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ON NUCLEAR STATES OF $\bar{K}$ MESONS

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The search for nuclear states of $\bar{K}$ mesons poses interesting problems for the nuclear and low-energy hadron physics: the behavior of tightly bound nuclear systems with strongly correlated impurities, the new kind of binding mechanisms and the extension of effective low-energy theories to the strange sector. These problems are briefly presented and a method of variational calculation of the binding energies is discussed.

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INTRODUCTION

It has been known for a long time that low-energy $\bar{K}$ mesons are attracted by nuclei. This attraction, tested in kaonic atoms, has been attributed to excitations of nucleons by the meson to the $\Lambda(1405)$ baryon. The latter is commonly interpreted as a $\bar{K}N$ quasi-bound state which decays to the hyperon Σ and π meson. These excitations may generate binding of $\bar{K}$ mesons in nuclei via a multiple scattering process. The Σ, π channel becomes the main decay mode of such nuclear states. Some theories, supported by experimental indications [1], predict a very strong binding and in such circumstances the nuclear $\bar{K}$ and nuclear-few-$\bar{K}$ states offer a new physics that is interesting at least in three aspects:

- The binding is generated by exciting nucleons to the S-wave $\Lambda(1405)$ and P-wave $\Sigma(1385)$ states.
- The bound states may involve very high nuclear densities.
- There is a possibility that such states live long enough to allow precise experimental studies of the expected rich spectroscopy. Another point of interest is its astrophysical content.

There are uncertainties and different opinions related to these problems, some are presented below. The last section presents a method flexible enough to calculate nuclear states in the $\bar{K}$-few-N systems.

1. ATOMIC STATES OF K^- MESON

Atomic levels are shifted and broadened by nuclear interactions. The optical potential U_K used to describe these changes is, in general, attractive and strongly absorptive. The standard, first-order expression

$$U_K(r) = \frac{2\pi}{\mu_{KN}} f_{KN} \rho(r) \tag{1}$$

is parameterized by an effective scattering length f_{KN}. Here ρ denotes the nuclear density, μ_{KN} — the reduced mass. At distant nuclear surface where the interaction is tested (in high atomic angular momentum states) the f_{KN} is expected to be a free but off-shell elastic $\bar{K}N$ scattering amplitude. The f_{KN} needed involves the sub-$\bar{K}N$-threshold energy region as both particles are bound

$$f_{KN} = f_{KN}(-E_B - E_{\mathrm{recoil}}), \tag{2}$$

where E_B is the nucleon separation energy and E_{recoil} is the recoil energy of the pair relative to the rest of the system. In atomic states the energies in Eq. (2) span the region from a few to about 50 MeV. There, the amplitude is dominated by $\Lambda(1405)$. Roughly

$$f_{KN}(E) \simeq f_{Kp}(E) \simeq \gamma^2/(E - E^*), \tag{3}$$

where γ is a coupling constant and $E^* = E_r - i\Gamma_r/2$ is the complex resonance energy. Below the resonance energy E_r this amplitude is negative and the optical potential becomes attractive. This kind of attractive mechanism does not depend on the structure and origin of the resonant state. It operates also in the case of $\Sigma(1385)$ which is of quark origin and makes an analogy to the «level repulsion» rule of atomic physics. The resonance is an external state to the $\bar{K}N$ system and generates an attraction if $E < E_r$ and repulsion if $E > E_r$.

The extraction of f_{KN} from atomic data, given in the Table, indicates two problems. The upper level widths and the hydrogen $1S$ level determine $\mathrm{Im}\, f_{KN}$ essentially in the Born approximation and indicate strong energy dependence. Comparison of the hydrogen level and the «all data» best fits indicates sign change of $\mathrm{Re}\, f_{KN}$. Both these findings are due to the resonant behavior of the amplitude (3). The last two lines reflect comparable fits by a deep or a shallow potential. These two possibilities are to some extent motivated by two models

Effective f_{KN} scattering amplitudes extracted from K^- atoms. Separation energies refer to protons as $\Lambda(1405)$, $I = 0$, couples to K^-p

Atom	E_B, MeV	f_{KN}, fm	State
H	0	0.47(9)–i0.31(12)	$1S$ hydrogen
Al, Si, P, S, Cl	8–11	–i0.6(1)	$4F$ upper widths
C	16	–i1.2(2)	$3D$ upper width
All data	6–50	$\sim$–0.4–i0.5	Lower levels [4]
All data	6–50	$\sim$–1.5–i0.8	Lower levels [5]

of $\Lambda(1405)$. The old phenomenological description [2] based on multichannel K matrix and dispersion relations yields $E_r \simeq 1410$ MeV and generates attractive U_K of large values. On the other hand, modern effective QCD models produce $E_r \simeq 1420$ MeV and weaker nuclear effects [3, 4]. The question of $\Lambda(1405)$ position and shape has not been solved, as yet, and becomes one of the main uncertainties in this field.

2. NUCLEAR STATES OF $\bar{K}$ MESONS

The optical potential for K mesons is tested at the nuclear surface and for meson energies close to zero. Extrapolation to higher densities and to energies of nuclear bound states presents difficulties. One way is to use a $\bar{K}N$ model amplitude $\hat{f}_0$, extrapolate it off the energy shell, and solve the multiple scattering equations in the nuclear medium with two-body Green's functions

$$\hat{f} = \hat{f}_0 + \hat{f}_0[G(E, U_K, k_{\mathrm{Fermi}}) - G_0(E)]\hat{f}. \tag{4}$$

The first attempt based on free amplitudes of [2] found a self-consistent solution with $\mathrm{Re}\,U_K(0) \sim -90$ MeV $+ V_{\mathrm{coul}}$ at the nuclear matter density [6]. States bound in such a potential well would reduce the phase space for the $\bar{K}N \to \Sigma\pi$ transitions and the nuclear levels might be as narrow as 20 MeV. On the other hand, a similar type of approach with the chiral $\hat{f}_0$ generates much weaker $\mathrm{Re}\,U_K(0) \sim -50$ MeV and larger widths. Another calculation, based on the Relativistic Mean Field method, yields deep potentials $\mathrm{Re}\,U_K(0) \sim -180$ MeV [7] and much broader states. The dependence on the input $\bar{K}N$ interactions is sizable, but there is another weak point in all these methods. It is the single nucleon approach which misses important $\bar{K}NN$ correlations. The significance of these has been realized in [8]. It is the latter work that initiated active experimental search for the $\bar{K}$-few-N systems. These are discussed now.

Few Nucleon Systems. Akaishi and Yamazaki have indicated that deeply bound states may be formed already in the K^-pp system [8]. The bindings as high as 100 MeV were expected and the experiment [1] supports these estimates. Such states are very difficult to get in experiments and related uncertainties are very large. Another interesting result is the expectation that K^-K^-pp system may be compressed so strongly that its central density is several times larger than the density of nuclear matter. These calculations involved some simplifications: a one-channel approximation and a questionably weak NN repulsion. Nevertheless, a subsequent solution of Faddeev equations with separable interactions in the K^-pp system supports large binding, at least in the $E_r = 1405$ MeV model [9]. Precise calculations, with the proper NN repulsion, are difficult for the $\bar{K}NN$ as well as for systems with larger number of nucleons. To remedy some problems a kind of variational method was introduced [10].

Variational Method. The variational method presented here allows one to find a satisfactory description of the $\bar{K}N$ and NN and $\bar{K}NN$ correlations at short distances. It allows one also to introduce many channels and the P-wave interactions due to $\Sigma(1385)$. It consists of two steps:

(1) The meson wave function $\chi_K(\mathbf{x}, \mathbf{x}_i)$ and complex energy $E(x_i)$ are found for a system of $\bar{K}$ interacting with nucleons fixed at positions x_i.

(2) Next, the nucleon degrees of freedom are allowed and the trial $\bar{K}$-few-N wave function is used in the form $\Psi = \chi_K(\mathbf{x}, \mathbf{x}_i)\chi_N(\mathbf{x}_i)$. The total Hamiltonian involves the meson and nucleon kinetic energies, NN and $\bar{K}N$ interactions. The minimal energy is found by varying parameters which enter χ_N. The width is determined as the average $\Gamma/2 = \langle\Psi|\,\mathrm{Im}\,E(x_i)|\Psi\rangle$.

The first step was used in [11] to solve the scattering problem, now it is used to find the binding energy. Here, we present it briefly for the $\bar{K}NN$ system with a one channel, separable S-wave $\bar{K}N$ interaction. Consider the scattering of a light meson bound on two identical fixed nucleons. Let $\upsilon(\mathbf{x}-\mathbf{x}_i)$ be the potential form factor in coordinate space which defines the amplitude at each scatterer by

$$\psi_i = \int \mathbf{dx}\,\upsilon(\mathbf{x}-\mathbf{x}_i)\chi_K(\mathbf{x}, \mathbf{x}_1, \mathbf{x}_2). \tag{5}$$

To find equations for ψ_i one introduces matrix elements of the propagator

$$G_{i,j}(\mathbf{x}_i, \mathbf{x}_j) = \int \mathbf{dy}\,\mathbf{dx}\,\upsilon(\mathbf{x}-\mathbf{x}_i)\,\frac{\exp\left(\mathbf{ip}\mid \mathbf{x}-\mathbf{y}\mid\right)}{4\pi\mid \mathbf{x}-\mathbf{y}\mid}\,\upsilon(\mathbf{y}-\mathbf{x}_j). \tag{6}$$

The diagonal value, $G_{i,j}$, determines the meson nucleon scattering matrix f. The multiple scattering equation can be expressed in terms of scattering amplitudes f_i at each nucleon i and propagators describing the passage from the nucleon i to the other nucleon j. One arrives at a standard set of equations

$$\psi_i + \sum_{j\neq i} t_j G_{i,j}\psi_j = 0. \tag{7}$$

With two amplitudes ψ_i these reduce to

$$\psi_1 + tG\psi_2 = 0, \quad \psi_2 + tG\psi_1 = 0, \tag{8}$$

where $G = G_{1,2}$. When the determinant $D = 1 - (tG)^2$ is put to zero, the binding «momenta» $p(r)$ may be obtained. The solution of interest corresponding to $1 + tG = 0$ is symmetric, $\psi_2 = \psi_1$, and describes the meson in the S state with respect to the NN center of mass. It exists for all internucleon distances r provided there exists a singularity in $f(E)$ below the $\bar{K}N$ threshold as happens in the $\Lambda(1405)$ case. In some energy region $f = \gamma^2/(E - E^*)$ and the eigenvalue $p(r)$ is given by the condition $1 + tG = 0$, which takes the form

$E = E^* - \gamma^2 G(r,p)$. The solution $E_B(r) - i\Gamma(r)/2$ depends on the NN separation r. As $\mathrm{Re}\, G(r,p)$ close to the resonance is positive, the binding of $\bar{K}$ to fixed NN pair is stronger than the binding of $\bar{K}$ to a nucleon. Asymptotically for $r \to \infty$ one obtains $G \to 0$ and $E(r) \to E^*$, i.e., the $\bar{K}$ meson becomes bound to one of the nucleons. The lifetime of $\bar{K}NN$ becomes equal to the lifetime of $\Lambda(1405)$. Hence, the separation energy is understood here as the energy needed to split $\bar{K}NN \to \Lambda(1405)N$. The difference between the binding at a given separation r and its asymptotic value generates a potential $V_K(r)$, which contracts the nucleons to a smaller radius. It is defined as $\mathrm{Re}\, V_K(r) = E_B(r) - E_B(\infty)$ and generates the bound states. On the other hand, a part of the binding is hidden in $E_B(\infty)$ that is in the structure of $\Lambda(1405)$.

In the $\bar{K}NN$ case this method compares well with the solution of Faddeev equations for the same input. Till now it has been applied also to $\bar{K}NNN$ and $\bar{K}NNNN$ systems [10]. It generates a limited number of states bound via $\Lambda(1405)$ and a large number of states containing $\Sigma(1385)$ and nucleons interacting in S and in P waves*.

To summarize: this field presents a number of challenging problems both for theorists and experimentalists and is likely to develop in many laboratories. One additional problem is — would N. N. Bogolyubov be interested to look into it. I believe so. The still unsolved questions are: how many K mesons could be attached to a nucleus and is there a kaon condensation in nuclear systems. These are likely to be studied with his methods.

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