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BOGOLIUBOV COMPENSATION APPROACH IN QCD AND IN THE ELECTROWEAK THEORY

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We apply the Bogoliubov compensation principle to the gauge electroweak interaction to demonstrate a spontaneous generation of anomalous three-boson gauge-invariant effective interaction. The nontrivial solution of compensation equations uniquely defines values of parameters of the theory and the form factor of the anomalous interaction. The contribution of this interaction to running EW coupling $\alpha_{\rm ew}(p^2)$ gives its observable value $a_{\rm ew}(M_W^2) = 0.0374$ in satisfactory agreement to the experiment.

The same approach is applied to QCD to demonstrate a spontaneous generation of three-gluon gauge-invariant effective interaction which contributes significantly in the infrared region. Then we consider a possibility of a spontaneous generation of an effective interaction, leading to $SU(2) \times SU(2)$ Nambu–Jona-Lasinio model. The nontrivial solution of the compensation equation gives unique definition of the form factor of the effective interaction. The resulting theory contains two parameters: average low-energy value of α_s and current light quark mass m_0. All other low-energy parameters — the pion decay constant, mass of the π meson, mass of the σ meson and its width, the constituent quark mass, the quark condensate — are expressed in terms of the two input parameters in satisfactory correspondence to experimental data and chiral phenomenology. The same method is applied for a description of ρ and a_i mesons. The results agree with data up to accuracy of the approximation being used. Thus, we conclude that the Bogoliubov compensation principle works quite effectively in application to elementary particle physics.

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In previous works [1–6] N. N. Bogoliubov's compensation principle [7,8] was applied to studies of spontaneous generation of effective nonlocal interactions in renormalizable gauge theories. In view of this, one performs «add and subtract» procedure for the effective interaction with a form factor. Then one assumes the presence of the effective interaction in the interaction Lagrangian and the same term with the opposite sign is assigned to the newly defined free Lagrangian.

Now we shall firstly demonstrate an application of the principle to the electroweak theory. We start with EW Lagrangian with $N_{\rm gen} = 3$ lepton and colour quark doublets with gauge group $SU(2)$. That is, we restrict the gauge sector to triplet of W_μ^a only:

$$L = \sum_{k=1}^{3}\biggl(\frac{\imath}{2}\Bigl(\bar\psi_k\gamma_\mu\partial_\mu\psi_k - \partial_\mu\bar\psi_k\gamma_\mu\psi_k\Bigr) - m_k\bar\psi_k\psi_k + \frac{g}{2}\,\bar\psi_k\gamma_\mu\tau^a W_\mu^a\psi_k\biggr) + \\ + \sum_{k=1}^{3}\biggl(\frac{\imath}{2}\Bigl(\bar q_k\gamma_\mu\partial_\mu q_k - \partial_\mu\bar q_k\gamma_\mu q_k\Bigr) - M_k\bar q_k q_k + \frac{g}{2}\,\bar q_k\gamma_\mu\tau^a W_\mu^a q_k\biggr) - \\ - \frac{1}{4}\left(W_{\mu\nu}^a W_{\mu\nu}^a\right), \quad W_{\mu\nu}^a = \partial_\mu W_\nu^a - \partial_\nu W_\mu^a + g\,\epsilon_{abc}W_\mu^b W_\nu^c, \quad (1)$$

where we use the standard notations and ψ_k and q_k correspond to left leptons and quarks, respectively. We write here masses for leptons and quarks, bearing in mind the ready Higgs phenomenology. In accordance to the Bogoliubov approach in application to QFT, we look for a nontrivial solution of a compensation equation, which is formulated on the basis of the Bogoliubov add–subtract procedure. We have $L = L_0 + L_{\rm int}$, where

$$L_0 == \sum_{k=1}^{3}\biggl(\frac{\imath}{2}\Bigl(\bar\psi_k\gamma_\mu\partial_\mu\psi_k - \partial_\mu\bar\psi_k\gamma_\mu\psi_k\Bigr) - m_k\bar\psi_k\psi_k + \frac{\imath}{2}\Bigl(\bar q_k\gamma_\mu\partial_\mu q_k - \\ \partial_\mu\bar q_k\gamma_\mu q_k\Bigr) - M_k\bar q_k q_k\biggr) - \frac{1}{4}\,W_{\mu\nu}^a W_{\mu\nu}^a + \frac{G}{3!}\,\epsilon_{abc}\,W_{\mu\nu}^a\,W_{\nu\rho}^b\,W_{\rho\mu}^c, \quad (2)$$

$$L_{\rm int} = \frac{g}{2}\sum_{k=1}^{3}\biggl(\bar\psi_k\gamma_\mu\tau^a W_\mu^a\psi_k + \bar q_k\gamma_\mu\tau^a W_\mu^a q_k\biggr) - \frac{G}{3!}\epsilon_{abc}W_{\mu\nu}^a W_{\nu\rho}^b W_{\rho\mu}^c. \quad (3)$$

Here the notation $(G/3!)\,\epsilon_{abc}\,W_{\mu\nu}^a\,W_{\nu\rho}^b\,W_{\rho\mu}^c$ means corresponding nonlocal vertex in the momentum space

$$(2\pi)^4 G\,\epsilon_{abc}(g_{\mu\nu}(q_\rho pk - p_\rho qk) + g_{\nu\rho}(k_\mu pq - q_\mu pk) + g_{\rho\mu}(p_\nu qk - k_\nu pq) + \\ + q_\mu k_\nu p_\rho - k_\mu p_\nu q_\rho)\,F(p,q,k)\,\delta(p+q+k) + \ldots, \quad (4)$$

where $F(p,q,k)$ is a form factor and $p,\mu,a;\ q,\nu,b;\ k,\rho,c$ are respectively incoming momenta, Lorentz indices and weak isotopic indices of W bosons.

Effective interaction $-(G/3!)\epsilon_{abc}\,W_{\mu\nu}^a\,W_{\nu\rho}^b\,W_{\rho\mu}^c$; $G = g\,\lambda/M_W^2$ is usually called anomalous three-boson interaction and it is considered for long time on phenomenological grounds.

Let us consider expression (2) as the new free Lagrangian L_0, whereas expression (3) as the new interaction Lagrangian $L_{\rm int}$. Then compensation conditions will consist in demand of full connected three-boson vertices of the structure (4), following from Lagrangian L_0, to be zero. This demand gives a nonlinear equation for form factor F.

According to terminology by Bogoliubov, such equations are called compensation equations. In a study of these equations the existence of a perturbative trivial solution (in our case $G = 0$) is always evident, but, in general, a nonperturbative nontrivial solution may also exist.

The goal of a study is a quest of an adequate approach, the first nonperturbative approximation of which describes the main features of the problem. Improvement of a precision of results is to be achieved by corrections to the initial first approximation.

Now in first approximation we come to the following equation for $F(x)$:

$$F(z) \,=\, 1 + \frac{85\, g\,\sqrt{N}\,\sqrt{z}}{96\,\pi}\Biggl(\ln\, z + 4\gamma + 4\ln\, 2 + \frac{1}{2}\, G^{31}_{15}\Bigl(z_0\,\Big|^{0}_{0,0,1/2,-1,-1/2}\Bigr) - \\ - \frac{3160}{357}\Biggr) + \frac{2}{3\,z}\int\limits_0^z F(t)\,t\,dt - \frac{4}{3\sqrt{z}}\int\limits_0^z F(t)\sqrt{t}\,dt - \frac{4\sqrt{z}}{3}\int\limits_z^{z_0} F(t)\frac{dt}{\sqrt{t}} + \\ + \frac{2\,z}{3}\int\limits_z^{z_0} F(t)\,\frac{dt}{t}\; z \,=\, \frac{G^2\,N\,x^2}{1024\,\pi^2}. \quad (5)$$

Here $x = p^2$, $N = 2$. We introduce here an effective cutoff Y, which bounds a «low-momentum» region where our nonperturbative effects act and consider the equation at interval $[0, Y]$ under condition $F(Y) = 0$. We solve equation (5) and obtain

$$F(z) \,=\, \frac{1}{2}\, G^{31}_{15}\Bigl(z\,\Big|^{0}_{1,\,1/2,\,0,\,-1/2,\,-1}\Bigr) - \frac{85\, g\sqrt{N}}{512\,\pi}\, G^{31}_{15}\Bigl(z\,\Big|^{1/2}_{1,\,1/2,\,1/2,\,-1/2,\,-1}\Bigr) + \\ + C_1\, G^{10}_{04}\Bigl(z\,\Big|_{1/2,\,1,\,-1/2,\,-1}\Bigr) + C_2\, G^{10}_{04}\Bigl(z\,\Big|_{1,\,1/2,\,-1/2,\,-1}\Bigr), \quad (6)$$

where $G^{nm}_{qp}\Bigl(z\,\Big|^{a_1,\ldots,a_q}_{b_1,\ldots,b_p}\Bigr)$ is a Meijer function [9]. Constants C_1, C_2 are defined by boundary conditions. With $N = 2$ this gives

$$g(z_0) = -0.4301; \quad z_0 = 205.4254; \quad C_1 = 0.00369; \quad C_2 = 0.00582. \quad (7)$$

Note that there is also a solution with large positive $g(z_0)$, which will be considered further on.

We use the Schwinger–Dyson equation for W-boson polarization operator to obtain a contribution of additional effective vertex to the running EW coupling constant $\alpha_{\rm ew}$. The contribution under discussion reads after angular integrations

$$\Delta\Pi_{\mu\nu}(x) \,=\, (g_{\mu\nu}\, p^2 - p_\mu p_\nu)\,\Pi(x), \quad x \,=\, p^2, \quad y' \,=\, q^2 + \frac{3x}{4},$$

$$\Pi(x) \;=\; -\,\frac{g\,G\,N}{32\,\pi^2}\Bigg(\int\limits_{3x/4}^{x}\frac{F(y')dy'}{y'-x/2}\left(16\frac{y'^3}{x^2}-48\frac{y'^2}{x}+45y-\frac{27}{2}x\right)+ \\ +\int\limits_{x}^{Y}\frac{F(y')dy'}{y'-x/2}\left(-\,3y'\,+\,\frac{5}{2}\,x\right)\Bigg),\quad g\;=\;g(Y). \tag{8}$$

So we have modified one-loop expression for $\alpha_{\rm ew}(p^2)$, $x = p^2$

$$\alpha_{\rm ew}(x) \;=\; \frac{6\,\pi\,\alpha_{\rm ew}(x'_0)}{6\,\pi\,+\,5\,\alpha_{\rm ew}(x'_0)\ln{(x/x'_0)}\,+\,6\,\pi\,\Pi(x)},\quad \alpha_{\rm ew}(x_0)\;=\;\frac{g(Y)^2}{4\,\pi}, \tag{9}$$

where $x'_0 \;=\; 4/3\,x_0$ means a normalization point such that $\Pi(x'_0) \;=\; 0$. Using expression (9), we calculate behaviour of $\alpha_{\rm ew}(x)$ down to values of $x \;=\; M_W^2$ and obtain $\alpha_{\rm ew}(M_W^2) \;=\; 0.0374$ ($\alpha_{\rm ew}^{\rm exp}(M_W^2) = \alpha(M_W)/\sin^2_W = 0.0337$). It is only 10% larger than the experimental value. We consider this result as strong confirmation of the approach.

Let us consider a contribution of effective interaction in (3) to $g-2$ anomaly Δa. Considering contributions of the the new three-boson interaction to $W\,W\,H$ interaction, we have from its conventional definition

$$G = \frac{g\lambda}{M_W^2},\quad \lambda = -0.0151, \tag{10}$$

which agrees with experimental limitations. Then for mass of Higgs $M_H \;=\; 114$ GeV we obtain $\Delta a \;=\; 3.34\cdot 10^{-9}$, which comfortably fits into error bars for well-known deviation [13] $\Delta a \;=\; (3.02\,\pm\,0.88)\cdot 10^{-9}$. With M_H growing, Δa slowly decreases inside the error bars down to $2.67\cdot 10^{-9}$ for $M_H = 300$ GeV [6].

We apply the same procedure to QCD Lagrangian with $N_f = 3$ colour quarks with gauge group $SU(3)$ and study a possibility of a spontaneous generation of effective interaction $-\,(G_g/3!)\,f_{abc}\,F^a_{\mu\nu}\,F^b_{\nu\rho}\,F^c_{\rho\mu}$, which is usually called anomalous three-gluon interaction. The second solution of Eq. (5) for $N = 3$ reads

$$z_0 = 0.01784,\quad g(z_0) = 2.92145,\quad C_1 = -18.8241,\quad C_2 = 56.2171. \tag{11}$$

Then we again use the Schwinger–Dyson equation for gluon polarization operator to obtain a contribution of additional effective vertex to the running strong coupling constant α_s.

So we have modified one-loop expression for $\alpha_s(p^2)$

$$\alpha_s(x) \;=\; \frac{4\,\pi\,\alpha_s(x'_0)}{4\,\pi\,+\,9\,\alpha_s(x'_0)\ln{(x/x'_0)}\,+\,4\,\pi\,\Pi(x)},\quad x = p^2, \tag{12}$$

where x_0' means a normalization point such that $\Pi(x_0') = 0$. We normalize the running coupling by conditions

$$\alpha_s(x_0) \,=\, \frac{g(Y)^2}{4\,\pi} \,=\, 0.679185, \quad \alpha_s(x_0') \,=\, \frac{4\alpha_s(x_0)\pi(1+\Pi(x_0))}{4\pi + 9\,\alpha_s(x_0)\ln(4/3)}. \qquad (13)$$

Applying the standard transformation, we have

$$\alpha_s(x) = \frac{4\pi}{9\,\ln{(x/\Lambda^2)}(1 + 2\,g\,\sqrt{3}(\alpha_s(x_0')\,\ln{(x/\Lambda^2)})^{-1}\Pi(x))}, \qquad (14)$$
$$\Pi(x) = 0 \text{ for } x \geqslant x_0'.$$

From here we have also $x_0/\Lambda^2 = 1.1707002$, $G_g = 6.62198/\Lambda^2$. Using expressions (12), (13), (14) and normalization at point M_τ, we calculate behaviour of $\alpha_s(x)$. We also use the Shirkov–Solovtsov [10] method to eliminate the ghost pole, which means the following substitution in (14):

$$\alpha_s(x) \,=\, \frac{4\pi}{9}\biggl(\frac{1}{\ln{(x/\Lambda^2)}} \,-\, \frac{\Lambda^2}{x-\Lambda^2}\biggr)\times$$
$$\times\left(1 + \frac{2\,g\,\sqrt{3}}{\alpha_s(x_0')}\biggl(\frac{1}{\ln{(x/\Lambda^2)}} \,-\, \frac{\Lambda^2}{x-\Lambda^2}\biggr)\Pi(x)\right)^{-1}, \quad (15)$$

$$\alpha_s^{(2)} \simeq 0.67 \text{ for } Q \,<\, 250 \text{ MeV}.$$

As the next step we apply the Bogoliubov compensation principle to studies of spontaneous generation of effective nonlocal Nambu–Jona-Lasinio interaction. The NJL model [11,12] proves to be effective in description of low-energy hadron physics. However, the problem how to calculate parameters of the model (G_i, Λ_i; ...) from the fundamental QCD was not solved for a long time. For the purpose we start with the conventional QCD Lagrangian with two light quarks and use again the Bogoliubov *add–subtract* procedure with test interaction

$$\frac{G_1}{2}\Bigl(\bar\psi\tau^b\gamma_5\psi\,\bar\psi\tau^b\gamma_5\psi \,-\, \bar\psi\,\psi\,\bar\psi\,\psi\Bigr) +$$
$$+\,\frac{G_2}{2}\Bigl(\bar\psi\tau^b\gamma_\mu\psi\,\bar\psi\tau^b\gamma_\mu\psi + \bar\psi\tau^b\gamma_5\gamma_\mu\psi\bar\psi\tau^b\gamma_5\gamma_\mu\psi\Bigr). \quad (16)$$

Here notation, e.g., $(G_1/2)\bar\psi\,\psi\,\bar\psi\,\psi$ means corresponding nonlocal vertex in the momentum space with form factor $F_1(p_i)$.

Then we again come to the corresponding compensation equation (see [3]), which has the following nontrivial solution:

$$F_1(z) = C_1\,G^{40}_{06}\left(z|1,\frac{1}{2},\frac{1}{2},0,a,b\right) + C_2\,G^{40}_{06}\left(z|1,\frac{1}{2},b,a,\frac{1}{2},0\right) + \\ + C_3\,G^{40}_{06}\left(z|1,0,b,a,\frac{1}{2},\frac{1}{2}\right), \quad a = -\frac{1-\sqrt{1-64u_0}}{4}, \quad F_1(u_0) = 1, \quad (17)$$

$$b = -\frac{1+\sqrt{1-64u_0}}{4}, \quad \beta = \frac{(G_1^2+6G_1G_2)N}{16\pi^4}, \quad z = \frac{\beta x^2}{64}, \quad u_0 = \frac{\beta m_0^4}{64}.$$

Constants C_i are defined by boundary conditions. So we have the unique solution. The value of parameter u_0 and ratio of two constants G_i are also fixed: $u_0 = 1.92\cdot 10^{-8} \simeq 2\cdot 10^{-8}$; $G_1 = (6/13)\,G_2$. We would draw attention to a natural appearance of small quantity u_0.

Thus, we come to effective nonlocal NJL interaction, which we use to obtain the description of low-energy hadron physics [2, 3, 5]. In this way we obtain expressions for all the quantities under study. Analysis shows that the optimal set of low-energy parameters corresponds to $\alpha_s = 0.67$ and $m_0 = 20.3$ MeV. We present a set of calculated parameters for these conditions including quark condensate, constituent quark mass m, parameters of light mesons:

$$\begin{gathered} \alpha_s = 0.67, \quad m_0 = 20.3\text{ MeV}, \quad m_\pi = 135\text{ MeV}, \quad g = 3.16, \\ m_\sigma = 492\text{ MeV}, \quad \Gamma_\sigma = 574\text{ MeV}, \quad f_\pi = 93\text{ MeV}, \\ m = 295\text{ MeV}, \quad \langle\bar q\,q\rangle = -(222\text{ MeV})^3, \quad G_1^{-1} = (244\text{ MeV})^2, \\ M_\rho = 926.3\text{ MeV}, \quad \Gamma_\rho = 159.5\text{ MeV}, \quad M_{a_1} = 1174.8\text{ MeV}, \\ \Gamma_{a_1} = 350\text{ MeV}, \quad \frac{\Gamma(a_1\to\sigma\,\pi)}{\Gamma_{a_1}} = 0.23. \end{gathered} \quad (18)$$

We use input quantity m_0, while all the other quantities are calculated. The overall accuracy may be estimated to be of the order of $10{-}15\%$. The worse accuracy occurs in value M_ρ (20%).

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