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BOGOLIUBOV GROUP VARIABLES IN RELATIVISTIC MODELS

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Poincare-invariant systems are considered. Quantization is made in the presence of two-periodic classical field. Using of Bogoliubov group variables permits one to develop scheme of perturbation theory taking into account conservations laws.

The problem of accurate account of conservation laws in quantum field theory when methods of approximate calculations are used has been solved by N.N.Bogoliubov in 1950.

The main idea of Bogoliubov method consists in transformation which separates new values from field variables. Those variables have sence of parameters of invariance group for the system Hamiltonian.

New variables turn out to be cyclic. And derivatives with respect to new variables coincided with system momentum, so operators of Hamiltonian and momentum commuted and momentum conservation law was accomplished at once.

Nowadays there is enough quantity of works that specialize and develope the original idea. All of them underline essencial role of Hamiltonian, however they are not applicalbe for the accurate account of conservation laws connected with Poincare invariance for example, because Hamiltonian structure as generator of time translations becomes clear only after solution of equation of motion, so the task was a rather indefinite.

In the present talk we hope to fill up the gap keeping efficiency of original method.

We define group variables together with developing of perturbation theory, and to specify formulae of variables substitutions step by step, in accordance with forthcoming to accurate solutions of field equations.

Let's variables $x^{'}$ are connected with x by Poincare transformation:

$$x^{\alpha} = A^{\alpha}_{\beta}(\phi)x^{'\beta} + \tau^{\beta}.$$

We define Bogoliubov transformation as following:

$$f(x) = gv(x^{'}) + u(x^{'}),$$

dimensionless parameter g is assumed to be large, and (ϕ, τ) are independent new variables.

The problem is how to formulate invariant conditions, which we have to impose on functions $u(x^{'})$. The substitution $f(x) \rightarrow \{u, (\phi, \tau)\}$ enlarge the number of independent variables, so those conditions are necessary.

We consider systems in which there are Poincare-invariant symplectic forms that look like the following:

$$\omega(f(x), g(x)) = \int_D (f_n(x)g(x) - f(x)g_n(x))dx,$$

here D is some space-like surface and $f_n(x)$ means normal derivative of $f(x)$ in the surface D.

We chose some functions $N^a(x^{'})$ (a is the number of group parameters) and impose on function $u(x)$ the following conditions:

$$\omega(N^a, u) = 0.$$

Using this condition one can obtain equations, which define group variables as functional of $f(x)$ and $f_n(x)$ on the D in the differential form:

$$\frac{\delta\tau^a}{\delta f(x)} = -\frac{1}{g} Q^a_b \tilde{N}^b_n(x^{'}), \qquad \frac{\delta\tau^a}{\delta f_n(x)} = \frac{1}{g} Q^a_b \tilde{N}^b(x^{'}),$$

where Q^a_b are the solution of the equation:

$$Q^a_b = \delta^a_b - \frac{1}{g} R^a_c Q^c_b.$$

Here $\tilde{N}^a$ is a linear combination of N^a, such that the equation $\omega(\tilde{N}^a, v_b) = 0$ holds true; and R^a_c is a c-number, calculated with the help of $v(x^{'})$ and $u(x^{'})$.

It is possible to define on D operators $\hat{q}(x)$ and $\hat{p}(x)$:

$$\hat{p}(x) = \frac{1}{\sqrt{2}}\Big(f_n(x) + i\frac{\delta}{\delta f(x)}\Big), \qquad \hat{q}(x) = \frac{1}{\sqrt{2}}\Big(f(x) - i\frac{\delta}{\delta f_n(x)}\Big).$$

They are hermitian in the space following scalar product:

$$< F_1|F_2 >= \int Df\, Df_n\, F_1\,[f, f_n] F_2[f, f_n]$$

and satisfy the formal commutation relation:

$$[\hat{q}(x), \hat{p}(x^{'})] = i\delta(x - x^{'}).$$

So we can treat $\hat{q}(x)$ and $\hat{p}(x)$ as operators of coordinate and momentum of oscillators of field and we can develop the secondary quantization scheme. However there is another pair of selfconjugated operators which satisfy the same commutation relations. So the number of possible field states turns out to be doubled, so we have to reduce the number of possible field states. The following scheme is proposed: we use Bogoliubov transformation and, in spite of appearance of exceed states, we will develop scheme of perturbation theory. After that reduction of states number is made, so it will depend on dynamic system equations.

Now we can quantize and substitute $f(x^{'})$, and $f_n(x^{'})$ as following:

$$f(x^{'}) \longrightarrow \hat{q}(x), \qquad f_n(x^{'}) \longrightarrow \hat{p}(x).$$

In the terms of new variables $\hat{q}(x)$ and $\hat{p}(x)$ are the series with respect to inverted powers of coupling constant. Hence integrals of motion of the system can be represented as series with respect to inverted powers of coupling constant:

$$O = g^2 O_{-2} + gO_{-1} + O_0 + \frac{1}{g} O_1 + ...$$

In this series operators O_{-2} are C-numbers and operators O_{-1} are linear with respect to $u(x^{'})$, $u_n(x^{'})$, $\partial/\partial u(x^{'})$, $\partial/\partial u_n(x^{'})$. There are not normalizible eigenvectors of these operators, so it is required to set them to zero for perturbation theory construction. Let's explore if it is possible. Suppose that some boundary conditions are accomplished, and the following equation is perfomed:

$$\Box F(x^{'}) + V^{'}(F) = 0,$$

then operators O_{-1} are equal to zero.

Here $F(x^{'})$ is connected linearly with a classical component $v(x^{'})$.

Hereinafter we assume $F(x)$ to be the solution of the Cauchy problem with given data on D.

As we have mentioned above the number of possible field states is doubled over against real situation. That is why states number reduction is necessary.

Primarily let's analyze the number of independent variables. Original number of independent variables was ∞. After defining the Bogoliubov group variables (they are considered to be independent) the number became $\infty + 10$. This number was doubled due to determination of creation-annihilation operators: $(\infty{+}10){*}2 = 2 * \infty + 20$. Additional conditions reduced the number of independent variables on 10, that is at present time the number of possible field states is $2 * \infty + 10$.

Let's separate from field variables $u(x^{'})$ ten variables r_a which have no any physical sense and are connected with the method of perturbation scheme realization. Then the state number is $2 * \infty$, and field is described via $w(x^{'})$ variables which are determined as the following:

$$u(x^{'}) = w(x^{'}) + \tilde{N}^a(x^{'}) r_a, \qquad u_n(x^{'}) = w_n(x^{'}) + \tilde{N}^a_n(x^{'}) r_a.$$

Necessary reduction of the states number can be made by the following way: let's suppose that field condition is defined by functionals of $w(x^{'})$ and $w_n(x^{'})$, in which $\delta/\delta w(x^{'})$ and $\delta/\delta w_n(x^{'})$ become following ones:

$$\frac{\delta}{\delta w(x^{'})} \longrightarrow \frac{\delta}{\delta w(x^{'})} - i w_n(x^{'}), \qquad \frac{\delta}{\delta w_n(x^{'})} \longrightarrow -i w(x^{'}).$$

After reduction independent variables become:

(10 group parameters)+(10 variables r_a)+
+($\infty - 10$–dimensioned function w space).

The variables r_a have not physical sense. They have appeared as a rest of the state space reduction in terms of Bogoliubov group variables. Separation of these variables is connected with integrals of motion structure in the zero-point order, so it is dynamic by nature.

After reduction of the number of independent variables integrals of motion at the zero-point order with respect to g are:

$$O_0 = i\Im^{\alpha}\frac{\partial}{\partial\tau^{\alpha}}.$$

So completely relativistic-invariant description of the quantum system with nonzero classical component has been given.

All calculations have been done in the zero-point order. The higher order calculations can be done the same way.

Proposed scheme can be developed for the concrete system with a given interaction if the system allows symplectic structure, for example systems of interacting fields, general relativity and so on.

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