*ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА*
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# FURTHER REMARKS ON ELECTROWEAK MOMENTS OF BARYONS AND MANIFESTATIONS OF BROKEN $SU(3)$ SYMMETRY

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The role of nonvalence, e.g., sea quarks and/or meson degrees of freedom in static and quasistatic baryon electroweak observables, is discussed within the phenomenological sum rule approach. The inclusion of nonvalence degrees of freedom in the analysis of baryon magnetic moments is shown to explain extremely strong violation of the standard $SU(6)$-symmetry based quark-model prediction for the magnetic moment ratio $R_{\Sigma/\Lambda} = (\Sigma^+ + 2\Sigma^-)/(-\Lambda) \simeq 0.23$, while the value $R_{\Sigma/\Lambda}(SU(6)) = 1$ corresponds to the nonrelativistic quark model. We also obtain $F/D = 0.72$ for the quark-current-baryon couplings $SU(3)_f$ ratio. The implications for «strangeness» magnetism of the nucleon, and for the weak axial-to-vector coupling constant relations measured in the lowest octet baryon $\beta$ decays are discussed.

В рамках подхода, основанного на феноменологических правилах сумм, обсуждается влияние невалентных степеней свободы, т. е. морских кварков и/или мезонов, на статические и квазистатические электрослабые моменты барионов. Учет невалентных степеней свободы при анализе магнитных моментов барионов объясняет чрезвычайно сильное нарушение предсказания стандартной модели кварков, основанной на использовании $SU(6)$-симметрии, для отношения магнитных моментов $R_{\Sigma/\Lambda} = (\Sigma^+ + 2\Sigma^-)/(-\Lambda) \simeq 0{,}23$, тогда как в нерелятивистской модели кварков получается значение $R_{\Sigma/\Lambda}(SU(6)) = 1$. Получена также величина $F/D = 0{,}72$ для отношения констант связи тока кварков с барионами в $SU(3)_f$-симметрии. В связи с этим обсуждаются следствия для вкладов странных кварков в магнитные моменты нуклонов и для отношений аксиальных и векторных констант, получаемых из анализа $\beta$-распадов основного состояния октета барионов. Показано, что при извлечении величины отношений $(g_1/f_1)$-констант из данных по $\beta$-распадам гиперонов следует учитывать в анализе вклады от токов второго рода, т. е. индуцированные нарушениями симметрии псевдотензорные формфакторы, или, иначе, формфакторы типа так называемого «слабого электричества».

**1.** In this report we present some further consequences from sum rules for the static electroweak characteristics of baryons following mainly from the phenomenology of broken internal symmetries. The phenomenological sum rule technique was chosen to obtain a more reliable, though not very much detailed information about the hadron properties in question. The main focus was laid on the role of nonvalence degrees of freedom (the nucleon sea partons and/or peripheral meson currents) in parameterization and description of hadron magnetic moments and axial-vector coupling constants.

As is known, in the broken $SU(3)$-symmetry approach, based on the nonrelativistic quark model (NRQM) of the ground state octet baryons [1], where $B \leftrightarrow 2q_{\rm even} + q_{\rm odd}$, and the magnetic moments of constituent quarks in the corresponding baryons $B = \{P, N; \Sigma^{\pm}; \Xi^{0,-}; \Lambda\}$ satisfy the relation $\mu(u) : \mu(d) : \mu(s) = -2 : 1 : (m_d/m_s)$, one obtains the familiar expressions for magnetic moments

$$\begin{aligned} \mu(B) \equiv B &= (4/3)q_e - (1/3)q_0, \\ \Lambda &= s, \\ \mu(\Lambda\Sigma^0) &= (1/\sqrt{3})(u - d) \end{aligned} \tag{1}$$

(herewith, we use the particle and quark symbols for the corresponding magnetic moments). The most spectacular difficulty of the above parameterization is seen from comparing two ratios $R_{\Sigma/\Lambda}$ [2] and $R_{\Xi/\Lambda}$ with experimental values [3] — the first one is drastically broken, while both should be valid in the NRQM:

$$\begin{aligned} R_{\Sigma/\Lambda} &= \frac{\Sigma^+ + 2\Sigma^-}{-\Lambda} = \frac{s(\Sigma)}{s(\Lambda)} = 1 \text{ vs } 0.23 \text{ [3]}, \\ R_{\Xi/\Lambda} &= \frac{\Xi^0 + 2\Xi^-}{4\Lambda} = \frac{s(\Xi)}{s(\Lambda)} = 1 \text{ vs } 1.04 \text{ [3]}. \end{aligned} \tag{2}$$

Earlier we considered a number of consequences of sum rules for the static electroweak characteristics of baryons following from the theory of broken internal symmetries and common features of the quark models including corrections due to nonvalence degrees of freedom — the sea partons and/or the meson clouds at the periphery of baryons and no assumptions referring to the nonrelativistic quark dynamics were made.

Here, we list some of the earlier discussed [4–7] sum rules (we use the particle and quark symbols for the corresponding magnetic moments):

$$\alpha_D = \left.\frac{D}{F + D}\right|_{\rm mag} = \frac{1}{2}\left(1 - \frac{\Xi^0 - \Xi^-}{\Sigma^+ - \Sigma^- - \Xi^0 + \Xi^-}\right). \tag{3}$$

The $D$ and $F$ constants in Eq. (3) parameterize the «reduced» matrix elements of the quark current operators where $SU(3)$-symmetry-breaking effects are contained in the factorized effective coupling constants of the single-quark-type operators, while other contributions (e.g., representing the pion exchange current effects) are cancelled in all sum rules by construction. The ratio $u/d \neq -2$

$$\frac{u}{d} = \frac{\Sigma^+(\Sigma^+ - \Sigma^-) - \Xi^0(\Xi^0 - \Xi^-)}{\Sigma^-(\Sigma^+ - \Sigma^-) - \Xi^-(\Xi^0 - \Xi^-)} \tag{4}$$

is related to the chiral constituent quark model where a given baryon consists of three «dressed» massive constituent quarks. Owing to the virtual transitions

$q \leftrightarrow q + \pi(\eta), q \leftrightarrow K + s$, the «magnetic anomaly» is developing, i.e., $u/d = -1.80 \pm 0.02 \neq Q_u/Q_d = -2$.

The ratio $s/d \simeq 0.64$ demonstrating the $SU(3)$-symmetry breaking is evaluated via

$$\frac{s}{d} = \frac{\Sigma^+\Xi^- - \Sigma^-\Xi^0}{\Sigma^-(\Sigma^+ - \Sigma^-) - \Xi^-(\Xi^0 - \Xi^-)}. \tag{5}$$

Now, we list some consequences of the obtained sum rules. The numerical relevance of the adopted parameterization is seen from the results enabling even estimation from one of the obtained sum rules, namely,

$$\begin{gathered}(\Sigma^+ - \Sigma^-)(\Sigma^+ + \Sigma^- - 6\Lambda + 2\Xi^0 + 2\Xi^-) - \\ -(\Xi^0 - \Xi^-)(\Sigma^+ + \Sigma^- + 6\Lambda - 4\Xi^0 - 4\Xi^-) = 0,\end{gathered} \tag{6}$$

the necessary effect of the isospin-violating $\Sigma^0\Lambda$-mixing. By definition, the $\Lambda$ value entering into Eq. (6) should be «refined» from the electromagnetic $\Lambda\Sigma^0$ mixing affecting $\mu(\Lambda)_{\rm exp}$. Hence, the numerical value of $\Lambda$, extracted from Eq. (6), can be used to determine the $\Lambda\Sigma^0$-mixing angle through the relation

$$\sin\theta_{\Lambda\Sigma} \simeq \theta_{\Lambda\Sigma} = \frac{\Lambda - \Lambda_{\rm exp}}{2\mu(\Lambda\Sigma)} = (1.43 \pm 0.31) \cdot 10^{-2} \tag{7}$$

in accord with the independent estimate of $\theta_{\Lambda\Sigma}$ from the electromagnetic mass-splitting sum rule [8].

Naturally, our approach is free of the disbalance problem exemplified in Eqs. (2). With the parameters $u/d = -1.80$ and $\alpha_D = (D/(F+D))_{\rm mag} = 0.58$, defined without including the $\Lambda$-hyperon magnetic moment in fit and taking into account the $\Sigma^0 - \Lambda$ mixing, we obtain $R_{\Sigma/\Lambda} \simeq 0.27$ and $R_{\Xi/\Lambda} \simeq 1.13$, which turn out to be in excellent accord with the data if one takes also into account in Eqs. (2) the $\Lambda$ value corrected for mixing: $\Lambda_0 \simeq -0.567$ n.m. For further use, we also list below the limiting relations following from the neglect of the nonvalence degrees of freedom

$$\begin{gathered}\Sigma^+[\Sigma^-] = P[-P-N] + \left(\Lambda - \frac{N}{2}\right)\left(1 + \frac{2N}{P}\right), \\ \Xi^0[\Xi^-] = N[-P-N] + 2\left(\Lambda - \frac{N}{2}\right)\left(1 + \frac{N}{2P}\right), \\ \mu(\Lambda\Sigma) = -\frac{\sqrt{3}}{2}N.\end{gathered} \tag{8}$$

We stress that neither NR assumption nor explicit $SU(6)$-wave function are used this time. In this case the ratio $F/D = 0.64$ and it is definitely less than

$F/D = 0.72$, when nonvalence degrees of freedom are included. This is the demonstration of substantial influence of the nonvalence degrees of freedom on this important parameter.

**2.** One can note that the accordance of the ratios $R_{\Sigma/\Lambda,\Xi/\Lambda}$ with the data is valid in two, seemingly dual, parameterizations of the baryon magnetic moments. The first is specified by the renormalization of the constituent quark characteristics by the meson current effects resulting in $u/d \neq -2$, etc. However, one can follow a complementary view of the nucleon structure, keeping the constraint $u/d = -2$, and the OZI-rule violating the contribution of sea quarks parameterized as $\Delta(N) = \sum_{q=u,d,s} \mu(q)\langle N|\bar{s}s|N\rangle \neq 0$.

We have referred to this approach [5] as a correlated current-quark picture of nucleons and made use of it to estimate the contributions of the sea quarks to baryon magnetic moments. In particular, the following important sum rules were obtained (all quantities are in n.m.):

$$\begin{aligned} \Delta(N) &= \frac{1}{6}(3(P+N) - \Sigma^+ + \Sigma^- - \Xi^0 + \Xi^-) = -0.06 \pm 0.01, \\ \mu_N(\bar{s}s) &= \mu(s)\langle N|\bar{s}s|N\rangle = \left(1 - \frac{d}{s}\right)^{-1} \Delta(N) = 0.11, \end{aligned} \tag{9}$$

where the ratio $d/s = 1.55$ follows from the correspondingly modified Eq. (5) (that is with $Y$ replaced by $(Y - \Delta(N))$). By definition, $\mu_N(\bar{s}s)$ represents the contribution of strange («current») quarks to nucleon magnetic moments. Actually, our Eqs. (9) and (5) are equivalent, up to the common factor $-(1/3)$, which is the electric charge of the strange quark, to the half-sum of two relations in Ref. 9 that refer to $\mu_N(\bar{s}s)$ and where the ratios of effective magnetic moments of quarks in different baryons should be taken the same. Indeed, within the lattice QCD approach with a chosen extrapolation prescription to the chiral limit of small current quark masses [9], two sum rules were written down

$$\begin{aligned} G_M^s(0) &= -\left(1 - \frac{d}{s}\right)^{-1} \left[2P + N - \frac{u(P)}{u(\Sigma^+)}(\Sigma^+ - \Sigma^-)\right], \\ G_M^s(0) &= -\left(1 - \frac{d}{s}\right)^{-1} \left[P + 2N - \frac{u(N)}{u(\Xi^0)}(\Xi^0 - \Xi^-)\right], \end{aligned} \tag{10}$$

$$G_M^s(0) = (-0.16 \pm 0.18). \tag{11}$$

At last, as the representative of the approach pretending to be the limit of the QCD with a large number of colours $N_C \to \infty$, we write also the sum rule of the chiral soliton model [10]

$$G_M^s(0) = \frac{1}{3}(N - \Sigma^+ - 4\Sigma^- + \Xi^0 - 3\Xi^-) = +0.32. \tag{12}$$

Within still rather large experimental uncertainties, the latest value of the SAMPLE Collaboration [11]

$$G_M^s(0)|_{\rm exp} = 0.01 \pm 0.29 \pm 0.31 \pm 0.07, \tag{13}$$

where the three errors are statistical, systematic, and theoretical, respectively, does not contradict any of the model values mentioned above.

It is quite natural to expect that we have now the evident constraint $G_M^s(0) \to 0$ in the limit when we neglect all nonvalence quark contributions to baryon magnetic moments, that is when all the relations of Eq. (8) are put into any of the sum rules for $\mu_N(s\bar{s})$. We notice that our relation for $\mu_N(s\bar{s})$ and $G_M^s(0)$ satisfies this constraint identically, and the lattice QCD relations [9] require the «environment» influence to be absent, i.e., $(u(P)/u(\Sigma^+) = (u(N)/u(\Xi^0) = 1$, while the chiral soliton relation [10] requires the fulfillment of the substantially stronger additional assumption $\Lambda = -(N/2)$ which is equivalent to exact $SU(3)$-symmetry relations for magnetic moments. This peculiarity makes the last relation less attractive and, theoretically, more subject to doubts compared to the first two predicting the negative value of $G_M^s(0)$.

**3.** To estimate a possible influence of the $SU(3)$ breaking in the ratio of the weak axial-to-vector coupling constants, we adopt the following prescription suggested by the success of our parameterization of the baryon magnetic moment values within the constituent quark model. In essence, we assume that the leading symmetry breaking effect is produced by different renormalization of the $\bar{q}qW$-strangeness-conserving and strangeness-nonconserving vertices with the participation of the constituent quarks. We note further that in all but one [12] analyses of the hyperon $\beta$ decays, the absence of the «weak electricity» form factor $g_2(Q^2)$ due to the induced second-class weak current has been postulated from the very beginning. However, the fit to all $\Sigma^- \to ne\bar{\nu}$ decay data of Ref. 12 with $g_2 \neq 0$ yields $g_a = (g_1/f_1) - 0.20 \pm 0.08$ and $(g_2/f_1) = +0.56 \pm 0.37$. It seems that one cannot then define $(F/D)_{\Delta S=1}$ because data for all other hyperons have been treated under the assumption $g_2 = 0$.

Having in mind the evidence of a potentially important correlation between the values of the axial-to-vector coupling $(g_1/f_1)$ and the «weak electricity»-to-vector $(g_2/f_1)$ coupling ratio, observed in the $\Sigma^- \to Ne\nu$ decay [12], we parameterize $(g_i/f_1), i = 1, 2$ in the strangeness-violating $\beta$ decays by their (different) $F_i$ and $D_i$ parameters in the expression

$$\frac{g_1}{f_1}(F_1, D_1) + r_2\frac{g_2}{f_1}(F_2, D_2) = \frac{g_1}{f_1}(F_1^{\rm eff}, D_1^{\rm eff}) \tag{14}$$

with the same correlation coefficient $r_2 \simeq -0.25$, quoted in the recent review [13] for both $\Sigma^-$ and $\Lambda$ semileptonic decays but not measured in the $\Xi^{0,-}$ decays yet. The $F_1^{\rm eff}$ and $D_1^{\rm eff}$ will then play the role of the «effective» parameters defined from data with the *ad hoc* constraint $g_2 = 0$. Taking $F_1 + D_1 = 1.26$ and $F_1/D_1|_{\Delta S=0} = 0.72$ we find $F_2$ and $D_2$ from the known data on the $\Sigma^- \to N$ and $\Lambda \to P$ semileptonic decays

$$F_1 - D_1 + r_2(F_2 - D_2) = -0.34 \pm 0.02, \tag{15}$$

$$F_1 + (1/3)D_1 + r_2[F_2 + (1/3)D_2] = 0.718 \pm 0.015 \tag{16}$$

to obtain «effective» parameters for the $\Xi^-$ and $\Xi^0$ decays equal to $0.19 \pm 0.03$ $(0.25 \pm 0.05)$ and $1.25 \pm 0.03$ $(1.32 \pm 0.20)$, respectively. The presently measured «effective» parameters [13] are given in the parentheses and they are seen to be within one standard deviation from the calculated ones. We also notice that the ratio of $|(g_2/f_1)|$ in the $\Sigma^-$ and $\Lambda$ decays is close to that calculated within the dynamical model of Ref. 14; however, the same type ratios including the $\Xi^{-,0}$-decay constants are completely different. Accumulation of new data announced in [13] and their improved analysis is, therefore, of great interest.

**4.** To conclude, besides the importance of resolution of the problem on the presence and quantitative role of the weak second-class current and the corresponding form factors in the hyperon $\beta$-decay observable, one can also mention major theoretical interest in the careful study of the strangeness-conserving $\Sigma^\pm \to \Lambda e^\pm \nu(\bar{\nu})$ transitions which would not only prove (or disprove) hypotheses about the dependence of $(F/D)$ ratios on $\Delta S$, labelling the transitions, but also would provide information on the isospin breaking effects underlying the $\Lambda - \Sigma^0$-mixing.

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