ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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S. M. Bilenky

Joint Institute for Nuclear Research, Dubna

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The neutrinoless double β decay of nuclei is reviewed. We discuss neutrino mixing and 3×3 Pontecorvo–Maki–Nakagawa–Sakata (PMNS) neutrino mixing matrix. Basic theory of neutrinoless double β decay is presented in some detail. Results of different calculations of nuclear matrix element are discussed. Experimental situation is considered. Appendix is dedicated to E. Majorana (brief biography and his paper in which the theory of Majorana particles is given).

Представлен обзор проблемы безнейтринного двойного бета-распада ядер. Подробно обсуждаются смешивание нейтрино и матрица смешивания ПМНС. Детально рассматривается теория безнейтринного двойного бета-распада. Обсуждаются результаты различных вычислений ядерных матричных элементов. Обсуждаются опыты по поиску безнейтринного двойного бета-распада. Приложение посвящено Э. Майоране. Дается его краткая биография, и излагается статья, в которой предложена теория частиц Майораны.

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INTRODUCTION

Observation of neutrino oscillations in atmospheric, solar, reactor and accelerator neutrino experiments is one of the most important recent discoveries in particle physics. Small neutrino masses cannot be naturally explained by the Standard Higgs mechanism. A new, beyond the Standard Model mechanism of the generation of neutrino masses is required. The most plausible seesaw mechanism of the neutrino mass generation is based on the assumption of the violation of the total lepton number at a large scale and Majorana nature of neutrinos with definite masses.

After it was established that neutrino masses are different from zero, the problem of the nature of neutrinos with definite masses ν_i (Dirac or Majorana?) is the most important one. Investigation of the neutrino oscillations cannot allow one to answer this fundamental question. The observation of the neutrinoless double β decay ($0\nu\beta\beta$ decay) of some even–even nuclei would be a proof that ν_i are Majorana particles.

The neutrinoless double β decay is extremely rare process. First, this is a process of the second order of the perturbation theory in the Fermi constant. And, second, this process is possible due to helicity flip. Thus, the matrix element of the process is proportional to the effective Majorana mass $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$ (m_i is the mass of the neutrino ν_i). Smallness of the neutrino masses is an additional reason for smallness of the probability of the $0\nu\beta\beta$ decay.

Very high values for the lower bounds of the half-lives of the $0\nu\beta\beta$ decay of different nuclei were reached in the Heidelberg–Moscow [1], IGEX [2], CUORICINO [3] and other experiments. However, in order to reach the values of the half-lives of the $0\nu\beta\beta$ decay which are expected on the basis of the neutrino oscillation data in the case if the neutrino mass spectrum follows the inverted hierarchy, new challenging experiments with a sensitivity to $|m_{\beta\beta}|$ about two orders of magnitude better than the today's sensitivity are required. It is expected that such a sensitivity will be reached in several future experiments.

In Sec. 1 we will consider in some detail neutrino mixing. In Sec. 2 we will discuss the standard (Type I) seesaw mechanism of the neutrino mass generation. In Sec. 3 we will consider general properties of the neutrino mixing matrix and obtain its standard parametrization. Then in Sec. 4 we will discuss briefly the present status of neutrino oscillations. In Sec. 5 we will present a quite detailed derivation of the matrix element of the $0\nu\beta\beta$ decay. Then in Sec. 6 we will consider effective Majorana mass under different assumptions about neutrino mass spectrum. In Secs. 7 and 8 we will discuss the present-day situation with the calculations of nuclear matrix elements of the $0\nu\beta\beta$ decay and experiments on the search for neutrinoless double β decay. In Appendix we will present a short biography of E. Majorana and briefly discuss his 1937 paper in which the theory of the Majorana particles was developed and a possibility of the existence of such particles was discussed.

For different aspects of the $0\nu\beta\beta$ decay, see reviews [4–10].

1. NEUTRINO MIXING

We will consider here the neutrinoless double β decay under two general assumptions.

1. *The neutrino interaction is the Standard Model electroweak interaction*. The Lagrangian of the standard charged current (CC) interaction has the form

$$\mathcal{L}_I^{\mathrm{CC}}(x) = -\frac{g}{2\sqrt{2}}\, j_\alpha^{\mathrm{CC}}(x) W^\alpha(x) + \mathrm{h.c.} \tag{1}$$

Here $W^\alpha(x)$ is the field of the charged $W^\pm$ vector bosons; g is the constant of the electroweak interaction and

$$j_\alpha^{\mathrm{CC}}(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x)\gamma_\alpha l_L(x) + j_\alpha^h(x) \tag{2}$$

is the sum of the leptonic and hadronic charged current. The hadronic charged current is given by the expression

$$j^h_\alpha(x) = 2(\bar{u}_L(x)\gamma_\alpha\ d^{\rm mix}_L(x) + \bar{c}_L(x)\gamma_\alpha\ s^{\rm mix}_L(x) + \bar{t}_L(x)\gamma_\alpha\ b^{\rm mix}_L(x)), \tag{3}$$

where

$$d^{\rm mix}_L(x) = \sum_{q=d,s,b} V_{uq}\ q_L,\ s^{\rm mix}_L(x) = \sum_{q=d,s,b} V_{cq}\ q_L,\ b^{\rm mix}_L(x) = \sum_{q=d,s,b} V_{tq}\ q_L. \tag{4}$$

In (4) the matrix V is the 3×3 Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix [11, 12].

The interaction (1) perfectly describes the data of numerous experiments on the study of the weak decays, neutrino reactions, etc.

2. *The neutrino mixing takes place*. Neutrino fields $\nu_{lL}(x)$ in the leptonic current (2) are mixed fields:

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\nu_{iL}(x). \tag{5}$$

Here $\nu_i(x)$ is the field of neutrino with mass m_i and U is the 3×3 Pontecorvo–Maki–Nakagawa–Sakata [13, 14] neutrino mixing matrix.

The hypothesis of the neutrino mixing was confirmed by the observation of the neutrino oscillations in experiments with the atmospheric, solar, reactor and accelerator neutrinos. All existing neutrino oscillation data are described if we assume that the number of massive neutrinos is equal to the established number of flavor neutrinos (three).

Quarks are charged particles; the quarks and antiquarks have the same masses and their charges differ in sign. Thus, the quark fields $q(x)$ are complex Dirac fields.

The electric charges of neutrinos are equal to zero. For neutrinos there are two fundamentally different possibilities.

- If the total lepton number $L = L_e + L_\mu + L_\tau$ is conserved, neutrino fields $\nu_i(x)$ are complex four-component *Dirac fields*. In this case neutrinos ν_i and antineutrinos $\bar{\nu}_i$ have the same mass and different lepton numbers ($L(\nu_i) = -L(\bar{\nu}_i) = 1$).
- If there are no conserved lepton numbers, neutrino fields $\nu_i(x)$ are two-component *Majorana fields*. In this case $\nu_i \equiv \bar{\nu}_i$.

Investigation of the neutrino oscillations does not allow one to distinguish between these two possibilities [15, 16]. In order to reveal the Majorana nature of ν_i it is necessary to observe processes in which the total lepton number is violated. *Neutrinoless double β decay of some nuclei is the only such process the study of which allows one to reach the necessary sensitivity.*

The nature of neutrinos with definite masses and the form of the neutrino mixing is determined by *the neutrino mass term of the Lagrangian*. We will consider now possible mass terms for neutrinos (see [17–19]).

A neutrino mass term is the Lorentz-invariant product of the left-handed and right-handed components of neutrino fields. The three left-handed current fields $\nu_{lL}(x)$, components of $SU(2)$ doublets, must enter into any neutrino mass term. If we assume that three right-handed singlet fields $\nu_{lR}(x)$ also enter into the Lagrangian, in this case we can build the following neutrino mass term:

$$\mathcal{L}^D(x) = -\bar{\nu}_L(x)\, M^D\, \nu_R(x) + \mathrm{h.c.}, \tag{6}$$

where

$$\nu_L = \begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix}, \quad \nu_R = \begin{pmatrix} \nu_{eR} \\ \nu_{\mu R} \\ \nu_{\tau R} \end{pmatrix} \tag{7}$$

and M^D is the 3×3 neutrino mass matrix. It is obvious that the total Lagrangian with the neutrino mass term (6) is invariant under the global gauge transformations

$$\nu_L(x) \to \mathrm{e}^{i\,\Lambda}\nu_L(x),\ \nu_R(x) \to \mathrm{e}^{i\,\Lambda}\nu_R(x),\ l_{L,R}(x) \to \mathrm{e}^{i\,\Lambda}\, l_{L,R}(x),\ q(x) \to q(x), \tag{8}$$

where Λ is an arbitrary constant phase. The invariance under the transformation (8) means that *the total lepton number L is conserved*.

The mass term (6) can be easily diagonalized. For a complex matrix M^D we have

$$M^D = U\ m\ V^\dagger, \tag{9}$$

where U and V are unitary 3×3 matrices and m is a diagonal 3×3 matrix ($m_{ik} = m_i\delta_{ik}$, $m_i > 0$). From (6) and (9) we find

$$\mathcal{L}^D(x) = -\bar{\nu}^m(x)\, m\, \nu^m(x) = -\sum_{i=1}^{3} m_i\, \bar{\nu}_i(x)\, \nu_i(x), \tag{10}$$

where

$$\nu_L^m = U^\dagger \nu_L = \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix}, \quad \nu_R^m = V^\dagger \nu_L = \begin{pmatrix} \nu_{1R} \\ \nu_{2R} \\ \nu_{3R} \end{pmatrix}. \tag{11}$$

The expression (10) is the sum of standard mass terms for the Dirac fields $\nu_i(x)$ with masses m_i. From (11) we find that the flavor fields $\nu_{lL}(x)$ are connected with the left-handed components of the Dirac neutrino fields $\nu_{iL}(x)$ by the mixing relation

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x). \tag{12}$$

We assumed that not only left-handed fields $\nu_{lL}(x)$ but also the right-handed fields $\nu_{lR}(x)$ enter into the total Lagrangian. In the original Glashow, Weinberg and Salam papers [20–22], in which the Standard Model was proposed, it was assumed that only $\nu_{lL}(x)$ fields, components of the lepton $SU(2)$ doublets, enter into the Lagrangian. In the seventies, after the success of the theory of the two-component neutrino, it was natural to make this simplest assumption. In such a Standard Model with a $SU(2)$ Higgs doublet, neutrinos are massless particles. We can, however, generalize the original SM and to build a model in which neutrino masses and neutrino mixing are generated by the spontaneous violation of the symmetry in the same way as masses and mixing of quarks and leptons. In such a model the neutrino mass term is the Dirac mass term (6).

We know from experiment that neutrino masses are many orders of magnitude smaller than masses of quarks and leptons. For example, for the particles of the third family

$$m_t \simeq 173 \text{ GeV}, \quad m_b \simeq 4.2 \text{ GeV}, \quad m_\tau \simeq 1.78 \text{ GeV}, \quad m_3 \leqslant 2 \cdot 10^{-9} \text{ GeV}. \quad (13)$$

In the framework of the SM there is no natural explanation of such a big difference between masses of neutrinos and other fundamental fermions belonging to the same family. It is very implausible that small neutrino masses are generated by the SM Higgs mechanism.

The small Dirac neutrino masses can be generated, however, in some models beyond the SM, for example, in the model with large extra dimensions [23]. In such a model the Newton law at small distances r has the form $F = \frac{1}{M^{2+n}} \frac{m_1 m_2}{r^{2+n}}$, where n is the number of the extra dimensions and M is a new scale ($\sim (1{-}10)$ TeV). Dirac neutrino masses in the model with extra dimensions are given by the expression

$$m_i \simeq k_i v \ \beta.$$

Here $v \simeq 250$ GeV is the electroweak scale and $\beta = M/M_P \simeq (10^{-15}{-}10^{-16})$ is a suppression factor ($M_P \sim 1.2 \cdot 10^{19}$ GeV is the Planck mass).

We will build now a neutrino mass term, assuming that fields $\nu_{lL}(x)$ and $\nu_{lR}(x)$ enter into the Lagrangian and there are no conserved lepton numbers. Let us consider the conjugated fields

$$(\nu_L)^c = C(\bar{\nu}_L)^T, \quad (\nu_R)^c = C(\bar{\nu}_R)^T, \qquad (14)$$

where C is the matrix of the charge conjugation which satisfies the relations

$$C\gamma_\alpha^T C^{-1} = -\gamma_\alpha, \quad C^T = -C. \qquad (15)$$

It is easy to show that $(\nu_L)^c$ ($(\nu_R)^c$) is the right-handed (left-handed) component of the conjugated field.

In fact, for the left-handed and right-handed components we have

$$\gamma_5\,\nu_L = -\nu_L, \quad \gamma_5\,\nu_R = \nu_R. \tag{16}$$

From these relations we find

$$\bar{\nu}_L\,\gamma_5 = \bar{\nu}_L, \quad \bar{\nu}_R\,\gamma_5 = -\bar{\nu}_R. \tag{17}$$

Now, taking into account that $C\gamma_5^T C^{-1} = \gamma_5$, we have

$$\gamma_5\;(\nu_L)^c = (\nu_L)^c, \quad \gamma_5\;(\nu_R)^c = -(\nu_R)^c. \tag{18}$$

From (18) we conclude that $(\nu_L)^c$ and $(\nu_R)^c$ are the right-handed and left-handed components.

The most general neutrino mass term, which can be built from the flavor left-handed fields $\nu_{lL}(x)$ and sterile fields $\nu_{lR}(x)$*, has the form

$$\mathcal{L}^{D+M} = -\frac{1}{2}\,\bar{\nu_L}\,M_L^M(\nu_L)^c - \bar{\nu}_L\,M^D\,\nu_R - \frac{1}{2}\,\overline{(\nu_R)^c}\,M_R^M\nu_R + \text{h.c.}, \tag{19}$$

where columns $\nu_{L,R}$ are given by (7) and $M^M_{L,R}$ and M^D are nondiagonal complex 3×3 matrices. It is easy to show that $M^M_{L,R}$ are symmetrical matrices. In fact, taking into account Fermi–Dirac statistics of the fields $\nu_{L,R}$, we have

$$\bar{\nu}_{L,R}\,M^M_{L,R}C\bar{\nu}^T_{L,R} = -\bar{\nu}_{L,R}\,(M^M_{L,R})^T C^T\,\bar{\nu}^T_{L,R} = \bar{\nu}_{L,R}\,(M^M_{L,R})^T\;C\bar{\nu}^T_{L,R}. \tag{20}$$

From this relation we find

$$M^M_{L,R} = (M^M_{L,R})^T. \tag{21}$$

It is obvious that the first and the third terms of the expression (19) are not invariant under the global gauge transformations $\nu_{L,R} \to \mathrm{e}^{i\Lambda}\nu_{L,R}$. Thus, in the case of the mass term (19) the total lepton number L is not conserved.

The first and the third terms of the expression (19) are called the left-handed and right-handed *Majorana mass terms*, respectively. The second term is the Dirac mass term. The mass term $\mathcal{L}^{D+M}$ is usually called *the Dirac and Majorana neutrino mass term* [24, 25].

We will show now that in the case of the mass term (19) *neutrinos with definite masses are Majorana particles*.

The mass term $\mathcal{L}^{D+M}$ can be presented in the following form:

$$\mathcal{L}^{D+M} = -\frac{1}{2}\,\bar{n}_L\,M^{D+M}(n_L)^c + \text{h.c.} \tag{22}$$

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*Neutrino fields that do not enter into the Lagrangian of the standard elecroweak interaction are called sterile.

Here

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix} \tag{23}$$

and

$$M^{D+M} = \begin{pmatrix} M_L^M & M^D \\ (M^D)^T & M_R^M \end{pmatrix} \tag{24}$$

is a symmetrical 6×6 matrix.

A symmetrical matrix M can be presented in the form

$$M = U\, m\, U^T, \tag{25}$$

where U is a unitary matrix and m is a diagonal matrix with positive diagonal elements.

From (22) and (25) we have

$$\mathcal{L}^{D+M} = -\frac{1}{2}\,\overline{U^\dagger n_L}\, m\, (U^\dagger\, n_L)^c - \frac{1}{2}\,\overline{(U^\dagger n_L)^c}\, m\, U^\dagger\, n_L = \\ = -\frac{1}{2}\,\bar{\nu}^m\, m\, \nu^m = -\frac{1}{2}\sum_{i=1}^{6} m_i\, \bar{\nu}_i\, \nu_i. \tag{26}$$

Here

$$\nu^m = U^\dagger n_L + (U^\dagger n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \\ \vdots \\ \nu_6 \end{pmatrix}. \tag{27}$$

From (26) and (27) we conclude that

- The field $\nu_i(x)$ $(i = 1, 2, \ldots, 6)$ is the field of neutrinos with mass m_i.
- The field $\nu_i(x)$ satisfies *the Majorana condition*

$$\nu_i(x) = \nu_i^c(x) = C\bar{\nu}_i^T(x). \tag{28}$$

Taking into account the unitarity of the matrix U, from (27) we find

$$n_L = U\, \nu_L^m. \tag{29}$$

From (29) we obtain the following *mixing relations in the general Dirac and Majorana case*:

$$\nu_{lL} = \sum_{i=1}^{6} U_{li}\, \nu_{iL}, \quad (\nu_{lR})^c = \sum_{i=1}^{6} U_{\bar{l}i}\, \nu_{iL}, \tag{30}$$

where U is a unitary 6×6 mixing matrix and ν_i is the field of the Majorana neutrino with mass m_i.

Let us discuss the meaning of the Majorana condition (28). A non-Hermitian field $\nu(x)$ can be presented in the following general form:

$$\nu(x) = \int \frac{1}{(2\pi)^{3/2}} \frac{1}{\sqrt{2p_0}} \left(c_r(p)\; u^r(p)\,\mathrm{e}^{-ipx} + d_r^\dagger(p)\,u^r(-p)\,\mathrm{e}^{ipx}\right) d^3p, \quad (31)$$

where $c_r(p)$ is the operator of absorption of neutrino with momentum p and helicity r and $d_r^\dagger(p))$ is the operator of creation of antineutrino with momentum p and helicity r and $u^r(-p) = C(\bar{u}^r(p))^T$. If the field $\nu(x)$ satisfies the Majorana condition (28), we find

$$c_r(p) = d_r(p). \quad (32)$$

Thus, if $\nu(x)$ is the Majorana field, *the neutrinos and antineutrinos are identical particles.* In other words, the Majorana field is the field of truly neutral particles. There is no notion of particles and antiparticles in the case of the Majorana field*.

We will finish this section with the following remarks:

1. Dirac and Majorana mass term can be generated only in theories beyond the SM.

2. If we assume that only left-handed fields $\nu_{lL}(x)$ enter into the mass term and the lepton number is not conserved, we come to the following (Majorana) mass term [26]:

$$\mathcal{L}^M = -\frac{1}{2}\,\bar{\nu_L}\,M_L^M(\nu_L)^c + \mathrm{h.c.}, \quad (33)$$

where M_L^M is 3×3 symmetrical matrix. After the diagonalization, the mass term (33) takes the standard form

$$\mathcal{L}^M = -\frac{1}{2}\sum_{i=1}^{3} m_i \bar{\nu}_i \nu_i \quad (34)$$

and we come to the Majorana mixing

$$\nu_{lL} = \sum_{i=1}^{3} U_{li}\,\nu_{iL}. \quad (35)$$

Here U is a 3×3 mixing matrix and ν_i is the Majorana field with the mass m_i which satisfies the condition (28). Notice that Higgs triplet is needed for the generation of the mass term $\mathcal{L}^M$.

*In the case of the Majorana field there are no conserved charges which allow one to distinguish particles and antiparticles.

3. From the Majorana condition (28) we have

$$\nu_{iR} = \nu_{iR}^c = (\nu_{iL})^c. \tag{36}$$

Thus, in the case of the Majorana field, right-handed and left-handed components are connected by the relation (36). In the case of the Dirac field, right-handed and left-handed components are independent. This is the major difference between Majorana and Dirac fields.

Right-handed components of neutrino fields enter into the Dirac mass term. If neutrinos are massless there are no mass term in the Lagrangian. This is the reason for the well-known theorem [27] which states that it is impossible to distinguish massless Dirac and Majorana neutrinos in the case of left-handed interaction.

4. The Dirac and Majorana mass term opens a possibility of the existence of the sterile neutrinos. If masses m_i are small, in this case in addition to the mixed flavor left-handed neutrinos ν_e, ν_μ and ν_τ mixed left-handed antineutrinos $\bar{\nu}_{lL}$, quanta of mixed right-handed fields ν_{lR}, must exist. Because right-handed fields do not enter into the standard CC and NC interactions, $\bar{\nu}_{lL}$ have no electroweak interaction. They are called sterile neutrinos. Let us notice that the existing LSND indication in favor of the sterile neutrinos [28] was not confirmed by the MiniBooNE experiment [29].

5. In the case of the Dirac and Majorana mass term, there are additional sterile right-handed fields ν_{lR} and many parameters in the mass matrix. This mass term opens a possibility to explain the smallness of the neutrino masses. This (so-called seesaw) possibility will be considered in the next section.

2. SEESAW MECHANISM OF THE NEUTRINO MASS GENERATION

The most popular mechanism of the generation of small neutrino masses is the seesaw mechanism [30]. In order to explain the main idea of this mechanism, we consider the simplest case of one generation. The Dirac and Majorana mass term is given in this case by the expression

$$\mathcal{L}^{D+M} = -\frac{1}{2}\, m_L \bar{\nu}_L (\nu_L)^c - m_D \bar{\nu}_L \nu_R - \frac{1}{2}\, m_R \overline{(\nu_L)^c} \nu_R + \text{h.c.} \tag{37}$$

We will assume that m_L, m_D and m_R are real parameters. Let us write Eq. (37) in the matrix form. We have

$$\mathcal{L}^{D+M} = -\frac{1}{2}\, \bar{n}_L\, M^{D+M} (n_L)^c + \text{h.c.} \tag{38}$$

Here

$$M^{D+M} = \begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \tag{39}$$

and

$$n_L = \begin{pmatrix} \nu_L \\ (\nu_R)^c \end{pmatrix}. \tag{40}$$

The real symmetrical matrix M^{D+M} can be presented in the form

$$M^{D+M} = O\, m'\, O^T, \tag{41}$$

where

$$O = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{42}$$

and $m'_{ik} = m'_i \delta_{ik}$, m'_i being an eigenvalue of the matrix M^{D+M}. We have

$$m'_{1,2} = \frac{1}{2}\,(m_R + m_L) \mp \frac{1}{2}\,\sqrt{(m_R - m_L)^2 + 4\, m_D^2}. \tag{43}$$

From (41), (42) and (43) for the mixing angle θ we obtain the following relations:

$$\tan\, 2\theta = \frac{2m_D}{m_R - m_L}, \quad \cos\, 2\theta = \frac{m_R - m_L}{\sqrt{(m_R - m_L)^2 + 4\, m_D^2}}. \tag{44}$$

The eigenvalues $m'_{1,2}$ can be positive or negative. Let us write down

$$m'_i = m_i \eta_i, \tag{45}$$

where $m_i = |m'_i|$ and $\eta_i = \pm 1$.

From (41) and (45) we find

$$M^{D+M} = U m U^T, \tag{46}$$

where

$$U = O\,\sqrt{\eta} \tag{47}$$

is a unitary matrix. Using the general results of the previous section, we easily bring the mass term (48) to the standard form

$$\mathcal{L}^{D+M} = -\frac{1}{2}\bar{\nu}^m \nu^m = -\frac{1}{2}\sum_{i=1,2} m_i \bar{\nu}_i \nu_i. \tag{48}$$

Here

$$\nu^m = U^\dagger n_L + (U^\dagger n_L)^c = \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}, \tag{49}$$

ν_i being the Majorana field with the mass m_i. From (49) we have

$$n_L = U\nu_L^m. \tag{50}$$

Thus, the fields ν_L and $(\nu_R)^c$ are connected with the fields ν_{1L} and ν_{2L} by the following mixing relations:

$$\begin{aligned} \nu_L &= \cos\theta\sqrt{\eta_1}\,\nu_{1L} + \sin\theta\sqrt{\eta_2}\,\nu_{2L}, \\ (\nu_R)^c &= -\sin\theta\sqrt{\eta_1}\,\nu_{1L} + \cos\theta\sqrt{\eta_2}\,\nu_{2L}. \end{aligned} \tag{51}$$

Neutrino masses are many orders of magnitude smaller than masses of leptons and quarks which are generated by the standard Higgs mechanism of the electroweak symmetry breaking. This fact is commonly considered as an evidence in favor of a nonstandard mechanism of neutrino mass generation. The seesaw mechanism connects smallness of neutrino masses with the violation of the total lepton number at a very large scale.

The standard (type I) seesaw mechanism [30] is based on the following assumptions:

1. There is no left-handed Majorana mass term in the Lagrangian ($m_L = 0$).

2. The Dirac mass term is generated by the Higgs mechanism (m_D is of the order of the mass of a charged lepton or quark).

3. The constant m_R, which characterizes the right-handed Majorana mass term, the source of the violation of the total lepton number, is much larger than m_D:

$$m_R \gg m_D. \tag{52}$$

From (43), (44) and (52) we have

$$m_1 \simeq \frac{m_D}{m_R}\, m_D \ll m_D, \quad m_2 \simeq m_R, \quad \tan\theta \simeq \frac{m_D}{m_R} \ll 1. \tag{53}$$

Thus, the seesaw mechanism generates Majorana neutrino mass m_1 which is much smaller than a Dirac mass of a lepton or quark. As a consequence of the seesaw mechanism, a heavy Majorana particle with a mass $m_2 \simeq m_R$ must exist.

Let us consider now the case of the three families. The seesaw mixing matrix has in this case the form

$$M^{\text{seesaw}} = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \tag{54}$$

Here m_D is a complex 3×3 matrix, M_R is a symmetrical complex matrix and $m_D \ll M_R$.

Let us introduce the matrix M by the relation

$$U^T M^{\text{seesaw}} U = M, \tag{55}$$

where U is a unitary matrix. We will show now the matrix U can be chosen in such a form that M is the block-diagonal matrix.

Notice that in the case of one generation up to terms linear in $m_D/m_R \ll 1$ we have

$$U^{(2)} \simeq \begin{pmatrix} 1 & m_D/m_R \\ -m_D/m_R & 1 \end{pmatrix}. \tag{56}$$

Let us consider the matrix

$$U \simeq \begin{pmatrix} 1 & A \\ -A^\dagger & 1 \end{pmatrix}, \tag{57}$$

where A is a 3×3 matrix and $A_{ik} \ll 1$. It is easy to see that up to terms linear in A, $U^\dagger U \simeq 1$. The nondiagonal element of the symmetrical matrix $U^T\, M^{\text{seesaw}}\, U$ in the approximation linear over A is equal to

$$m_D^T - M_R A^\dagger. \tag{58}$$

If we choose

$$A^\dagger = M_R^{-1} m_D^T, \tag{59}$$

the matrix $U^T\, M^{\text{seesaw}}\, U$ takes a block-diagonal form

$$U^T\, M^{\text{seesaw}}\, U \simeq \begin{pmatrix} -m_D M_R^{-1} m_D^T & 0 \\ 0 & M_R \end{pmatrix}. \tag{60}$$

For the left-handed Majorana neutrino mass term from (60) we find

$$\mathcal{L}^M = -\frac{1}{2}\bar{\nu}_L M_L^M (\nu_L)^c + \text{h.c.}, \tag{61}$$

where

$$M_L^M = -m_D M_R^{-1} m_D^T \tag{62}$$

and ν_L is given by (7).

Equation (61) is the mass term for three light Majorana neutrinos. After the diagonalization of the total mass term, in addition to Majorana neutrino mass term, we will obtain a mass term for three heavy Majorana particles. Thus, in the case of the Dirac and Majorana mass term with the matrix (54) in the spectrum of masses there are

- three light Majorana neutrino masses;
- three heavy Majorana masses, which are characterized by the scale of the violation of the total lepton number.

These are general features of the seesaw mechanism. The values of neutrino masses and mixing angles can be obtained only in the framework of a concrete model.

Thus, the seesaw mechanism connects smallness of the neutrino masses with violation of the total lepton number at a large scale*. The observation of the neutrinoless double β decay would be an evidence in favor of this mechanism. Let us notice that the existence of heavy Majorana particles, seesaw partners of neutrinos, could allow us to explain the baryon asymmetry of the Universe (see [31]).

3. NEUTRINO MIXING MATRIX

In this section we will consider the general properties of the unitary 3×3 Dirac (or Majorana) mixing matrix.

A unitary $n \times n$ matrix U is characterized by n^2 real parameters**. The number of the angles which characterize the unitary $n \times n$ matrix coincides with the number of parameters which characterize a real orthogonal $n \times n$ matrix O ($O^T O = 1$). Thus, for the number of the angles we have***

$$n_{\mathrm{ang}} = \frac{n(n-1)}{2}. \tag{63}$$

Other parameters of the matrix U are phases. The number of phases is equal to

$$n_{\mathrm{ph}} = n^2 - \frac{n(n-1)}{2} = \frac{n(n+1)}{2}. \tag{64}$$

The number of *physical phases* in the neutrino mixing matrix is smaller than n_{ph}. The neutrino mixing matrix enters into the charged current. Let us consider first the case of the Dirac neutrinos ν_i. Because phases of the Dirac fields $l_L(x)$ and $\nu_{iL}(x)$ are arbitrary, the matrices U and

$$U' = S^\dagger(\beta) U S(\alpha) \tag{65}$$

are equivalent. Here $S_{ll'}(\beta) = \mathrm{e}^{i\beta_l}\,\delta_{ll'}$, $S_{ik}(\alpha) = \mathrm{e}^{i\alpha_i}\,\delta_{ik}$ and β_l, α_i are real, arbitrary phases.

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*Usually it is assumed that this scale is about 10^{15}–10^{16} GeV.

**In fact, it can be presented in the form $U = \mathrm{e}^{iH}$, where H is the Hermitian matrix. The Hermitian matrix is characterized by $n + 2\left(\dfrac{n^2 - n}{2}\right) = n^2$ real parameters.

***The orthogonal matrix O can be presented in the form $O = \mathrm{e}^{A}$, where $A^T = -A$. Diagonal elements of the matrix A are equal to zero. The number of the real nondiagonal elements is equal to $\dfrac{n(n-1)}{2}$.

We can use this freedom in order to exclude $(2n-1)$ phases from the matrix U^*. Thus, in the case of the Dirac neutrinos the number of the physical phases in the mixing matrix U is equal to

$$\bar{n}_{\rm ph} = \frac{n(n+1)}{2} - (2n-1) = \frac{(n-1)(n-2)}{2}. \qquad (66)$$

In the case of the mixing of the three Dirac neutrinos, the mixing matrix is characterized by three mixing angles and one phase.

Let us consider now the case of the Majorana neutrinos ν_i. The Majorana condition

$$\nu_i^c(x) = \nu_i(x) \qquad (67)$$

does not allow one to include arbitrary phases into the Majorana fields. For the number of the physical phases we have in the Majorana case [15, 16]

$$\bar{n}^M_{\rm ph} = \frac{n(n+1)}{2} - n = \frac{n(n-1)}{2}. \qquad (68)$$

Thus, in the case of the three Majorana neutrinos, the mixing matrix is characterized by three mixing angles and three phases.

We will obtain now constraints on the neutrino mixing matrix which follow from the condition of the CP invariance in the lepton sector. Let us consider first the Dirac neutrinos ν_i. The condition of the CP invariance in the lepton sector has the form

$$V_{CP}\ \mathcal{L}_I^{\rm CC}(x)\ V_{CP}^{-1} = \mathcal{L}_I^{\rm CC}(x'). \qquad (69)$$

Here V_{CP} is the operator of the CP conjugation, $x' = (x^0, -\mathbf{x})$ and

$$\mathcal{L}_I^{\rm CC}(x) = -\frac{g}{\sqrt{2}} \sum_{l,i} \bar{l}_L(x)\gamma_\alpha U_{li}\nu_{iL}(x)W^{\alpha\dagger} - \\ - \frac{g}{\sqrt{2}} \sum_{l,i} \bar{\nu}_{iL}(x)\gamma_\alpha U_{li}^* l_L(x) W^\alpha \qquad (70)$$

is the Lagrangian of the CC interaction of neutrinos, leptons and W bosons. Taking into account arbitrariness of the phases of fermion fields, we can put CP phase factors of the lepton and neutrino fields equal to one. We have

$$V_{CP} l_L(x) V_{CP}^{-1} = \gamma^0 C \bar{l}_L^T(x'), \quad V_{CP}\nu_{iL}(x)V_{CP}^{-1} = \gamma^0 C \bar{\nu}_{iL}^T(x'). \qquad (71)$$

---

*We can always make one element of the matrix $S(\alpha)$ (or $S(\beta)$) equal to one. In fact, let us present the matrix $S(\alpha)$ in the form $S(\alpha) = \mathrm{e}^{i\alpha_n} S(\bar{\alpha})$, where $\bar{\alpha}_i = \alpha_i - \alpha_n$. The phase factor $\mathrm{e}^{i\alpha_n}$ can be, obviously, included into $S^\dagger(\beta)$. We have in this case $S^\dagger(\beta)\,\mathrm{e}^{i\alpha_n} = S^\dagger(\bar{\beta})$, where $\bar{\beta}_l = \beta_l - \alpha_n$.

From these relations we find

$$V_{CP}\bar{l}_L(x)V_{CP}^{-1} = -l_L^T(x')C^{-1}\gamma^0, \quad V_{CP}\bar{\nu}_{iL}(x)V_{CP}^{-1} = -\nu_{iL}^T(x')C^{-1}\gamma^0. \quad (72)$$

For the field of the charged $W^{\pm}$ vector bosons we have

$$V_{CP}W_\alpha(x)V_{CP}^{-1} = -\delta_\alpha W_\alpha^\dagger(x'), \quad (73)$$

where δ_α is a sign factor ($\delta_0 = 1,\ \delta_i = -1$). From all these relations we easily find

$$V_{CP}\mathcal{L}_I^{\mathrm{CC}}(x)V_{CP}^{-1} = -\frac{g}{\sqrt{2}}\sum_{l,i}\bar{\nu}_{iL}(x')\gamma_\alpha U_{li} l_L(x')W^\alpha(x') - \\ -\frac{g}{\sqrt{2}}\sum_{l,i}\bar{l}_L(x')\gamma_\alpha U_{li}^* \nu_{iL}(x')W^{\alpha\dagger}(x'). \quad (74)$$

Comparing (69) and (74), we come to the conclusion that in the case of the CP invariance in the lepton sector the Dirac mixing matrix is real:

$$U_{li} = U_{li}^*. \quad (75)$$

We will consider now the case of the Majorana fields [32–34]. The CP transformation of the Majorana field ν_i has the form

$$V_{CP}\nu_i(x)V_{CP}^{-1} = \eta_i^*\gamma^0 C\bar{\nu}_i^T(x') = \eta_i^*\gamma^0\nu_i(x'), \quad (76)$$

where η_i^* is a phase factor. Unlike the Dirac fields, it cannot be included in the field. We will show now that the phase factor η_i can take the values $\pm i$. In fact, from (76) by the Hermitian conjugation and multiplication from the right by the matrix γ^0 we find

$$V_{CP}\bar{\nu}_i(x)V_{CP}^{-1} = \eta_i\bar{\nu}_i(x')\gamma^0. \quad (77)$$

From this relation we have

$$V_{CP}C\ \bar{\nu}_i^T(x)V_{CP}^{-1} = \eta_i C\gamma^{0T}C^{-1}C\bar{\nu}_i^T(x') = -\eta_i\gamma^0 C\bar{\nu}_i^T(x'). \quad (78)$$

Finally, taking into account the Majorana condition, we find

$$V_{CP}\nu_i(x)V_{CP}^{-1} = -\eta_i\gamma^0\nu_i(x'). \quad (79)$$

If we compare now (76) and (79), we conclude that

$$\eta_i^* = -\eta_i, \quad \eta_i^2 = -1. \quad (80)$$

Thus, the CP parity of a Majorana field can take values $\pm i$.

From (69), (71) and (76) we find that in the case of the CP invariance in the lepton sector the Majorana mixing matrix satisfies the condition

$$U_{li}\eta_i^* = U_{li}^*. \tag{81}$$

Finally, we will obtain the standard parametrization of the 3×3 Dirac mixing matrix. Let us consider two systems of orthogonal and normalized vectors $|i\rangle$ and $|\nu_l\rangle$ ($i = 1, 2, 3,\ l = e, \mu, \tau$). We have

$$\langle k|i\rangle = \delta_{ik}, \quad \langle l'|l\rangle = \delta_{l'l}. \tag{82}$$

Vectors $|\nu_l\rangle$ and $|i\rangle$ are connected by the relation

$$|\nu_l\rangle = \sum_i U_{li}^* |i\rangle. \tag{83}$$

From (82) it is obvious that U is a unitary matrix.

In the most general case, vectors $|\nu_l\rangle$ can be obtained from vectors $|i\rangle$ by three Euler rotations. The first rotation will be performed at the angle θ_{12} around the vector $|3\rangle$. New orthogonal and normalized vectors are

$$\begin{aligned} |1\rangle^{(1)} &= c_{12}\ |1\rangle + s_{12}\ |2\rangle, \\ |2\rangle^{(1)} &= -s_{12}\ |1\rangle + c_{12}\ |2\rangle, \\ |3\rangle^{(1)} &= |3\rangle, \end{aligned} \tag{84}$$

where $c_{12} = \cos\theta_{12}$ and $s_{12} = \sin\theta_{12}$. In the matrix form, (84) can be written as follows:

$$|\nu\rangle^{(1)} = U^{(1)}\ |\nu\rangle. \tag{85}$$

Here

$$|\nu\rangle^{(1)} = \begin{pmatrix} |1\rangle^{(1)} \\ |2\rangle^{(1)} \\ |3\rangle^{(1)} \end{pmatrix}, \quad |\nu\rangle = \begin{pmatrix} |1\rangle \\ |2\rangle \\ |3\rangle \end{pmatrix} \tag{86}$$

and

$$U^{(1)} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{87}$$

We will perform now the second rotation at the angle θ_{13} around the vector $|2\rangle^{(1)}$. At this step we will introduce the CP phase δ. We have

$$|\nu\rangle^{(2)} = U^{(2)}\ |\nu\rangle^{(1)}. \tag{88}$$

Here

$$U^{(2)} = \begin{pmatrix} c_{13} & 0 & s_{13}\,\mathrm{e}^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,\mathrm{e}^{-i\delta} & 0 & c_{13} \end{pmatrix}. \tag{89}$$

Finally, let us perform the rotation around the vector $|1\rangle^{(2)}$ at the angle θ_{23}. We have

$$|\nu^{\mathrm{mix}}\rangle = U^{(3)}|\nu\rangle^{(2)}. \tag{90}$$

Here

$$|\nu^{\mathrm{mix}}\rangle = \begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} \tag{91}$$

and

$$U^{(3)} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}. \tag{92}$$

From (85), (88) and (90) we find

$$|\nu^{\mathrm{mix}}\rangle = U^{*}\ |\nu\rangle, \tag{93}$$

where

$$\begin{aligned} U &= \left(U^{(3)}\ U^{(2)}\ U^{(1)}\right)^{*} = \\ &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}\,\mathrm{e}^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}\,\mathrm{e}^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned} \tag{94}$$

This is the so-called standard parametrization of the 3×3 Dirac mixing matrix. This matrix is characterized by three mixing angles θ_{12}, θ_{23} and θ_{13} and the CP phase δ. From (94) we have

$$U = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}\,\mathrm{e}^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{13}s_{23} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}\,\mathrm{e}^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}\,\mathrm{e}^{i\delta} & c_{13}c_{23}. \end{pmatrix}. \tag{95}$$

The 3×3 Majorana mixing matrix is characterized by three mixing angles and three CP phases. It can be presented in the form

$$U^{M} = US^{M}(\alpha), \tag{96}$$

where the matrix U is given by (94) and

$$S^M(\alpha) = \begin{pmatrix} e^{i\alpha_1} & & \\ & e^{i\alpha_2} & \\ & & 1 \end{pmatrix}, \tag{97}$$

where $\alpha_{1,2}$ are additional Majorana phases.

4. ON NEUTRINO OSCILLATIONS

The most important manifestation of the neutrino mixing is neutrino oscillations. Neutrino oscillations are based on the fact that in processes of neutrino production and neutrino detection due to Heisenberg uncertainty principle small neutrino mass-squared differences cannot be resolved. As a result, in a weak decay

$$a \to b + l^+ + \nu_l, \tag{98}$$

together with the lepton l^+, a «mixed» left-handed flavor neutrino ν_l is produced. The state of ν_l is *a coherent superposition* of the states of neutrinos with definite masses

$$|\nu_l\rangle = \sum_i U_{li}^* |\nu_i\rangle, \tag{99}$$

where $|\nu_i\rangle$ is the state of neutrino with mass m_i and momentum p_i.

If at $t = 0$ flavor neutrino ν_l is produced, at the time t for the neutrino state we have

$$|\nu_l\rangle_t = e^{-iHt}\,|\nu_l\rangle = \sum_{i=1}^{3} e^{-iE_i t}\, U_{li}^* |\nu_i\rangle = \sum_{l'} |\nu_{l'}\rangle \sum_{i=1}^{3} U_{l'i}\, e^{-iE_i t}\, U_{li}^*. \tag{100}$$

Thus, the probability of the transition $\nu_l \to \nu_{l'}$ during the time interval t is given by the expression

$$P(\nu_l \to \nu_{l'}) = \left| \sum_{i=1}^{3} U_{l'i}\, e^{-i\,(E_i - E_k)t}\, U_{li}^* \right|^2, \tag{101}$$

where k is fixed. If all phase differences are small ($|E_i - E_k|\,t \ll 1$) or/and there is no mixing ($U_{li}^* = \delta_{li}$), in this case it will be no neutrino oscillations ($P(\nu_l \to \nu_{l'}) \simeq \delta_{l'l}$). Thus, neutrino oscillations are effect of the neutrino mixing and relatively large phase difference(s).

Assuming that $\mathbf{p}_i = \mathbf{p}$, we obtain the standard expression for the phase difference

$$(E_i - E_k)t \simeq \frac{\Delta m_{ki}^2}{2E} L. \tag{102}$$

Here $\Delta m^2_{ki} = m^2_i - m^2_k$ and $L \simeq t$ is the distance between a neutrino source and neutrino detector.

The transition probability $P(\nu_l \to \nu_{l'})$ depends on six parameters (two mass-squared differences Δm^2_{23} and Δm^2_{12}, three mixing angles θ_{23}, θ_{12} and θ_{13} and CP phase δ). However, from analysis of the data of neutrino oscillation experiments it follows that the parameter $\sin^2\theta_{13}$ and the ratio $\Delta m^2_{12}/\Delta m^2_{23}$ are small:

$$\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \simeq 3 \cdot 10^{-2}, \quad \sin^2\theta_{13} \lesssim 5 \cdot 10^{-2}. \tag{103}$$

If we neglect contribution of the small parameters to the transition probabilities, we will find that in the atmospheric and accelerator long baseline region of the values of the parameter L/E the two-neutrino $\nu_\mu \rightleftarrows \nu_\tau$ oscillations, driven by Δm^2_{23}, take place. From (101) and (102), for the probability of ν_μ to survive, we obtain the following expression (see [18]):

$$P(\nu_\mu \to \nu_\mu) \simeq 1 - \frac{1}{2}\ \sin^2 2\theta_{23}\left(1 - \cos\,\Delta m^2_{23}\frac{L}{2E}\right). \tag{104}$$

In the KamLAND region $\nu_e \rightleftarrows \nu_{\mu,\tau}$ oscillations, driven by Δm^2_{12}, take place in the leading approximation. For the probability of $\bar\nu_e$ to survive, we obtain the following expression (see [18]):

$$P(\bar\nu_e \to \bar\nu_e) \simeq 1 - \frac{1}{2}\sin^2 2\theta_{12}\left(1 - \cos\,\Delta m^2_{12}\frac{L}{2E}\right). \tag{105}$$

In the leading approximation the probability of solar ν_e to survive in matter is also given by the two-neutrino expression. It depends on $\tan^2\theta_{12}$, Δm^2_{12} and electron number density in the sun.

We will present now the results of the analysis of the experimental data. From the analysis of the data of the *atmospheric Super-Kamiokande experiment*, for the parameters Δm^2_{23} and $\sin^2 2\theta_{23}$ the following 90% CL ranges were obtained [35]:

$$1.5 \cdot 10^{-3} \leqslant \Delta m^2_{23} \leqslant 3.4 \cdot 10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.92. \tag{106}$$

The results of the atmospheric Super-Kamiokande experiment were confirmed by the K2K [36] and MINOS [37] accelerator long-baseline neutrino oscillation experiments. From the analysis of the MINOS data for the neutrino oscillation parameters the following values were found [37]:

$$\Delta m^2_{23} = (2.43 \pm 0.13) \cdot 10^{-3}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{23} > 0.90\ (90\%\ \mathrm{CL}). \tag{107}$$

From the global analysis of the data of the reactor KamLAND experiment and data of the solar neutrino experiments, for the parameters Δm^2_{12} and $\tan^2\theta_{12}$ the following values were obtained [38]:

$$\Delta m^2_{12} = (7.59^{+0.21}_{-0.21}) \cdot 10^{-5}\ \mathrm{eV}^2, \quad \tan^2\theta_{12} = 0.47^{+0.06}_{-0.05}. \tag{108}$$

In the reactor CHOOZ experiment [39] no indications in favor of $\bar{\nu}_e \to \bar{\nu}_e$ transitions, driven by Δm^2_{23}, were found. From the exclusion plot, obtained from the data of this experiment, for the parameter $\sin^2\theta_{13}$ the following upper bound can be inferred:

$$\sin^2\theta_{13} \lesssim 5 \cdot 10^{-2}. \tag{109}$$

At present, a stage of the high-precision neutrino oscillation experiments starts. In the future DOUBLE CHOOZ [40], Daya Bay [41] and RENO [42] reactor neutrino experiments, sensitivities to the parameter $\sin^2 2\theta_{13}$ will be 10–20 times better than in the CHOOZ experiment. The same sensitivity is planned to be reached in the accelerator T2K experiment [43]. In this experiment, parameters Δm^2_{23} and $\sin^2 2\theta_{23}$ will be measured with the accuracies $\delta(\Delta m^2_{23}) \sim 10^{-4}$ eV^2 and $\delta(\sin^2 2\theta_{23}) \sim 10^{-2}$, correspondingly. High-precision neutrino oscillation experiments are planned at the future Super Beam [44], Beta-beam [45], and Neutrino Factory facilities [46].

5. BASIC ELEMENTS OF THE THEORY OF $0\nu\beta\beta$ DECAY

In this section we will consider the neutrinoless double β decay of even–even nuclei [4, 5]

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{110}$$

We will assume that

- The Hamiltonian of the weak interaction is given by the SM.
- The neutrino mixing takes place.
- Neutrinos with definite masses ν_i *are Majorana particles*.

For the effective Hamiltonian of the process we have

$$\mathcal{H}_I(x) = \frac{G_F}{\sqrt{2}} 2 \sum_i \bar{e}_L(x)\gamma_\alpha\, U_{li}\, \nu_{iL}(x)\, j^\alpha(x) + \text{h.c.} \tag{111}$$

Here G_F is the Fermi constant; $j^\alpha(x)$ is the hadronic charged current, and the field $\nu_i(x)$ satisfies the condition

$$\nu_i^c(x) = C\bar{\nu}_i^T(x) = \nu_i(x). \tag{112}$$

The neutrinoless double β decay is the second order in G_F process with the virtual neutrinos. The matrix element of the process is given by the following expression:

$$\langle f|S^2|i\rangle = 4\frac{(-i)^2}{2\,!} \left(\frac{G_F}{\sqrt{2}}\right)^2 \times$$
$$\times N_{p_1}N_{p_2} \int \sum_i \bar{u}_L(p_1)\,\mathrm{e}^{ip_1x_1}\gamma_\alpha\, U_{ei}\langle 0|T(\nu_{iL}(x_1)\ \nu^T_{iL}(x_2)|0\rangle\gamma^T_\beta\ U_{ei}\bar{u}^T_L(p_2)\times$$
$$\times\, \mathrm{e}^{ip_2x_2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\ d^4x_1\,d^4x_2 - (p_1 \rightleftarrows p_2). \tag{113}$$

Here p_1 and p_2 are electron momenta; $J^\alpha(x)$ is the weak charged current in the Heisenberg representation; N_i and N_f are the states of the initial and the final nuclei with 4-momenta $P_i = (E_i, \mathbf{p}_i)$ and $P_f = (E_f, \mathbf{p}_f)$, respectively, and $N_p = \dfrac{1}{(2\pi)^{3/2}\sqrt{2p^0}}$ is the standard normalization factor.

Let us consider the neutrino propagator. From the Majorana condition (112) we find

$$\langle 0|T(\nu_{iL}(x_1)\nu_{iL}^T(x_2)|0\rangle = -\frac{1-\gamma_5}{2}\langle 0|T(\nu_i(x_1)\bar{\nu}_i(x_2))|0\rangle \, \frac{1-\gamma_5}{2}\, C. \quad (114)$$

Further, we have

$$\langle 0|T(\nu_i(x_1)\bar{\nu}_i(x_2))|0\rangle = \frac{i}{(2\pi)^4}\int \mathrm{e}^{-iq\ (x_1-x_2)}\frac{\gamma q + m_i}{q^2 - m_i^2}\, d^4q. \quad (115)$$

Thus, for the neutrino propagator we find the following expression*:

$$\langle 0|T(\nu_{iL}(x_1)\bar{\nu}_{iL}(x_2))|0\rangle = \\ = -\frac{i}{(2\pi)^4}\int \mathrm{e}^{-iq\ (x_1-x_2)}\frac{m_i}{q^2 - m_i^2}\, d^4q\ \frac{1-\gamma_5}{2}C. \quad (116)$$

The neutrino propagator is proportional to m_i. It is obvious from (115) that this is connected with the fact that only left-handed neutrino fields enter into the Hamiltonian of the weak interaction. In the case of massless neutrinos ($m_i = 0$, $i = 1, 2, 3$), in accordance with the theorem on the equivalence of the theories with massless Majorana and Dirac neutrinos, the matrix element of the neutrinoless double β decay is equal to zero.

Let us consider the second term of the matrix element (113). It is easy to show that

$$\bar{u}_L(p_1)\gamma_\alpha(1-\gamma_5)\gamma_\beta C\bar{u}_L^T(p_2) = \bar{u}_L(p_2)C^T\gamma_\beta^T(1-\gamma_5^T)\gamma_\alpha^T\bar{u}_L^T(p_1) = \\ = -\bar{u}_L(p_2)\gamma_\beta(1-\gamma_5)\gamma_\alpha C\bar{u}_L^T(p_1). \quad (117)$$

If we take into account (117) and the relation

$$T(J^\beta(x_2)J^\alpha(x_1)) = T(J^\alpha(x_1)J^\beta(x_2)), \quad (118)$$

*Notice that in the case of the Dirac neutrinos $\langle 0|\nu_{iL}(x_1)\nu_{iL}^T(x_2)|0\rangle = \dfrac{1-\gamma_5}{2}\times \langle 0|\nu_i(x_1)\nu_i^T(x_2)|0\rangle \dfrac{1-\gamma_5^T}{2} = 0$. The neutrinoless double β decay is obviously forbidden in the Dirac case.

we can show that the second term of the matrix element (113) is equal to the first one. Thus, for the matrix element we obtain the following expression:

$$\langle f|S^2|i\rangle = -4\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\int \bar{u}_L(p_1)\,\mathrm{e}^{ip_1x_1}\gamma_\alpha\frac{i}{(2\pi)^4}\times$$

$$\times\sum_i U_{ei}^2 m_i\int\frac{\mathrm{e}^{-iq(x_1-x_2)}}{p^2-m_i^2}\,d^4q\times$$

$$\times\frac{1-\gamma_5}{2}\gamma_\beta C\;\bar{u}_L^T(p_2)\,\mathrm{e}^{ip_2x_2}\langle N_f|T(J^\alpha(x_1)J^\beta(x_2))|N_i\rangle\,d^4x_1\,d^4x_2. \quad (119)$$

Initial nuclei in the process (110) are ^{76}Ge, ^{136}Xe, ^{130}Te, ^{100}Mo and other heavy nuclei. The calculation of the nuclear part of the matrix element of the $0\nu\beta\beta$ decay is a complicated nuclear problem. In such a calculation different approximations are used. We will present now the matrix element of the $0\nu\beta\beta$ decay in a form which is appropriate for such approximate calculations.

Let us perform in (119) the integration over the time variables x_2^0 and x_1^0. The integral over x_2^0 can be presented in the form

$$\int\limits_{-\infty}^{\infty}\cdots dx_2^0 = \int\limits_{-\infty}^{x_1^0}\cdots dx_2^0 + \int\limits_{x_1^0}^{\infty}\cdots dx_2^0. \quad (120)$$

After the integration over q^0 in the neutrino propagator, in the region $x_1^0 > x_2^0$ we find*

$$\frac{i}{(2\pi)^4}\int\frac{\mathrm{e}^{-iq(x_1-x_2)}}{q^2-m_i^2}d^4q = \frac{1}{(2\pi)^3}\int\frac{\mathrm{e}^{-iq_i^0(x_1^0-x_2^0)+i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}}{2q_i^0}d^3q, \quad (121)$$

where

$$q_i^0 = \sqrt{\mathbf{q}^2+m_i^2}. \quad (122)$$

In the region $x_1^0 < x_2^0$ we have

$$\frac{i}{(2\pi)^4}\int\frac{\mathrm{e}^{-iq(x_1-x_2)}}{q^2-m_i^2}d^4q = \frac{1}{(2\pi)^3}\int\frac{\mathrm{e}^{-iq_i^0(x_2^0-x_1^0)+i\mathbf{q}(\mathbf{x}_2-\mathbf{x}_1)}}{2q_i^0}d^3q. \quad (123)$$

For the operators $J^\alpha(x)$ from the invariance under the translations we have

$$J^\alpha(x) = \mathrm{e}^{iHx^0}J^\alpha(\mathbf{x})\,\mathrm{e}^{-iHx^0}, \quad (124)$$

*It is assumed that in the propagator $m_i^2 = m_i^2 - i\epsilon$.

where H is the total Hamiltonian. From this relation we find

$$\langle N_f|J^\alpha(x_1)J^\beta(x_2)|N_i\rangle =$$
$$= \sum_n \mathrm{e}^{i(E_f-E_n)x_1^0}\,\mathrm{e}^{i(E_n-E_i)x_2^0}\ \langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2))|N_i\rangle, \quad (125)$$

where $|N_n\rangle$ is the vector of the state of the intermediate nucleus with 4-momentum $P_n = (E_n,\ \mathbf{p}_n)$. In (125) the sum over the total system of the states $|N_n\rangle$ is assumed.

Taking into account that at $\pm\infty$ the interaction is turned off, we have

$$\int\limits_{-\infty}^{0} \mathrm{e}^{iax_2^0}\,dx_2^0 \to \int\limits_{-\infty}^{0} \mathrm{e}^{i(a-i\epsilon)x_2^0}\,dx_2^0 = \lim_{\epsilon\to 0}\frac{-i}{a-i\epsilon} \quad (126)$$

and

$$\int\limits_{0}^{-\infty} \mathrm{e}^{iax_2^0}\ dx_2^0 \to \int\limits_{0}^{\infty} \mathrm{e}^{i(a+i\epsilon)x_2^0}\,dx_2^0 = \lim_{\epsilon\to 0}\frac{i}{a+i\epsilon}. \quad (127)$$

From (126) and (127) we find

$$\int\limits_{-\infty}^{\infty} dx_1^0 \int\limits_{-\infty}^{x_1^0} dx_2^0 \sum_n \langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2)|N_i\rangle\times$$
$$\times\ \mathrm{e}^{i(E_f-E_n)x_1^0+i(E_n-E_i)x_2^0}\,\mathrm{e}^{i(p_1^0x_1^0+p_2^0x_2^0)}\mathrm{e}^{iq_i^0(x_2^0-x_1^0)} =$$
$$= -i\sum_n \frac{\langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2))|N_i}{E_n+p_2^0+q_i^0-E_i-i\epsilon}2\pi\delta(E_f+p_1^0+p_2^0-E_i). \quad (128)$$

Taking into account all these relations, for the matrix element of the neutrinoless double β decay we obtain the following expression:

$$\langle f|S^2|i\rangle = 2i\left(\frac{G_F}{\sqrt{2}}\right)^2 N_{p_1}N_{p_2}\bar{u}(p_1)\gamma_\alpha\gamma_\beta(1+\gamma_5)C\bar{u}^T(p_2)\times$$
$$\times \int d^3x_1\,d^3x_1\,\mathrm{e}^{-i\mathbf{p}_1\mathbf{x}_1-i\mathbf{p}_2\mathbf{x}_2}\sum_i U_{ei}^2 m_i\frac{1}{(2\pi)^3}\int \frac{\mathrm{e}^{i\mathbf{q}(\mathbf{x}_1-\mathbf{x}_2)}}{q_i^0}\,d^3q\times$$
$$\times\left[\sum_n \frac{\langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2)|N_i\rangle}{E_n+p_2^0+q_i^0-E_i-i\epsilon}+\right.$$
$$\left.+\sum_n\frac{\langle N_f|J^\beta(\mathbf{x}_2)|N_n\rangle\langle N_n|J^\alpha(\mathbf{x}_1)|N_i\rangle}{E_n+p_1^0+q_i^0-E_i-i\epsilon}\right]2\pi\delta(E_f+p_1^0+p_2^0-E_i). \quad (129)$$

Equation (129) is the exact expression for the matrix element of $0\nu\beta\beta$ decay in the second order of the perturbation theory. We will consider major $0^+ \to 0^+$ transitions of even–even nuclei. For such transitions the following approximations are standard:

1. *Small neutrino masses can be safely neglected in* q_i^0. The averaged momentum of the virtual neutrino is given by the relation $q \simeq 1/r$, where r is the average distance between two nucleons in nucleus. Taking into account that $r \simeq 10^{-13}$ cm, we have $q \simeq 100$ MeV. Neutrino masses are smaller than 2.2 eV. Thus, we have $q_i^0 = \sqrt{\mathbf{q}^2 + m_i^2} \simeq q$.

2. *Long-wave approximation.* We have $p_k x_k \leqslant p_k R$, where $R \simeq 1.2A^{1/3} \cdot 10^{-13}$ cm is the radius of nucleus ($k = 1, 2$). Taking into account that $p_k \lesssim 1$ Mev, we have $p_k x_k \ll 1$. Thus, $\mathrm{e}^{-i\mathbf{p}_1\mathbf{x}_1 - i\mathbf{p}_2\mathbf{x}_2} \simeq 1$; i.e., two electrons are emitted in S states.

3. *Closure approximation.* Energy of the virtual neutrino is much larger than the excitation energy $(E_n - E_i)$. Thus, we can change the energy of the intermediate states E_n by average energy $\overline{E}$. In this (closure) approximation we have

$$\frac{\langle N_f|J^\alpha(\mathbf{x}_1)|N_n\rangle\langle N_n|J^\beta(\mathbf{x}_2))|N_i\rangle}{E_n + p_2^0 + q_i^0 - E_i - i\epsilon} \simeq \frac{\langle N_f|J^\alpha(\mathbf{x}_1)J^\beta(\mathbf{x}_2))|N_i\rangle}{\overline{E} + p_2^0 + q - E_i - i\epsilon}. \qquad (130)$$

4. *The impulse approximation for the hadronic charged current* $J^\alpha(\mathbf{x})$. Taking into account the major terms, the hadronic charged current takes the form*

$$J^\alpha(\mathbf{x}) \simeq \sum_n \delta(\mathbf{x} - \mathbf{r}_n)\tau_+^n[g_V(q^2)g^{\alpha 0} + g_A(q^2)\sigma_i^n g^{\alpha i}]. \qquad (131)$$

Here $g_V(q^2)$ and $g_A(q^2)$ are vector and axial form factors; σ_i and τ_i are Pauli matrices; $\tau_+ = 1/2\ (\tau_1 + i\tau_2)$ and index n runs over all nucleons in a nucleus. We have $g_V(0) = 1, \quad g_A(0) = g_A \simeq 1.27$.

It is obvious that $\tau_+^n\, \tau_+^n = 0$. Thus, in the impulse approximation the hadronic currents satisfy the relation

$$J^\alpha(\mathbf{x}_1)J^\beta(\mathbf{x}_2) = J^\beta(\mathbf{x}_2)J^\alpha(\mathbf{x}_1). \qquad (132)$$

Further, the matrix $\gamma_\alpha\gamma_\beta$ in the leptonic part of the matrix element (129) can be presented in the form

$$\gamma_\alpha\gamma_\beta = g_{\alpha\beta} + \frac{1}{2}(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha). \qquad (133)$$

*The pseudoscalar term in the one-nucleon matrix element of the hadronic charged current induces a tensor term in the current. From numerical calculations it follows that its contribution to the matrix element can be significant (see [47]).

It follows from (132) that the second term of (133) does not give contribution to the matrix element. From (131) we have

$$J^{\alpha}(\mathbf{x}_1)J_{\alpha}(\mathbf{x}_2) = \sum_{n,m} \tau_+^n \tau_+^m \delta(\mathbf{x}_1 - \mathbf{r}_n)\ \delta(\mathbf{x}_2 - \mathbf{r}_m)(g_V^2(q^2) - g_A^2(q^2)\boldsymbol{\sigma}^n \cdot \boldsymbol{\sigma}^m). \tag{134}$$

Neglecting nuclei recoil, we obtain in the laboratory frame

$$M_i = M_f + p_2^0 + p_1^0,$$

where M_i and M_f are masses of the initial and final nuclei. From this relation we find

$$q + p_{1,2}^0 + \overline{E} - M_i = q \pm \left(\frac{p_1^0 - p_2^0}{2}\right) + \overline{E} - \frac{M_i + M_f}{2}. \tag{135}$$

The term $\left(\frac{p_1^0 - p_2^0}{2}\right)$ is much smaller than all other terms in the right-hand side of this relation. Neglecting this term, we have

$$q + p_{1,2}^0 + \overline{E} - M_i \simeq q + \overline{E} - \frac{M_i + M_f}{2}. \tag{136}$$

Further, taking into account that $g_V(q^2) \simeq \frac{1}{1 + q^2/0.71\ \mathrm{GeV}^2}$ and $g_A(q^2) \simeq \frac{1}{1 + q^2/M_A^2}$, where $M_A \simeq 1\ \mathrm{GeV}^2$, we can neglect q^2-dependence of the form factors. After the integration in the matrix element (129) over $\mathbf{x}_1$ and $\mathbf{x}_2$, for the neutrino propagator we find the following expression:

$$\frac{1}{(2\pi)^3} \int \frac{\mathrm{e}^{i\mathbf{q}\mathbf{r}_{nm}}\, d^3q}{q(q + \overline{E} - 1/2(M_i + M_f))} = \frac{1}{4\pi R} H(r_{nm}, \overline{E}), \tag{137}$$

where

$$H(r, \overline{E}) = \frac{2R}{\pi r} \int_0^{\infty} \frac{\sin qr\, dq}{q + \overline{E} - 1/2(M_i + M_f)}. \tag{138}$$

Here R is the nuclei radius and $\mathbf{r}_{nm} = \mathbf{r}_n - \mathbf{r}_m$.

Taking into account all these relations, from (129) for the matrix element of $0\nu\beta\beta$ decay we obtain the following expression:

$$\langle f|S^2|i\rangle = -i \left(\frac{G_F}{\sqrt{2}}\right)^2 \frac{1}{(2\pi)^3}\ \frac{1}{\sqrt{p_1^0 p_2^0}} m_{\beta\beta}\ g_A^2 \frac{1}{R} \bar{u}(p_1)(1 + \gamma_5) C \bar{u}^T(p_2) \times \\ \times M^{0\nu} \delta(p_1^0 + p_2^0 + M_f - M_i), \tag{139}$$

where

$$m_{\beta\beta} = \sum_i U_{ei}^2 m_i \tag{140}$$

is the effective Majorana mass and

$$M^{0\nu} = M^{0\nu}_{\rm GT} - \frac{1}{g_A^2} M^{0\nu}_F \tag{141}$$

is the nuclear matrix element. Here

$$M^{0\nu}_F = \langle \Psi_f | \sum_{n,m} H(r_{n,m}, \overline{E}) \tau^n_+ \tau^m_+ |\Psi_i\rangle \tag{142}$$

is the Fermi matrix element and

$$M^{0\nu}_{\rm GT} = \langle \Psi_f | \sum_{n,m} H(r_{n,m}, \overline{E}) \tau^n_+ \tau^m_+ \boldsymbol{\sigma}^n \cdot \boldsymbol{\sigma}^m) |\Psi_i\rangle \tag{143}$$

is the Gamow–Teller matrix element. In (142) and (143), $|\Psi_{i,f}\rangle$ are wave functions of the initial and final nuclei.

From (139) we conclude that matrix element of $0\nu\beta\beta$ decay is a product of the effective Majorana mass $m_{\beta\beta}$, the electron matrix element and the nuclear matrix element which includes neutrino propagator (neutrino potential). Taking into account that $\overline{E} - 1/2(M_i + M_f)$ is much smaller than $\bar{q}$, for the neutrino propagator we obtain the following approximate relation:

$$H(r) \simeq \frac{2R}{\pi} \int_0^\infty \frac{\sin\, qr}{qr}\, dq = \frac{R}{r}. \tag{144}$$

Using the standard rules, from (139) we can easily obtain the decay rate of the $0\nu\beta\beta$ decay. The electron part of the decay probability is given by the trace

$$\mathrm{Tr}(1+\gamma_5)(\gamma \cdot p_2 - m_e)(1-\gamma_5)(\gamma \cdot p_1 + m_e) = 8 p_1 p_1. \tag{145}$$

Taking into account the final-state electromagnetic interaction of the electrons and nucleus for the decay rate of the $0\nu\beta\beta$ decay we find the following expression:

$$d\Gamma^{0\nu} = |m_{\beta\beta}|^2 |M^{0\nu}|^2 \frac{1}{(2\pi)^5} G_F^4 \frac{1}{R^2} g_A^4 (E_1 E_2 - p_1 p_2 \cos\,\theta) \times \\ \times F(E_1, (Z+2)) F(E_2, (Z+2)) p_1 p_2 \sin\,\theta\, d\theta\, dE_2, \tag{146}$$

where $E_{1,2} \equiv p^0_{1,2}$ is electron total energy ($E_2 = M_i - M_f - E_1$), θ is the angle between electron momenta $\mathbf{p}_1$ and $\mathbf{p}_2$, and

$$F(Z) \simeq \frac{2\pi\eta}{1 - \mathrm{e}^{-2\pi\eta}} \tag{147}$$

is the Fermi function ($\eta = Z\alpha(m_e/p)$).

From (146) it follows that for the ultrarelativistic electrons θ-dependence of the decay rate is given by the factor $(1 - \cos\,\theta)$. Thus, ultrarelativistic electrons cannot be emitted in the same direction. This is connected with the fact that the helicity of the high-energy electrons, produced in the weak interaction, is equal to -1. If electrons are emitted in the same direction, the projection of their total angular momentum onto the direction of the momentum is equal to -1. It is obvious that such electrons cannot be produced in $O^+ \to O^+$ transition.

From expression (146) for the total decay rate we obtain the following expression:

$$\Gamma^{0\nu} = \frac{1}{T^{0\nu}_{1/2}} = |m_{\beta\beta}|^2 |M^{0\nu}|^2 G^{0\nu}(Q, Z), \tag{148}$$

where*

$$G^{0\nu}(Q, Z) = \frac{1}{2(2\pi)^5} G_F^4 \frac{1}{R^2} g_A^4 \int\limits_0^Q dT_1 \int\limits_0^\pi \sin\,\theta\, d\theta (E_1 E_2 - p_1 p_2 \cos\,\theta) p_1 p_2 \times \\ \times F(E_1, (Z+2)) F(E_2, (Z+2)). \tag{149}$$

Here $T_1 = E_1 - m_e$, $Q = M_i - M_f - 2m_e$ is the total released kinetic energy and $T^{0\nu}_{1/2}$ is the half-life of the $0\nu\beta\beta$ decay. In Table 1 we present numerical values of $G^{0\nu}(Q, Z)$ for some nuclei [48].

Table 1. **The values of the factor $G^{0\nu}(Q, Z)$ for some nuclei**

Nucleus	$G^{0\nu}(Q, Z)$, 10^{-25} $\mathrm{y}^{-1} \cdot \mathrm{eV}^{-2}$
^{76}Ge	0.30
^{100}Mo	2.19
^{130}Te	2.12
^{136}Xe	2.26

The total rate of the $0\nu\beta\beta$ decay is the product of three factors:
1. The modulus squared of the effective Majorana mass.
2. Square of nuclear matrix element.
3. The known factor $G^{0\nu}(Q, Z)$.

We have considered in some detail neutrinoless double β decay of nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^-. \tag{150}$$

There could be other second order in the Fermi constant G_F processes with the virtual Majorana neutrinos in which the total lepton number is changed by two.

*An additional factor $1/2$ is due to the fact that in the final state we have two identical electrons.

The examples are the decays

$$K^- \to \pi^+ + \mu^- + e^- \tag{151}$$

and

$$K^+ \to \pi^- + \mu^+ + \mu^+, \tag{152}$$

the process

$$\mu^- + (A, Z) \to (A, Z-2) + e^+, \tag{153}$$

and others.

The leptonic part of the operator which gives contribution to matrix elements of (150), (151) and other similar processes is given by

$$\sum_i T(\bar{l}_L(x_1)\gamma_\alpha U_{li}\ \langle 0|T(\nu_{iL}(x_1)\nu_{iL}^T(x_2))|0\rangle\, U_{l'i}\gamma_\beta^T \bar{l}_L'^T(x_2)), \quad l, l' = e, \mu, \tag{154}$$

where Majorana neutrino propagator is given by the expression (116). Taking into account that $m_i^2 \ll p^2$, we can neglect m_i^2 in the denominator of the propagator. Thus, the matrix element of a process in which a lepton pair (ll') is produced, is proportional to

$$m_{ll'} = \sum_i U_{li}\, U_{l'i}\, m_i. \tag{155}$$

Analogously, matrix elements of the processes (152), (153) and other similar processes are proportional to $m_{ll'}^*$.

The sensitivities to the parameter $|m_{ll'}|$ of the experiments on the search for the processes (151)–(153) and other similar processes are much worse than the sensitivity of the experiments on the search for $0\nu\beta\beta$ decay to the parameter $|m_{\beta\beta}|$.

For example, in the experiment [52] on the search for the process $\mu^-\mathrm{Ti} \to e^+\mathrm{Ca}$ the following upper bound was obtained:

$$\frac{\Gamma(\mu^-\mathrm{Ti} \to e^+\mathrm{Ca})}{\Gamma(\mu^-\mathrm{Ti} \to \mathrm{all})} \leqslant 1.7 \cdot 10^{-12}. \tag{156}$$

For the probability of the decay $K^+ \to \pi^-\mu^+\mu^+$, the following upper bound was reached [53]:

$$\frac{\Gamma(K^+ \to \pi^-\mu^+\mu^+)}{\Gamma(K^+ \to \mathrm{all})} \leqslant 3 \cdot 10^{-9}. \tag{157}$$

From these data the following upper bounds can be found (see [7]):

$$|m_{\mu e}| \leqslant 82\ \mathrm{MeV}, \quad |m_{\mu\mu}| \leqslant 4 \cdot 10^4\ \mathrm{MeV}. \tag{158}$$

These values must be compared with the sensitivity of the experiments on the search for $0\nu\beta\beta$ decay to the effective Majorana mass (in today's experiments $|m_{\beta\beta}| \simeq 0.2-1.3$ eV (see below)).

The effective Majorana mass is determined by neutrino masses and neutrino mixing angles. Information about the neutrino mixing angles θ_{ik} and neutrino mass-squared differences Δm^2_{ik} was obtained from the data of the neutrino oscillation experiments. Taking into account these data, we will consider now possible values of the effective Majorana mass.

6. EFFECTIVE MAJORANA MASS

From neutrino oscillation data it follows that one mass-squared difference (solar) is much smaller than the other one (atmospheric). For three massive neutrinos two types of neutrino mass spectra are possible in this case:

1. Normal spectrum

$$m_1 < m_2 < m_3, \quad \Delta m^2_{12} \ll \Delta m^2_{23}. \tag{159}$$

2. Inverted spectrum*

$$m_3 < m_1 < m_2, \quad \Delta m^2_{12} \ll |\Delta m^2_{13}|. \tag{160}$$

In the case of the normal spectrum the neutrino masses $m_{2,3}$ are connected with the lightest mass m_1 and two neutrino mass-squared differences Δm^2_{12} and Δm^2_{23} by the following relations:

$$m_2 = \sqrt{m_1^2 + \Delta m^2_{12}}, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{12} + \Delta m^2_{23}}. \tag{161}$$

In the case of the inverted spectrum we have

$$m_1 = \sqrt{m_3^2 + |\Delta m^2_{13}|}, \quad m_2 = \sqrt{m_3^2 + |\Delta m^2_{13}| + \Delta m^2_{12}}. \tag{162}$$

It is obvious that effective Majorana mass is determined not only by the lightest neutrino mass and neutrino mass-squared differences, but also by the character of the neutrino mass spectrum.

*In order to have the same notation Δm^2_{12} for the solar-KamLAND neutrino mass-squared difference and to determine this quantity as a positive one, the neutrino masses are usually labeled differently in the cases of the normal and inverted neutrino mass spectra. In the case of the normal spectrum $\Delta m^2_{23} > 0$ and in the case of the inverted spectrum $\Delta m^2_{13} < 0$. Thus, with such a notation the character of the neutrino mass spectrum is determined by the sign of the larger (atmospheric) neutrino mass-squared difference. It is clear, however, that the sign of the atmospheric mass-squared difference has no physical meaning: it is a convention based on the labeling of the neutrino masses and determination of the neutrino mass-squared difference ($\Delta m^2_{ik} = m^2_k - m^2_i$). In both cases of the neutrino mass spectrum for the mixing angles the same notations can be used.

Usually, the following three typical neutrino mass spectra are considered*:
1. Hierarchy of the neutrino masses

$$m_1 \ll m_2 \ll m_3. \tag{163}$$

2. Inverted hierarchy of the neutrino masses

$$m_3 \ll m_1 < m_2. \tag{164}$$

3. Quasi-degenerate neutrino mass spectrum

$$m_1 \simeq m_2 \simeq m_3, \quad m_1(m_3) \gg \sqrt{\Delta m_{23}^2} \left(\sqrt{|\Delta m_{13}^2|}\right). \tag{165}$$

We will discuss now the possible values of the effective Majorana mass in the case of these three neutrino mass spectra.

1. Hierarchy of the Neutrino Masses. In this case we have

$$m_1 \ll \sqrt{\Delta m_{12}^2}, \quad m_2 \simeq \sqrt{\Delta m_{12}^2}, \quad m_3 \simeq \sqrt{\Delta m_{23}^2}. \tag{166}$$

Thus, in the case of neutrino mass hierarchy the neutrino masses m_2 and m_3 are determined by the neutrino mass-squared differences Δm_{12}^2 and Δm_{23}^2, correspondingly, and the lightest mass is very small. Neglecting the contribution of m_1 to the effective Majorana mass and using the standard parametrization of the neutrino mixing matrix, we find

$$|m_{\beta\beta}| \simeq \left|\sin^2\theta_{12}\sqrt{\Delta m_{12}^2} + e^{2i\alpha}\sin^2\theta_{13}\sqrt{\Delta m_{23}^2}\right|. \tag{167}$$

Here α is a Majorana phase difference.

The first term in Eq. (167) is small because of the smallness of Δm_{12}^2. The contribution of the «large» Δm_{23}^2 to $|m_{\beta\beta}|$ is suppressed by the small factor $\sin^2\theta_{13}$. Using the values (107) and (108) and the CHOOZ bound (109), we have

$$\sin^2\theta_{12}\sqrt{\Delta m_{12}^2} \simeq 2.8\cdot 10^{-3}\ \text{eV}, \quad \sin^2\theta_{13}\sqrt{\Delta m_{23}^2} \lesssim 2.5\cdot 10^{-3}\ \text{eV}. \tag{168}$$

Thus, if the value of the parameter $\sin^2\theta_{13}$ is close to the CHOOZ bound, the first term and the modulus of the second term of (167) are approximately equal

*Let us notice that these three neutrino mass spectra correspond to different mechanisms of neutrino mass generation. Masses of quarks and charged leptons satisfy hierarchy of the type (163). Hierarchy of neutrino masses is a typical feature of GUT models (like $SO(10)$) in which quarks and leptons are unified. Inverted spectrum and quasi-degenerate spectrum require specific symmetries of the neutrino mass matrix.

and at $\alpha \simeq \pi/2$ the terms in the expression (167) practically cancel each other. In this case the Majorana mass $|m_{\beta\beta}|$ will be close to zero.

Even without this possible cancelation the effective Majorana mass in the case of the neutrino mass hierarchy is very small. In fact, from (167) and (168) we have the following upper bound:

$$|m_{\beta\beta}| \leqslant \left(\sin^2\theta_{12}\sqrt{\Delta m^2_{12}} + \sin^2\theta_{13}\sqrt{\Delta m^2_{23}}\right) \lesssim 5.3\cdot 10^{-3}\ \text{eV}. \qquad (169)$$

This bound is significantly smaller than the expected sensitivity of the future experiments on the search for $0\nu\beta\beta$ decay (see later).

2. Inverted Hierarchy of the Neutrino Masses. For the neutrino masses we have in this case

$$m_3 \ll \sqrt{|\Delta m^2_{13}|},\ m_1 \simeq \sqrt{|\Delta m^2_{13}|},\ m_2 \simeq \sqrt{|\Delta m^2_{13}|}\left(1+\frac{\Delta m^2_{12}}{2\,|\Delta m^2_{13}|}\right). \qquad (170)$$

In the expression for the effective Majorana mass $|m_{\beta\beta}|$ the lightest mass m_3 is multiplied by the small parameter $\sin^2\theta_{13}$. Neglecting the contribution of this term and also neglecting the small term $\dfrac{\Delta m^2_{12}}{2|\Delta m^2_{13}|}$ in (170), we find

$$|m_{\beta\beta}| \simeq \sqrt{|\Delta m^2_{13}|}(1-\sin^2 2\theta_{12}\,\sin^2\alpha)^{1/2}, \qquad (171)$$

where α is the difference of the Majorana phases of the elements U_{e2} and U_{e1}. The phase difference α is the only unknown parameter in the expression for $|m_{\beta\beta}|$ in the case of the inverted hierarchy. From (171) we find

$$\cos 2\,\theta_{12}\sqrt{|\Delta m^2_{13}|} \leqslant |m_{\beta\beta}| \leqslant \sqrt{|\Delta m^2_{13}|}. \qquad (172)$$

The upper and lower bounds of the inequality (172) correspond to the CP invariance in the lepton sector. In fact, the elements of the first row of the neutrino mixing matrix can be written in the form $U_{ei} = |U_{ei}|\,\mathrm{e}^{i\alpha_i}$. In the case of the CP invariance, the elements of the neutrino mixing matrix satisfy the condition (81). From this condition we have

$$\mathrm{e}^{2i\alpha_i} = \eta_i, \qquad (173)$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino with mass m_i. For the phase difference $\alpha = \alpha_2 - \alpha_1$ we have

$$\mathrm{e}^{2i\alpha} = \eta_2\eta_1^*. \qquad (174)$$

If $\eta_2 = \eta_1$, we obtain $\alpha = 0, \pi$ (the upper bound in the inequality (172)). If $\eta_2 = -\eta_1$, we have $\alpha = \pm\pi/2$ (the lower bound in the inequality (172)).

From (107) and (108) we find the following range of the possible values of the effective Majorana mass:

$$1.8 \cdot 10^{-2} \leqslant |m_{\beta\beta}| \leqslant 4.9 \cdot 10^{-2} \text{ eV}. \tag{175}$$

Thus, in the case of the inverted hierarchy of the neutrino masses the lower bound of the effective Majorana mass is different from zero.

The anticipated sensitivities to the effective Majorana mass of the next generation of the experiments on the search for the $0\nu\beta\beta$ decay are in the range (175) (see below). Thus, the future $0\nu\beta\beta$-decay experiments will probe the Majorana nature of neutrinos with definite masses in the case of the inverted hierarchy of the neutrino masses.

3. Quasi-Degenerate Neutrino Mass Spectrum. Neglecting the small contribution of $\sin^2\theta_{13}$, for the effective Majorana mass we obtain in the case of the quasi-degenerate neutrino mass spectrum the following expression:

$$|m_{\beta\beta}| \simeq m_{\min}\,(1 - \sin^2 2\theta_{12}\,\sin^2\alpha)^{1/2}, \tag{176}$$

where $m_{\min}$ is the lightest neutrino mass and α is the Majorana phase difference. Thus, $|m_{\beta\beta}|$ depends in this case on two unknown parameters: $m_{\min}$ and α. From (176) we obtain the following range for the effective Majorana mass:

$$\cos 2\theta_{12} m_{\min} \leqslant |m_{\beta\beta}| \leqslant m_{\min}. \tag{177}$$

If $0\nu\beta\beta$ decay is observed and the effective Majorana mass turns out to be relatively large ($|m_{\beta\beta}| \gg \sqrt{\Delta m^2_{23}|}$), it would be an evidence that neutrinos are Majorana particles and the spectrum of their mass is quasi-degenerate. In this case we could conclude that

$$|m_{\beta\beta}| \leqslant m_{\min} \leqslant 2.8\,|m_{\beta\beta}|. \tag{178}$$

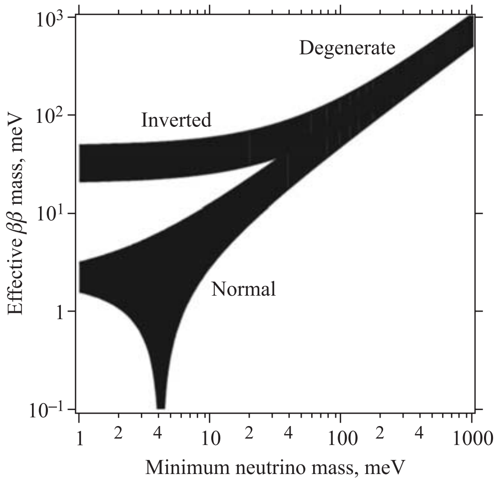
Information about the lightest neutrino mass can be obtained from experiments on the measurement of the end-point part of the β spectrum of tritium. From existing data of the Mainz [49] and Troitsk [50] tritium experiments the upper bound was found:

$$m_{\min} < 2.2 \text{ eV}. \tag{179}$$

The sensitivity of the future KATRIN experiment [51] is expected to be

$$m_{\min} \simeq 0.2 \text{ eV}. \tag{180}$$

We have considered three neutrino mass spectra with special values of the lightest neutrino mass $m_{\min}$. In the figure the effective Majorana mass for the normal and inverted neutrino mass spectra as a function of $m_{\min}$ is presented. Uncertainties



The effective Majorana mass for the normal and inverted neutrino mass spectra as a function of minimal neutrino mass

of the parameters Δm^2_{12}, Δm^2_{23} and $\tan^2\theta_{12}$ and possible values of the Majorana phase difference α are taken into account in the figure.

In conclusion let us notice that if in the KATRIN (or other) experiments the neutrino mass is measured and in the $0\nu\beta\beta$-decay experiments, sensitive to the effective Majorana mass in the range (177), a positive signal is not observed, it would be an evidence that neutrinos with definite masses are Dirac particles.

7. NUCLEAR MATRIX ELEMENTS OF $0\nu\beta\beta$ DECAY

The effective Majorana mass $|m_{\beta\beta}|$ is not directly measurable quantity. From the measurement of the half-life of the $0\nu\beta\beta$ decay only *the product of the effective Majorana mass and the nuclear matrix element* can be obtained. In order to determine the effective Majorana mass, we need to know nuclear matrix elements of the $0\nu\beta\beta$ decay (NME).

The calculation of NME is a complicated nuclear many-body problem. Two main approaches are used: Nuclear Shell Model (NSM) [54] and Quasiparticle Random Phase Approximation (QRPA) [55, 56].

The Nuclear Shell Model is attractive from physical point of view: there are many spectroscopic data in favor of shell structure of nuclei (spins and parities of nuclei, binding energies of magic nuclei, etc.). It is based on the assumption that there exists spherically symmetrical averaged nucleon field (usually oscillator potential) and one-particle states in this field are used as a basis for the description of valence nucleons. An effective interaction between nucleons is taken into account in the Hamiltonian. Because of computational difficulties, rather a limited number of one-particle states can be used in the NSM calculations. However, all possible distributions of valence nucleons over these states are taken into account.

The neutrinoless double β decay of a nucleus is due to transition of two neutrons into two protons with the emission of two electrons. The operator of the transition of two neutrons into two protons can be presented in the form of the sum of products of an operator of the absorption of two neutrons in a state with total momentum J and parity π and an operator of creation of two protons with the same momentum and parity:

$$M = \sum (P^{J^\pi})^\dagger P^{J^\pi}. \tag{181}$$

It was found [57] that the dominant contribution to the NME comes from the 0^+ state of the neutron–neutron pair. Sizable contribution is given also by the 2^+ state. It is, however, smaller and has opposite sign. The contributions of other states are negligibly small. The dominance of the contribution of the 0^+ state corresponds to the pairing content of the initial and final wave functions. Let us notice that if seniority of the initial and final wave functions is equal to zero, NME would be maximal.

Further, it was found [58] that the major contribution to NME comes from pairs of neutrons at the distance $r \leqslant 2{-}3$ fm. In order to take into account strong repulsion of nucleons at small distances ($\leqslant 1$ fm), an additional r-dependence (so-called short-range correlations) is introduced in the expression for the NME.

This additional r-dependence is parameterized by Jastrow-type function [59]

$$f(r) = 1 - \mathrm{e}^{-ar^2}(1 - br^2), \quad a = 1.1\ \mathrm{fm}^{-2}, \quad b = 0.68\ \mathrm{fm}^{-2}. \tag{182}$$

Recently it was proposed to take into account the short-range correlations by a Unitary Correlation Operator Method (UCOM) [60]. In this method the correlated wave function is obtained by a unitary transformation of uncorrelated wave function.

In Table 2 we present the NME values of nuclear matrix elements of the $0\nu\beta\beta$ decay $M^{0\nu}$ which were calculated with Jastrow-like and UCOM short-range correlations.

Notice that except double-magic nucleus $^{48}\mathrm{Ca}$ NSM nuclear matrix elements of the $0\nu\beta\beta$ decay for all considered nuclei are practically the same (they differ by not more than $\sim 20\%$).

There are two groups which are performing the QRPA calculation of NME at present: Tübingen group [61–63] and Jyväskylä group [64–67]. The QRPA method allows one to include pairing correlations in nuclear wave functions through the introduction of quasiparticles (particle–hole pairs). Two parameters

Table 2. **The NSM values of nuclear matrix elements of the $0\nu\beta\beta$ decay [58]**

Nuclei transition	$M^{0\nu}$ (UCOM)	$M^{0\nu}$ (Jastrow)
$^{48}\mathrm{Ca} \to {}^{48}\mathrm{Ti}$	0.85	0.64
$^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se}$	2.81	2.30
$^{82}\mathrm{Se} \to {}^{82}\mathrm{Kr}$	2.64	2.18
$^{124}\mathrm{Sn} \to {}^{124}\mathrm{Te}$	2.62	2.10
$^{128}\mathrm{Te} \to {}^{128}\mathrm{Te}$	2.88	2.34
$^{130}\mathrm{Te} \to {}^{130}\mathrm{Xe}$	2.65	2.12
$^{136}\mathrm{Xe} \to {}^{136}\mathrm{Ba}$	2.19	1.76

g_{pp} and g_{ph} of the model characterize particle–particle and particle–hole interactions. The constant g_{ph} is obtained from the fit of the energy of the giant Gamow–Teller resonance. The Tübingen group determines the value of the constant g_{pp} from the measured half-life of the $2\nu\beta\beta$ decay of the corresponding nucleus. The Jyväskylä group determines the constant g_{pp} from data on the β decay of nuclei which are close to the nuclei of the interest for the $0\nu\beta\beta$ decay. They also use the value of the constant g_{pp}, obtained from the half-life of the $2\nu\beta\beta$ decay.

In the QRPA approach the mean nuclear field is described by the Woods–Saxon potential. The number of basic one-particle states which can be used in the QRPA is much larger than in the NSM. This is an important advantage of the QRPA approach. However, only limited excitations can be taken into account.

Like in the NSM case, in the QRPA approach the dominant contribution to NME gives 0^+ state of neutron pairs. However, in the QRPA not only 2^+ state but also other states give significant contribution.

In both approaches, major contribution to NME comes from neutron pairs at a distance smaller than 2–3 fm. The short-range correlations, taking into account nucleon repulsion at short distances, are introduced in the QRPA expression for NME via the Jastrow-type function (182) and through Unitary Correlation Operator Method procedure. Recently [63] the short-range correlations were calculated directly from different nucleon–nucleon potentials by the coupled cluster method (CCM) [68].

In Table 2 the results of the calculations of the QRPA nuclear matrix elements by the Tübingen group are presented. The short-range correlations were calculated by the CCM method. For comparison in Table 2 the results of the calculation of NME with the Jastrow-type short-range correlations are also presented.

The uncertainties of NME in Table 3 are mainly due to different values of the axial constant g_A which are used in the calculations. Upper bounds of NME correspond to the free nucleon value $g_A = 1.25$ and lower bounds correspond to quenched in the nuclear matter value $g_A = 1$.

Table 3. **The values of QRPA nuclear matrix elements of the $0\nu\beta\beta$ decay with CCM and Jastrow short-range correlations [63]**

Nucleus	$M^{0\nu}$ (Jastrow)	$M^{0\nu}$ (CCM)
^{76}Ge	3.33–4.68	4.07–6.64
^{82}Se	2.82–4.17	3.53–5.92
^{96}Zr	1.01–1.34	1.43–2.12
^{100}Mo	2.22–3.53	2.94–5.56
^{100}Mo	2.22–3.53	2.94–5.56
^{116}Cd	1.83–2.93	2.30–4.14
^{128}Te	2.46–3.77	3.21–5.65
^{130}Te	2.27–3.38	2.92–5.04
^{136}Xe	1.17–2.22	1.57–3.24

Table 4. **The values of NME calculated in the framework of QRPA by the Jyväskylä group [64]**

Nucleus	$g_{\rm pp}$	g_A	$M^{0\nu}$ (Jastrow)	$M^{0\nu}$ (UCOM)
^{76}Ge	1.02	1.00	5.08	6.56
	1.06	1.25	4.03	5.36
^{82}Se	0.96	1.00	3.54	4.60
	1.00	1.25	2.78	3.72
^{96}Zr	1.06	1.00	3.13	4.31
	1.11	1.25	2.07	3.12
^{100}Mo	1.07	1.00	3.53	4.85
	1.00	1.25	2.74	3.93
^{116}Cd	0.82(β)	1.25	3.98	4.93
	0.97	1.00	3.68	4.68
	1.01	1.25	3.03	3.94
^{128}Te	0.86(β)	1.25	4.07	5.51
	0.89	1.00	4.23	5.84
	0.92	1.25	3.38	4.79
^{130}Te	0.84	1.00	4.06	5.44
	0.90	1.25	2.99	4.22
^{136}Xe	0.74	1.00	2.86	3.72
	0.83	1.25	2.05	2.80

The results of the calculations of nuclear matrix elements of the $0\nu\beta\beta$ decay performed by the Jyväskylä group are presented in Table 4. The short-range correlations were taken into account by Jastrow and UCOM procedures.

It is difficult to expect that outcome of the many-body nuclear calculations, based on different assumptions, will be the same. However, from the results presented in Tables 2–4 we can conclude the following:

1. The values of the nuclear matrix elements of the $0\nu\beta\beta$ decay of different nuclei obtained in the latest QRPA and NSM calculations are qualitatively compatible.

2. NSM nuclear matrix elements of ^{76}Ge, ^{82}Se and ^{130}Te are by a factor of (1.5–2) lower than QRPA nuclear matrix elements.

3. There is no doubt that traditional methods of the calculation of NME will be improved and, apparently, new methods will appear. However, it will be very important to find a way to test the calculations.

If neutrinoless double β decay is discovered and half-lives of *different nuclei* are measured, from the ratios of measured half-lives in this case it will be possible to test different models of the calculation of NME [69]. If, for example, half-lives of the $0\nu\beta\beta$ decay of ^{76}Ge and ^{130}Te are measured, the ratio of half-lives will be practically equal to the inverse ratio of the corresponding phase-space factors in the case of NSM nuclear matrix elements and could be significantly different from this ratio in the case of QRPA nuclear matrix elements.

8. EXPERIMENTS ON THE SEARCH FOR $0\nu\beta\beta$ DECAY

At present there exist data of many experiments on the search for neutrinoless double β decay. The most stringent lower bound on the half-lives of the $0\nu\beta\beta$ decay of different nuclei was obtained in the Heidelberg–Moscow [1] and IGEX [2] experiments, and in the recent CUORICINO [3] and NEMO [70] experiments.

In the Heidelberg–Moscow and IGEX experiments, two electrons with total energy $Q_{\beta\beta} = 2039$ keV which are produced in the $0^+ \to 0^+$ transition $^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + e^- + e^-$ were searched for. In the Heidelberg–Moscow experiment the source (and detector) consists of five crystals of 86% enriched ^{76}Ge with total mass 10.96 kg. In the IGEX experiment $\sim$ 7 kg of enriched ^{76}Ge was used. Low background level (~ 0.06 counts/(keV $\cdot$ kg $\cdot$ y)) and high energy resolution ($\sim$ 3 keV) were reached in the germanium experiments.

For the half-life of ^{76}Ge in the Heidelberg–Moscow experiment the following lower bound was obtained [1]:

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.9 \cdot 10^{25}\ \mathrm{y}. \tag{183}$$

From this result the following upper bound on the effective Majorana mass was inferred: $|m_{\beta\beta}| < 0.35$ eV.

In the IGEX experiment it was found [2] that

$$T^{0\nu}_{1/2}(^{76}\mathrm{Ge}) > 1.57 \cdot 10^{25}\ \mathrm{y}. \tag{184}$$

From these results, assuming different NME, the bound $|m_{\beta\beta}| < 0.33-1.35$ eV was found.

In the cryogenic experiment CUORICINO [3] the search for the $0\nu\beta\beta$ decay of ^{130}Te was performed. An array of 62 TeO_2 crystals with a total active mass of 40.7 kg was cooled to 8–10 mK in a dilution refrigerator. Since the heat capacity is proportional to T^3, an increase of temperature due to tiny release of energy in the $0\nu\beta\beta$ decay can be recorded by special thermometers.

No evidence for the $0\nu\beta\beta$ decay of ^{130}Te was obtained in the CUORICINO experiment. For the half-life of ^{130}Te a limit

$$T^{0\nu}_{1/2}(^{130}\mathrm{Te}) > 3.0 \cdot 10^{24}\ \mathrm{y} \tag{185}$$

was obtained [3]. From this limit, using the values of the NME, calculated in the latest papers, the following upper bound was inferred: $|m_{\beta\beta}| < 0.19{-}0.68$ eV.

In the NEMO3 experiment [70] the cylindrical source was divided into sectors with enriched ^{100}Mo (6914 g), ^{82}Se (932 g), ^{116}Cd (405 g), ^{130}Te (454 g), ^{150}Nd (34 g), ^{96}Zr (94 g) and ^{48}Ca (7 g). For the detecting of the two electrons drift cells and plastic scintillator were used. No $0\nu\beta\beta$ decay was observed. In Table 5 the results of the NEMO3 experiment are presented.

Table 5. **Lower bounds of the half-lives of the $0\nu\beta\beta$ decay of different nuclei, obtained in the NEMO3 experiment [70]**

Nucleus	$T^{0\nu}_{1/2}$ (90% CL), y	$\|m_{\beta\beta}\|$, eV
^{100}Mo	$\geqslant 5.8 \cdot 10^{23}$	$\leqslant 0.6{-}1.3$
^{82}Se	$\geqslant 2.1 \cdot 10^{23}$	$\leqslant 1.2{-}2.2$
^{96}Zr	$\geqslant 8.6 \cdot 10^{21}$	$\leqslant 7.4{-}20.1$
^{48}Ca	$\geqslant 1.3 \cdot 10^{22}$	$\leqslant 29.7$
^{150}Nd	$\geqslant 1.8 \cdot 10^{22}$	$\leqslant 4.0{-}6.3$

Several new experiments on the search for the $0\nu\beta\beta$ decay are at preparation at present. In these new experiments it is planned to reach the sensitivity $|m_{\beta\beta}| \simeq$ a few 10^{-2} eV, corresponding to the inverted hierarchy of the neutrino mass spectrum.

In the future GERDA experiment [71], array of enriched Ge crystals will be cooled and shielded by liquid argon (or nitrogen) of very high radiopurity. In Phase I of the GERDA experiment, 5 detectors from the Heidelberg–Moscow experiment (active mass 11.9 kg) and 3 detectors from the IGEX experiment (active mass 6 kg) will be used. The expected background at this phase of the experiment will be $\sim 10^{-2}$ counts/(kg $\cdot$ keV $\cdot$ y). The expected sensitivity will be $T_{1/2}(^{76}\mathrm{Ge}) \simeq 3 \cdot 10^{25}$ y at 90% CL. Nonobservation of the neutrinoless double β decay at this phase of the experiment would allow one to obtain the upper bound $|m_{\beta\beta}| \lesssim 0.27$ eV (with QRPA NME).

During Phase II of the GERDA experiment, additional 22 kg of the enriched Ge will be used (total active mass of the enriched Ge will be about 40 kg). The expected background is 10^{-3} counts/(kg $\cdot$ keV $\cdot$ y). The sensitivity $T_{1/2}(^{76}\mathrm{Ge}) \simeq 1.4 \cdot 10^{26}$ y (at 90% CL) is planned to be reached. This sensitivity corresponds to the sensitivity to the effective Majorana mass $|m_{\beta\beta}| \simeq 0.11$ eV (QRPA NME).

If the goals of Phase I and Phase II are achieved and the level of the background 10^{-4} counts/(kg $\cdot$ keV $\cdot$ y) is reached, it is planned (in cooperation with the Majorana collaboration) to build ~ 1 t germanium detector with the aim to investigate the region of the inverted neutrino mass hierarchy.

As is well known, the group of participants of the Heidelberg–Moscow experiment claimed that it found an evidence for neutrinoless double β decay of $^{76}\mathrm{Ge}$ [72]. For the half-life of the decay the authors obtained the following 3σ range: $T_{1/2}(^{76}\mathrm{Ge}) = (1.30{-}3.55) \cdot 10^{25}$ y. These values correspond to the following range for the effective Majorana mass: $|m_{\beta\beta}| = 0.24{-}0.58$ eV (with NME calculated in [73]). There is no detailed analysis of the systematic errors in [72] (see [74]). The only way to confirm or refute the claim is to perform more sensitive than the Heidelberg–Moscow experiment (preferably $^{76}\mathrm{Ge}$ experiment in order to avoid the NME problem). One of the aims of the GERDA experiment is to check the claim made in [72].

In the proposed Majorana experiment [75], an array of enriched Ge crystals will be installed inside of high-purity electroformed copper cryostat. It is expected that the background in the Majorana experiment will be a factor of 150 lower than in the Heidelberg–Moscow and IGEX experiments. Staged approach based on the 60 kg enriched Ge array (60/120/180 kg) is planned. The expected sensitivity at the first stage of the experiment ($T_{1/2}(^{76}\mathrm{Ge}) \simeq 5.5 \cdot 10^{26}$ y) will allow one to check the claim made in the papers [72].

In the cryogenic CUORE experiment [76], an array of 19 towers made from $5 \times 5 \times 5$ cm $\mathrm{TeO_2}$ crystals is used as a source (detector). The total number of the crystals in the experiment is equal to 988. The total mass of the crystals is 741 kg of $\mathrm{TeO_2}$ (204 kg of $^{130}\mathrm{Te}$). In the CUORICINO experiment one similar tower of a mass 40.7 kg was used.

The expected background in the CUORE experiment is 0.01 counts/(kg $\cdot$ keV $\cdot$ y). The expected sensitivity to the half-life is $T_{1/2}(^{130}\mathrm{Te}) \simeq 2.5 \cdot 10^{26}$ y. With the present-day values of NME, the following sensitivity to the effective Majorana mass will be achieved: $|m_{\beta\beta}| \simeq (4.7{-}5.3) \cdot 10^{-2}$ eV.

In the future EXO experiment [77], the $0\nu\beta\beta$ decay of $^{136}\mathrm{Xe}$ will be sought for. Because there is no need to grow crystals and procedure of enrichment is relatively simple, Xe is ideal for a large-scale (one ton or more) neutrinoless double β-decay experiment. Ion $^{136}\mathrm{Ba}^{++}$, produced in the decay $^{136}\mathrm{Xe} \to {}^{136}\mathrm{Ba}^{++} + e^- + e^-$, by the capture of an electron can be transferred to the ion $^{136}\mathrm{Ba}^{+}$ which is stable in Xe. The EXO collaboration plans to identify $^{136}\mathrm{Ba}^{+}$

ion by optical pumping with lasers. Single ion can be detected by this technique (via photon rate 10^7 s^{-1}). When the program of the $^{136}\mathrm{Ba}^+$ tagging will be realized, the background in the experiment on the search for the $0\nu\beta\beta$ decay will be drastically reduced.

At present the EXO collaboration is constructing 200 kg liquid xenon TPC with Xe enriched to 80% in $^{136}\mathrm{Xe}$. No $^{136}\mathrm{Ba}^+$ tagging will be done at this stage. In this experiment the sensitivity $|m_{\beta\beta}| \simeq 1.5 \cdot 10^{-1}$ eV is anticipated.

We have discussed experiments on the search for neutrinoless double β decay which will be done in the coming years. There are several other experiments which are in R&D stage: Super-NEMO ($^{150}\mathrm{Nd}$ or $^{82}\mathrm{Se}$) [78], MOON ($^{100}\mathrm{Mo}$) [79], SNO ++ ($^{150}\mathrm{Nd}$) [80], COBRA ($^{116}\mathrm{Cd}$, $^{130}\mathrm{Te}$) [81], CANDLES ($^{48}\mathrm{Nd}$) [82], DCBA ($^{150}\mathrm{Nd}$) [83], CAMEO ($^{116}\mathrm{Cd}$) [84], XMASS ($^{136}\mathrm{Xe}$) [85], and others.

CONCLUSION

The observation of the neutrino oscillations in experiments with atmospheric, solar, reactor, and accelerator neutrinos proves that neutrino masses are different from zero and that the states of flavor neutrinos ν_e, ν_μ, ν_τ are mixtures of states of neutrinos with different masses. There are two general possibilities for neutrinos with definite masses: they can be 4-component Dirac particles, possessing conserved total lepton number which distinguishes neutrinos and antineutrinos, or purely neutral 2-component Majorana particles with identical neutrinos and antineutrinos.

It will be extremely important for the further development of the theory of the neutrino masses and mixing to answer the fundamental question: are neutrinos with definite masses Dirac or Majorana particles?

Neutrino masses are many orders of magnitude smaller than masses of their family partners, leptons and quarks. This fact tell us that *neutrino masses and masses of leptons and quarks are of different origin.* The most natural possibility of the explanation of the smallness of the neutrino masses gives us the seesaw mechanism of the neutrino mass generation. This beyond-the-Standard-Model mechanism connects smallness of neutrino masses with the violation of the total lepton number at a large scale and Majorana nature of neutrino masses. If it is established that neutrinos with definite masses are Majorana particles, it will be a strong argument in favor of the seesaw origin of neutrino masses.

Investigation of the neutrinoless double β decay of nuclei is the only practical way which could allow one to prove that neutrinos are Majorana particles. This is simply connected with the fact that there are a huge number of parent nuclei in a source. However, even if neutrinos are Majorana particles, the probability of the $0\nu\beta\beta$ decay is extremely small. There are two reasons for that:

• The $0\nu\beta\beta$ decay is the second order in the Fermi constant process.

• The $0\nu\beta\beta$ decay is possible due to neutrino helicity flip. In the case of neutrino mixing this means that the matrix element of the process is proportional to effective Majorana mass $m_{\beta\beta} = \sum_i U_{ei}^2 m_i$. Smallness of neutrino masses is additional suppression factor in the decay probability.

Experiments on the measurement of the half-lives of such a rare process as neutrinoless double β decay with severe requirements on background and energy resolution are extremely difficult. A big progress was achieved. However, future experiments with about one ton detectors, which will allow one to reach the region of values of the effective Majorana mass, which is predicted from neutrino oscillation data in the case of the inverted mass hierarchy, is definitely a challenge. Taking into account importance of the problem of the nature of massive neutrinos, there is no doubt that goals of future experiments will be achieved.

In this review we considered $0\nu\beta\beta$ decay, driven by the left-handed SM weak interaction and Majorana neutrino masses. If total lepton number is not conserved and neutrinos with definite masses are Majorana particles, such a mechanism of the $0\nu\beta\beta$ decay obviously must exist. In the literature many other possible mechanisms of the $0\nu\beta\beta$ decay were considered (for references see, for example, [8]). We shortly discuss here a mechanism due to the exchange of a heavy SUSY neutralino. Let us assume that there exist an R parity and a lepton-number-violating interaction, which induces the transition $d \to u + \tilde{e}$ ($\tilde{e}$ is the selectron). In combination with the standard SUSY interaction, which induces transition $\tilde{e} \to e + \chi$ (χ is the neutralino), these two interactions in the case of the virtual neutralino provide the $0\nu\beta\beta$ transition $n + n \to p + p + e + e$. If the constants of the SUSY interactions are of the order of the electroweak constant g and if masses of SUSY particles are characterized by a scale Λ, in this case a contribution of these interactions to the matrix element of the $0\nu\beta\beta$ decay is proportional to

$$M_{\rm SUSY} \sim G_F^2 \, \frac{m_W^4}{\Lambda^5}. \tag{186}$$

This contribution must be compared with the contribution to the matrix element of the $0\nu\beta\beta$ decay of the standard small Majorana neutrino mass mechanism

$$M_0 \sim G_F^2 \, \frac{|m_{\beta\beta}|}{\langle q^2 \rangle}. \tag{187}$$

Taking into account that $|m_{\beta\beta}| \lesssim 1$ eV and $\langle q^2 \rangle \simeq 100$ MeV^2, we come to the conclusion that for $\Lambda \simeq 1$ TeV $M_{\rm SUSY}$ can be comparable with M_0 if a hypothetical SUSY interaction, which does not conserve R parity and the lepton number, is characterized by the electroweak constant g (for more detail, see [86]).

Appendix

ETTORE MAJORANA

Great Italian physicist Ettore Majorana was born in Catania (Scicily, Italy) on August 5, 1906. His father was an engineer, specialist in telecommunication. There were five children in the family*.

In 1921 the family moved to Rome. In 1923 E. Majorana finished High School and entered the Engineer Faculty of Rome University.

Among his fellow-students and friends were E. Segre and E. Amaldi. In 1927 Segre and later Amaldi transferred to the Physics Faculty and started to work with E. Fermi who was appointed in 1926 as a Professor of theoretical physics at Rome University.

E. Majorana was famous at the Engineer Faculty for his extraordinary ability of solving difficult mathematical problems. E. Segre convinced E. Majorana to meet and to speak with Fermi. At that time Fermi was developing the statistical model which is known as Thomas–Fermi model. He explained Majorana the model and showed him the table with numerical values of the screening potential which he calculated numerically.

Next morning Majorana returned back to the Institute of Physics with his own table of values of the potential. He transformed the second-order nonlinear Thomas–Fermi equation into the Riccati equation and solved it numerically. Majorana and Fermi results coincided.

A few days later E. Majorana became student of the Physics Faculty. He impressed everybody by his lively mind and broad interests. He was a very critical person. For his criticism he was called in the Fermi group «Great Inquisitor».

In 1929 Majorana received diploma. His theses were devoted to the investigation of the structure of nuclei and to the theory of the alpha decay. His supervisor was Fermi.

After doctorate Majorana visited the Institute of Physics for a few hours every day. He spend most of his time in library, working and studying Dirac, Heisenberg, Pauli, Weil and Wigner papers.

At that time Fermi and his group worked on problems of atomic and molecular physics. Majorana wrote six papers on the subject. These papers demonstrated profound Majorana's ability of using symmetry properties of the states. This allowed him to simplify the problem and to choose the suitable approximation (which is normal now but was not usual at that time). These papers also demonstrated perfect Majorana's knowledge of experimental data.

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*For a detailed biography of E. Majorana, see E. Amaldi [87].

In 1932 Majorana received teaching diploma («libero docente»). Committee (Fermi, Lo Surdo, Persico) concluded that «the candidate has a complete mastery in theoretical physics».

At the end of 1931–beginning of 1932 Fermi and his group started to concentrate their efforts on nuclear physics.

After the discovery of the neutron by Chadwick (1932) Majorana was one of the first who came to an idea that constituents of nuclei are protons and neutrons. He started to develop the theory of nuclear forces. Majorana proposed the theory of space exchange forces between p and n (Majorana potential).

Fermi was very interested in the idea and tried to convince Majorana to publish his results. However, Majorana refused and even did not allow Fermi to mention them in his talk at a conference in Paris. E. Fermi managed, however, to persuade Majorana to go to Leipzig where W. Heisenberg was working and to Copenhagen where N. Bohr was working.

E. Majorana was abroad during seven months, starting from January 1933. Heisenberg, who worked at that time on the theory of nuclear forces, discussed with Majorana his paper on nuclear theory. He convinced Majorana to publish it.

After returning from Germany E. Majorana started to come to the Institute of Physics at via Panisperna rather rarely and after some months did not come at all.

He was at home and became interested in political economy, philosophy, construction of ships, medicine. He even wrote a paper on statistical laws in physics and social sciences, which was discovered and published after his disappearance.

Meanwhile new talented physicists grew up in Italy (Wick, Racah, Giovanni Gentili Jr. and others). It was time to create a new chair in theoretical physics. This chair was created at the University of Palermo and at the beginning of 1937 a competition for the chair was announced.

It was a problem to convince Majorana to take part in the competition. Finally, Fermi, Amaldi and Segre managed to convince him.

Majorana had no publications during several years. He sent to «Nuovo Cimento» his most important paper «Symmetrical Theory of the Electron and the Positron» in which the theory of the Majorana particles was proposed.

After that the following happened. By the request of Senator Giovanni Gentili E. Majorana for his extraordinary abilities without competition was appointed as a professor at Napoli University.

In January 1938 E. Majorana came to Napoli. In Napoli he had rather lonely life. He went to the University only when he had lectures (on quantum mechanics). After lectures he visited Professor Carrelli with whom he became friendly and discussed different problems in physics. He never mentioned what he was doing. He discussed his neutrino theory and Carrelli had an impression that Majorana considered this theory as his most important contribution to physics.

On March 23, 1938 E. Majorana decided to go to Palermo. On March 25 Carrelli received a telegram from Majorana from Palermo. He asked Carrelli not to worry about a letter which he would receive. In the letter which came soon, Majorana wrote that he found his life useless and decided to commit suicide. Carrelli called Fermi and Fermi called Luciano, Ettore's brother. Luciano immediately went to Napoli. He understood that on the evening of March 25 Ettore took boat to Napoli. He was seen sleeping in his cabin when the boat was entering into the Napoli bay. He did not arrive in Napoli. His body was never found.

During several months there was an investigation conducted by family and by the police. Vatican tried to find out whether he entered some monastery. No traces were found.

I will finish with two citations: «There are various kinds of scientists in the world. The second- and third-rate ones do their best but do not get very far. There are also first-rate people who make very important discoveries which are of capital importance for the development of the science. Then there are geniuses like Galileo and Newton. Ettore Majorana was one of these. Majorana had greater gifts than anyone else in the world; unfortunately, he lacked one quality which other men generally have: plain common sense» (E. Fermi from Cocconi memories).

«E. Majorana was very critical to himself and other people. He was permanently unhappy with himself. He was a pessimist but had very acute sense of humor. He was conditioned by complicated and absolutely nontrivial living rules. ... E. Majorana was quite rich and I cannot avoid thinking that his life might not have finished so tragically should he have been obliged to work for a living. For that reason and also because he did not like to publish the results of all investigations he had made, Majorana contribution to physics is much less than it could be» (B. Pontecorvo [88]).

In conclusion I will discuss briefly the content of Majorana's paper «Symmetrical Theory of the Electron and the Positron» [89].

E. Majorana was not satisfied with the then existing theory of electrons and positrons in which positrons were considered as holes in the Dirac sea of the states of electrons with negative energies. He wanted to formulate the symmetrical theory in which there is no notion of states with negative energies.

Let us consider the Dirac equation for a complex field $\psi(x)$

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\ \psi(x) = 0, \tag{188}$$

where m is the mass of the particles-quanta of the field. The conjugated field

$$\psi^{c}(x) = C\bar{\psi}^{T}(x) \tag{189}$$

(C is the matrix of the charge conjugation) obviously satisfies the same equation

$$(i\gamma^{\alpha}\partial_{\alpha} - m)\psi^{c}(x) = 0. \tag{190}$$

Let us present the field $\psi(x)$ in the form

$$\psi(x) = \frac{\chi_1 + i\chi_2}{\sqrt{2}}, \tag{191}$$

where

$$\chi_1(x) = \frac{\psi(x) + \psi^c(x)}{\sqrt{2}}, \quad \chi_2(x) = \frac{\psi(x) - \psi^c(x)}{\sqrt{2}i}. \tag{192}$$

It is obvious from (188), (190) and (192) that the fields $\chi_{1,2}(x)$ satisfy the Dirac equations

$$(i\gamma^\alpha \partial_\alpha - m)\ \chi_{1,2}(x) = 0. \tag{193}$$

The fields $\chi_{1,2}(x)$ satisfy also *additional (Majorana) conditions*

$$\chi^c_{1,2}(x) = \chi_{1,2}(x). \tag{194}$$

Majorana used the representation in which γ^α are imaginary matrices (Majorana representation). In this representation $\psi^c(x) = \psi^*(x)$ and $\chi_1(x)$ and $\chi_2(x)$ are real and imaginary parts of the field $\psi(x)$.

Majorana built quantum field theory of the fields $\chi_{1,2}(x)$. First of all it is easy to show that there are no electromagnetic currents for the fields $\chi_{1,2}(x)$. In fact, taking into account (193), we have

$$\begin{aligned} j^\alpha_i(x) = \bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = -\chi^T_i(x)(\gamma^\alpha)^T\bar{\chi}_i(x)^T = \\ = -\bar{\chi}_i(x)\gamma^\alpha\chi_i(x) = 0 \quad (i = 1, 2). \end{aligned} \tag{195}$$

Therefore, $\chi_{1,2}(x)$ are fields of particles with electric charge and magnetic moment equal to zero.

For the operator of the energy and momentum, Majorana obtained the following expressions:

$$P^i_\alpha = \int \sum_r p_\alpha (a^i_r(p))^\dagger\ a^i_r(p)\ d^3p \quad (i = 1, 2), \tag{196}$$

where operators $a^i_r(p)$ and $(a^i_r(p))^\dagger$ satisfy usual anticommutation relations.

Thus, $(a^i_r(p))^\dagger$ $(a^i_r(p))$ is the operator of the creation (absorption) of a particle with momentum p and helicity r. There are no states with negative energies and quanta of the fields $\chi_{1,2}(x)$ are neutral particles (which are identical to their antiparticles).

In the case of the complex field $\psi(x) = \dfrac{\chi_1 + i\chi_2}{\sqrt{2}}$ the current $j^i_\alpha(x) = \bar{\psi}_i(x)\gamma^\alpha\psi_i(x)$ is different from zero. After quantization Majorana came to symmetrical theory of particles and antiparticles with operators of total momentum and total charge given by the following expressions:

$$P^{\alpha} = \int \sum_{r} p^{\alpha}[c_r^{\dagger}(p)c_r(p) + d_r^{\dagger}(p)\ d_r(p)]d^3p, \tag{197}$$

$$Q = e \int \sum_{r} [c_r^{\dagger}(p)c_r(p) - d_r^{\dagger}(p)\ d_r(p)]d^3p. \tag{198}$$

Here $c_r^{\dagger}(p)(c_r(p))$ is the operator of the creation (absorption) of particle with charge e, momentum p and helicity r and $d_r^{\dagger}(p)(d_r(p))$ is the operator of the creation (absorption) of antiparticle with charge $-e$, momentum p and helicity r. Correspondingly,

$$|p\rangle_a = c_r^{\dagger}(p)|0\rangle, \quad |p\rangle_{\bar{a}} = d_r^{\dagger}(p)|0\rangle \tag{199}$$

are states of particle with charge e, helicity r and mass m and antiparticle with charge $-e$, helicity r and the same mass m.

Majorana wrote in the paper [89]: «A generalization of Jordan–Wigner quantization method allows one not only to give symmetrical form to the electron–positron theory but also to construct an essentially new theory for particles without electric charge (neutrons and hypothetical neutrinos)». And further in the paper: «Although it is perhaps not possible now to ask experiment to choose between the new theory and that in which the Dirac equations are simply extended to neutral particles, one should keep in mind that the new theory is introducing in the unexplored field a smaller number of hypothetical entities».

Soon after the Majorana paper, Racah [90] and Furry [91] proposed the methods which could allow one to test whether neutrino is a Majorana or Dirac particle. The so-called Racah chain of reactions

$$(A, Z) \to (A, Z+1) + e^- + \nu, \quad \nu + (A', Z') \to (A', Z'+1) + e^- \tag{200}$$

is allowed in the case of the Majorana neutrino and is forbidden in the case of the Dirac neutrino. Of course, in 1937 Racah could not know that even in the case of the Majorana neutrino the chain (200) is strongly suppressed due to neutrino helicity.

In 1938 Furry considered neutrinoless double β decay of nuclei

$$(A, Z) \to (A, Z+2) + e^- + e^- \tag{201}$$

induced by the Racah chain with virtual neutrinos.

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