SUPERSYMMETRY IN SUPERSPACE: 35 YEARS OF THE RESEARCH ACTIVITY AT LTP

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This is an overview of the all-years research activity in supersymmetry in the Markov–Ogievetsky group at the Bogoliubov Laboratory of Theoretical Physics. It started shortly after the discovery of supersymmetry and is lasting for more than 30 years. In this survey, the main emphasis is made on the superspace geometric approaches and unconstrained superfield formulations. Alongside such milestones as the geometric formulation of supergravity and the harmonic superspace approach to extended supersymmetry, some other developments largely contributed by the Dubna theorists are briefly accounted for.

Дан обзор исследований по суперсимметрии за все годы в группе Маркова–Огиевецкого в Лаборатории теоретической физики им. Н. Н. Боголюбова. Они начались вскоре после открытия суперсимметрии и продолжаются уже более 30 лет. Главный упор в обзоре сделан на геометрические методы в суперпространстве и суперполевые формулировки без связей. Наряду с такими широкоизвестными результатами, как геометрическая формулировка $N = 1$ супергравитации и подход гармонического суперпространства в расширенных суперсимметриях, обсуждается ряд других достижений дубненской группы, получивших мировое признание.

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INTRODUCTION

Supersymmetry (SUSY) is a remarkable new type of relativistic symmetry which combines into irreducible multiplets the particles with different spin and statistics: bosons (integer spins, Bose–Einstein statistics) and fermions (half-integer spins, Fermi–Dirac statistics). Since it transforms bosons into fermions and vice versa, the corresponding (super)algebras and (super)groups involve both bosonic and fermionic generators. To avoid a contradiction with the fundamental spin-statistics theorem, the fermionic generators should obey the anticommutation relations in contrast to the bosonic ones which still satisfy the commutation relations. Correspondingly, the group parameters associated with the fermionic generators should be anticommuting (Grassmann) numbers.

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The actual interest in supersymmetries arose after the appearance of the papers [1–3] where self-consistent fermionic extensions of the 4D Poincaré algebra were discovered and their field-theoretic realizations were found. The simplest ($N = 1$) Poincaré supersymmetry, besides the standard Poincaré group generators $P_m, L_{[m,n]}$ ($m, n = 0, 1, 2, 3$, P_m being the 4-translation generators; and $L_{[m,n]}$, Lorentz group ones), involves the fermionic Weyl generators $Q_\alpha, \bar{Q}_{\dot\alpha}$ ($\alpha, \dot\alpha = 1, 2$) which transform as $(1/2, 0)$ and $(0, 1/2)$ of the Lorentz group and satisfy the following anticommutation relations:

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2(\sigma^m)_{\alpha\dot\alpha} P_m,$$
$$\{Q_\alpha, Q_\beta\} = \{\bar{Q}_{\dot\alpha}, \bar{Q}_{\dot\beta}\} = 0, \tag{1}$$
$$(\sigma^m)_{\alpha\dot\alpha} = (1, \boldsymbol{\sigma})_{\alpha\dot\alpha}.$$

The $N > 1$ extended supersymmetry involves N copies of the fermionic generators, each satisfying relations (1)

$$\{Q_\alpha^i, \bar{Q}_{\dot\alpha\, k}\} = 2\delta_k^i (\sigma^m)_{\alpha\dot\alpha} P_m,$$
$$\{Q_\alpha^i, Q_\beta^k\} = \{\bar{Q}_{\dot\alpha\, i}, \bar{Q}_{\dot\beta\, k}\} = 0. \tag{2}$$

Here $i = 1, \ldots, N$ is the index of a fundamental representation of the internal automorphism symmetry (or R symmetry) group $U(N)$.

The possibility of achieving a nontrivial junction of internal symmetry with the Poincaré symmetry by placing the fermionic generators into nontrivial representations of the internal symmetry and thus of evading the Coleman–Mandula theorem [4] is one of the remarkable new opportunities suggested by supersymmetry. Nowadays, it has a lot of theoretical manifestations and applications, in particular, in String Theory. Another nice new feature follows directly from relations (1) and (2). Since the anticommutator of global supersymmetry transformations produces a shift of x^m ($P_m = -i(\partial/\partial x^m)$), it is clear that the anticommutator of two *local* supersymmetry transformations inevitably produces a *local* shift of x^m. The gauge theory of local x^m translations (or R^4 diffeomorphisms) is the Einstein gravity. Hence, any theory invariant under local supersymmetry transformations should include gravity. Since the generators $Q_\alpha, \bar{Q}_{\dot\alpha}$ carry the spinor index of the Lorentz group, the associated gauge fields should be, first, fermions and, second, carry an extra vector index m, i.e., be represented by the Rarita–Schwinger field ψ_m^α, $\bar{\psi}_m^{\dot\alpha}$. So these massless gauge fields should carry spin 3/2 (or helicity $\pm 3/2$ on the mass shell) and form, together with the graviton h_{mn}, an irreducible supermultiplet (in the general case of local N extended supersymmetry this supermultiplet contains more fields, with a nontrivial assignment with respect to the R-symmetry group). Such an extension of gravity is the supergravity theory. The supergravity as a gauge theory of linearly realized

local supersymmetry was constructed in [5]*. Supergravity theories are the only possible self-consistent field theories of an interacting spin 3/2 field (with a finite number of gauge fields).

The discovery of supersymmetry at the beginning of the seventies was, to some extent, an expectable event for Victor Isaakovich Ogievetsky. At that time, V. I. Ogievetsky was a member of Markov's group at LTP of JINR (Dubna). The symmetries and their applications in the particle physics and cosmology constituted the traditional line of research of the Academician Moisei Aleksandrovich Markov (see, e.g., [7] and refs. therein) and it was among the basic ones in the scientific activity of his research group at LTP. V. I. Ogievetsky was one of the leading experts in the theory of symmetries, therefore it is not too astonishing that the pioneering papers [1–3] received a quick enthusiastic respond just in Markov's group, among the theorists (including young researchers) concentrated around V. I. Ogievetsky. Later on, after M. A. Markov has delegated his duties of the group leader to V. I. Ogievetsky, this research group transformed into the sector «Supersymmetry» of LTP. It was headed by V. I. Ogievetsky for a long time. M. A. Markov never lost the interest in the investigations on supersymmetry performed in his former group, and his well-wishing support in this respect can hardly be exaggerated.

There was a special reason why the concept of supersymmetry turned out, in a sense, expectable for V. I. Ogievetsky and attracted his attention so strongly.

In the sixties, V. I. Ogievetsky and I. V. Polubarinov (also a member of Markov's group) put forward a new viewpoint on the gauge fields (which on their own were a rather exotic concept at that time) based on the so-called «spin principle» [8–11]. They introduced an important notion of the spin of an interacting field and argued that the gauge invariance was just the device to ensure some massless interacting fields to have a definite spin. They showed that requiring a massless vector field to have spin 1 uniquely leads to Yang–Mills theory, while requiring a massless tensor field h_{mn} to possess spin 2 in interaction (actually with an admixture of spin 0) yields the Einstein theory.

In lectures [11] Ogievetsky and Polubarinov posed a question about the existence of the theory of interacting massless spin-vector field, such that the latter carried the definite spin 3/2 in interaction. In other words, they proposed to search for a theory in which the Rarita–Schwinger field played a role of a gauge field, with the corresponding gauge invariance being intended to eliminate a superfluous spin 1/2 carried by an interacting spin-vector field. They were not

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*First gauge theory of $N = 1$ Poincaré supersymmetry, with the latter being nonlinearly realized as spontaneously broken symmetry, was constructed by Volkov and Soroka in [6]. Thus, these authors were first to realize that «supersymmetry plus gravity equal supergravity» and that the spin 3/2 field is the corresponding gauge field.

able to find a satisfactory solution to this problem [12]. Now we know that this mysterious gauge invariance is the local supersymmetry, while the corresponding gauge theory is supergravity.

Ogievetsky quickly realized that supersymmetry is potentially capable of providing an answer to his and Polubarinov's query about a self-consistent spin 3/2 theory. And it was Ogievetsky who initiated (with blessing of M. A. Markov) the study of this new type of symmetry at LTP in the first half of the seventies. This review is a brief (and inevitably biased) account of the history of these studies for more than 30 years which passed since we became aware of supersymmetry, with focusing on the milestones. Many of the results reviewed below were paralleled and in some cases rediscovered by other groups. Because of the lack of space, I mainly cite the relevant works of the Dubna group and frequently omit references to some important parallel studies. I apologize for this incompleteness of the reference list.

1. FIRST STUDIES: 1974–1980

1.1. Superspace: What It Is and How It Helps. Any symmetry implies some framework within which it admits a concise and suggestive realization. For instance, Poincaré symmetry can be naturally realized on Minkowski space and fields given in it. For supersymmetry, such a natural framework is superspace, an extension of some bosonic space by anticommuting fermionic (Grassmann) coordinates. For the $N = 1$ Poincaré supersymmetry (1) it was actually introduced in one of the pioneering papers, [2], as a coset of the $N = 1$ Poincaré supergroup over its bosonic Lorentz subgroup. However, the fermionic coset parameters, in the spirit of the nonlinear realizations method, were treated in [2] as Nambu–Goldstone fields «living» on Minkowski space. The treatment of the fermionic coordinates on equal footing with x^m as *independent* coordinates was suggested by Salam and Strathdee [13] who considered fields on such an extended space and showed that these fields naturally encompass the irreducible multiplets of $N = 1$ supersymmetry ($N = 1$ supermultiplets). They named this space *superspace* and fields on it *superfields*.

In $N = 1$ superspace

$$(x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}), \tag{3}$$

$N = 1$ supersymmetry (1) acts as shifts of Grassmann coordinates

$$\theta^{\alpha\prime} = \theta^\alpha + \epsilon^\alpha, \quad \bar{\theta}^{\dot{\alpha}\prime} = \bar{\theta}^{\dot{\alpha}} + \bar{\epsilon}^{\dot{\alpha}}, \quad x^{m\prime} = x^m + i(\theta\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\theta}), \tag{4}$$

where $\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}}$ are the mutually conjugated Grassmann transformation parameters associated with the generators Q_α and $\bar{Q}_{\dot{\alpha}}$. It is easy to check that the Lie bracket of two such transformations of x^m yields a constant shift of x^m, in accord with

relation (1). A general $N = 1$ superfield is an unconstrained function $\Phi(x, \theta, \bar{\theta})$ which transforms as

$$\delta\Phi(x, \theta, \bar{\theta}) = -\delta\theta^\alpha \frac{\partial}{\partial\theta^\alpha}\Phi(x, \theta, \bar{\theta}) - \delta\bar{\theta}^{\dot{\alpha}}\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\Phi(x, \theta, \bar{\theta}) - \delta x^m \frac{\partial}{\partial x^m}\Phi(x, \theta, \bar{\theta}) \equiv$$
$$\equiv i\left(\epsilon^\alpha Q_\alpha + \bar{\epsilon}_{\dot{\alpha}}\bar{Q}^{\dot{\alpha}}\right)\Phi(x, \theta, \bar{\theta}). \quad (5)$$

The generators $Q_\alpha, \bar{Q}_{\dot{\alpha}}$ can be checked to satisfy the anticommutation relation (1).

The crucial feature of superfields is that they concisely encompass finite-component off-shell field multiplets of the given supersymmetry. As discovered by Salam and Strathdee, this key property is related to the fact that θ^α and $\bar{\theta}^{\dot{\alpha}}$ are anticommuting variables:

$$\{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}^{\dot{\alpha}}, \bar{\theta}^{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}^{\dot{\beta}}\} = 0. \quad (6)$$

These relations imply, in particular,

$$(\theta^1)^2 = (\theta^2)^2 = 0 \quad \text{(and c.c.)}. \quad (7)$$

Then, expanding $\Phi(x, \theta, \bar{\theta})$ in a series over all possible monomials constructed from θ^α and $\bar{\theta}^{\dot{\alpha}}$, one observes that this series terminates at the monomial $\theta^1\theta^2\bar{\theta}^{\dot{1}}\bar{\theta}^{\dot{2}} \sim (\theta)^2(\bar{\theta})^2$, where $(\theta)^2 = \epsilon_{\alpha\beta}\theta^\alpha\theta^\beta$, $(\bar{\theta})^2 = \overline{(\theta)^2}\,^*$. As a result, $\Phi(x, \theta, \bar{\theta})$ contains $(8+8)$ fields: 8 bosonic fields and 8 fermionic fields:

$$\Phi(x, \theta, \bar{\theta}) = \phi(x) + \theta^\alpha \chi_\alpha(x) + \bar{\theta}_{\dot{\alpha}}\bar{\chi}^{\dot{\alpha}}(x) + \theta\sigma^n\bar{\theta}\,A_n(x) +$$
$$+ (\bar{\theta})^2\theta^\alpha\omega_\alpha(x) + (\theta)^2\bar{\theta}_{\dot{\alpha}}\bar{\omega}^{\dot{\alpha}}(x) + (\theta)^2(\bar{\theta})^2 D(x). \quad (8)$$

The precise transformation laws of the component fields can be easily deduced from (5). These fields still form a *reducible* representation of $N = 1$ super-symmetry. To make $\Phi(x, \theta, \bar{\theta})$ carry an irreducible supermultiplet, one needs to impose on this superfield proper constraints covariant under $N = 1$ supersymmetry. These constraints involve the covariant spinor derivatives,

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i(\sigma^m\bar{\theta})_\alpha\partial_m,$$
$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} - i(\theta\sigma^m)_{\dot{\alpha}}\partial_m, \quad (9)$$
$$\{D_\alpha, \bar{D}_{\dot{\alpha}}\} = -2i(\sigma^m)_{\alpha\dot{\alpha}}\partial_m.$$

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*We use the standard two-dimensional spinor notation, with $\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}$, $\epsilon_{12} = 1$, $\epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^\alpha_\gamma$ (and the same for dotted indices).

These operators anticommute with the generators $Q_\alpha, \bar{Q}_{\dot\alpha}$, so the result of their action on $\Phi(x, \theta, \bar\theta)$ is again a superfield. The covariant constraints singling out two irreducible multiplets contained in a general unconstrained $\Phi(x, \theta, \bar\theta)$ are as follows:

$$\text{(a) } \bar{D}_{\dot\alpha}\Phi_{(1)}(x, \theta, \bar\theta) = 0 \quad (\text{or } D_\alpha\bar\Phi_{(1)} = 0\,),$$

$$\text{(b) } (D)^2\Phi_{(2)}(x, \theta, \bar\theta) = (\bar{D})^2\Phi_{(2)}(x, \theta, \bar\theta) = 0. \tag{10}$$

Using the appropriate projection operators, the general real superfield $\Phi(x, \theta, \bar\theta)$ can be decomposed into the irreducible pieces as follows:

$$\Phi = \Phi_{(1)} + \bar\Phi_{(1)} + \Phi_{(2)}. \tag{11}$$

This decomposition is an analog of the well-known decomposition of 4D vector field into the longitudinal and transversal parts (spins 0 and 1). In the case of supersymmetry, the notion of spin is generalized to the *superspin*. The constrained superfields $\Phi_{(1)}$ and $\Phi_{(2)}$ can be shown to possess definite superspins, 0 and 1/2, respectively. The superfield constraint (10a) admits a nice geometric solution. Namely, making the complex change of the superspace coordinates

$$(x^m, \theta^\alpha, \bar\theta^{\dot\alpha}) \Rightarrow (x_L^m = x^m + i\theta\sigma^m\bar\theta, \theta^\alpha, \bar\theta^{\dot\alpha}), \tag{12}$$

one finds that $\bar{D}_{\dot\alpha}$ is «short» in this new («left-chiral») basis

$$\bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}}, \tag{13}$$

and (10a) becomes the Grassmann Cauchy–Riemann condition stating that $\Phi_{(1)}$ is independent of the half of Grassmann coordinates in this basis:

$$\frac{\partial}{\partial\bar\theta^{\dot\alpha}}\Phi_{(1)}(x, \theta, \bar\theta) = 0 \Rightarrow \Phi_{(1)}(x, \theta, \bar\theta) = \varphi(x_L^m, \theta^\alpha). \tag{14}$$

It is easy to directly check that the set (x_L^m, θ^α) is closed under the supertranslations (4) and so forms a complex invariant space of the $N = 1$ Poincaré supergroup, chiral superspace. The θ expansion of the superfield $\varphi(x_L, \theta)$, *chiral $N = 1$ superfield* [14], directly yields the scalar $N = 1$ supermultiplet of fields:

$$\varphi(x_L^m, \theta^\alpha) = \varphi(x_L) + \theta^\alpha\psi_\alpha(x_L) + (\theta)^2 F(x_L), \tag{15}$$

where $\varphi(x_L)$ and $F(x_L)$ are two complex scalar fields and $\psi_\alpha(x_L)$ is a two-component left-handed Weyl spinor.

The basic advantages of using off-shell superfields are as follows.

First of all, their SUSY transformation laws do not depend on the dynamics, i.e., are the same whatever the invariant action of the involved fields is.

An important property of superfields is the presence of the so-called auxiliary fields in their θ expansion, which is necessary for the off-shell closure of the SUSY algebra on the component fields. In the example (15) it is just the field $F(x_L)$. Ascribing the canonical dimensions 1 and $3/2$ to the «physical fields» φ and ψ_α and taking into account that $[\theta] = -1/2$, one finds that $[F] = 2$, whence it follows that F should enter any $D = 4$ action without derivatives. In other words, its equation of motion is always algebraic and serves to express F in terms of the physical fields (or to put F equal to a constant or zero). Since SUSY mixes this algebraic equation with those for physical fields, it closes on the physical fields only modulo their equations of motion. As a result, the realization of SUSY on the physical fields depends on the choice of the invariant action, and for this reason it proves very difficult to construct invariant actions with making use of the physical fields only.

On the other hand, any product of superfields, with or without x- or spinor derivatives on them, is again a superfield. The second crucial property of the off-shell superfields is that the component field appearing as a coefficient of the highest-degree θ monomial always transforms as a total x derivative of the lower-order component fields. Hence, its integral over the Minkowski space is SUSY *invariant* and so is a candidate for an invariant action. Forming products of some basic elementary superfields and using the property that these products are superfields on their own, one can be sure that the (composite) component fields appearing as coefficients of the highest-order θ monomials in these products are transformed by a total derivative. So the invariant actions can be constructed as Minkowski space integrals of these composite fields. In other words, the superfield approach provides a universal way of searching for supersymmetric actions.

The remarkable features of the superfield approach listed above led V. I. Ogievetsky to rapidly realize how indispensable it promises to be for exploring geometric and quantum properties of supersymmetric theories. In the middle of the seventies, he started to actively work on the superspace methods, together with his disciples Luca Mezincescu from Bucharest and Emery Sokatchev from Sofia.

1.2. Action Principle in Superspace. In [15], Ogievetsky and Mezincescu proposed an elegant way of writing down the invariant superfield actions. As mentioned above, the invariant actions can be constructed as the x integrals of the coefficients of the highest-degree θ monomials in the appropriate products of the involved superfields. The question was how to extract these components in a manifestly supersymmetric way. Ogievetsky and Mezincescu proposed to use the important notion of Berezin integral [16] for this purpose. In fact, Berezin integration is equivalent to the Grassmann differentiation and, in the case of

$N = 1$ superspace, is defined by the rules

$$\int d\theta_\alpha \, \theta^\beta = \delta_\alpha^\beta, \quad \int d\theta^\alpha \, 1 = 0, \quad \{d\theta_\alpha, d\theta_\beta\} = \{\theta_\alpha, d\theta_\beta\} = 0. \quad (16)$$

It is easy to see that, up to the appropriate normalization,

$$\int d^2\theta \, (\theta)^2 = 1, \quad \int d^2\bar\theta \, (\bar\theta)^2 = 1, \quad \int d^2\theta d^2\bar\theta \, (\theta)^4 = 1, \quad (17)$$

and, hence, Berezin integration provides the efficient and manifestly supersymmetric way of singling out the coefficients of the highest-order θ monomials. For example, the simplest invariant action of chiral superfields can be written as

$$S \sim \int d^4x d^4\theta \, \varphi(x_L, \theta)\bar\varphi(x_R, \bar\theta), \quad x_R^m = \overline{(x_L^m)} = x^m - i\theta\sigma^m\bar\theta. \quad (18)$$

Using (15) and (17), it is easy to integrate over $\theta, \bar\theta$ in (18) and, discarding total x derivatives, to obtain the component form of the action

$$S \sim \int d^4x \left(\partial^m\bar\varphi\partial_m\varphi - \frac{i}{2}\psi\sigma^m\partial_m\bar\psi + F\bar F \right). \quad (19)$$

It is just the free action of the massless scalar $N = 1$ multiplet. One can easily generalize it to the case with interaction by choosing the Lagrangian as an arbitrary function $K(\bar\varphi, \varphi)$ and adding independent potential terms

$$\sim \int d^4x_L d^2\theta \, P(\varphi) + \text{c.c.}, \quad (20)$$

which in components produce mass terms, scalar potentials, and fermionic Yukawa coupling for the physical fields after elimination of the auxiliary fields $F, \bar F$ in a sum of the superfield kinetic and potential terms. The sum of (18) and the superpotential term (20) with $P(\varphi) \sim g\varphi^3 + m\varphi^2$ corresponds to the Wess–Zumino model [17] which was the first example of nontrivial $N = 1$ supersymmetric model and the only renormalizable model of scalar $N = 1$ multiplet. Ogievetsky and Mezincescu argued in [15] that the representation of the action of the Wess–Zumino model in terms of Berezin integral was very useful and suggestive while developing the superfield perturbation theory for it*.

In 1975, Ogievetsky and Mezincescu wrote a comprehensive review on the basics of supersymmetry and superspace approach [18]. Until present it remains one of the best introductory reviews in the field.

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*One can show that all quantum corrections have the form of the integral over the whole $N = 1$ superspace, so the superpotential term (and, hence, the parameters g and m) is not renormalized. This statement is the simplest example of the so-called nonrenormalization theorems.

1.3. Superfields with Higher Superspins and New Supergauge Theories.
The next benchmark became Sokatchev's work [19] where the general classification of $N = 1$ superfields with respect to superspin was given, and the corresponding irreducibility superfield constraints (generalizing (10)) together with the relevant projection operators on definite superspins were given in an explicit form. In the pioneering paper [13], the decomposition into the superspin-irreducible parts was discussed in detail only for a scalar $N = 1$ superfield. Higher superspins are carried by superfields with external Lorentz indices. Like in the case of bosonic gauge theories, the requirement of preserving definite superspins by interacting superfields was expected to fully determine the structure of the corresponding action and the gauge group intended to make harmless extra superspins carried by the given off-shell superfield. In fulfilling this program of research, the formalism of the projection operators of [19] proved to be indispensable.

An $N = 1$ superextension of the Yang–Mills theory was constructed in [20]. It was shown that the fundamental object (prepotential) carrying the irreducible field content of the off-shell $N = 1$ vector multiplet (gauge field $b_m(x)$, gaugino $\psi_\alpha(x), \bar{\psi}_{\dot\alpha}(x)$ and the auxiliary field $D(x)$, all taking values in the adjoint representation of gauge group) is the real scalar superfield $V(x, \theta, \bar\theta)$ with certain gauge freedom. The latter, in the Abelian case, is given by the transformations

$$V'(x, \theta, \bar\theta) = V(x, \theta, \bar\theta) + \frac{i}{2}\left(\Lambda(x_L, \theta) - \bar\Lambda(x_R, \bar\theta)\right), \qquad (21)$$

where Λ and $\bar\Lambda$ are mutually conjugated superfield parameters «living» as unconstrained functions on the left and right $N = 1$ chiral subspaces. Any component in $V(x, \theta, \bar\theta)$ which undergoes an additive shift by a gauge parameter, can be fully removed by fixing this parameter; proceeding in this way, one can show that the maximally reduced form of $V(x, \theta, \bar\theta)$ (Wess–Zumino gauge) is as follows:

$$V(x, \theta, \bar\theta) = \theta\sigma^n\bar\theta\, A_n + (\bar\theta)^2\theta^\alpha\psi_\alpha + (\theta)^2\bar\theta_{\dot\alpha}\bar\psi^{\dot\alpha} + (\theta)^2(\bar\theta)^2 D, \qquad (22)$$

$$\delta A_n = \partial_n\lambda_0, \quad \lambda_0 \equiv -\frac{1}{2}(\Lambda + \bar\Lambda)|_{\theta=\bar\theta=0}.$$

The fields in (22) are recognized as the irreducible off-shell $N = 1$ vector multiplet (superspin 1/2).

Ogievetsky and Sokatchev asked whether there exist more complicated superfield gauge theories, with the prepotentials having extra Lorentz indices and so carrying other definite superspins in interaction. Using the formalism of the projection operators developed in [19], they firstly tried to construct a self-contained theory of spinor gauge superfield $\Psi_\alpha(x, \theta, \bar\theta), \bar\Psi_{\dot\alpha}(x, \theta, \bar\theta)$ [21] as an alternative to the standard $N = 1$ gauge theory, with the gauge vector being in the same irreducible multiplet with a massless spin 3/2 field. They constructed a self-consistent free action for such a spinor superfield, but failed to promote some

important gauge symmetry of it to a non-Abelian interacting case. The reason for this failure was realized later on: a self-consistent theory of interacting massless Rarita–Schwinger field should be supergravity which necessarily includes Einstein gravity as a subsector.

Searching for a self-consistent theory of massless vector superfield (carrying superspins 3/2 and 1/2) turned out to be more suggestive. This superfield $H^n(x, \theta, \bar{\theta})$ encompasses, in its component field expansion, massless tensor field e_a^n and spin-vector field ψ_α^n,

$$H^n = \theta\sigma^a\bar{\theta}e_a^n + (\bar{\theta})^2\theta^\alpha\psi_\alpha^n + (\theta)^2\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}n} + \dots,$$

which could naturally be identified with the graviton and gravitino fields. In [21], Ogievetsky and Sokatchev put forward the hypothesis that the correct «minimal» $N = 1$ superfield supergravity should be a theory of gauge axial-vector superfield $H^m(x, \theta, \bar{\theta})$ generated by the conserved supercurrent. The latter unifies into an irreducible $N = 1$ supermultiplet the energy-momentum tensor and spin-vector current associated with the supertranslations (see [23, 24] and refs. therein). Ogievetsky and Sokatchev relied upon the clear analogy with the Einstein gravity which can be viewed as a theory of massless tensor field generated by the conserved energy-momentum tensor. The whole Einstein action and its non-Abelian 4D diffeomorphism gauge symmetry can be uniquely restored step-by-step, starting with a free action of symmetric tensor field and requiring its source (constructed from this field and its derivatives, as well as from matter fields) to be conserved [10]. In [22], this Noether procedure was applied to the free action of $H^m(x, \theta, \bar{\theta})$. The first-order coupling of H^m to the conserved supercurrent of the matter chiral superfield was restored, and superfield gauge symmetry generalizing bosonic diffeomorphism symmetry was identified at the linearized level. The geometric meaning of this supergauge symmetry and its full non-Abelian form were revealed by Ogievetsky and Sokatchev later, in the remarkable papers [25,26]. Before dwelling on this, let me mention a few important parallel investigations on $N = 1$ SUSY performed in our Sector approximately at the same time, i.e., in the second half of the seventies and beginning of the eighties.

1.4. General Relation between Linear and Nonlinear Realizations of $N = 1$ **SUSY.** One of the first known realizations of $N = 1$ SUSY was its nonlinear (Volkov–Akulov) realization [2]

$$
\begin{aligned}
y^{m\,\prime} &= y^m + i[\lambda(y)\sigma^m\bar{\epsilon} - \epsilon\sigma^m\bar{\lambda}(y)], \\
\lambda^{\alpha\,\prime}(y\,\prime) &= \lambda^\alpha(y) + \epsilon^\alpha, \\
\bar{\lambda}^{\dot{\alpha}\,\prime}(y\,\prime) &= \bar{\lambda}^{\dot{\alpha}}(y) + \bar{\epsilon}^{\dot{\alpha}},
\end{aligned}
\tag{23}
$$

where the corresponding Minkowski space coordinate is denoted by y^m to distinguish it from x^m corresponding to the superspace realization (4). The main

difference between (23) and (4) is that (23) involves the $N = 1$ Goldstone fermion (goldstino) $\lambda(y)$ the characteristic feature of which is the inhomogeneous transformation law under supertranslations, which corresponds to the spontaneously broken SUSY. It is a field given on Minkowski space, while θ^α in (4) is an independent Grassmann coordinate, and $N = 1$ superfields support a linear realization of $N = 1$ SUSY. The invariant action of $\lambda, \bar\lambda$ is [2]:

$$S_{(\lambda)} = \frac{1}{f^2} \int d^4y \, \det E_m^a, \quad E_m^a = \delta_m^a + i \left(\lambda \sigma^a \partial_m \bar\lambda - \partial_m \lambda \sigma^a \bar\lambda \right), \qquad (24)$$

where f is a coupling constant ($[f] = -2$).

The natural question was what is the precise relation between the nonlinear and superfield (linear) realizations of the same $N = 1$ SUSY. We with my friend and co-worker Sasha Kapustnikov (now late) were the first to pose this question and present the explicit answer [27–29]. We showed that, given the Goldstone fermion $\lambda(y)$ with the transformation properties (23), the relation between two types of the $N = 1$ SUSY realizations, (4) and (23), is given by the following invertible change of the superspace coordinates:

$$x^m = y^m + i \left[\theta \sigma^m \bar\lambda(y) - \lambda(y)\sigma^m \bar\theta \right], \quad \theta^\alpha = \tilde\theta^\alpha + \lambda^\alpha(y), \quad \bar\theta^{\dot\alpha} = \tilde{\bar\theta}^{\dot\alpha} + \bar\lambda^{\dot\alpha}(y),$$
$$(25)$$

where

$$\tilde\theta^{\alpha\,\prime} = \tilde\theta^\alpha. \qquad (26)$$

Then the transformations (23) imply for $(x^m, \theta^\alpha, \bar\theta^{\dot\alpha})$ just the transformations (4) and, vice versa, (4) imply (23). Using (25), any linearly transforming superfield can be put in the new «splitting» basis

$$\Phi(x, \theta, \bar\theta) = \tilde\Phi(y, \tilde\theta, \tilde{\bar\theta}). \qquad (27)$$

Since $\tilde\theta^\alpha$ is «inert» under $N = 1$ SUSY, Eq. (26), the components of $\tilde\Phi$ transform as «sigma-fields»

$$\delta\phi(y) = -i[\lambda(y)\sigma^m \bar\epsilon - \epsilon\sigma^m \bar\lambda(y)] \, \partial_m\phi(y), \quad \text{etc.,} \qquad (28)$$

independently of each other, whence the term «splitting» for this basis. As demonstrated in [29], irrespective of the precise mechanism of generating goldstino in a theory with the linear realization of spontaneously broken $N = 1$ SUSY, the corresponding superfield action can be rewritten in the splitting basis (after performing integration over the inert Grassmann variables) as

$$S_{\text{lin}} \sim \int d^4y \, \det E_m^a \left[1 + \mathcal{L}(\sigma, \nabla_a\sigma, \ldots) \right]. \qquad (29)$$

Here $\mathcal{L}$ is a function of the «sigma» fields and their covariant derivatives $\nabla_a = E_a^m \partial_m$ only, while $\lambda^\alpha(y)$ is related to the goldstino of the linear realization

through a field redefinition. Thus, the Goldstone fermion is always described by the universal action (24), independently of details of the given dynamical theory with the spontaneous breaking of $N = 1$ supersymmetry, in the spirit of the general theory of nonlinear realizations.

The transformation (25), (27) can be easily generalized to chiral superfields and to higher N. It proved very useful for exhibiting the low-energy structure of theories with spontaneously broken SUSY and in some other problems. It was generalized to the case of local $N = 1$ SUSY in [30].

1.5. AdS$_4$ Superspace. Soon after the $N = 1$ Poincaré supersymmetry was discovered, there was found $N = 1$ superextension of another important $D = 4$ group, conformal group $SO(2,4) \sim SU(2,2)$. The latter was known to play an important role in quantum field theory (specifying the structure of the Green functions in some massless $D = 4$ models), as well as in gravity which, e.g., can be regarded as a theory following from the spontaneous breaking of the local conformal group with the Goldstone dilaton field as a «compensator» (see, e.g., [31]). This was the main motivation for considering $N = 1$ super-conformal group $SU(2,2|1)$ (and its higher N analogs $SU(2,2|N)$). Later on, the gauge versions of these symmetries were used to construct extended super-gravities.

An important property of the conformal group is that it admits a natural action in the conformally-flat $D = 4$ space-times, with the distances related to the Minkowski interval by a Weyl factor. The corresponding groups of motion are subgroups of the conformal group. This class of spaces includes anti-de Sitter and de Sitter spaces AdS$_4 \sim (SO(2,3)/SO(1,3))$ and dS$_4 \sim (SO(1,4)/SO(1,3))$. One could expect that the property of conformal flatness is generalized to su-perspaces. While the dS$_4$ spinor comprises eight independent components, no such doubling as compared to the Minkowski space occurs for AdS$_4$: the AdS$_4$ spinor is the Weyl one with two complex components. Keeping this in mind, the corresponding SUSY was expected to be similar to (1). There was an urgent necessity to construct a self-consistent superfield formalism for AdS$_4$ SUSY, and in 1978 we turned to this problem with my PhD student Alexander Sorin from the Dniepropetrovsk State University.

$N = 1$ AdS$_4$ superalgebra is $osp(1|4) \subset su(2,2|1)$, and it is defined by the following (anti)commutation relations:

$$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2(\sigma^m)_{\alpha\dot\alpha}P_m, \qquad \{Q_\alpha, Q_\beta\} = m(\sigma^{mn})_{\alpha\beta}L_{[m,n]},$$

$$[Q_\alpha, P_m] = \frac{m}{2}(\sigma_m)_{\alpha\dot\alpha}\bar{Q}^{\dot\alpha}, \qquad [P_m, P_n] = -im^2 L_{[m,n]}. \tag{30}$$

Here $(\sigma^{mn})_\alpha^\beta = (i/2)(\sigma^m\tilde\sigma^n - \sigma^n\tilde\sigma^m)_\alpha^\beta$, $(\tilde\sigma^m)^{\dot\alpha\beta} = \epsilon^{\dot\alpha\dot\omega}\epsilon^{\beta\gamma}(\sigma^m)_{\gamma\dot\omega}$, $m \sim r^{-1}$ is the inverse radius of AdS$_4$ and $L_{[m,n]}$ are generators of the Lorentz $SO(1,3)$

subgroup of $SO(2,3) \propto (P_m, L_{[m,n]})$. To Eqs. (30), one should add complex-conjugate relations and (trivial) commutators with $L_{[m,n]}$. In the limit $m \to 0$ ($r \to \infty$), relations (30) go over into (1).

In [32,33], for constructing $OSp(1|4)$ covariant superfield formalism we applied a powerful method of Cartan forms (viz. the coset method) which allowed us to find the true AdS_4 analogs of the general and chiral $N = 1$ superfields, as well as the vector and spinor covariant derivatives, invariant superspace integration measures, etc. Having developed the AdS_4 superfield techniques, we constructed the $OSp(1|4)$ invariant actions generalizing the actions of the Wess–Zumino model and $N = 1$ SYM theory. Just to give a feeling what such actions look like, I present here an analog of the free massless action (19) of $N = 1$ scalar multiplet, with the auxiliary fields eliminated by their equations of motion:

$$
S \sim \int d^4x \, a^4(x) \left(\nabla^m \bar{\varphi} \nabla_m \varphi - \frac{i}{4} \psi \sigma^m \nabla_m \bar{\psi} + \frac{i}{4} \nabla_m \psi \sigma^m \bar{\psi} + 2m^2 \, \varphi \bar{\varphi} \right).
$$
(31)

Here $a(x) = 2/(1 + m^2 x^2)$ is a scalar factor specifying the AdS_4 metric in a conformally-flat parametrization, $ds^2 = a^2(x)\eta_{mn}dx^m dx^n$, and $\nabla_m = a^{-1}\partial_m$. Taking into account that $m^2 = -(1/12)R$, where R is the scalar curvature of AdS_4, this action is the standard form of the massless scalar field action in a curved background.

In [33], we thoroughly studied the vacuum structure of the general massive AdS_4 Wess–Zumino model, which turned out to be much richer as compared to the standard «flat» Wess–Zumino model due to the presence of the «intrinsic» mass parameter m. We also showed that both the AdS_4 massless Wess–Zumino model and super YM model can be reduced to their flat $N = 1$ super Minkowski analogs via some superfield transformation generalizing the Weyl transformation

$$
\varphi(x) = a^{-1}(x)\tilde{\varphi}(x), \quad \psi^\alpha(x) = a^{-3/2}(x)\tilde{\psi}^\alpha(x),
$$
(32)

which reduces (31) to (19). The existence of the superfield Weyl transformation was an indication of the superconformal flatness of the AdS_4 superspace (although this property has been proven only recently [34]).

Afterwards, the simplest supermultiplets of $OSp(1|4)$ derived for the first time in [32] from the superfield formalism and the corresponding projection operators were used, e.g., in [35] to give an algebraic meaning to the superfield constraints of $N = 1$ supergravity. The interest in $OSp(1|4)$ supersymmetry and in the relevant model-building has especially grown up in recent years in connection with the famous Maldacena's AdS/CFT conjecture.

2. COMPLEX GEOMETRY OF $N = 1$ SUPERGRAVITY

Poincaré $N = 1$ supergravity (SG) was discovered in [5] as a theory of interacting gauge vierbein field $e_m^a(x) = \delta_m^a + \kappa h_m^a(x)$ (graviton, with κ being Einstein constant) and spin-vector field $\psi_m^\mu(x), \bar{\psi}_m^{\dot{\mu}}(x)$ (gravitino) and possessing, in addition to $D = 4$ diffeomorphisms, also a local supersymmetry. It was an urgent problem to find a complete off-shell formulation of $N = 1$ SG, i.e., to extend the set of physical fields e, ψ to an off-shell multiplet by adding the appropriate auxiliary fields and/or to formulate $N = 1$ SG in superspace, making all its symmetries manifest.

One of the approaches to $N = 1$ SG in superspace was based on considering the most general differential geometry in $N = 1$ superspace. One defines supervielbeins, supercurvatures and supertorsions, which are covariant under arbitrary $N = 1$ superdiffeomorphisms, and then imposes the appropriate constraints, so as to end up with the minimal set of the off-shell $N = 1$ superfields encompassing the irreducible field content of SG [36]. Another approach is to reveal the fundamental minimal gauge group of SG and the basic unconstrained SG prepotential, an analog of $N = 1$ SYM prepotential (21). This was just the strategy which Ogievetsky and Sokatchev follow in [25] to discover a beautiful geometric formulation of the conformal and «minimal» Einstein $N = 1$ SG.

It is based on a generalization of the notion of flat $N = 1$ chirality to the curved case. The flat chiral $N = 1$ superspace (x_L^m, θ_L^μ) possesses the complex dimension $(4|2)$ and includes the $N = 1$ superspace $(x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}})$ as a real $(4|4)$ dimensional hypersurface defined by the following embedding conditions:

(a) $x_L^m + x_R^m = 2x^m$,   (b) $x_L^m - x_R^m = 2i\theta\sigma^m\bar{\theta}$, $\theta_L^\mu = \theta^\mu$, $\bar{\theta}_R^{\dot{\mu}} = \bar{\theta}^{\dot{\mu}}$,   (33)

and $x_R^m = \overline{(x_L^m)}, \bar{\theta}_R^{\dot{\mu}} = \overline{(\theta_L^\mu)}$. It turned out that the underlying gauge group of conformal $N = 1$ SG is just the group of general diffeomorphisms of the chiral superspace:

$$\delta x_L^m = \lambda^m(x_L, \theta_L), \quad \delta\theta_L^\mu = \lambda^\mu(x_L, \theta_L), \tag{34}$$

with λ^m, λ^μ being arbitrary complex functions of their arguments. The fermionic part of the embedding conditions (33) remains unchanged while the bosonic one is generalized to

(a) $x_L^m + x_R^m = 2x^m$,   (b) $x_L^m - x_R^m = 2iH^m(x, \theta, \bar{\theta})$.   (35)

The basic gauge prepotential of conformal $N = 1$ SG is just the axial-vector superfield $H^m(x, \theta, \bar{\theta})$ in (35). It specifies the superembedding of real $N = 1$ superspace as a hypersurface into the complex chiral $N = 1$ superspace (x_L^m, θ_L^μ) and so possesses a nice geometric meaning. Through relations (35), the transformations (34) generate field-dependent nonlinear transformations of the $N = 1$

superspace coordinates $(x^m, \theta^\mu, \bar{\theta}^{\dot{\mu}})$ and of the superfield $H^m(x, \theta, \bar{\theta})$. The field content of H^m can be revealed in the WZ gauge which requires knowing only the linearized form of the transformations:

$$\delta^* H^m = \frac{1}{2i} \left[\lambda^m(x + i\theta\sigma\bar{\theta}, \theta) - \bar{\lambda}^m(x - i\theta\sigma\bar{\theta}, \bar{\theta}) \right] -$$
$$- \lambda(x + i\theta\sigma\bar{\theta}, \theta)\sigma^m\bar{\theta} - \theta\sigma^m\bar{\lambda}(x - i\theta\sigma\bar{\theta}, \bar{\theta}). \quad (36)$$

Here we took into account the presence of the «flat» part $\theta\sigma^m\bar{\theta}$ in $H^m = \theta\sigma^a\bar{\theta}(\delta_a^m + \kappa h_a^m) + \ldots$ An easy calculation yields the WZ gauge form of H^m as

$$H^m_{\text{WZ}} = \theta\sigma^a\bar{\theta}\, e_a^m + (\bar{\theta})^2\theta^\mu\psi_\mu^m + (\theta)^2\bar{\theta}_{\dot{\mu}}\bar{\psi}^{m\dot{\mu}} + (\theta)^2(\bar{\theta})^2 A^m. \quad (37)$$

Here one finds the vierbein e_a^m presenting the conformal graviton (gauge-independent spin 2 off-shell), the gravitino ψ_μ^m (spin $(3/2)^2$), and the gauge field A^m (spin 1) of the local $\gamma_5\, R$ symmetry, just $(8 + 8)$ off-shell degrees of freedom forming the superspin 3/2 $N = 1$ Weyl multiplet.

The Einstein $N = 1$ SG can now be deduced in two basically equivalent ways. The first one was used in the original paper [25] and it is to restrict the group (34) by the constraint

$$\partial_m\lambda^m - \partial_\mu\lambda^\mu = 0, \quad (38)$$

which is the infinitesimal form of the requirement that the integration measure of chiral superspace (x_L, θ^μ) is invariant. One can show that, with this constraint, the WZ form of H^m collects two extra scalar auxiliary fields, while A^m ceases to be gauge and also becomes an auxiliary field. On top of this, there disappears one fermionic gauge invariance (corresponding to conformal SUSY) and, as a result, spin-vector field starts to carry 12 independent components. So one ends up with the $(12 + 12)$ off-shell multiplet of the so-called «minimal» Einstein SG [37].

Another, more suggestive way to come to the same off-shell content is to use the compensator ideology which can be traced back to the interpretation of Einstein gravity as conformal gravity with the compensating (Goldstone) scalar field [31]. Since the group (34) preserves the chiral superspace, in the local case one can still define a chiral superfield $\Phi(x_L, \theta)$ as an unconstrained function on this superspace and ascribe to it the following transformation law:

$$\delta\Phi = -\frac{1}{3} \left(\partial_m\lambda^m - \partial_\mu\lambda^\mu \right) \Phi, \quad (39)$$

where the specific choice (–1/3) of the conformal weight of Φ is needed for constructing the invariant SG action. Assuming that the vacuum expectation value of Φ is nonvanishing and recalling the θ expansion

$$\Phi = \langle f \rangle + \tilde{f} + ig + \theta^\mu\chi_\mu + (\theta)^2(S + iP), \quad \langle f \rangle \neq 0, \quad (40)$$

one observes from the transformation law (39) that the fields $\tilde{f}, g$, and χ_μ can be gauged away, thus fully «compensating» dilatations, R transformations, and conformal supersymmetry. The fields S and P and the nongauge field A^m coming from H^m constitute the set of auxiliary fields. Together with other fields from the appropriate WZ gauge for $H^m(x, \theta, \bar\theta)$ they yield the required off-shell $(12 + 12)$ representation.

The basic advantage of the compensating method is that it allows one to easily write the action of the minimal Einstein SG as an invariant action of the compensator Φ in the background of the Weyl multiplet carried by H^m:

$$S_{\mathrm{SG}} = -\frac{1}{\kappa^2} \int d^4x \, d^2\theta \, d^2\bar\theta \, E \, \Phi(x_L, \theta)\bar\Phi(x_R, \bar\theta)+$$

$$+ \xi \left(\int d^4x_L \, d^2\theta \Phi^3(x_L, \theta) + \text{c.c.} \right). \quad (41)$$

Here E is a density constructed from H^m and its derivatives [26], such that its transformation cancels the total weight transformation of the integration measure $d^4x \, d^2\theta \, d^2\bar\theta$ and the product of chiral compensators. In components, the first term in (41) yields the minimal Einstein $N = 1$ SG action without cosmological term, while the second term in (41) is the superfield form of the cosmological term $\sim \xi$ *.

Later on, many other off-shell component and superfield versions of $N = 1$ SG were discovered. They mainly differ in the choice of the compensating supermultiplet. This variety of compensating superfields is related to the fact that the same on-shell scalar $N = 1$ multiplet admits variant off-shell representations.

The Ogievetsky–Sokatchev formulation of $N = 1$ SG was one of the main indications that the notion of chiral superfields and chiral superspace play the key role in $N = 1$ supersymmetry. Later it was found that the superfield constraints of $N = 1$ SG have the nice geometric meaning: they guarantee the existence of chiral $N = 1$ superfields in the curved case, once again pointing out the fundamental role of chirality in $N = 1$ theories. The constraints defining the $N = 1$ SYM theory can also be derived from requiring chiral representations to exist in the full interaction case. The parameters of the $N = 1$ gauge group are chiral superfields (see (21)), so this group manifestly preserves the chirality. The geometric meaning of $N = 1$ SYM prepotential $V(x, \theta, \bar\theta)$ was discovered in [38]. By analogy with $H^n(x, \theta, \bar\theta)$, the superfield V specifies a real $(4|4)$ dimensional hypersurface, this time in the product of $N = 1$ chiral superspace and the internal coset space G^c/G, where G^c is complexification of the gauge group G. At last,

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*The original Ogievetsky–Sokatchev differential geometry formalism and invariant action [26] amount to some specific gauge choice in (41).

chiral superfields provide the most general description of $N = 1$ matter since all variant off-shell representations of $N = 1$ scalar multiplet are related to chiral multiplet via duality transformation.

Soon after revealing the nice geometric formulation of $N = 1$ SG described above, there arose a question as to how it can be generalized to the most interesting case of extended supergravities and, first of all, to $N = 2$ supergravity. To answer this question, it proved necessary to understand what the correct generalization of $N = 1$ chirality to $N \geqslant 2$ SUSY is and to invent a new sort of superspaces, the harmonic ones.

3. EXTENDED SUSY AND HARMONIC SUPERSPACE

3.1. Difficulties. The basic problem with extended superspace $(x^m, \theta_i^\alpha, \bar\theta^{\dot\alpha i})$ was that the corresponding superfields, due to a large number of Grassmann co-ordinates, contain too many irreducible supermultiplets. So they should be either strongly constrained or subjected to some powerful gauge groups, with a priori unclear geometric meaning. Another problem was that some constraints imply the equations of motion for the involved fields before assuming any invariant action for them. For instance, in the $N = 2$ case $(i = 1, 2)$ the simplest matter multiplet (analog of $N = 1$ chiral multiplet) is the hypermultiplet which is represented by a complex $SU(2)$ doublet superfield $q^i(x, \theta, \bar\theta)$ subjected to the constraints

$$D_\alpha^{(i} q^{k)} = \bar D_{\dot\alpha}^{(i} q^{k)} = 0. \tag{42}$$

Here $^{(\ )}$ means symmetrization and $D_\alpha^i, \bar D_{\dot\alpha}^k$ are $N = 2$ spinor covariant derivatives satisfying the relations

$$\{D_\alpha^i, D_\beta^k\} = \{\bar D_{\dot\alpha}^k, \bar D_{\dot\beta}^i\} = 0, \quad \{D_\alpha^i, \bar D_{\dot\beta}^k\} = 2i\epsilon^{ik}(\sigma^m)_{\alpha\dot\beta}\partial_m. \tag{43}$$

Using (43), it is a direct exercise to check that (42) gives rise to the equations of motion for the physical component fields in $q^i = f^i + \theta^{i\alpha}\psi_\alpha + \bar\theta_{\dot\alpha}^i \bar\chi^{\dot\alpha} + \dots$, viz.,

$$\Box f^i = 0, \quad \partial_m \psi \sigma^m = \sigma^m \partial_m \bar\chi = 0. \tag{44}$$

This phenomenon is a reflection of the «no-go» theorem [39] stating that no off-shell representation for hypermultiplet in its «complex form» (i.e., with bosonic fields arranged into $SU(2)$ doublet) can be achieved with any *finite* number of auxiliary fields. It remained to explore whether there exists a reasonable way to evade this theorem and to write a kind of off-shell action for the hypermultiplet.

It was as well unclear how to construct a geometric unconstrained formulation of the $N = 2$ SYM theory, similar to the prepotential formulation of $N = 1$ SYM. The differential geometry constraints defining this theory were given in [40]

$$\{\mathcal{D}_\alpha^{(i}, \mathcal{D}_\beta^{k)}\} = \{\bar{\mathcal{D}}_{\dot\alpha}^{(k}, \bar{\mathcal{D}}_{\dot\beta}^{i)}\} = \{\mathcal{D}_\alpha^{(i}, \bar{\mathcal{D}}_{\dot\beta}^{k)}\} = 0, \tag{45}$$

where $\mathcal{D}_\alpha^i = D_\alpha^i + i\mathcal{A}_\alpha^i$ is a gauge-covariantized spinor derivative. Luca Mezincescu was the first to find the solution of these constraints in the Abelian case through an unconstrained prepotential [41]. However, the latter possesses a non-standard dimension –2, and the corresponding gauge freedom does not admit a geometric interpretation. So it remained to see whether something like a nice geometric interpretation of the $N = 1$ SYM gauge group and prepotential V can be revealed in the $N = 2$ case (and higher N cases). The same problem existed for superfield $N = 2$ SG.

In [42], Galperin, Ogievetsky, and me observed that extended SUSY, besides standard chiral superspaces generalizing the $N = 1$ one, also admit some other type of invariant subspaces which we called «Grassmann-analytic». Like in the case of chiral superspaces, these subspaces are revealed by passing to some new basis in the general superspace, such that spinor covariant derivatives with respect to some fraction of Grassmann variables become «short» in it. Then one can impose Grassmann Cauchy–Riemann conditions with respect to these variables, with preserving full SUSY. In the $N = 2$ case, allowing the $U(2)$ automorphism symmetry to be broken down to $O(2)$, and making the appropriate shift of x^m, one can define the complex «$O(2)$ analytic subspace»

$$(\tilde{x}^m, \ \theta_\alpha^1 + i\theta_\alpha^2, \ \bar{\theta}_{\dot\alpha}^1 + i\bar{\theta}_{\dot\alpha}^2), \tag{46}$$

which is closed under $N = 2$ SUSY, and the related Grassmann-analytic superfields. It was natural to assume that this new type of analyticity plays a fundamental role in extended SUSY, similarly to chirality in the $N = 1$ case. In [43], we found that the hypermultiplet constraints (42) imply that different components of $N = 2$ superfield q^i «live» on different $O(2)$-analytic subspaces. Since (42) is $SU(2)$ covariant, it was tempting to «$SU(2)$-covariantize» the $O(2)$ analyticity.

All these problems were solved with invention of the harmonic superspace [44–46].

3.2. $N = 2$ **Harmonic Superspace.** $N = 2$ harmonic superspace (HSS) is defined as the product

$$(x^m, \theta_{\alpha\,i}, \bar{\theta}_{\dot\beta}^k) \otimes S^2. \tag{47}$$

Here, $S^2 \sim SU(2)_A/U(1)$, with $SU(2)_A$ being the automorphism group of the $N = 2$ superalgebra. The internal 2-sphere S^2 is represented in a parametrization-independent way by the lowest (isospinor) $SU(2)_A$ harmonics

$$S^2 \in (u_i^+, u_k^-), \quad u^{+i}u_i^- = 1, \quad u_i^\pm \to e^{\pm i\lambda}u_i^\pm. \tag{48}$$

It is assumed that nothing depends on the $U(1)$ phase $e^{i\lambda}$, so one effectively deals with the 2-sphere $S^2 \sim SU(2)_A/U(1)$. The superfields given on (47) (harmonic $N = 2$ superfields) are assumed to be expandable into the harmonic series on S^2,

with the set of all symmetrized products of u_i^+, u_i^- as the basis. These series are fully specified by the $U(1)$ charge of the given superfield.

The main advantage of HSS is the existence of an invariant subspace in it, the $N = 2$ analytic HSS with a half of the original odd coordinates

$$\left(x_A^m, \theta_\alpha^+, \bar{\theta}_{\dot{\alpha}}^+, u^{\pm i}\right) \equiv \left(\zeta^M, u_i^\pm\right), \tag{49}$$

$$x_A^m = x^m - 2i\theta^{(i}\sigma^m\bar{\theta}^{k)}u_i^+u_k^-, \quad \theta_\alpha^+ = \theta_\alpha^i u_i^+, \quad \bar{\theta}_{\dot{\alpha}}^+ = \bar{\theta}_{\dot{\alpha}}^i u_i^+.$$

It is $SU(2)$ covariantization of the $O(2)$ analytic superspace (46). It is closed under $N = 2$ SUSY transformations and is real with respect to the special involution which is the product of the ordinary complex conjugation and the antipodal map (Weyl reflection) of S^2. All $N = 2$ supersymmetric theories have the off-shell formulations in terms of unconstrained superfields given on (49), the *Grassmann analytic* $N = 2$ superfields.

N=2 Matter is represented by n hypermultiplet superfields $q_a^+(\zeta, u)$ $(\overline{(q_a^+)} = \Omega^{ab}q_b^+, \quad \Omega^{ab} = -\Omega^{ba}; \ a, b = 1, \dots, 2n)$ with the following general off-shell action:

$$S_q = \int du\, d\zeta^{(-4)} \left\{q_a^+ D^{++}q^{+a} + L^{+4}(q^+, u^+, u^-)\right\}. \tag{50}$$

Here, $du\, d\zeta^{(-4)}$ is the appropriate (charged!) measure of integration over the analytic superspace (49), $D^{++} = u^{+\,i}(\partial/\partial u^{-i}) - 2i\theta^+\sigma^m\bar{\theta}^+(\partial/\partial x^m)$ is the analytic basis form of one of three harmonic derivatives, which one can define on S^2 (it is distinguished in that it preserves the harmonic Grassmann analyticity), and the indices are raised and lowered by the $Sp(n)$ totally skew-symmetric tensors Ω^{ab}, Ω_{ab}, $\Omega^{ab}\Omega_{bc} = \delta_c^a$. The interaction Lagrangian L^{+4} is an arbitrary function of its arguments, the only restriction is its harmonic $U(1)$ charge $+4$ which is needed for the whole action to be neutral. The crucial feature of the general q^+ action (50) is an infinite number of auxiliary fields coming from the harmonic expansion on S^2. This allowed one to circumvent the no-go theorem about the nonexistence of the off-shell formulations of the $N = 2$ hypermultiplet in its complex form. The on-shell constraints (42) (and their nonlinear generalizations) amount to *both* the harmonic analyticity of q^+ (which is a kinematic property like $N = 1$ chirality) and to the dynamical equations of motion following from the action (50). After eliminating infinite sets of auxiliary fields by their equations of motion, one gets the most general self-interaction of n hypermultiplets. It yields in the bosonic sector the generic sigma model with $4n$-dimensional hyper-Kähler (HK) target manifold in accord with the theorem of Alvarez–Gaumé and Freedman about the one-to-one correspondence between $N = 2$ supersymmetric sigma models and HK manifolds [47]. In general, the action (50) and the corresponding HK sigma model possess no any isometries. The object L^{+4} is the HK potential, analog of the Kähler potential of $N = 1$ supersymmetric sigma models: Taking one or

another specific L^{+4}, one gets the explicit form of the relevant HK metric after eliminating auxiliary fields from (50). So, the general hypermultiplet action (50) provides an efficient universal tool of the *explicit* construction of the HK metrics.

$N = 2$ *Super Yang–Mills Theory* has as its fundamental geometric object the analytic harmonic connection $V^{++}(\zeta, u)$ which covariantizes the analyticity-preserving harmonic derivative:

$$D^{++} \to \mathcal{D}^{++} = D^{++} + igV^{++}, \quad (V^{++})' = \frac{1}{ig}e^{i\omega}\left(D^{++} + igV^{++}\right)e^{-i\omega}, \quad (51)$$

where g is a coupling constant and $\omega(\zeta, u)$ is an arbitrary analytic gauge parameter containing infinitely many component gauge parameters in its combined θ, u-expansion. The harmonic connection V^{++} contains infinitely many component fields, however almost all of them can be gauged away by $\omega(\zeta, u)$. The rest of the $(8 + 8)$ components is just the off-shell content of $N = 2$ vector multiplet. More precisely, in the WZ gauge V^{++} has the following form:

$$
\begin{aligned}
V_{\text{WZ}}^{++} = {} & (\theta^+)^2 w(x_A) + (\bar{\theta}^+)^2 \bar{w}(x_A) + \\
& + i\theta^+ \sigma^m \bar{\theta}^+ V_m(x_A) + (\bar{\theta}^+)^2 \theta^{+\alpha} \psi_\alpha^i(x_A) u_i^- + \\
& + (\theta^+)^2 \bar{\theta}_{\dot\alpha}^+ \bar{\psi}^{\dot\alpha i}(x_A) u_i^- + (\theta^+)^2 (\bar{\theta}^+)^2 D^{(ij)}(x_A) u_i^- u_j^-. \quad (52)
\end{aligned}
$$

Here, $V_m, w, \bar{w}, \psi_i^\alpha, \bar{\psi}^{\dot\alpha i}, D^{(ij)}$ are the gauge field, complex physical scalar field, doublet of gaugini and the triplet of auxiliary fields, respectively. All the geometric quantities of the $N = 2$ SYM theory (spinor and vector connections, covariant superfield strengths, etc.), as well as the invariant action, admit a concise representation in terms of $V^{++}(\zeta, u)$. In particular, the closed V^{++} form of the $N = 2$ SYM action was found in [48].

$N = 2$ *Conformal Supergravity* (Weyl) multiplet is represented in HSS by the analytic vielbeins covariantizing D^{++} with respect to the analyticity-preserving diffeomorphisms of the superspace $\left(\zeta^M, u^{\pm i}\right)$:

$$
D^{++} \to \mathcal{D}^{++} = u^{+i}\frac{\partial}{\partial u^{-i}} + H^{++M}(\zeta, u)\frac{\partial}{\partial \zeta^M} + H^{++++}(\zeta, u)u^{-i}\frac{\partial}{\partial u^{+i}},
$$

$$
\delta \zeta^M = \lambda^M(\zeta, u), \quad \delta u_i^+ = \lambda^{++}(\zeta, u)u_i^-, \quad (53)
$$

$$
\delta H^{++M} = \mathcal{D}^{++}\lambda^M - \delta_{\mu+}^M \theta^{\mu+}\lambda^{++}, \quad \delta H^{++++} = \mathcal{D}^{++}\lambda^{++}, \quad \mu \equiv (\alpha, \dot\alpha),
$$

$$
\delta\mathcal{D}^{++} = -\lambda^{++}D^0, \quad D^0 \equiv u^{+i}\frac{\partial}{\partial u^{+i}} - u^{-i}\frac{\partial}{\partial u^{-i}} + \theta^{\mu+}\frac{\partial}{\partial\theta^{\mu+}}.
$$

The vielbein coefficients H^{++M}, H^{++++} are unconstrained analytic superfields involving an infinite number of the component fields which come from the harmonic expansions. Most of them, like in V^{++}, can be gauged away by the

analytic parameters λ^M, λ^{++}, leaving in the WZ gauge just the $(24+24)$ $N=2$ Weyl multiplet. The invariant actions of various versions of $N=2$ *Einstein* SG are given by a sum of the action of $N=2$ vector compensating superfield $H^{++5}(\zeta, u), \delta H^{++5} = \mathcal{D}^{++}\lambda^5(\zeta, u)$, and that of matter compensator superfields, both in the background of $N=2$ conformal SG. The superfield $H^{++5}(\zeta, u)$ and extra gauge parameter $\lambda^5(\zeta, u)$ have, respectively, the geometric meaning of the vielbein coefficient associated with an extra coordinate x^5 (central charge coordinate) and the shift along this coordinate. Nothing is assumed to depend on this coordinate. The most general off-shell version of $N=2$ Einstein SG is obtained by choosing the superfield $q^{+a}(\zeta, u)$ as the matter compensator. It involves an infinite number of auxiliary fields and yields all the previously known off-shell versions with finite sets of auxiliary fields via appropriate superfield duality transformations. Only this version allows for the most general SG-matter coupling. The latter gives rise to a generic quaternion-Kähler sigma model in the bosonic sector, in accordance with the theorem of Bagger and Witten [49].

More references to the HSS-oriented works of the Dubna group can be found in the book [46].

3.3. Some Further Developments. Here we sketch a few basic directions in which the HSS method was developed after its invention in [44]. It can be generalized to $N \geqslant 2$. It was used to construct, for the first time, an unconstrained off-shell formulation of the $N=3$ super YM theory (equivalent to $N=4$ YM on shell) in the harmonic $N=3$ superspace with the purely harmonic part $SU(3)/[U(1) \times U(1)]$, $SU(3)$ being the automorphism group of $N=3$ SUSY [50]. The corresponding action is written in the analytic $N=3$ superspace and has a nice form of the superfield Chern–Simons term. The $N=4$ HSS with the harmonic part $SU(4)/[U(1) \times SU(2) \times SU(2)]$ was employed to give a new geometric interpretation of the on-shell constraints of $N=4$ super YM theory [51]. In [52, 53], the bi-harmonic superspace with two independent sets of $SU(2)$ harmonics was introduced and shown to provide an adequate off-shell description of $N=(4,4)$, 2D sigma models with torsion. $N=4$, 1D HSS was used in [70] to construct a new super KdV hierarchy, $N=4$ supersymmetric one. Various versions of HSS in diverse dimensions were also explored in [55]. The current important applications of the HSS approach involve the quantum off-shell calculations in $N=2$ and $N=4$ gauge theories (see, e.g., [56,57]), classifying «short» and «long» representations of various superconformal groups in diverse dimensions in the context of the AdS/CFT correspondence [58], study of the domain-wall solutions in the hypermultiplet models [59], description of self-dual supergravities [60], etc. The Euclidean version of $N=2$ HSS was applied in [61–63] to construct string theory-motivated non-anticommutative (nilpotent) deformations of $N=(1,1)$ hypermultiplet and gauge theories. Using the HSS approach, an example of renormalizable $N=(1,0)$ supersymmetric 6D gauge

theory was constructed [64]. Quite recently, harmonic superspace methods allowed one to construct $N = 6$ and $N = 5$ superextensions of 3D Chern–Simons action [65]. The application of the harmonic superspace methods to extended supersymmetric quantum mechanics was initiated in [66–68] and recently continued in [69–71]. In particular, it was argued in [70] that the $N = 4$, 1D harmonic superspace provides a unified description of all known off-shell multiplets of $N = 4$ supersymmetric mechanics. The corresponding $N = 4$, 1D superfields are related to each other and to the so-called «root» $N = 4$ superfield via gauging the appropriate isometries of the superfield actions by nonpropagating «topological» $N = 4$ gauge multiplets. These relations are a manifestly supersymmetric superfield form of the similar relations established in [72] at the component level.

No doubt, the HSS method as the most appropriate approach to the off-shell theories with extended supersymmetries will be widely used and advanced in future studies including those to be carried out in Dubna.

4. OTHER SUSY-RELATED ACTIVITIES

Besides the mainstream SUSY researches outlined in the previous Sections, there were several important pioneering achievements of Dubna group in the fields related to some other applications of supersymmetry.

First of all, these are the issues related to two-dimensional supersymmetric integrable systems. In [73], together with Sergey Krivonos, we constructed an integrable $N = (2, 2)$ extension of the Liouville theory which was unknown before. The first example of $N = (4, 4)$ integrable system, the $N = (4, 4)$ WZW–Liouville theory*, was presented in [74] and further studied (at the classical and quantum levels) in [75, 76]. In [74], we, independently of the authors of [77], discovered $N = (4, 4)$ twisted multiplet and in fact gave the first example of supersymmetric WZW model (at once with $N = (4, 4)$ supersymmetry). Superfield actions of $N = 4$ and higher N supersymmetric and superconformal quantum mechanics were pioneered in our papers [78–80]. New integrable super KdV and NLS type hierarchies were discovered in [70, 81–85]. The manifestly $N = 2$ supersymmetric superfield Hamiltonian reduction as a powerful method of constructing $N = 2$ super W algebras and integrable systems was developed in [86].

Interesting exercises in the superstring theory were undertaken in the unpublished papers [87], following [88]. There, we considered a generalization of the standard Green–Schwarz superstring to certain supergroup manifolds using the powerful techniques of Cartan 1-forms, found the conditions for kappa invariance

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*WZW (Wess–Zumino–Witten) stands for sigma models on group manifolds with torsion.

of the relevant curved actions (with specific nontrivial examples), constructed Hamiltonian formalism for these models and showed their classical integrability. It is curious that this study was fulfilled about 10 years prior to emerging the current vast interest in such constructions within the AdS/CFT paradigm.

Another, more recent activity associated with superbranes was related to their superfield description as systems realizing the concept of Partial Breaking of Global Supersymmetry (PBGS) pioneered by Bagger and Wess [89] and Hughes and Polchinsky [90]. In this approach, the physical worldvolume superbrane degrees of freedom are accommodated by Goldstone superfields, on which the worldvolume SUSY is realized by linear transformations and so is manifest. The rest of the full target SUSY is realized nonlinearly, a la Volkov–Akulov. In components, the corresponding Goldstone superfield actions yield a static-gauge form of the relevant Green–Schwarz-type worldvolume actions. In the cases when Goldstone supermultiplets are vector ones, the Goldstone superfield actions simultaneously provide appropriate supersymmetrizations of the Born–Infeld action. The references to works of the Dubna group on various aspects of the PBGS approach and superextensions of the Born–Infeld theory can be found, e.g., in the review papers [91, 92]. Among the most sound results obtained on this way I would like to distinguish the interpretation of the hypermultiplet as a Goldstone multiplet supporting partial breaking of $N = 1$, 10D SUSY [93], the construction of $N = 2$ extended Born–Infeld theory with partially broken $N = 4$ SUSY [94, 95], as well as of $N = 3$ superextension of Born–Infeld theory with the use of the $N = 3$ HSS approach [96]. Closely related issues of the twistor-harmonic description of superbranes in diverse dimensions were addressed in [97, 98]. Recently, the superfield PBGS approach was generalized to partially broken AdS supersymmetries (see [99, 100] and refs. therein).

One more related direction concerns the description of higher spins and the derivation of the higher-spin dynamics from quantization of some extended objects. A new superfield approach to the higher-spin multiplets based on nonlinear realizations of the generalized 4D superconformal group $OSp(1|8)$ has been developed in [101]. It was argued that the higher-spin generalization of $N = 1$ supergravity should be based on the preservation of the chiral coset manifolds of $OSp(1|8)$. A new concept of the double (bosonic and fermionic) supersymmetry was proposed in [102]. The corresponding superparticle model was constructed and shown to lead, upon quantization, to a new $SU(3, 2|1)$ superconformal superfield system of equations for the massless higher-spin $N = 1$ supermultiplets.

One of the current research activities is the study of models of supersymmetric quantum mechanics with extended $N = 4$ and $N = 8$ SUSY. It continues and advances the directions initiated by the papers [78–80] and aims at further understanding of the structure of the parent higher-dimensional supersymmetric field theories, as well as of the AdS$_2$/CFT$_1$ version of general string/gauge corre-

spondence. For some of the latest developments in this area see, e.g., [103–106] (along with the already cited [66–70]).

At last, let me mention recent papers [107, 108] which treat supersymmetric versions of the quantum-mechanical Landau problem on a plane and two-sphere, as well as the closely related issue of «fuzzy» supermanifolds (which are non-anticommutative versions of the classical supermanifolds, such that their super-space coordinates form a superalgebra isomorphic to that of superisometries of the classical supermanifold). This direction of research looks very prospective from the physical point of view, since it is expected to give rise to a deeper understanding of quantum Hall effect and superextensions thereof, equally as of the relationships of these models with superparticles and superbranes.

CONCLUSIONS

For more than 30 years which passed since the discovery of supersymmetry, the latter grew up into one of the underlying concepts of the modern high-energy theoretical physics. In spite of the fact that the experimental confirmation of supersymmetry (superparticles) is still lacking*, the theoretical consequences of this concept are so impressive (ultraviolet-finite quantum filed theory models, superbranes, AdS/CFT, solving the hierarchy problem in GUT, etc.) that it is impossible to admit that it could be fake. The natural arena for supersymmetry is superspace and superfields. As I tried to show above, the Dubna group largely contributed to the development of the geometric superfield methods. This contri-bution has been widely recognized over the world, it suffices to say that the basic paper on the harmonic superspace (second Ref. in [44]) has now more than 500 citations in the SLAC database. Keeping in mind the current reserach activity of the Dubna theorists in various aspects of supersymmetry, there are enough reasons to believe that in the future the Dubna group will preserve its leading positions in the field as well.

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