ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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The fundamental importance of searching for neutrinoless double-beta decay ($0\nu\beta\beta$-decay) is widely recognized. Observation of the decay would tell us that the total lepton number is not conserved and that, consequently, neutrinos are massive Majorana fermions. A brief history of the double-beta decay is presented. The $0\nu\beta\beta$-decay is discussed in the context of neutrino oscillation data. The perspectives of the experimental $0\nu\beta\beta$-decay searches are analyzed. The importance of reliable determination of the $0\nu\beta\beta$-decay nuclear matrix elements is pointed out. The problem of distinguishing of the light-neutrino exchange, heavy-neutrino exchange and the trilinear R-parity breaking supersymmetric ($R\!\!\!/_p$ SUSY) mechanisms of the $0\nu\beta\beta$-decay is addressed. Further, the process of resonant neutrinoless double-electron capture ($0\nu\varepsilon\varepsilon$) is revisited. Arguments are presented that an experimental search for the $0\nu\varepsilon\varepsilon$ might be feasible.

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INTRODUCTION

Neutrinos are one of the fundamental particles which make up the Universe. The properties of the neutrinos have been the most important issues in particle physics, astrophysics, and cosmology. After neutrino oscillations discovery, the physics community worldwide is embarking on the next challenging problem, finding whether neutrinos are indeed Majorana particles (i.e., identical to its own antiparticle) as many particle models suggest or Dirac particles (i.e., is different from its antiparticle). This problem is directly related to the issue of the total lepton number conservation. Lepton Number (LN) conservation is one of the most obscure sides of the Standard Model (SM) not supported by an underlying principle. It follows from an accidental interplay between gauge symmetry and the field content. However, nonzero neutrino masses, as indicated by the recent neutrino oscillations experiments, have proved that the success of the SM should be viewed as that of a low-energy effective theory. It is not unreasonable to expect that in some extensions of the SM, LN conservation may not hold. More specifically, once LN is broken, neutrinos are not protected from getting nonzero Majorana masses after electroweak symmetry breaking. Indeed, a Majorana mass term for the neutrinos violates total lepton number.

A viable scenario is that the neutrino masses are generated at some high-energy scale. This is well motivated by the observed properties of the light neutrinos including tiny masses, large mixings, and the fact that neutrinos are the only electrically neutral fundamental fermions. A Majorana type of the neutrino mass matrix induces a class of LN violating (LNV) processes [1] like neutrino–antineutrino oscillations [2, 3], semileptonic decays of mesons, muon-to-positron conversion in nuclei [4], neutrinoless double-beta decay, muonic analogue of the neutrinoless double-beta decay [5], etc. Probabilities and decay rates of these processes are given in terms of the neutrino mass matrix elements, and a semirealistic event rate has been estimated. The LNV process has been sought in many experiments. Over the years the possibility of LN nonconservation has been attracting a great deal of theoretical and experimental efforts since any positive experimental LNV signal would request physics beyond the SM.

The total LN violating neutrinoless double-beta decay ($0\nu\beta\beta$-decay) [6–9],

$$(A, Z) \to (A, Z + 2) + e^- + e^-, \tag{1}$$

is the most powerful tool to clarify if the neutrino is a Dirac or a Majorana particle. From the experimental point of view, LN violation in $0\nu\beta\beta$-decay is observed through the appearance of two electrons in the final state with no missing energy. The search for the $0\nu\beta\beta$-decay represents the new frontiers of neutrino physics, allowing one, in principle, to fix the neutrino mass scale, the neutrino nature and possible CP violation effects. Many next-generation $0\nu\beta\beta$-decay experiments are in preparation or under consideration. Observing $0\nu\beta\beta$-decay would tell us that the total LN is not a conserved quantity and that neutrinos are massive Majorana fermions.

Recently, there has been an increased theoretical and experimental interest to another LNV process, which is the resonant neutrinoless double-electron capture ($0\nu\varepsilon\varepsilon$) [10–14]. In this reaction two bound electrons from the atomic shell are captured by two protons, thereby lowering the charge of the final nucleus by two units:

$$(A, Z) + e_b^- + e_b^- \to (A, Z - 2)^{**}. \tag{2}$$

Here, the two asterisks denote the possibility of leaving the system in an excited nuclear and/or atomic state, the latter being characterized by two vacancies in the electron shell of the otherwise neutral atom. Already long ago Bernabéu, De Rujula, and Jarlskog [15] pointed out that this process might be as important as the $0\nu\beta\beta$-decay in the case of resonant enhancement of the decay rate. This might happen, if there is a small energy difference of initial and final atoms. However, the question remains, which atomic systems are favorable for the detection of the $0\nu\varepsilon\varepsilon$, what would mean that neutrino is a Majorana particle. The reaction in (2) could in principle be detected by monitoring the X-rays or Auger electrons emitted from excited electron shell of the atom and the electromagnetic decay of the excited nucleus (in case of a nonground-state transition).

In this contribution the development in the field of the $0\nu\beta\beta$-decay is reviewed. The light and heavy neutrino exchange mechanisms as well as R-parity breaking mechanisms of the $0\nu\beta\beta$-decay are analyzed. The problem of a reliable determination of the $0\nu\beta\beta$-decay nuclear matrix elements is discussed. Further, the process of resonant neutrinoless double electron capture is revisited for those cases where the two participating atoms are nearly degenerate in mass. The theoretical framework is the formalism of an oscillation of two atoms with different total LN number (and parity), one of which can be in an excited state so that mass degeneracy is realized.

1. A BRIEF HISTORY OF DOUBLE-BETA DECAY

1.1. The Early Period. Double-beta decay, namely the two-neutrino double-beta decay ($2\nu\beta\beta$-decay)

$$(A, Z) \to (A, Z+2) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e \tag{3}$$

was first considered in publication [16] of Maria Goeppert-Mayer in 1935. It was Eugene Wigner, who suggested this problem to the author of [16] about one year after the Fermi weak-interaction theory appeared. In the work of Maria Goeppert-Mayer [16] an expression for the $2\nu\beta\beta$-decay rate was derived and a half-life of 10^{17} y was estimated by assuming a Q-value of about 10 MeV.

Two years later (1937), Ettore Majorana formulated theory of neutrinos (neutrino ν and antineutrino $\overline{\nu}$ are indistinguishable) and suggested antineutrino induced β^--decay for experimental verification of this hypothesis [17]. Giulio Racah was the first, who proposed testing Majorana's theory with the $0\nu\beta\beta$-decay for processes with real neutrinos [18]. In 1939, Wolfgang Furry discussed a double-beta decay without emission of neutrino ($0\nu\beta\beta$-decay with virtual neutrino) [19]. In 1952, Henry Primakoff [20] calculated the electron–electron angular correlations and electron energy spectra for both the $2\nu\beta\beta$-decay and the $0\nu\beta\beta$-decay, producing a useful tool for distinguishing between the two processes.

At that time nothing was known about the chirality suppression of the $0\nu\beta\beta$-decay. It was believed that due to a considerable phase-space advantage the $0\nu\beta\beta$-decay mode dominates the double-beta decay rate. Starting in 1950, this phenomenon was exploited in early geochemical, radiochemical and counter experiments. It was found that the measured lower limit on the $\beta\beta$-decay half-life far exceeds the values expected for this process, $T_{1/2} \sim 10^{12}$–10^{15} y. In 1955, the Raymond Davis experiment [21], which searched for the antineutrinos from reactor via nuclear reaction $\overline{\nu}_e + {}^{37}\text{Cl} \to {}^{37}\text{Ar} + e^-$, produced a zero result.

The above experiments were interpreted as a proof that the neutrino was not a Majorana particle, but a Dirac particle. This prompted the introduction of the lepton number to distinguish the neutrino from its antiparticle. The assumption of lepton number conservation allows the $2\nu\beta\beta$-decay but forbids the $0\nu\beta\beta$-decay, in which lepton number is changed by two units.

In 1949, Fireman reported observation of the $\beta\beta$-decay of ^{124}Sn in a laboratory experiment [22], but disclaimed it later [23]. The first geochemical observation of the $\beta\beta$-decay, with an estimated half-life $T_{1/2}(^{130}\mathrm{Te}) = 1.4 \cdot 10^{21}$ y, was announced by Ingram and Reynolds in 1950 [24].

1.2. The Period of Scepticism. Shortly after Lee and Yang formulated the parity violation in the weak interaction, it has been established by two epochal experiments. In 1957, Wu et al. discovered the asymmetry in the angular distribution of the β-particles emitted relative to the spin orientation of the parent nucleus ^{60}Co. A year later Goldhaber et al. found the complete polarization of neutrinos by measuring the photon spin direction determined by the de-excitation of a $^{152}\mathrm{Eu}^*$ nucleus after K-capture. In 1958, seemingly confused situation was simplified in the form of the vector–axial vector (V–A) theory of weak interactions describing maximal parity violation in agreement with available data. In order to account for the chiral symmetry breaking of the weak interaction, only left-handed fermions participate and the mediating particles must be vectors of spin 1 and left-handed, as well.

The maximal parity violation is easily realized in the lepton sector by using the two-component theory of a massless neutrino, proposed in 1957 by L. Landau, T. D. Lee, C. N. Yang, and A. Salam. (This idea was first developed by H. Weyl in 1929, but it was rejected by Pauli in 1933 on the grounds that it violates parity.) In this theory, neutrinos are left-handed and antineutrinos are right-handed, leading automatically to the V–A couplings.

With the discovery of parity violation, it became apparent that the Majorana/Dirac character of the electron neutrino was still in question. The particles that participate in the $0\nu\beta\beta$-decay reaction at nucleon level are right-handed antineutrino $\overline{\nu}_e$ and left-handed neutrino ν_e:

$$n \to p + e^- + \overline{\nu}_e^{\mathrm{RH}}, \quad \nu_e^{\mathrm{LH}} + n \to p + e^-. \tag{4}$$

Thus even if the neutrino is a (massless) Majorana particle, the absence of the $0\nu\beta\beta$-decay, as the first neutrino has the wrong helicity for absorption on a neutron, implies neither a Dirac electron neutrino nor a conserved lepton number.

The requirement that both the lepton number conservation and the γ_5 invariance of the weak current had to be violated, in order the $0\nu\beta\beta$-decay to occur, discouraged experimental searches.

1.3. The Period of GUTs. The maximal violation of parity (and of charge-conjugation) symmetry is accommodated in the SM, which describes jointly weak and electromagnetic interactions. This model was developed largely upon the empirical observations of nuclear beta decay during the latter half of the past century. Despite the phenomenological success of the SM, the fundamental origin of parity violation has been found unknown. In spite of the fact that the SM represents the simplest and the most economical theory, it has not been considered as the ultimate theory of nature. It was assumed likely to describe the effective interaction at low energy of an underlying more fundamental theory.

With the development of modern gauge theories at the beginning of 1970s, perceptions began to change. In the SM, it became apparent that the assumption of lepton number conservation led to the neutrino being strictly massless, thus preserving the γ_5-invariance of the weak current. With the development of Grand Unified Theories (GUTs) of the electroweak and strong interactions, the prejudice has grown that lepton number conservation is not the result of an exact global symmetry. The modern GUTs and supersymmetric (SUSY) extensions of the SM suppose that the conservation laws of the SM may be violated to some small degree. The lepton number may only appear to be conserved at low energies because of the large grand unified mass scale $\Lambda_{\rm GUT}$ governing its breaking. Within the proposed see-saw mechanism one expects the neutrino to acquire a small Majorana mass of a size $\sim$ (light mass)$^2/\Lambda_{\rm GUT}$, where «light mass» is typically that of a quark or charged lepton. The considerations of a sensitivity of the $0\nu\beta\beta$-decay experiments to a neutrino mass $m_\nu \sim 1$ eV became the genesis of a new interest to double-beta decay.

Neutrino masses require either the existence of right-handed neutrinos or require violation of the lepton number (LN) so that Majorana masses are possible. So, one is forced to go beyond the minimal models again, whereby LF and/or LN violation can be allowed in the theory. A good candidate for such a theory is the left-right symmetric model of Grand Unification (GUT) inaugurated by Salam, Pati, Mohapatra, and Senjanović [25] (especially models based on $SO(10)$ which have first been proposed by Fritzsch and Minkowski [26]) and its supersymmetric version [27]. The left-right symmetric models, representing generalization of the $SU(2)_L \otimes U(1)$ SM, predict not only that the neutrino is a Majorana particle, that means it is up to a phase identical with its antiparticle, but automatically predict the neutrino has a mass and a weak right-handed interaction.

In the left-right symmetric models, the LN conservation is broken by the presence of the Majorana neutrino mass. The LN violation is also inbuilt in those SUSY theories where R-parity, defined as $R_p = (-1)^{3B+L+2S}$ (S, B, and L are the spin, baryon and lepton number, respectively) is not a conserved quantity anymore.

The $0\nu\beta\beta$-decay which involves the emission of two electrons and no neutrinos, has been found as a powerful tool to study the LN conservation. Schechter and Valle proved that the $0\nu\beta\beta$-decay takes place only if the neutrino is a Majorana particle with nonzero mass [28]. It was recognized that the GUTs and

R-parity violating SUSY models offer a plethora of the $0\nu\beta\beta$-decay mechanisms triggered by exchange of neutrinos, neutralinos, gluinos, leptoquarks, etc. [30].

Another approach in handling the $0\nu\beta\beta$-decay problem based on consideration of particles other than the nucleons present in the nuclear soup was proposed as a remark by the genius of Pontecorvo [29]. He introduced the double-beta decay of pions in flight between nucleons. This idea was revived in the context of R-parity violating interactions [30], i.e., scalar, pseudoscalar, and tensor currents arising out of neutralino and gluino exchange. It was found that for this type of interactions the pion-exchange mechanism clearly dominates over the conventional two-nucleon mechanism.

The experimental effort concentrated on high $Q_{\beta\beta}$ isotopes, in particular on ^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{130}Te, ^{136}Xe, and ^{150}Nd [7]. In 1987, the first actual laboratory observation of the $2\nu\beta\beta$-decay was done for ^{82}Se by M. Moe and collaborators, who used a time projection chamber. Within the next few years, experiments employing counters were able to detect $2\nu\beta\beta$-decay of many nuclei. In addition, the experiments searching for the signal of the $0\nu\beta\beta$-decay pushed by many orders of magnitude the experimental lower limits for the $0\nu\beta\beta$-decay half-life of different nuclei.

1.4. The Period of Massive Neutrinos — the Current Period. Since 1998 we have a convincing evidence about neutrino masses due to SuperKamiokande, SNO, KamLAND and other experiments. Contrary to the implications of some popular press reports, most physicists have been expecting such results for several years. Nonzero neutrino mass can be accommodated by fairly straightforward extensions of the SM of particle physics. Earlier measurements of neutrinos produced in the Sun, in the atmosphere, and by accelerators suggested that neutrinos might oscillate from one «flavor» (electron-, muon-, and tau-) to another expected consequence of nonzero mass. Neutrino mass gives additional data in constructing the Grand Unified Theory (GUT) of physics. It also provides additional data for cosmologists and establishes perspectives for observation of the $0\nu\beta\beta$-decay.

So far, the $2\nu\beta\beta$-decay has been recorded for ten nuclei (^{48}Ca, ^{76}Ge, ^{82}Se, ^{96}Zr, ^{100}Mo, ^{116}Cd, ^{128}Te, ^{130}Te, ^{150}Nd, ^{238}U) [7, 9]. In addition, the $2\nu\beta\beta$-decay of ^{100}Mo and ^{150}Nd to 0^+ excited state of the daughter nucleus has been observed and the two-neutrino double–electron capture process in ^{130}Ba has been recorded. Experiments studying $2\nu\beta\beta$-decay are presently approaching a qualitatively new level, when high-precision measurements are performed not only for half-lives but also for all other observables of the process. As a result, a trend is emerging towards thorough investigation of all aspects of $2\nu\beta\beta$-decay, and this will furnish very important information about the values of nuclear matrix elements, the parameters of various theoretical models, and so on. In this connection, one may expect advances in the calculation of nuclear matrix elements and in the understanding of the nuclear-physics aspects of double-beta decay.

Neutrinoless double-beta decay has not yet been confirmed. The strongest limits on the half-life of the $0\nu\beta\beta$-decay were set in Heidelberg–Moscow [31], NEMO3 [32], and CUORICINO [33] experiments:

$$\begin{aligned} T_{1/2}^{0\nu\beta\beta}(^{76}\mathrm{Ge}) &\geqslant 1.9\cdot 10^{25}\ \mathrm{y}, \\ T_{1/2}^{0\nu\beta\beta}(^{100}\mathrm{Mo}) &\geqslant 5.8\cdot 10^{23}\ \mathrm{y}, \\ T_{1/2}^{0\nu\beta\beta}(^{130}\mathrm{Te}) &\geqslant 3.0\cdot 10^{24}\ \mathrm{y}. \end{aligned} \tag{5}$$

The recent claim for an observation of the $0\nu\beta\beta$-decay of ^{76}Ge with $T_{1/2}^{0\nu} = 2.23^{+0.44}_{-0.31}\cdot 10^{25}$ y [34] implies $m_{\beta\beta} \simeq 0.18{-}0.30$ eV by assuming the renormalized QRPA (RQRPA) nuclear matrix element and its uncertainty of [35]. The goal of the upcoming GERDA experiment [36] is to put this claim to a test by improving the sensitivity limit of the detection by more than an order of magnitude. The next generation experiments, which will be performed using several other candidate nuclei, will eventually be able to achieve this goal as well [9].

1.5. The Period of Majorana Neutrinos? There is a hope that *the period of Majorana neutrinos* is not far. This period should start by a direct and undoubtable observation of the $0\nu\beta\beta$-decay. It would establish that neutrinos are Majorana particles, and a measurement of the decay rate, when combined with neutrino oscillation data and a reliable calculation of nuclear matrix elements, would yield insight into all three neutrino mass eigenstates.

2. NEUTRINOLESS DOUBLE-BETA DECAY

In the SM there are no gauge invariant interactions that can lead to nonzero neutrino mass. As a result, all lepton flavors as well as total lepton number are exactly conserved. It is not unreasonable to expect that in some extensions of the SM, the lepton number conservation may not hold. More specifically, once lepton number is broken, neutrinos are not protected from getting nonzero Majorana masses after electroweak symmetry breaking. Indeed, a Majorana mass term for the neutrinos violates total lepton number. A viable scenario is that the neutrino masses are generated at some high-energy scale. This is well motivated by the observed properties of the light neutrinos including tiny masses, large mixings, and the fact that neutrinos are the only electrically neutral fundamental fermions.

2.1. Effective Mass of Majorana Neutrinos. Neutrino oscillations have been observed in solar, atmospheric, and long-baseline reactor and accelerator experiments. The data of these experiments are well fitted in the framework of three-neutrino mixing scheme,

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li}\ \nu_{iL}(x); \quad l = e, \mu, \tau. \tag{6}$$

Here, $\nu_i(x)$ is the field of the neutrino with mass m_i ($i = 1, 2, 3$), and $\nu_{lL}(x)$ is a flavor neutrino field which enters into the standard charged and neutral currents

$$\begin{aligned} j_\alpha^{CC}(x) &= 2 \sum_l \bar{\nu}_{lL}(x)\, \gamma_\alpha\, l_L(x), \\ j_\alpha^{NC}(x) &= \sum_l \bar{\nu}_{lL}(x)\, \gamma_\alpha\, \nu_{lL}(x). \end{aligned} \tag{7}$$

Here, U is the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) [2,37] mixing matrix. For massive Dirac neutrinos the PMNS matrix U^D in the standard parameterization has the form

$$U^D = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}\,\mathrm{e}^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}\,\mathrm{e}^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}\,\mathrm{e}^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}\,\mathrm{e}^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}\,\mathrm{e}^{i\delta} & c_{23}c_{13} \end{pmatrix}. \tag{8}$$

Here $s_{ij} = \sin\theta_{ij}$, $c_{ij} = \cos\theta_{ij}$, θ_{ij} ($i < j$) is the neutrino mixing angle and δ is the unknown CP-violating phase. The best-fit values for oscillation parameters from global data including solar, atmospheric, reactor (KamLAND and CHOOZ) and accelerator (K2K and MINOS) experiments are as follows [38]:

$$\begin{aligned} \Delta m_{21}^2 &= 7.65 \cdot 10^{-5}\ \mathrm{eV}^2, \quad \sin^2\theta_{12} = 0.30, \\ |\Delta m_{31}^2| &= 2.40 \cdot 10^{-3}\ \mathrm{eV}^2, \quad \sin^2\theta_{23} = 0.50, \quad \sin^2\theta_{13} = 0.01, \end{aligned} \tag{9}$$

where $\Delta m_{jk}^2 = m_j^2 - m_k^2$.

At present, the structure of the neutrino mass spectrum is not known as well. Two types of spectra are possible:

1. Normal spectrum:

$$m_1 < m_2 < m_3; \quad \Delta m_{21}^2 \ll \Delta m_{32}^2. \tag{10}$$

2. Inverted spectrum:

$$m_3 < m_1 < m_2; \quad \Delta m_{21}^2 \ll |\Delta m_{31}^2|. \tag{11}$$

We note that it is common to label neutrino masses differently in the case of the normal and the inverted spectra. For both spectra we have $m_2 > m_1$. But in the case of the normal spectrum, m_3 is the mass of the heaviest neutrino and in

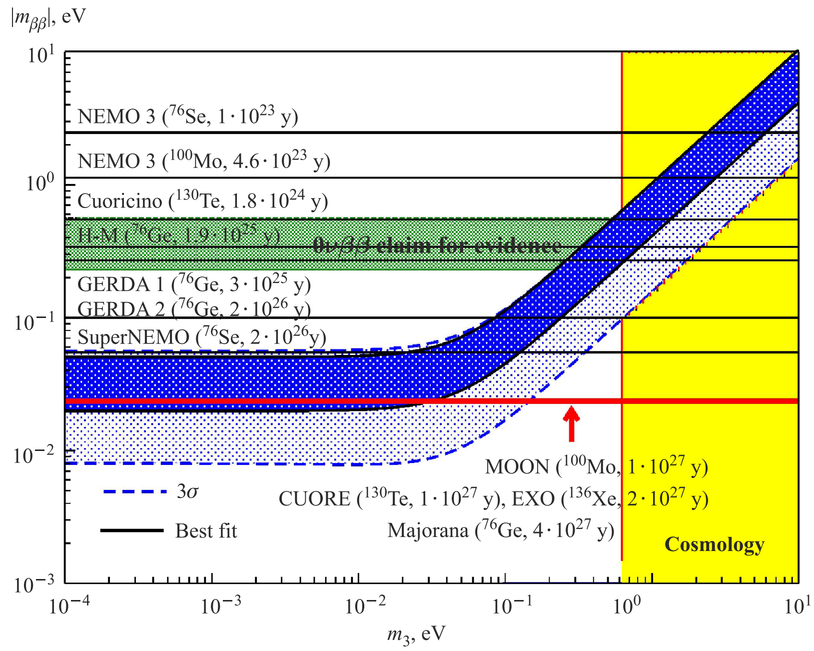

Fig. 1. The effective Majorana neutrino mass $m_{\beta\beta}$ as a function of the lightest neutrino mass m_3 for the inverted hierarchy of neutrino masses

the case of the inverted hierarchy, m_3 is the mass of the lightest neutrino. This convention allows one to keep the same notation of the mixing angles for both spectra. Existing oscillation data are compatible both with normal and inverted spectra (see Fig. 1).

The lightest neutrino mass $m_0 = m_1(m_3)$, which determines the absolute values of neutrino masses, is currently also unknown. From an analysis of the data of the Mainz [39] and Troitsk [40] tritium experiments, it was found $m_0 \leqslant 2.3$ eV. A more stringent bound on the sum of neutrino masses can be found from the measurement of the matter power spectrum $P(k)$. Depending on the data which were taken into account, the cosmological upper bound on the sum of neutrino masses was obtained as (see [41, 42] and references therein) $\sum\limits_i m_i \leqslant 0.5{-}1.7$ eV.

An important evidence that masses and mixing of neutrinos are of a nature beyond the SM would be that massive neutrinos are Majorana particles. If ν_i are Majorana particles:

1. Neutrino fields $\nu_i(x)$ satisfy the Majorana conditions

$$\nu_i^c(x) = \nu_i(x), \tag{12}$$

where $\nu_i^c(x) = C\,\bar{\nu}_i^T(x)$ is the conjugated field (C is the charge conjugation matrix).

2. The neutrino mixing matrix has the form [44]

$$U = U^D\, S(\alpha), \tag{13}$$

where $S(\alpha)$ is a diagonal phase matrix. In the case of three-neutrino mixing, the matrix $S(\alpha)$ is characterized by two Majorana CP-violating phases. The matrix $S(\alpha)$ can be presented in the form

$$S_{ik} = \mathrm{e}^{i\alpha_i}\, \delta_{ik}; \quad \alpha_3 = 0. \tag{14}$$

The unitary matrix U^D, which is characterized by the three mixing angles θ_{12}, θ_{23}, θ_{13} and one phase δ, was already introduced in Eq. (8).

If in the lepton sector CP invariance holds, for the Majorana mixing matrix we have [43]

$$U_{li} = U_{li}^*\, \eta_i, \tag{15}$$

where $\eta_i = \pm i$ is the CP parity of the Majorana neutrino ν_i. The condition (15) can be presented in the form

$$U_{li}^2 = |U_{li}|^2\, \mathrm{e}^{i\,(\pi/2)\,\rho_i}, \tag{16}$$

where $\rho_i = \pm 1$.

Investigations of neutrino oscillations in vacuum and in matter do not allow one to distinguish massive Dirac from massive Majorana neutrinos [44–46]. In order to reveal the Majorana nature of ν_i, it is necessary to study processes in which the total lepton number is violated. Because the standard electroweak interaction conserves helicity, the probabilities of such processes are proportional to the squares of the neutrino masses, and, consequently, they are strongly suppressed. The best sensitivity on small Majorana neutrino masses can be reached in the investigation of neutrinoless double-beta decay ($0\nu\beta\beta$) of some even–even nuclei.

By assuming the dominance of the light neutrino mass mechanism the inverse value of the $0\nu\beta\beta$-decay half-life for a given isotope (A, Z) is given by [9]

$$\left(T_{1/2}^{0\nu}\right)^{-1} = |m_{\beta\beta}|^2\, |M^{0\nu}|^2\, G_{0\nu}(Q_{\beta\beta}, Z). \tag{17}$$

Here, $G_{0\nu}(Q_{\beta\beta}, Z)$ is the known phase space factor, which includes the fourth power of axial-coupling constant g_A and the inverse square of the nuclear radius R^{-2}, compensated by the factor R in the nuclear matrix element (NME) $M^{0\nu}$. $M^{0\nu}$ consists of Fermi, Gamow–Teller and tensor parts as

$$M^{0\nu} = -\frac{M_F}{g_A^2} + M_{\mathrm{GT}} + M_T \tag{18}$$

and depends on the nuclear structure of the particular isotope under study. Under the assumption of the mixing of three massive Majorana neutrinos, the effective Majorana neutrino mass $m_{\beta\beta}$ takes the form

$$m_{\beta\beta} = U_{e1}^2\, m_1 + U_{e2}^2\, m_2 + U_{e3}^2\, m_3. \tag{19}$$

It contains the usual 3-neutrino mixing angles plus a CP-violating phase, which appears in oscillations, and two additional Majorana phases, ϕ_1, ϕ_2.

The data from neutrino oscillation experiments allow ranges of possible values of the effective Majorana mass for different neutrino mass spectra to be predicted. The value of the effective Majorana mass, as it appears in Eq. (19), contains several dependences on phases and masses. Because of experimental uncertainties, different mass scenarios, like the normal ($m_3 \gg m_2 \gg m_1$) or inverted ($m_2 > m_1 \gg m_3$) hierarchy scenario, or degenerate ($m_3 \approx m_2 \approx m_1$) or nondegenerate cases, can presently still be entertained, which allow a wide range of possible mass values for $m_{\beta\beta}$, even zero in the most extreme and unfortunate situation of the normal hierarchy scenario [47]. Though, even in that case the $0\nu\beta\beta$ will still be allowed due to a contribution from the mass term in the neutrino propagator [48], which one usually neglects. But, its decay rate would be utterly unobservable.

The main aim of the experiments on the search for $0\nu\beta\beta$-decay is the measurement of the effective neutrino Majorana mass $m_{\beta\beta}$. Many new projects for measurements of $0\nu\beta\beta$-decay have been proposed with a sensitivity corresponding to $m_{\beta\beta}$ predicted under the assumption of inverted hierarchy of neutrino masses. The GERDA/MAJORANA (^{76}Ge), SuperNEMO (^{82}Se), CUORE (^{130}Te), COBRA(^{116}Cd), LUCIFER(^{82}Se), EXO(^{136}Xe), Kamland-ZEN(^{136}Xe) and other experiments hope to probe $m_{\beta\beta}$ down to 10–50 meV, what is the region of the inverted hierarchy of neutrino masses (see Fig. 2). These experiments would require about 1 t of radioactive isotope and 5–10 y of measurements. There is already first proposal for the $0\nu\beta\beta$-decay experiment with sensitivity to normal hierarchy of neutrino masses (see Fig. 2). The Super-KamLAND-ZEN would need about 40 t of ^{136}Xe.

2.2. Nuclear Matrix Elements. From the measurement of half-life of the $0\nu\beta\beta$-decay only the product,

$$|m_{\beta\beta}|\ |M^{0\nu}(A,Z)|, \tag{20}$$

of effective neutrino mass and nuclear matrix element can be determined. Clearly, the accuracy of the determination of $|m_{\beta\beta}|$ from the measured $0\nu\beta\beta$-decay half-life is mainly given by our knowledge of nuclear matrix elements. Without accurate calculation of the $0\nu\beta\beta$-decay NMEs, it is not possible to reach qualitative conclusions about neutrino masses, the type of neutrino mass spectrum and CP violation.

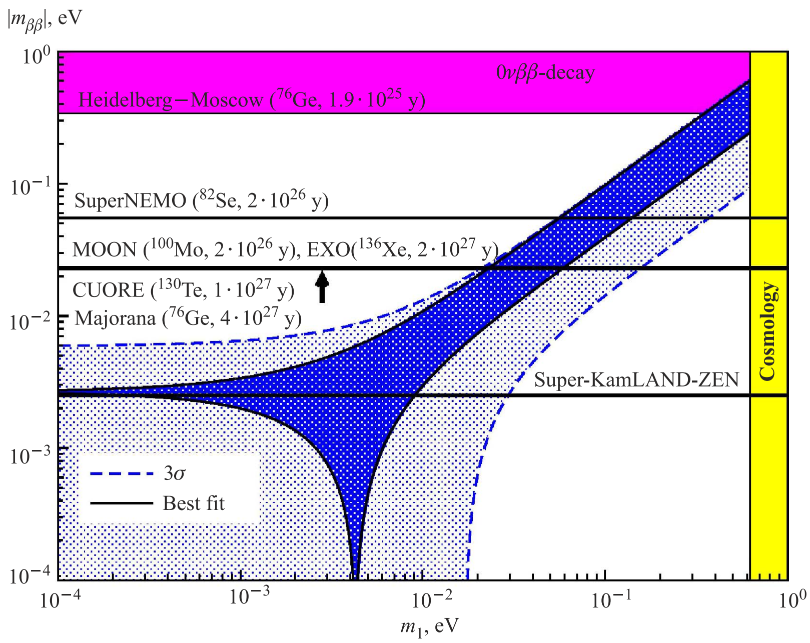

Fig. 2. The effective Majorana neutrino mass $m_{\beta\beta}$ as a function of the lightest neutrino mass m_1 for the normal hierarchy of neutrino masses

Interpreting existing results as a measurement of $m_{\beta\beta}$ and planning new experiments depends crucially on the knowledge of the corresponding nuclear matrix elements (NMEs) that govern the decay rate. The NMEs for $0\nu\beta\beta$-decay must be evaluated using tools of nuclear structure theory. Unfortunately, there are no observables that could be directly linked to the magnitude of $0\nu\beta\beta$-decay nuclear matrix elements and that could be used to determine them in an essentially model-independent way. The calculation of the $0\nu\beta\beta$-decay NMEs is a difficult problem because ground and many excited states of open-shell nuclei with complicated nuclear structure have to be considered. Accurate determination of the NMEs, and a realistic estimate of their uncertainty, is of great importance. Nuclear matrix elements need to be evaluated with uncertainty of less than 30% to establish the neutrino mass spectrum and CP-violating phases of the neutrino mixing matrix.

The two main approaches used for evaluation of double-beta decay NMEs are the Quasiparticle Random Phase Approximation (QRPA) [35, 49] and the Large Scale Shell Model (LSSM) [50]. Both methods have the same starting point, namely a Slater determinant of independent particles. However, there are substantial differences between both approaches, in fact, the kind of correlations they include are complementary. The QRPA treats a large single-particle model space, but truncates heavily the included configurations. The LSSM, by a contrast, treats a small fraction of this model space, but allows the nucleons to correlate in arbitrary ways. Matrix elements for the double-beta decay are calculated also by angular momentum projected (with real quasiparticle transformation) Hartree–Fock–Bogoliubov (P-HFB) wave functions [51], the Interacting Boson Model (IBM) [52] and by Energy Density Functional Method (EDF) [53]. In the P-HFB, the nucleon pairs different from 0^+ in the intrinsic coordinate system are strongly suppressed compared to the results of the LSSM and the QRPA. The approaches LSSM and QRPA show also, that other neutron pairs contribute strongly, which cannot be included into real P-HFB. The IBM is also restrictive: It allows only that 0^+ and 2^+ neutron pairs are changed into proton pairs.

Comparing $0\nu\beta\beta$-decay nuclear matrix elements calculated using different methods gives some insight in the advantages or disadvantages of different candidate nuclei. However, matrix elements are not quite the relevant quantities. Experimentally, half-lives are measured or constrained, and the effective Majorana neutrino mass $m_{\beta\beta}$ is the ultimate goal. For $m_{\beta\beta}$ equal to 50 meV the calculated half-lives for double β-decaying nuclei of interest are presented in Fig. 3. We see that the spread of half-lives for given isotope is up to the factor of 4–5.

The improvement of the calculation of the $0\nu\beta\beta$-decay NMEs is a very important and challenging problem. The uncertainty associated with the calculation of the $0\nu\beta\beta$-decay NMEs can be diminished by suitably chosen nuclear probes. A complementary experimental information from related processes like charge-exchange reactions, muon capture and charged current (anti)neutrino-nucleus re-

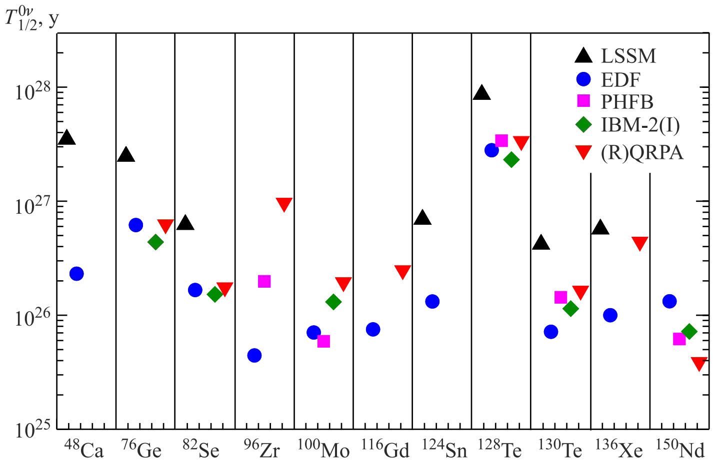

Fig. 3. The calculated $0\nu\beta\beta$-decay half-lives by assuming $m_{\beta\beta}$ = 50 meV and NMEs of different approaches

actions is highly required. A direct confrontation of nuclear structure models with data from these processes might improve quality of nuclear structure models [54]. The constrained parameter space of nuclear models is a promising way to reduce uncertainty in the calculated $0\nu\beta\beta$-decay NMEs [55].

Recently, there has been a significant progress in understanding the source of the spread of calculated NMEs. Nevertheless, there is no consensus as yet among nuclear theorists about their correct values, and corresponding uncertainty. But, a recent development in the field is encouraging. There is a reason to be hopeful that the uncertainty will be reduced.

3. EXOTIC MECHANISMS OF THE $0\nu\beta\beta$-DECAY

In connection with the neutrino oscillations much attention is attracted to the light neutrino mass mechanism of the $0\nu\beta\beta$-decay. However, the observation of the $0\nu\beta\beta$-decay will not mean that this is the dominant mechanism of this process. Many extensions of the SM generate Majorana neutrino masses and offer a plethora of $0\nu\beta\beta$-decay mechanisms like the exchange of SUSY superpartners with R-parity violating, leptoquarks, right-handed W-bosons or Kaluza–Klein excitations, among others, which have been discussed in the literature.

The heavy neutrino exchange and trilinear R-parity breaking mechanisms of the $0\nu\beta\beta$-decay cannot be distinguished kiematically from the light neutrino exchange mechanism. The half-life for a given isotope (A, Z) can be written as

$$\left(T^{0\nu}_{1/2}\right)^{-1} = |\eta^{\rm LNV}|^2 \, |M^{0\nu}|^2 \, G_{0\nu}(Q_{\beta\beta}, Z). \tag{21}$$

The effective lepton number violating parameters of interest $\eta^{\rm LNV}$ associated with these mechanisms together with corresponding nuclear matrix elements $M^{0\nu}$ are presented briefly below.

3.1. Heavy Neutrino Exchange. We assume that the neutrino mass spectrum includes heavy Majorana states N_k with masses M_k much larger than the typical energy scale of the $0\nu\beta\beta$-decay. These heavy states can mediate this process as the previous light neutrino exchange mechanism. The difference is that the neutrino propagators in this case can be contracted to points and, therefore, the corresponding effective transition operators are local unlike in the light neutrino exchange mechanism with long-range internucleon interactions.

The corresponding LNV parameter is given by

$$\eta_{_N} = \sum_k^{\rm heavy} |U_{ek} U_{ek}| \, \xi_k \, \frac{m_p}{M_k}. \tag{22}$$

Here, m_p is the mass of proton. We assume that the mass of heavy neutrinos M_k is large in comparison with their average momenta ($M_k \gg 1$ GeV). U_{ek} are elements of the neutrino mixing matrix associated with left-handed interaction. ξ_k are CP-violating phases.

Separating the Fermi (F), Gamow–Teller (GT) and the tensor (T) contributions we write down

$$\mathcal{M}^{0\nu}_{N} = -\frac{M_{F(N)}}{g_A^2} + M_{\mathrm{GT}(N)} + M_{T(N)} = \\ = \langle 0_i^+ | \sum_{kl} \tau_k^+ \tau_l^+ \left[\frac{H_F^{(N)}(r_{kl})}{g_A^2} + H_{\mathrm{GT}}^{(N)}(r_{kl})\sigma_{kl} - H_T^{(N)}(r_{kl}) S_{kl} \right] |0_f^+\rangle, \quad (23)$$

where

$$S_{kl} = 3(\boldsymbol{\sigma}_k \cdot \hat{\mathbf{r}}_{kl})(\boldsymbol{\sigma}_l \cdot \hat{\mathbf{r}}_{kl}) - \sigma_{kl}, \quad \sigma_{kl} = \boldsymbol{\sigma}_k \cdot \boldsymbol{\sigma}_l. \quad (24)$$

The radial parts of the exchange potentials are

$$H_{F,\mathrm{GT}}^{(N)}(r_{kl}) = \frac{2}{\pi} \frac{R}{m_p m_e} \int\limits_0^\infty j_0(q r_{kl}) h_{F,\mathrm{GT}}(q^2) q^2 \, dq, \\ H_T^{(N)}(r_{kl}) = \frac{2}{\pi} \frac{R}{m_p m_e} \int\limits_0^\infty j_2(q r_{kl}) h_T(q^2) q^2 \, dq. \quad (25)$$

3.2. R-Parity Breaking SUSY Mechanism. In the SUSY models with R-parity nonconservation there are present the LNV couplings which may trigger the $0\nu\beta\beta$-decay. Recall, that R-parity is a multiplicative quantum number defined by $R = (-1)^{2S+3B+L}$ (S, B, L are spin, baryon, and lepton numbers). Ordinary particles have $R = +1$; while their superpartners, $R = -1$. The LNV couplings emerge in this class of SUSY models from the R-parity breaking part of the superpotential

$$W_{R\!\!\!/_p} = \lambda_{ijk} L_i L_j E_k^c + \lambda'_{ijk} L_i Q_j D_k^c + \mu_i L_i H_2, \quad (26)$$

where L, Q stand for lepton and quark $SU(2)_L$ doublet left-handed superfields; while E^c, D^c, for lepton and down quark singlet superfields. Here, we concentrate only on the trilinear λ'-couplings.

At the quark level there are basically two types of $R\!\!\!/_p$ SUSY mechanisms: the short-range mechanism with the exchange of heavy superpartners (gluino, neutralinos, selectron, and squarks) [30, 56, 57] and the long-range mechanism involving both the exchange of heavy squarks and light neutrino [59], which we call squark–neutrino mechanism.

Assuming the dominance of gluino exchange, we obtain for the LNV parameter the following simplified expression:

$$\eta_{\lambda'} = \frac{\pi \alpha_s}{6} \frac{\lambda'^2_{211}}{G_F^2 m_{\tilde{d}_R}^4} \frac{m_p}{m_{\tilde{g}}} \left[1 + \left(\frac{m_{\tilde{d}_R}}{m_{\tilde{u}_L}} \right)^2 \right]^2 . \quad (27)$$

Here, G_F is the Fermi constant; $\alpha_s = g_3^2/(4\pi)$ is $SU(3)_c$ gauge coupling constant; $m_{\tilde{u}_L}$, $m_{\tilde{d}_R}$, and $m_{\tilde{g}}$ are masses of the u-squark, d-squark, and gluino, respectively.

At the hadron level there is a dominance of the pion-exchange mode. Enhancement of the pion exchange mode with respect to the conventional two-nucleon mechanism is due to the long-range character of nuclear interaction and the details of the bosonization of the $\pi^- \to \pi^+ + e^- + e^-$ vertex.

We denote the $0\nu\beta\beta$-decay nuclear matrix element to be substituted in Eq. (21) as $\mathcal{M}_\lambda^{0\nu}$. We have [30]

$$\mathcal{M}_\lambda^{0\nu} = c^{1\pi}\left(M_T^{1\pi} + M_{\rm GT}^{1\pi}\right) + c^{2\pi}\left(M_T^{2\pi} + M_{\rm GT}^{2\pi}\right) \tag{28}$$

with

$$\begin{aligned} c^{1\pi} &= -\frac{2}{9}\frac{\sqrt{2}f_\pi m_\pi^4}{m_p^3 m_e(m_u+m_d)}\frac{g_s F_P}{g_A^2}, \\ c^{2\pi} &= \frac{1}{18}\frac{f_\pi^2 m_\pi^4}{m_p^3 m_e(m_u+m_d)^2}\frac{g_s^2}{g_A^2}. \end{aligned} \tag{29}$$

Here, g_S and F_P stand for the standard pion–nucleon coupling constant ($g_s = 13.4$) and the nucleon pseudoscalar constant (we take the bag model value $F_P \approx 4.41$ from [60]), respectively; $f_\pi = 0.668 m_\pi$ and m_π is the mass of pion; m_u and m_d denote current quark masses. The partial nuclear matrix elements of the $R\!\!\!/_p$ SUSY mechanism for the $0\nu\beta\beta$ process are:

$$\begin{aligned} M_{\rm GT}^{k\pi} &= \langle 0_f^+ | \sum_{k\neq l} \tau_k^+\tau_l^+ \; H_{\rm GT}^{k\pi}(r_{kl})\; \boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j,\; |0_i^+\rangle, \\ M_T^{k\pi} &= \langle 0_f^+ | \sum_{k\neq l} \tau_k^+\tau_l^+ \; H_T^{k\pi}(r_{kl})\; S_{kl}\; |0_i^+\rangle \end{aligned} \tag{30}$$

with

$$\begin{aligned} H_{\rm GT}^{1\pi}(r_{kl}) &= -\frac{2}{\pi}R\int_0^\infty j_0(qr_{kl})\frac{q^4/m_\pi^4}{1+q^2/m_\pi^2}f_A^2(q^2)\,dq, \\ H_T^{1\pi}(r_{kl}) &= \frac{2}{\pi}R\int_0^\infty j_2(qr_{kl})\frac{q^4/m_\pi^4}{1+q^2/m_\pi^2}f_A^2(q^2)\,dq, \\ H_{\rm GT}^{2\pi}(r_{kl}) &= -\frac{4}{\pi}R\int_0^\infty j_0(qr_{kl})\frac{q^4/m_\pi^4}{(1+q^2/m_\pi^2)^2}f_A^2(q^2)\,dq, \\ H_T^{2\pi}(r_{kl}) &= \frac{4}{\pi}R\int_0^\infty j_2(qr_{kl})\frac{q^4/m_\pi^4}{(1+q^2/m_\pi^2)^2}f_A^2(q^2)\,dq. \end{aligned} \tag{31}$$

The two-nucleon exchange potentials are expressed in momentum space as the momentum dependence of nucleon form factors ($f_A(q^2)$) is taken into account.

3.3. Squark Mixing Mechanism. In the case of squark–neutrino mechanism [59] due to the chiral structure of the $R\!\!\!/_p$ SUSY interactions, the amplitude of $0\nu\beta\beta$-decay does not vanish in the limit of zero neutrino mass unlike the ordinary Majorana neutrino exchange mechanism proportional to the light neutrino mass. Instead, the squark–neutrino mechanism is roughly proportional to the momentum of the virtual neutrino which is of the order of the Fermi momentum of the nucleons inside of nucleus $p_F \approx 100$ MeV. This is a manifestation of the fact that the LNV necessary for $0\nu\beta\beta$-decay is supplied by the $R\!\!\!/_p$ SUSY interactions instead of the Majorana neutrino mass term and therefore this mechanism is not suppressed by the small neutrino mass. The corresponding SUSY LNV parameter is defined as

$$\eta_{\tilde{q}} = \sum_k \frac{\lambda'_{11k}\lambda'_{1k1}}{2\sqrt{2}G_F} \sin 2\theta^d_{(k)} \left(\frac{1}{m^2_{\tilde{d}_1(k)}} - \frac{1}{m^2_{\tilde{d}_2(k)}} \right). \tag{32}$$

Here we use the notation $d_{(k)} = d, s, b$. This LNV parameter vanishes in the absence of $\tilde{q}_L - \tilde{q}_R$ — mixing, when $\theta^d = 0$.

At the hadron level we assume dominance of the pion-exchange mode. Then, the nuclear matrix element associated with squark–neutrino mechanism can be written as a sum of GT and tensor contributions

$$\mathcal{M}^{0\nu}_{\tilde{q}} = M_{\mathrm{GT}(\tilde{q})} - M_{T(\tilde{q})}. \tag{33}$$

The exchange potentials are given by

$$\begin{aligned} H^{(\tilde{q})}_{\mathrm{GT}}(r_{kl}) &= \frac{2}{\pi} R \int_0^\infty \frac{j_0(qr_{kl}) h^{\tilde{q}}_{\mathrm{GT}}(q^2) q^2}{|q|(|q| + E_n - (E_i + E_f)/2)}\, dq, \\ H^{(\tilde{q})}_{T}(r_{kl}) &= \frac{2}{\pi} R \int_0^\infty \frac{j_2(qr_{kl}) h^{\tilde{q}}_{T}(q^2) q^2}{|q|(|q| + E_n - (E_i + E_f)/2)}\, dq \end{aligned} \tag{34}$$

with

$$h^{\tilde{q}}_{T,\mathrm{GT}}(\mathbf{q}^2) = \frac{1}{3}, \frac{1}{4g_A^2} g_A^2(\mathbf{q}^2) \frac{m_\pi^4}{m_e(m_u + m_d)} \frac{\mathbf{q}^2}{(\mathbf{q}^2 + m_\pi^2)^2}. \tag{35}$$

4. DISTINGUISHING THE $0\nu\beta\beta$-DECAY MECHANISMS

An uncontroversial detection of the $0\nu\beta\beta$-decay will prove the total lepton number to be broken in nature, and neutrinos to be Majorana particles. However, it will immediately generate questions: What is the mechanism that triggers the decay? What happens if several mechanisms are active for the decay? There is a general consensus that a measurement of the $0\nu\beta\beta$-decay in one isotope does not allow us to determine the underlying physics mechanism. Complementary measurements in different isotopes are very important.

Possibilities to disentangle at least some of the possible mechanisms include the analysis of angular correlations between the emitted electrons [61], study of the branching ratios of $0\nu\beta\beta$-decays to ground and excited states [62], a comparative study of the $0\nu\beta\beta$-decay and neutrinoless electron capture with emission of positron ($0\nu EC\beta^+$) [63], and analysis of possible links with other lepton-flavor violating processes (e.g., $\mu \to e\gamma$) [64]. Unfortunately, the search for the $0\nu EC\beta^+$-decay is complicated due to small rates and the experimental challenge to observe the produced X-rays or Auger electrons, and most double-beta experiments of the next generation are not sensitive to electron tracks or transitions to excited states.

Recently, it has been shown that by exploiting the fact that the associated nuclear matrix elements are target-dependent, given definite experimental results on a sufficient number of targets, one can determine or sufficiently constrain all lepton violating parameters including the mass term [65]. Specifically, a possibility to extract value on the effective Majorana neutrino mass, $m_{\beta\beta}$, was discussed by assuming the claim of evidence of the $0\nu\beta\beta$-decay of ^{76}Ge [34] as a function of half-life data for the two promising nuclei (^{100}Mo and ^{130}Te). It was shown that in an analysis including two and three nuclear systems there are 2 and 4 different possible solutions for $|m_{\beta\beta}|$, respectively. One of the solutions leads to small values of $|m_{\beta\beta}|$, when all mechanisms add up coherently. This is compatible also with inverted ($m_i < 50$ meV) or normal ($m_i \approx$ few meV) hierarchy of neutrino masses. Other solutions, however, allow quite large values of $|m_{\beta\beta}|$, even larger than 1 eV. These can, of course, be excluded by cosmology and tritium β-decay experiments. It may not, however, be possible to exclude these solutions, if the claim of evidence for ^{76}Ge would be ruled out by future experiments, since then, the values we obtain become smaller than those of the other experiments.

It is thus important that experiments involving as many different targets as possible to be pursued. Furthermore, in the presence of interference between the various mechanisms, the availability of reliable nuclear matrix elements becomes more imperative. In Table 1 nuclear matrix elements $M^{0\nu}_{\nu}$ (light neutrino mass mechanism), $M^{0\nu}_{N}$ (heavy neutrino mass mechanism), $M^{0\nu}_{\lambda'}$ (trilinear R-parity breaking SUSY mechanism) and $M^{0\nu}_{\tilde{q}}$ (squark mixing mechanism) are presented. They have been obtained within the Self-Consistent Renormalized Quasiparticle Random Phase Approximation SRQRPA [66, 67]. The SRQRPA takes into account the Pauli exclusion principle and conserves the mean particle number in correlated ground state. In the calculation of the $0\nu\beta\beta$-decay NMEs the two-nucleon short-range correlations derived from the same potential as residual interactions, namely from the CD-Bonn potential [35], were considered.

Table 1. **Nuclear matrix elements $M_{\nu}^{0\nu}$, $M_{N}^{0\nu}$, $M_{\lambda'}^{0\nu}$, and $M_{\tilde{q}}^{0\nu}$ for the $0\nu\beta\beta$-decays of ^{76}Ge, ^{100}Se, ^{100}Mo, and ^{130}Te within the Self-Consistent Renormalized Quasiparticle Random Phase Approximation (SRQRPA). $G^{0\nu}(E_0, Z)$ is the phase-space factor. $g_A = 1.25$ is assumed**

Nucleus	$G^{0\nu}(E_0, Z)$, y^{-1}	NN pot.	$\lvert M_{\nu}^{0\nu}\rvert$	$\lvert M_{N}^{0\nu}\rvert$	$\lvert M_{\lambda'}^{0\nu}\rvert$	$\lvert M_{\tilde{q}}^{0\nu}\rvert$
^{76}Ge	$7.98 \cdot 10^{-15}$	Argonne	5.44	264.9	699.6	717.8
		CD-Bonn	5.82	411.5	595.6	727.6
^{82}Se	$3.53 \cdot 10^{-14}$	Argonne	5.29	262.9	697.7	710.2
		CD-Bonn	5.66	408.4	594.4	719.9
^{100}Mo	$5.73 \cdot 10^{-14}$	Argonne	4.79	259.8	690.3	82.6
		CD-Bonn	5.15	404.3	588.6	690.5
^{130}Te	$5.54 \cdot 10^{-14}$	Argonne	4.18	239.7	626.0	620.4
		CD-Bonn	4.70	384.5	540.3	640.7

5. RESONANT NEUTRINOLESS DOUBLE-ELECTRON CAPTURE

Recently, a new theoretical scheme for the description of neutrinoless double-electron capture has been proposed [11]. The final atomic state with the minimum mass difference with the original atom leads to a new phenomenon of *oscillations plus de-excitation* of atoms:

$$\begin{aligned} (\mathcal{A}, \mathcal{Z}) &\leftrightarrow (\mathcal{A}, \mathcal{Z}+2)^{**}, \\ (\mathcal{A}, \mathcal{Z}) &\leftrightarrow (\mathcal{A}, \mathcal{Z}-2)^{**}, \end{aligned} \tag{36}$$

which originates in the mixing of a pair of neutral atoms $(\mathcal{A}, \mathcal{Z})$ and $(\mathcal{A}, \mathcal{Z} \pm 2)^{**}$ differing by two units of the lepton charge. The first atom $(\mathcal{A}, \mathcal{Z})$ is in the ground state and the second atom $(\mathcal{A}, \mathcal{Z} \pm 2)^{*}$ can be in the excited state with respect to both atomic and nuclear structures. The underlying mechanism is transition of two protons and two bound electrons to two neutrons $p + p + e_b^- + e_b^- \leftrightarrow n + n$. The signature of oscillations can be electromagnetic de-excitation of the unstable nuclei and the atomic shell with electron holes.

Analysis of the mixing and oscillations-plus-de-excitation phenomenon in the atoms, based on the formalism of [11], indicates the possibility of a resonant enhancement of the $0\nu\varepsilon\varepsilon$-decay. The transition rate near the resonance is described by the Breit–Wigner formula. This gives evidence that the search for oscillations plus de-excitation of quasi-stable atoms can uncover the processes with violation of the total lepton number. For this purpose, selection was performed among the nuclei and their excitations, registered in the database of the Brookhaven National Laboratory, as well as all possible ways to capture the electrons were considered to find the atomic pairs with the smallest mass difference. The favorable transitions are as follows [12]: ^{106}Cd $\rightarrow$ ^{106}Pd**, ^{124}Xe $\rightarrow$ ^{124}Te**,

^{136}Ce $\to$ ^{136}Ba**, ^{152}Gd $\to$ ^{152}Sm**, ^{156}Dy $\to$ ^{156}Gd**, ^{164}Er $\to$ ^{164}Dy**, ^{168}Yb $\to$ ^{168}Er**, ^{180}W $\to$ ^{180}Hf**, ^{184}Os $\to$ ^{184}W**, ^{190}Pt $\to$ ^{190}Os** and some others. In this contribution we discuss the $0\nu\varepsilon\varepsilon$-decays of ^{152}Gd and ^{164}Er in detail.

Resonant enhancement of neutrinoless double-electron capture is possible if the initial atom and the final excited atom have close masses. The degeneracy is controlled by a parameter $\Delta = Q_{\varepsilon\varepsilon} - B_{2h} - E_\gamma$. The first term, Q-value, defines the difference between the masses of initial and final ground-state atoms, B_{2h} is the excitation energy of electron shell of the daughter atom with two electron holes, and E_γ is the nuclear excitation energy. The degeneracy parameter Δ enters the decay rate [4–6]

$$\Gamma^{\varepsilon\varepsilon} = |V_{\varepsilon\varepsilon}|^2 \frac{\Gamma}{\Delta^2 + \Gamma^2/4} = m_e \left|\frac{m_{\beta\beta}}{1\ eV}\right|^2 \left|\frac{V_{\varepsilon\varepsilon}}{m_{\beta\beta}}\right|^2 R, \tag{37}$$

where Γ is the sum of the widths of the excited electron shell and excited state of the daughter nuclide, R is the resonance enhancement factor. Equation (37) can be obtained in the second order of perturbation theory for the standard β-decay Hamiltonian with massive Majorana neutrinos. The value $V_{\varepsilon\varepsilon}$ has the meaning of the transition amplitude between two atoms with violation of lepton number. In the total decay amplitude, $V_{\varepsilon\varepsilon}$ factorizes when the resonance conditions are satisfied. $V_{\varepsilon\varepsilon}$ is proportional to the effective Majorana neutrino mass $m_{\varepsilon\varepsilon}$, to the wave functions of the captured electrons averaged over the volume of nucleus, and to the nuclear matrix element $M^{0\nu}$. In the case of capture of two s-orbital electrons,

$$V_{\varepsilon\varepsilon} = m_{\beta\beta}\, \frac{G_\beta^2}{4\pi} \frac{\sqrt{2}\overline{f}_a \overline{f}_b}{\sqrt{(1+\delta_{ab})4\pi}} \frac{g_A^2}{R_{\rm nucl}} M^{0\nu}. \tag{38}$$

Here, $G_\beta = G_F \cos\theta_C$, θ_C is the Cabibbo angle; g_A is the axial-vector nucleon coupling constant; $R_{\rm nucl}$ is nuclear radius; $\overline{f}_a$ is the averaged upper bispinor component of the $n_a s_{1/2}$ electron; n_a is the principal quantum number. Electron wave functions entering Eq. (38) are approximated by the solutions of the Dirac equation in a screened Coulomb potential. The screened nuclear charge is chosen in such a way as to reproduce the known values of hole energies [69]. The normalization is that of [70]. The explicit form of $M^{0\nu}$ for $0^+ \to 0^+$ transition can be found in [35].

5.1. The $0\nu\varepsilon\varepsilon$ of ^{152}Gd and ^{164}Er. The $0\nu\varepsilon\varepsilon$ of ^{152}Gd and ^{164}Er assumes 0^+ ground state to 0^+ ground state nuclear transition. The Quasiparticle Random Phase Approximation (QRPA) [35] was used to calculate corresponding nuclear matrix elements [68].

For $A = 152$ system, the single-particle model space consists of $3 - 5\hbar\omega$ oscillator shells plus $0i_{11/2}$ and $0i_{13/2}$ levels both for protons and for neutrons. In the case of $A = 164$ system we extended this model space with

$1g_{7/2}$ and $1g_{9/2}$ orbits. The single particle energies are obtained by using a Coulomb-corrected Woods–Saxon potential. Two-body G-matrix elements were derived from the Charge-Dependent Bonn (CD-Bonn) one-boson exchange potential within the Brueckner theory. The pairing interactions are adjusted to fit the empirical pairing gaps. The particle–particle and particle–hole channels of the G-matrix interaction of the nuclear Hamiltonian H are renormalized by introducing the parameters $g_{\rm pp}$ and $g_{\rm ph}$, respectively. The calculation is carried out for $g_{\rm ph} = 1.0$. The particle–particle strength parameter $g_{\rm pp}$ of the QRPA is fixed by the assumption that the matrix element $M^{2\nu}_{\rm GT}$ of the $2\nu\varepsilon\varepsilon$ process is within the range $(0, 0.10)$ MeV^{-1}. Recall that $M^{2\nu}_{\rm GT}$ for double-beta decaying nuclei from the region (^{128,130}Te, ^{136}Xe, and ^{150}Nd) does not exceed the above range by assuming weak-axial coupling constant g_A to be unquenched ($g_A = 1.269$) or quenched ($g_A = 1.0$). Then calculated ranges of the $0\nu\varepsilon\varepsilon$ NMEs for ground state to ground state transitions ^{152}Gd $\to$ ^{152}Sm and ^{164}Er $\to$ ^{164}Dy are

$$M^{0\nu}(^{152}{\rm Gd}) = (7.00, 7.39), \quad M^{0\nu}(^{164}{\rm Er}) = (5.92, 6.27). \tag{39}$$

Nuclei ^{152}Gd, ^{152}Sm, ^{164}Er, ^{164}Dy are deformed and the calculation performed within the spherical QRPA approach is a good approximation, if the deformations of initial and final nuclei are comparable [71]. However, this uncertainty in calculated NMEs is not the main source of uncertainty in the calculation of the $0\nu\varepsilon\varepsilon$ half-life.

In Table 2 we present the maximum $T^{\rm max}_{1/2}$ and minimum $T^{\rm min}_{1/2}$ values of the $0\nu\varepsilon\varepsilon$ half-lives for the most favorable cases of the atomic electrons capture. The minimum value of half-life is obtained in the case of complete degeneracy of the atomic masses. Radiative width of two electronic holes is evaluated using the measured values [72], it is also assumed $M^{0\nu}(^{152}{\rm Gd}) \simeq 7.0$ and $M^{0\nu}(^{164}{\rm Er}) \simeq 6.0$.

The maximum value of the half-life is obtained by substituting in Eq. (37) the effective degeneracy parameter

$$\Delta^2_{\rm eff} = (M^{**}_{A,Z-2} - M_{A,Z})^2 + \Delta M^2_{\rm exp}. \tag{40}$$

The last term takes into account the experimental error of measurement of $M^{**}_{A,Z-2} - M_{A,Z}$. After a more accurate measurement, the half-life can only decrease when compared with $T^{\rm max}_{1/2}$.

The results presented in Table 2 show that for $m_{\beta\beta} = 50$ meV some half-lives are predicted to be as low as 10^{25} y in the unitary limit. This value is lower than or comparable with the half-lives of $0\nu\beta\beta$-decays of nuclei of experimental interest. Because of the uncertainty in the atomic masses, the range of allowed $0\nu\varepsilon\varepsilon$ half-lives is broad, and reaches several orders of magnitude.

5.2. The $0\nu\varepsilon\varepsilon$ Experiments. Already long time ago Bernabéu, De Rujula, and Jarlskog [15] pointed out to the possibility of a resonant enhancement of the

Table 2. **The calculated $0\nu\varepsilon\varepsilon$ half-lives of ^{152}Gd and ^{164}Er***

$(n2jl)$ a	$(n2jl)$ b	E_a, keV	E_b, keV	E_C, keV	Γ_{ab}, eV	$M_{A,Z}-M^*_{A,Z-2}$, keV	$T^{\min}_{1/2}$, y	$T^{\max}_{1/2}$, y
^{152}Gd → ^{152}Sm								
110	210	46.83	7.74	0.34	23	–0.8±2.5±2.5	$5\cdot10^{25}$	$2\cdot10^{29}$
110	211	46.83	7.31	0.32	23	–1.3±2.5±2.5	$5\cdot10^{27}$	$5\cdot10^{31}$
110	310	46.83	1.72	0.11	32	–7.1±2.5±2.5	$2\cdot10^{26}$	$4\cdot10^{31}$
110	311	46.83	1.54	0.10	25	–7.3±2.5±2.5	$1\cdot10^{28}$	$5\cdot10^{33}$
110	410	46.83	0.35	0.04	24	–8.5±2.5±2.5	$4\cdot10^{26}$	$2\cdot10^{32}$
^{164}Er → ^{164}Dy								
210	210	9.05	9.05	0.22	8.6	–5.5±3.1±2.5	$2\cdot10^{26}$	$4\cdot10^{32}$
210	211	9.05	8.58	0.23	8.3	–5.9±3.1±2.5	$7\cdot10^{27}$	$2\cdot10^{34}$
210	310	9.05	2.05	0.11	1.8	–12.6±3.1±2.5	$5\cdot10^{27}$	$1\cdot10^{33}$
210	311	9.05	1.84	0.09	1.0	–12.8±3.1±2.5	$2\cdot10^{28}$	$2\cdot10^{35}$
211	211	8.58	8.58	0.27	8.0	–6.4±3.1±2.5	$5\cdot10^{28}$	$8\cdot10^{35}$

*The first and second columns show the quantum numbers of the electron holes. Here, n is the principal quantum number, j is the total angular momentum, and l is the orbital momentum. Shown in the columns three, four and five are the hole energies and their Coulomb interaction energy. The column six shows the radiation widths of the excited electron shells. The column seven shows the mass difference of initial and final atoms. The last two columns show the minimum and maximum half-lives of the $0\nu\varepsilon\varepsilon$ transitions.

$0\nu\varepsilon\varepsilon$ in the case of a mass degeneracy between the initial and final atoms. Proper candidates for experimental study of this total lepton number violating process were selected by using as criteria the natural abundance and the mass degeneracy Δ of involved atoms. Two of favored isotopes ^{74}Se and ^{112}Sn proposed in [15] were already experimentally investigated. Barabash et al. [14] established lower bounds $5.5\cdot10^{18}$ y [13] and $9.5\cdot10^{19}$ y for the $0\nu\varepsilon\varepsilon$ half-lives of ^{74}Se ($J=2^+$, 1204 keV) and ^{112}Sn ($J=0^+$, 1871 keV), respectively. Recently, a new ΔM measurement [73] has excluded a complete mass degeneracy for ^{112}Sn decay and have therefore disfavored significant resonant enhancement of the $0\nu\varepsilon\varepsilon$ mode for this transition.

The isotope ^{106}Cd belongs to the list of the favored candidates in [12, 15]. The initial focus on the $0\nu\varepsilon\varepsilon$ of ^{106}Cd (KL, 2741 keV) was driven by a good mass degeneracy of the participating states. The $0\nu\varepsilon\varepsilon$ resonant decay mode of ^{106}Cd (KL-capture) is expected as a transition to the excited 2741 keV state of ^{106}Pd, which is then depopulated either by emission of a 2741 keV γ-ray or by a 2229 keV and 512 keV γ-quanta cascade. The spin value of the final state of ^{106}Pd

(2741 keV) was unknown [15], and then assumed to be $J = (1,2)^+$ [74]. After measurements had begun, a new value for the spin of the 2741 keV level in ^{106}Pd of $J = 4^+$ was adopted [75], which following recent theoretical analysis dislikes this channel [12]. For the $0\nu\varepsilon\varepsilon$ of ^{106}Cd to the 2741 keV excited state of ^{106}Pd, the TGV collaboration established half-life limit $T^{\varepsilon\varepsilon}_{1/2}(^{106}\text{Cd}) > 1.1 \cdot 10^{20}$ y [76]. We note that in [12] ^{106}Cd with daughter excitation 2.737 MeV is listed among the best $0\nu\varepsilon\varepsilon$ candidates under the assumption that it is a 0^+ excited state of ^{106}Pd. This state, which angular momentum, parity and decay scheme is unknown yet, has been introduced into the BNL database [77] only recently.

In order to obtain more accurate predictions, the experimental accuracy of the atomic masses must be improved, which can easily be achieved with the use of the present-day Penning traps. Recently, in the search for the nuclide with the largest probability for $0\nu\varepsilon\varepsilon$, Eliseev and coauthors [78] have determined the Q-value between the ground states of ^{152}Gd and ^{152}Sm by Penning-trap mass-ratio measurements. The new Q-value of 55.70(18) keV is very close to the precisely calculated binding energy B_{2h} of two electron hole KL_1 in the daughter nuclide ^{152}Sm and results in a half-life of 10^{26} y for a 1 eV effective Majorana neutrino mass. With this smallest half-life among known $0\nu\varepsilon\varepsilon$-transitions, ^{152}Gd is a promising candidate for the search for the resonant $0\nu\varepsilon\varepsilon$-process.

We argue that accurate measurements of the mass differences between initial and final states of the nuclei are necessary, if future experiments of $0\nu\varepsilon\varepsilon$-decays with half-lives below 10^{27} y were to become a possibility.

The detection technique for identifying a $0\nu\varepsilon\varepsilon$ is rather different from the $0\nu\beta\beta$-decay, as there is no inherent background from the $2\nu\varepsilon\varepsilon$,

$$2e_b^- + (\mathcal{A}, \mathcal{Z}) \leftrightarrow (\mathcal{A}, \mathcal{Z}+2)^{**} + 2\nu_e. \tag{41}$$

This lepton number conserving decay is strongly suppressed due to almost vanishing phase space. We note that the existing and in preparation new generation of the $0\nu\beta\beta$-experiments have to cope with the $2\nu\beta\beta$-decay background. Further, by exploiting the coincidence technique, in particular if the de-excitation of the nucleus proceeds through a γ-ray cascade, a significantly improved signal-to-background ratio could be obtained, which would alleviate some of the demands on a low-background facility.

6. CONCLUSION AND OUTLOOK

The fundamental importance of the search for the neutrinoless double-beta decay ($0\nu\beta\beta$-decay) is widely accepted. After 70 years, the brilliant hypothesis of Ettore Majorana is likely to be valid and is strongly supported by the discovery of neutrino oscillations and by the construction of the Grand Unified Theories (GUT). The $0\nu\beta\beta$-decay is currently the most powerful tool to test if the neutrino is a Dirac or a Majorana particle. This issue is intimately related with the origin of neutrino masses, and thus has a strong impact on astrophysics and cosmology.

Neutrinoless double-beta decay allows us to determine whether the neutrino is a Majorana or a Dirac particle and gives also a value for the absolute scale of the neutrino masses and information on coupling constants and masses of physics beyond the SM for grand unified theories (GUTs), supersymmetry (SUSY) and models with extra dimensions.

Neutrinoless double-beta decay, if observed, can allow one to extract the absolute neutrino mass from the decay rate, provided that the corresponding nuclear matrix elements can be calculated reliably. But up to now there has been no complete agreement among different many-body methods of calculating these matrix elements. Thus, further tests of the reliability of the calculated nuclear wave functions are needed.

Recently, an increased attention is paid to the process of neutrinoless double electron capture for those cases where the two participating atoms are nearly degenerate in mass. The theoretical framework is the formalism of an oscillation of two atoms with different total lepton number (and parity), one of which can be in an excited state so that mass degeneracy is realized. Assuming an effective mass for the Majorana neutrino of 50 meV, some half-lives are predicted to be as low as 10^{25} y in the unitary limit. In order to obtain more accurate predictions for the $0\nu\varepsilon\varepsilon$ half-lives, precision mass measurements of the atoms involved are necessary, which can readily be accomplished by today's high precision Penning traps.

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