phi(x,t)$ localized at $t = 0$. We shall show that the localization is due to the interference of two complex solutions, each propagating causally

$$\Phi(x,t) = \Psi(x-t) + \Psi^*(x+t). \tag{1}$$

Here «*» denotes complex conjugation and we take $c = 1$. We show that both these wave packets are delocalized. They present long tails extending to arbitrary distances and decay according to a power law.

We start from the wave equation on the real line ($c = 1$).

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2}\right)\Phi(x,t) = 0. \tag{2}$$

The general complex solution of Eq. (2) is, by the Fourier transform, of the form

$$\Phi(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk \left\{\Phi_+(k)e^{-i\omega_k t} + \Phi_-(k)e^{i\omega_k t}\right\} e^{ikx}, \tag{3}$$

where $\omega_k = |k|$ and $\Phi_\pm(k)$ are arbitrary functions. We consider the special class of positive-frequency solutions to Eq. (2), i.e., $\Phi_-(k) \equiv 0$ and

$$\Phi_+(x,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, \Phi_+(k)e^{-i\omega_k t}e^{ikx}. \tag{4}$$

The positive-frequency solutions are determined by the initial condition $\Phi(x,0)$. Note that relation (4) leads to a complex field for $t \neq 0$, even if $\Phi_+(x,0)$ or $\Phi_+(k)$ are real.

Consider as an example a localized (rectangular) wave packet with centre x_0 and width 2b at time $t = 0$:

$$\Phi_{x_0,b} = (1/2b)\Theta(b - |x - x_0|). \tag{5}$$

The normalization has been chosen so that the integral of this function over x is equal to one. Then the function $\Phi(k)$ is:

$$\Phi_+(k) = \frac{1}{2\pi}\int_{-\infty}^{\infty} dk\, e^{-i\omega_k t}\Theta(b - |x - x_0|), \tag{6}$$

where $\Theta(b - |x - x_0|)$ is the step function, which is 0 for x negative, and 1 for x positive. Then the function $\Phi(x,t)$ in (4) is given by

$$\Phi(x,t) = \frac{1}{4\pi b}\int_{-\infty}^{\infty} dk \int_{x_0-b}^{x_0+b} dx' e^{-i|k|t+ik(x-x')}. \tag{7}$$

In agreement with (1) this is a sum of two functions corresponding to two wave packets moving in opposite directions,

$$\Phi(x,t) = \Psi(x - t) + \Psi^*(x + t), \tag{8}$$

where

$$\Psi(x) = \frac{1}{4\pi b}\int_{x_0-b}^{x_0+b} dx' \int_{0}^{\infty} dk\, e^{ik(x-x')}. \tag{9}$$

To evaluate the integral over k we introduce the usual regularisation by adding a positive infinitesimal to x, which leads to

$$\Psi(x) = \frac{1}{4\pi bi}\int_{x_0-b}^{x_0+b} \frac{dx'}{x - x' + i0}. \tag{10}$$

After integration over x' we obtain:

$$\Psi(x) = (i/4\pi b)\Big[\ln(x - x_0 + b + i0) - \ln(x - x_0 - b + i0)\Big]. \tag{11}$$

The logarithm of a complex number is given by

$$\ln(z) = \ln|z| + i(\arg(z) + 2\pi n), \tag{12}$$

where n is an integer. In order to have both terms in (11) on the same branch of the logarithm we take $n = 0$ for both of them (due to the difference of the two terms in (11) the result does not depend on the particular value of n). The argument of $x + i0$ can be expressed as

$$\arg(x + i0) = (\pi/2)(1 - \mathrm{sign}(x)), \tag{13}$$

where $\mathrm{sign}(x) = x/|x|$ is the sign of x. Then, inserting (12) and (13) into (11) we obtain

$$\Psi(x)=\frac{1}{8b}\Big(\mathrm{sign}(x-x_0+b)-\mathrm{sign}(x-x_0-b)\Big)+\frac{i}{4\pi b}\ln\left|\frac{x-x_0+b}{x-x_0-b}\right|. \tag{14}$$

We see that the function $\Psi(x)$ in (14) consists of a local real part (sign) and a nonlocal imaginary part (log). For $t \neq 0$ it is sufficient to replace x by $x - t$ in (8). A similar result is obtained for $\Psi^*(x+t)$. As a result, the function $\Phi(x+t)$ is also nonlocal because it is the superposition of the two complex functions $\Psi(x-t)$ and $\Psi^*(x+t)$ in (8), which describe nonlocal objects moving with the speed of the light in opposite directions. However, at $t = 0$ the imaginary parts cancel each other (see Fig.1), and we recover our localized initial condition (5), because only the real parts of these functions, which are local, remain. In all our figures time t is measured in seconds (s), the coordinate x is measured in «light second» (ls) and wave packet amplitudes are dimensionless.

At $t = 1s$ in Fig. 2, the overlapping is small and we have

$$|\Phi(x,t)| \approx |\Psi(x-t)| + |\Psi^*(x+t)|. \tag{15}$$

We see that the initial condition $\Phi(x,0)$ is local (Fig. 1) only because at $t = 0$ the nonlocal parts cancel each other by destructive interference. We may describe the appearance of nonlocality as a sort of «curtain effect». The nonlocal nature of each wave packet $\Psi(x-t)$ and $\Psi^*(x+t)$ is hidden behind a «curtain» at the initial time and emerges immediately afterwards. Each of the nonlocal wave packets is complex and propagates at the speed of light. We see that the localization of wave packets corresponding to positive frequency is unstable and involves complex space structures.

I mention only that the same results are obtained in relativistic quantum field theory. I want also to emphasize the analogy with EPR. The two waves which are formed are still correlated at arbitrary distance. In principle if you would measure one, you could predict that some observer sitting at arbitrary distance would find the other.

2. THE DYNAMICS OF DRESSING

Let us consider the well-known Friedrichs model. In this model we have a discrete state $|1\rangle$ representing a bare particle coupled to continuous states $|k\rangle$

corresponding to field modes [4]. The Hamiltonian is

$$\begin{aligned} H &= H_0 + \lambda V \\ &= |1\rangle\omega_1\langle 1| + \sum_k |k\rangle\omega_k\langle k| \\ &+ \lambda \sum_k V_k \left(|k\rangle\langle 1|\rangle + |1\rangle\langle k|\rangle\right). \end{aligned} \tag{16}$$

We will assume that

$$v_k = V_{-k} = \text{real} \tag{17}$$

and that the dispersion relation for the field modes (or «photons») is

$$\omega_k = c|k| \tag{18}$$

with $c = 1$.

We consider a one-dimensional system enclosed in a box of size L with usual periodic boundary conditions. We will consider the limit $L \to \infty$ where the spectrum of photon energies is continuous.

We further assume that

$$V_k \sim L^{1/2} \tag{19}$$

which corresponds to the usual volume dependence of matter-field interactions.

We may interpret our model as a simplified version of a two-level atom interacting with radiation. In this interpretation $|1\rangle$ represents the atom in its bare excited level, and no photons present, and $|k\rangle$ represents the atom in its ground level, together with a photon of momentum k. The interaction λV induces transitions from $|1\rangle$ to $|k\rangle$ or vice versa. We will often refer to $|1\rangle$ as the «bare particle», with the understanding that this particle is in an excited state, virtual processes are neglected.

We shall now consider in this simple case the dynamics of dressing and relate it to Hegerfeldt's theorem. We shall consider the simplest possible case which corresponds to the integrable Friedrichs model.

Let us consider the eigen states of the Hamiltonian. For this case we obtain

$$H|\phi_1\rangle = \tilde{\omega}_1|\phi_1\rangle. \tag{20}$$

The exact expression for $|\phi_1\rangle$ can be shown to be [4].

$$|\phi_1\rangle = N_1^{1/2}\left[|1\rangle - \sum_k \frac{\lambda V}{\omega_k - \tilde{\omega}_1}|k\rangle\right], \tag{21}$$

where N_1 is a normalization constant and the perturbed energy $\tilde{\omega}_1$ is the solution of the usual dispersion equation. Note that $|\phi_1\rangle$ has a cloud and a long tail. In other words, it decays in space in a polynomial way. So we have on one side, a localized unperturbed state and an exact state which has a long tail. Now we can consider the transformation of the localized state to the dressed state. This occurs in a way reminding very much the conclusion of paragraph 1. Indeed the bare state can be considered as a superposition of the virtual photon cloud corresponding to the dressing and interfering negatively with a compensating cloud. But this situation is not stable. The compensating cloud is dividing in two and escaping while the dressing remains. We have again a kind of curtain effect. The compensating cloud carries away the excess energy of the bare state. We remain with the dressed state (21). The existence and the motion of the compensating cloud has been verified by numerical simulations. It is an irreversible process in the sense that by taking away the excess energy we go to a more stable situation. The main point here is of course that we have a transition from a localized state into a delocalized state. The moving cloud can be observed at arbitrary distances. It is interesting to see that the dressing of the atom can be related to Hegerfeldt's theorem.

Let me indicate briefly a last example. That belongs to scattering theory. We send a photon wave packet on an excitable atom but the wave packet as we have seen is unstable. It leads to long tails. Therefore there will be interactions before the wave front of the photon wave packet reaches the particle. This curious result has also been verified by numerical simulation.

3. CONCLUDING REMARKS

Nonlocal structures simulate to some extent a superluminosity. The superluminosity in wave guide or tunneling has been observed by Enders and Nimtz [5]. We hope that our theory can be applied to their observations. Anyway this result leads to new questions in special relativity. There the description is in terms of points. But delocalized structures act more like a rigid body. This requires a reconsideration of synchronization.

I want to thank my colleagues Professors E. Karpov, G. Pronko, T. Petrosky, and G. Ordonez for their participation in this work. The main ideas are their.

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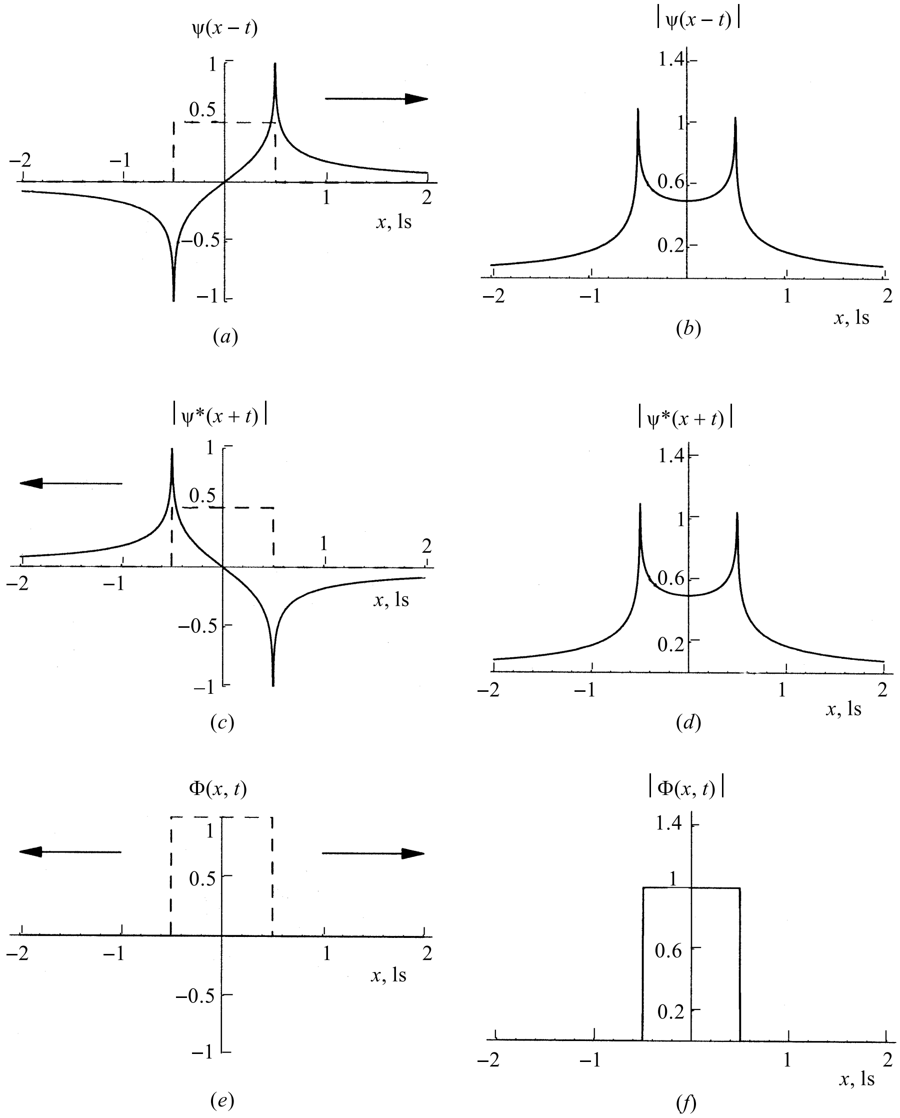

Fig. 1. The real part (dashed lines) and the imaginary part (solid lines) of $\Psi(x-t)$ (*a*), $\Psi^*(x+t)$ (*c*), and $\Phi(x,t) = \Psi(x-t) + \Psi^*(x+t)$ (*e*) as functions of x at $t = 0$; the absolute values $|\Psi(x-t)|$ (*b*), $|\Psi^*(x+t)|$ (*d*), and $|\Phi(x,t)| \neq |\Psi(x-t)| + |\Psi^*(x+t)|$ (*f*) as functions of x at $t = 0$

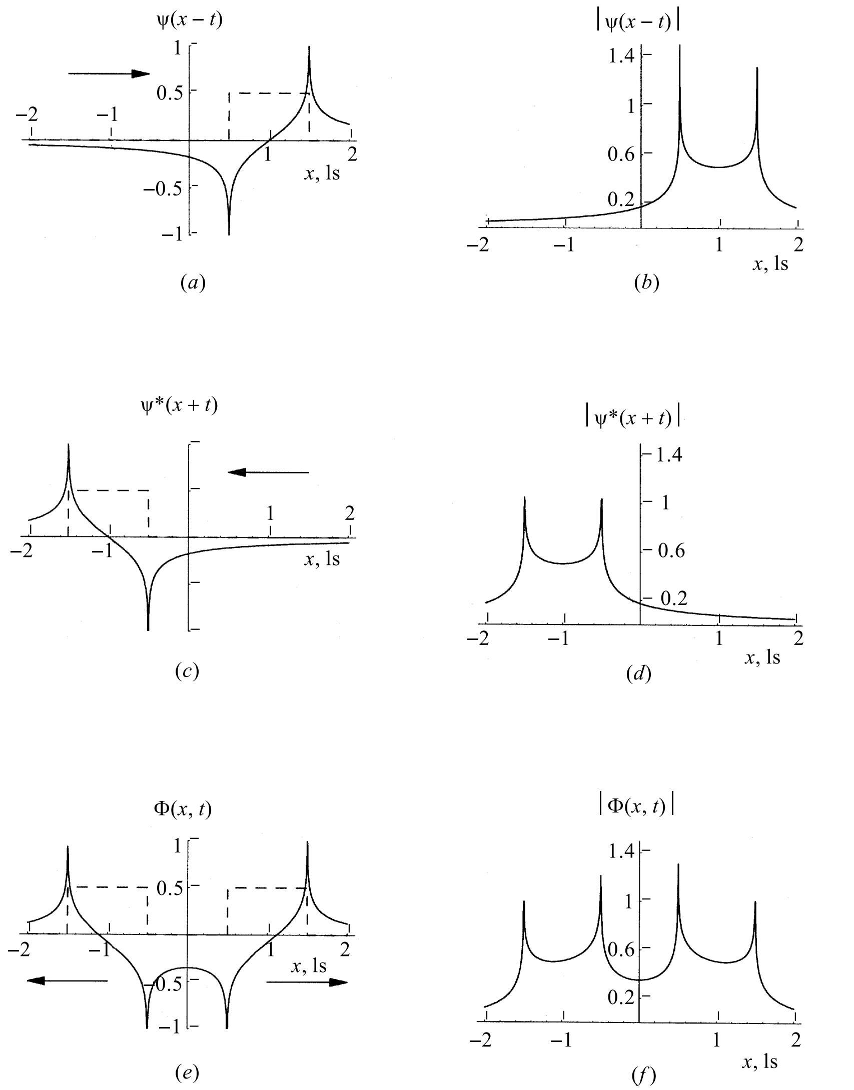

Fig. 2. The real part (dashed lines) and the imaginary part (solid lines) of $\Psi(x-t)$ (*a*), $\Psi^*(x+t)$ (*c*), and $\Phi(x,t) = \Psi(x-t) + \Psi^*(x+t)$ (*e*) as functions of x at $t = 1s$; the absolute values $|\Psi(x-t)|$ (*b*), $|\Psi^*(x+t)|$ (*d*), and $|\Phi(x,t)| \neq |\Psi(x-t)| + |\Psi^*(x+t)|$ (*f*) as functions of x at $t = 1s$

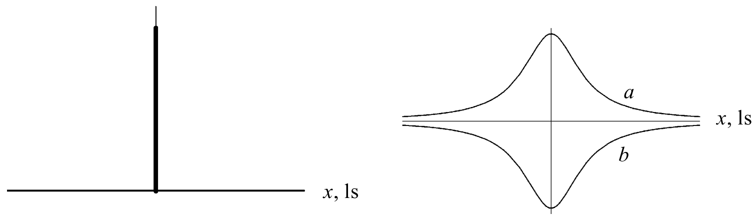

Fig. 3. Localized bare state (left), Virtual photon dressing (right) (*a*) and Compensating photons (right) (*b*)

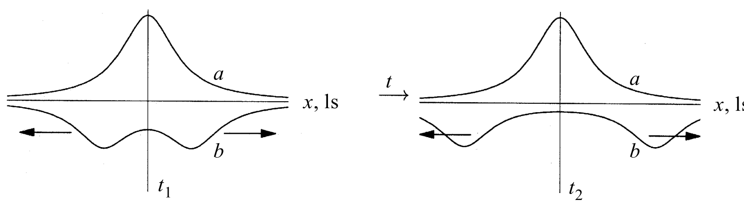

Fig. 4. Sticking cloud (*a*), Photon escaping (*b*), $t_1 < t_2$

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