BACKGROUND FIELD FORMALISM AND CONSTRUCTION OF EFFECTIVE ACTION FOR $\mathcal{N} = 2$, $d = 3$ SUPERSYMMETRIC GAUGE THEORIES

I. L. Buchbinder[1,*]*, N. G. Pletnev*[2,**]*, I. B. Samsonov*[3,***]

[1] Department of Theoretical Physics, Tomsk State Pedagogical University, Tomsk, Russia

[2]Department of Theoretical Physics, Institute of Mathematics, Novosibirsk, Russia

[3]Instituto Nazionale di Fisica Nucleare, Sezione di Padova, Padova, Italy

The background field method for three-dimensional Yang–Mills and Chern–Simons models in $\mathcal{N} = 2$ superspace is reviewed. Superfield proper time (heat kernel) techniques are developed, and exact expressions of heat kernels for constant backgrounds are presented. The background field method and heat-kernel techniques are applied for evaluating the low-energy effective actions in $\mathcal{N} = 2$ supersymmetric Yang–Mills and Chern–Simons models as well as in $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM theories.

PACS: 12.60.Jv

Dedicated to the 60 year jubilee
of Professor D. I. Kazakov

INTRODUCTION

It is our great pleasure to write the paper in honor of Professor D. I. Kazakov, the bright scientist and lecturer. Professor Kazakov was one of the pioneers in applications of superfield methods in supersymmetric quantum field theory [1–3] and made important contributions in the study of renormalizability properties of three-dimensional Chern–Simons-matter theories [4,5]. At present, the superfield methods are commonly recognized to be very effective for exploring quantum aspects of supersymmetric field theories, while the three-dimensional gauge theories have again become very hot topics recently. The present paper is devoted to a review of current state of the problem of low-energy effective action in three-dimensional extended supersymmetric gauge theories in the framework of $\mathcal{N} = 2$, $d = 3$ superspace.

*E-mail: joseph@tspu.edu.ru

**E-mail: pletnev@math.nsc.ru

***On leave from Tomsk Polytechnic University, 634050 Tomsk, Russia. E-mail: samsonov@mph.phtd.tpu.ru.

During last few years, quantum aspects of three-dimensional supersymmetric theories have been attracting considerable attention, mainly because of the progress in studying field theories modelling multiple M2 branes. The most important examples of such models are the Bagger–Lambert–Gustavsson (BLG) [6–11] and Aharony–Bergman–Jafferis–Maldacena (ABJM) [12] theories which are AdS/CFT-dual to the IIA superstring on the $AdS_4 \times CP^3$ background.

On the field theory side of the AdS/CFT correspondence, major attention is paid to the correlation functions of gauge invariant operators. In particular, in the ABJM model such correlation functions were studied in detail [13–24] (see [25] for a review). Another important object containing much information about quantum aspects of a field model is the low-energy effective action. For instance, the low-energy effective action of $\mathcal{N} = 4$, $d = 4$ SYM model is perfectly matched with the effective action of a probe D3 brane moving on the $AdS_5 \times S^5$ background [26–32]. It would be very interesting to observe similar matching between the effective action of an M2 brane on the $AdS_4 \times S^7$ background and the low-energy effective actions of ABJM-like models. Unfortunately, our current understanding of the latter is very insignificant in comparison with the four-dimensional case. To fill this gap, we need to develop the methods of quantum field theory for studying low-energy effective actions for various three-dimensional gauge theories.

An important feature of three-dimensional gauge field theory, in comparison with the four-dimensional case, is the possibility of having topological gauge-invariant mass of gauge fields which originates from the Chern–Simons term [33–35]. The Chern–Simons action, being considered by its own, does not describe propagating degrees of freedom, and its quantization may be useful rather for classifying topological invariants (see, e.g., [36]). However, in models with matter fields such Chern–Simons gauge fields are responsible for interactions which respect conformal invariance. The ABJM and BLG models fall exactly in this category as they represent specific examples of Chern–Simons field theories interacting with matter in a specific way such that the supersymmetry and conformal invariance get enhanced.

Let us comment on the renormalizability properties of three-dimensional gauge models. When the Chern–Simons term is present, one should care about renormalization of the Chern–Simons level k. Indeed, in the classical theory only integer values of k are compatible with large gauge invariance. In [37–42], it was proved that the Chern–Simons level may receive only integer shifts due to quantum corrections, so the gauge invariance is maintained at the quantum level. This result was confirmed by many subsequent computations for various three-dimensional gauge models with Chern–Simons term, both with and without supersymmetry (see, e.g., [43] for a review).

More generally, one can raise the issue of finding UV finite three-dimensional Chern–Simons-matter models which might be as interesting as the famous $\mathcal{N} = 4$,

$d = 4$ SYM model. This problem was extensively studied in the papers of Avdeev, Grigoriev, Kazakov, and Kondrashuk [4, 5] where RG-flows for various three-dimensional gauge models were studied. In particular, cancellations of two-loop divergences for $\mathcal{N} = 2$ Chern–Simons-matter models were found. For similar $\mathcal{N} = 3$ supersymmetric models, a nonrenormalization theorem was formulated which proves the all-loop UV finiteness of such theories [44]. These results show that not only ABJM and BLG models should remain superconformal on the quantum level, but the class of quantum-superconformal three-dimensional field theories is much wider [45–47]. Quantum aspects and, in particular, the problem of low-energy effective action in such three-dimensional superconformal theories deserves much attention.

As is well known, quantization procedure of gauge theories requires imposing a gauge which explicitly breaks the invariance of the effective action under classical gauge transformations. To keep track of the gauge invariance, one is to employ the background field method which was originally introduced by DeWitt [48] and developed in many subsequent papers [49–52]. The central idea of the background field method is a decomposition of the gauge fields into classical background and quantum fields (background-quantum splitting) and imposing the gauge conditions only on the quantum ones. After integrating out quantum fields, the path integral results in the gauge-invariant effective action depending on the background fields[*]. However, the background-quantum splitting can be very nontrivial for some gauge theories and, hence, the formulation of the background field method in any concrete theory demands a special study. For instance, construction of the background field method for $\mathcal{N} = 1$, $d = 4$ superfield gauge theories [54] is very specific whereas its analog in the $\mathcal{N} = 2$, $d = 4$ harmonic superspace [55] looks rather similar to the one for conventional Yang–Mills theories.

The low-energy effective action usually describes an effective dynamics of light degrees of freedom with the heavy ones integrated out. In SYM-like gauge field theories such a separation appears usually as a result of the Higgs mechanism of spontaneous gauge symmetry breaking. Unfortunately, for the ABJM-like models the Higgs mechanism works differently: once the matter fields acquire vacua, the Chern–Simons-matter model turns into a SYM model with higher derivative corrections [56–63]. Therefore we are led to study the low-energy effective action in the three-dimensional SYM models rather than in the ABJM model itself. To address the issue of effective action in the ABJM model, we need to quantize the gauge fields in the conformal phase when all fields remain massless. Then, to avoid IR divergences, a massive regulator (cut-off) is

---

[*]We stress that such an effective action, being gauge invariant, nevertheless depends on the choice of gauge fixing conditions (see the discussion of these aspects in [53]).

required.

In the present paper, after a short review of details of gauge theory in the $\mathcal{N} = 2$, $d = 3$ superspace given in Sec. 1, we develop the background field method for $\mathcal{N} = 2$ super Yang–Mills and Chern–Simons models. For these models, the quadratic-fluctuations operators are constructed for a general background and the structure of one-loop effective action is discussed (Sec. 2). Then, in Sec. 3, we compute the low-energy effective actions for pure SYM models with $\mathcal{N} = 2$, $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supersymmetry in the Coulomb branch. For pure $\mathcal{N} = 2$ Chern–Simons model, we compute the leading terms in the effective action with the lowest number of non-Abelian gauge superfields. For the latter model, we note that the ghost superfields at one loop produce the $\mathcal{N} = 2$ SYM-like term in the effective action. This is not surprising since the pure topological nature of classical Chern–Simons theory is broken explicitly at the quantum level. We believe that the presented $\mathcal{N} = 2$ superfield techniques for studying effective actions in the three-dimensional gauge models in the $\mathcal{N} = 2$, $d = 3$ superspace will be useful for further studies of other field theories modelling dynamics of M2 and D2 branes as well as appearing in phenomenological applications. Some open problems which deserve further studies are discussed in Conclusions. We follow the notations employed in our previous works [64–66].

1. SUPERSYMMETRIC GAUGE MODELS

1.1. $\mathcal{N} = 2$, $d = 3$ Superspace. The $\mathcal{N} = 2$, $d = 3$ superspace is parameterized by the coordinates $z^M = (x^m, \theta_\alpha, \bar\theta_\alpha)$ with $\bar\theta_\alpha = (\theta_\alpha)^*$. The supercovariant spinor derivatives read

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\bar\theta^\beta \partial_{\alpha\beta}, \quad \bar{D}_\alpha = -\frac{\partial}{\partial\bar\theta^\alpha} - i\theta^\beta \partial_{\alpha\beta}, \quad \{D_\alpha, \bar{D}_\beta\} = -2i\partial_{\alpha\beta}. \quad (1.1)$$

We use the following conventions for converting the vector and bi-spinor indices to each other:

$$x^{\alpha\beta} = (\gamma_m)^{\alpha\beta} x^m, \quad \partial_{\alpha\beta} = (\gamma^m)_{\alpha\beta}\partial_m, \quad (1.2)$$

where $(\gamma^0)_\alpha^\beta = -i\sigma_2$, $(\gamma^1)_\alpha^\beta = \sigma_3$, $(\gamma^2)_\alpha^\beta = \sigma_1$ are three-dimensional gamma matrices obeying standard orthogonality and completeness relations

$$(\gamma^m)_{\alpha\beta}(\gamma^n)^{\alpha\beta} = 2\eta^{mn}, \quad (\gamma^m)_{\alpha\beta}(\gamma_m)^{\rho\sigma} = (\delta_\alpha^\rho \delta_\beta^\sigma + \delta_\alpha^\sigma \delta_\beta^\rho). \quad (1.3)$$

These matrices are real and symmetric. We use «mostly minus» Minkowski space metric, $\eta_{mn} = \text{diag}(1, -1, -1)$.

The integration measure in the full $\mathcal{N} = 2$, $d = 3$ superspace is defined as

$$d^7 z \equiv d^3 x\, d^4\theta = \frac{1}{16} d^3 x\, D^2 \bar{D}^2, \quad \text{so that} \quad \int d^3 x\, f(x) = \int d^7 z\, \theta^2 \bar\theta^2 f(x), \quad (1.4)$$

for some field $f(x)$. Here we use the following conventions for contractions of spinor indices:

$$D^2 = D^\alpha D_\alpha, \quad \bar{D}^2 = \bar{D}^\alpha \bar{D}_\alpha, \quad \theta^2 = \theta^\alpha \theta_\alpha, \quad \bar{\theta}^2 = \bar{\theta}^\alpha \bar{\theta}_\alpha, \quad (1.5)$$

so that $(\theta^2)^* = -\bar{\theta}^2$, $(D^2)^* = -\bar{D}^2$.

The chiral subspace is parameterized by $z_+ = (x_+^m, \theta_\alpha)$, where $x_\pm^m = x^m \pm i\gamma_{\alpha\beta}^m \theta^\alpha \bar{\theta}^\beta$. The chiral superfields are defined as usual, $\bar{D}_\alpha \Phi = 0 \Rightarrow \Phi = \Phi(x_+^m, \theta_\alpha)$. The integration measure in the chiral superspace $d^5 z \equiv d^3 x\, d^2\theta$ is related to the full superspace measure (1.4) as

$$d^7 z = -\frac{1}{4} d^5 z\, \bar{D}^2 = -\frac{1}{4} d^5 \bar{z}\, D^2. \quad (1.6)$$

1.2. Gauge Superfields in $\mathcal{N} = 2$, $d = 3$ Superspace. Gauge fields in the $\mathcal{N} = 2$, $d = 3$ superspace are introduced as the connections for superspace derivatives (1.1),

$$\nabla_\alpha = D_\alpha + A_\alpha, \quad \bar{\nabla}_\alpha = \bar{D}_\alpha + \bar{A}_\alpha, \quad \nabla_m = \partial_m + A_m. \quad (1.7)$$

These gauge connections should obey the following superfield constraints [67–72]:

$$\{\nabla_\alpha, \bar{\nabla}_\beta\} = -2i(\gamma^m)_{\alpha\beta} \nabla_m + 2i\varepsilon_{\alpha\beta} G, \quad (1.8)$$

$$[\nabla_\alpha, \nabla_m] = -(\gamma_m)_{\alpha\beta} \bar{W}^\beta, \quad [\bar{\nabla}_\alpha, \nabla_m] = (\gamma_m)_{\alpha\beta} W^\beta, \quad (1.9)$$

$$[\nabla_m, \nabla_n] = iF_{mn}. \quad (1.10)$$

Here G, W_α, $\bar{W}_\alpha$, and F_{mn} are non-Abelian superfield strengths subject to Bianchi identities. In particular, the superfield G is Hermitian and is covariantly linear,

$$\nabla^\alpha \nabla_\alpha G = 0, \quad \bar{\nabla}^\alpha \bar{\nabla}_\alpha G = 0. \quad (1.11)$$

The superfields W_α and $\bar{W}_\alpha$ are covariantly (anti)chiral,

$$\nabla_\alpha \bar{W}_\beta = 0, \quad \bar{\nabla}_\alpha W_\beta = 0, \quad (1.12)$$

and satisfy «standard» Bianchi identity,

$$\nabla^\alpha W_\alpha = \bar{\nabla}^\alpha \bar{W}_\alpha. \quad (1.13)$$

These superfield strengths are expressed in terms of G as

$$\bar{W}_\alpha = \nabla_\alpha G, \quad W_\alpha = \bar{\nabla}_\alpha G. \quad (1.14)$$

We prefer to introduce the gauge prepotential in the so-called chiral representation in which the connection for the Grassmann derivative $\bar{D}_\alpha$ vanishes,

$$\nabla_\alpha = \mathrm{e}^{-2V} D_\alpha\, \mathrm{e}^{2V}, \quad \bar{\nabla}_\alpha = \bar{D}_\alpha, \quad V^\dagger = V. \quad (1.15)$$

In this representation the superfield strengths are expressed in terms of the pre-potential V as

$$G = \frac{i}{4}\bar{D}^\alpha(\mathrm{e}^{-2V}D_\alpha\,\mathrm{e}^{2V}),$$

$$\bar{W}_\alpha = \frac{i}{4}\nabla_\alpha\bar{D}^\beta(\mathrm{e}^{-2V}D_\beta\,\mathrm{e}^{2V}), \quad W_\alpha = -\frac{i}{8}\bar{D}^2(\mathrm{e}^{-2V}D_\alpha\,\mathrm{e}^{2V}).$$

(1.16)

They are covariant under the following gauge transformations:

$$\mathrm{e}^{2V} \to \mathrm{e}^{i\bar{\lambda}}\mathrm{e}^{2V}\,\mathrm{e}^{-i\lambda}, \tag{1.17}$$

or in the infinitesimal form

$$\delta V = -\frac{i}{2}L_V(\bar{\lambda}+\lambda) + \frac{i}{2}L_V\,\coth\,(L_V)(\bar{\lambda}-\lambda), \tag{1.18}$$

where λ and $\bar{\lambda}$ are chiral and antichiral superfields, respectively, and L_V denotes the commutator, e.g., $L_V\lambda = [V,\lambda]$.

In the Wess–Zumino gauge, the component field decomposition for V is given by

$$V = \theta^\alpha\bar{\theta}^\beta A_{\alpha\beta} + i\theta^\alpha\bar{\theta}_\alpha\phi + i\theta^2\bar{\theta}^\alpha\bar{\lambda}_\alpha - i\bar{\theta}^2\theta^\alpha\lambda_\alpha + \theta^2\bar{\theta}^2 D. \tag{1.19}$$

Here $A_{\alpha\beta}$ is a gauge vector field; ϕ is a real scalar; λ_α is a complex spinor, and D is a real auxiliary field.

1.3. Super Yang–Mills Model. The classical action of the $\mathcal{N}=2$ SYM can be written equivalently either in full $\mathcal{N}=2$ superspace or in the chiral subspace,

$$S_{\mathrm{SYM}}^{\mathcal{N}=2} = \frac{1}{g^2}\,\mathrm{tr}\int d^7z\,G^2 = -\frac{1}{2g^2}\,\mathrm{tr}\int d^5z\,W^\alpha W_\alpha, \tag{1.20}$$

where g is the dimensionfull coupling constant, $[g]=1/2$. Here the properties (1.11) and (1.14) were used, as well as the relation (1.6) has been applied.

The $\mathcal{N}=2$ supersymmetry of the action (1.20) can be extended to $\mathcal{N}=4$ by introducing the chiral superfield Φ in the adjoint representation of the gauge group,

$$S_{\mathrm{SYM}}^{\mathcal{N}=4} = \frac{1}{g^2}\mathrm{tr}\int d^7z\left(G^2 - \frac{1}{2}\,\mathrm{e}^{-2V}\,\bar{\Phi}\,\mathrm{e}^{2V}\,\Phi\right). \tag{1.21}$$

This action is invariant under non-Abelian gauge transformations

$$\Phi \to \mathrm{e}^{i\lambda}\,\Phi\,\mathrm{e}^{-i\lambda}, \quad \bar{\Phi} \to \mathrm{e}^{i\bar{\lambda}}\,\bar{\Phi}\,\mathrm{e}^{-i\bar{\lambda}}, \quad \mathrm{e}^{2V} \to \mathrm{e}^{i\bar{\lambda}}\,\mathrm{e}^{2V}\,\mathrm{e}^{-i\lambda}, \tag{1.22}$$

with λ and $\bar{\lambda}$ being (anti)chiral superfield gauge parameters, and under hidden $\mathcal{N}=2$ supersymmetry,

$$\mathrm{e}^{-2V}\,\delta_\epsilon\,\mathrm{e}^{2V} = \theta^\alpha\epsilon_\alpha\bar{\Phi}_c - \bar{\theta}^\alpha\bar{\epsilon}_\alpha\Phi_c, \quad \delta_\epsilon\Phi_c = -i\epsilon^\alpha\bar{\nabla}_\alpha G, \quad \delta_\epsilon\bar{\Phi}_c = -i\bar{\epsilon}^\alpha\nabla_\alpha G. \tag{1.23}$$

Here ϵ_α is the anticommuting complex parameter, and Φ_c, $\bar{\Phi}_c$ are covariantly (anti)chiral superfields,

$$\bar{\Phi}_c = \mathrm{e}^{-2V}\,\bar{\Phi}\,\mathrm{e}^{2V}, \quad \Phi_c = \Phi, \quad \nabla_\alpha \bar{\Phi}_c = 0, \quad \bar{\nabla}_\alpha \Phi_c = 0. \tag{1.24}$$

Similarly, the $\mathcal{N} = 8$ supersymmetric extension of (1.20) reads

$$S_{\mathrm{SYM}}^{\mathcal{N}=8} = \frac{1}{g^2}\mathrm{tr}\int d^7 z \left(G^2 - \frac{1}{2}\,\mathrm{e}^{-2V}\,\bar{\Phi}^i\,\mathrm{e}^{2V}\,\Phi_i \right) +$$
$$+ \frac{1}{12g^2}\left(\mathrm{tr}\int d^5 z\,\varepsilon^{ijk}\Phi_i[\Phi_j,\Phi_k] + \mathrm{c.c.}\right). \tag{1.25}$$

Here Φ_i, $i = 1,2,3$, is a triplet of chiral superfields. The transformations of hidden $\mathcal{N} = 6$ supersymmetry with the complex anticommuting parameter $\epsilon_{\alpha i}$ are given by

$$\mathrm{e}^{-2V}\delta_\epsilon\,\mathrm{e}^{2V} = \theta^\alpha \epsilon_{\alpha\,i}\bar{\Phi}_c^i - \bar{\theta}^\alpha \bar{\epsilon}_\alpha^i \Phi_{c\,i},$$
$$\delta_\epsilon \Phi_{c\,i} = -i\epsilon_i^\alpha \bar{\nabla}_\alpha G + \frac{1}{4}\varepsilon_{ijk}\bar{\nabla}^2(\bar{\theta}^\alpha \bar{\epsilon}_\alpha^j \bar{\Phi}_c^k), \tag{1.26}$$
$$\delta_\epsilon \bar{\Phi}_c^i = -i\bar{\epsilon}^{\alpha\,i}\nabla_\alpha G + \frac{1}{4}\varepsilon^{ijk}\nabla^2(\theta^\alpha \epsilon_{\alpha\,j}\Phi_{c\,k}).$$

We use the standard notations $\bar{\Phi}_c^i = \mathrm{e}^{-2V}\,\bar{\Phi}^i\,\mathrm{e}^{2V}$, $\Phi_c = \Phi$ for the covariantly (anti)chiral superfields.

1.4. Chern–Simons Model. The non-Abelian $\mathcal{N} = 2$ supersymmetric Chern–Simons action was constructed in [70],

$$S_{\mathrm{CS}}^{\mathcal{N}=2} = \frac{ik}{8\pi}\mathrm{tr}\int_0^1 dt\int d^7 z\,\bar{D}^\alpha(\mathrm{e}^{-2tV}\,D_\alpha\,\mathrm{e}^{2tV})\,\mathrm{e}^{-2tV}\,\partial_t\,\mathrm{e}^{2tV}. \tag{1.27}$$

Here t is an auxiliary real parameter and k is an integer (Chern–Simons level).

In the Abelian case, the integration over the parameter t can be explicitly done,

$$S_{\mathrm{CS}}^{\mathcal{N}=2} =$$
$$= \frac{k}{2\pi}\int d^7 z\,VG = \frac{k}{2\pi}\int d^3 x\left(\frac{1}{2}\varepsilon^{mnp}A_m\partial_n A_p + i\lambda^\alpha \bar{\lambda}_\alpha - 2\phi D\right). \tag{1.28}$$

In the non-Abelian case, the integration over t can also be performed for the variation of (1.27),

$$\delta S_{\mathrm{CS}}^{\mathcal{N}=2} = \frac{k}{\pi}\mathrm{tr}\int d^7 z\,G\Delta V, \tag{1.29}$$

where

$$\Delta V = \frac{1}{2}\, e^{-2V} \delta\, e^{2V} \tag{1.30}$$

is the so-called gauge-covariant variation.

The action (1.27) allows for the $\mathcal{N} = 4$ supersymmetric extension with a chiral superfield Φ in the adjoint representation,

$$S_{\mathrm{CS}}^{\mathcal{N}=4} = S_{\mathrm{CS}}^{\mathcal{N}=2} - \frac{ik}{4\pi}\mathrm{tr}\int d^5z\, \Phi^2 - \frac{ik}{4\pi}\mathrm{tr}\int d^5\bar{z}\, \bar{\Phi}^2. \tag{1.31}$$

The transformations of hidden $\mathcal{N} = 2$ supersymmetry with complex spinor parameter ϵ^α read

$$\Delta_\epsilon V = \epsilon^\alpha \bar{\theta}_\alpha \Phi_c - \epsilon^\alpha \theta_\alpha \bar{\Phi}_c, \quad \delta_\epsilon \Phi_c = -i\bar{\epsilon}^\alpha \bar{\nabla}_\alpha G, \quad \delta_\epsilon \bar{\Phi}_c = -i\epsilon^\alpha \nabla_\alpha G. \tag{1.32}$$

Note that (1.32) coincides with (1.23) only for real supersymmetry parameter, $\bar{\epsilon}_\alpha = \epsilon_\alpha$. Therefore the sum of the actions (1.21) and (1.31) has $\mathcal{N} = 3$ supersymmetry instead of $\mathcal{N} = 4$. This was first demonstrated in [73, 74] using component field approach.

It is well known that the sum of Chern–Simons (1.27) and Yang–Mills (1.20) actions describes topologically massive gauge theory,

$$S[V] = S_{\mathrm{CS}}^{\mathcal{N}=2} + S_{\mathrm{SYM}}^{\mathcal{N}=2}. \tag{1.33}$$

This can be most easily demonstrated for the corresponding Abelian equation of motion,

$$0 = \frac{\delta S}{\delta V} = \frac{i}{g^2}\bar{D}^\alpha D_\alpha G + \frac{k}{\pi}G. \tag{1.34}$$

This equation has the following differential consequence:

$$\frac{i\pi}{2kg^2}D^\alpha \bar{D}^2 D_\alpha G = \bar{D}^\alpha D_\alpha G. \tag{1.35}$$

Now, applying the identity

$$\frac{1}{16}(D^2\bar{D}^2 + \bar{D}^2 D^2 - 2D^\alpha \bar{D}^2 D_\alpha) = \square \tag{1.36}$$

and using the linearity (1.11) of the superfield strength G, we conclude that it obeys the Klein–Gordon equation with topological mass,

$$(\square + m^2)G = 0, \quad m^2 = \frac{k^2 g^4}{4\pi^2}. \tag{1.37}$$

In contrast with the massless case, this equation does not have constant field solutions. This makes the perturbation theory around classical solutions more complicate than in the pure SYM theory.

2. BACKGROUND FIELD METHOD IN $\mathcal{N} = 2$, $d = 3$ SUPERSPACE

Consider the $\mathcal{N} = 2$ super Yang–Mills–Chern–Simons model with the action (1.33). As we pointed out in Introduction, the background field method is based on a decomposition of initial gauge field into background and quantum fields. The form of such a splitting depends on the structure of gauge transformations. In the theory under consideration, it is convenient to decompose the gauge superfield V into the «background» V and «quantum» v superfields by the rule

$$\mathrm{e}^{2V} \to \mathrm{e}^{2V} \, \mathrm{e}^{2gv}, \tag{2.1}$$

so that

$$\nabla_\alpha = \mathrm{e}^{-2gv} \, \mathcal{D}_\alpha \, \mathrm{e}^{2gv}, \quad \bar{\nabla}_\alpha = \bar{\mathcal{D}}_\alpha, \tag{2.2}$$

where

$$\mathcal{D}_\alpha = \mathrm{e}^{-2V} \, D_\alpha \, \mathrm{e}^{2V}, \quad \bar{\mathcal{D}}_\alpha = \bar{D}_\alpha \tag{2.3}$$

are the background gauge-covariant spinor derivatives. There is a freedom in defining the gauge transformations for the background and quantum superfields. In particular, one can consider the so-called «background» gauge transformations

$$\mathrm{e}^{2V} \to \mathrm{e}^{i\bar{\lambda}} \, \mathrm{e}^{2V} \, \mathrm{e}^{-i\lambda}, \quad \mathrm{e}^{2gv} \to \mathrm{e}^{i\tau} \mathrm{e}^{2gv} \, \mathrm{e}^{-i\tau} \tag{2.4}$$

and the «quantum» ones,

$$\mathrm{e}^{2V} \to \mathrm{e}^{2V}, \quad \mathrm{e}^{2gv} \to \mathrm{e}^{i\bar{\lambda}} \, \mathrm{e}^{2gv} \, \mathrm{e}^{-i\lambda}. \tag{2.5}$$

Here λ and $\bar{\lambda}$ are (anti)chiral gauge parameters, while τ is real. The above decomposition of the gauge superfield V into the background V and quantum v superfields is analogous to the background-quantum splitting in $\mathcal{N} = 1$, $d = 4$ superfield Yang–Mills theory [54] but differs from the splitting in conventional Yang–Mills theory [48] and in $\mathcal{N} = 2$, $d = 4$ super Yang–Mills theory in harmonic superspace formulation [55].

To expand the action (1.33) in a series over quantum superfields, we compute the variational derivatives,

$$\frac{\delta S}{\Delta V} = \frac{i}{g^2} \mathcal{D}^\alpha \bar{\mathcal{D}}_\alpha G + \frac{k}{\pi} G, \tag{2.6}$$

$$\frac{\delta^2 S}{\Delta V(z_1) \Delta V(z_2)} = \left(\frac{1}{4g^2} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha - \frac{2i}{g^2} W^\alpha \mathcal{D}_\alpha + \frac{ik}{2\pi} \bar{\mathcal{D}}^\alpha \mathcal{D}_\alpha \right) \delta^7(z_1 - z_2). \tag{2.7}$$

This allows us to find the leading terms with respect to the quantum gauge superfields,

$$S[V, v] = S_0[V] + S_1[V, v] + S_2[V, v] + \ldots, \tag{2.8}$$

$$S_0[V] \equiv S[V], \tag{2.9}$$

$$S_1[V, v] = g \operatorname{tr} \int d^7 z \, v \frac{\delta S}{\Delta V} = \operatorname{tr} \int d^7 z \, v \left(\frac{i}{g} \mathcal{D}^\alpha \bar{\mathcal{D}}_\alpha G + \frac{gk}{\pi} G \right), \tag{2.10}$$

$$S_2[V, v] = \frac{g^2}{2} \operatorname{tr} \int d^7 z_1 \,^7 z_2 \, v(z_1) \, v(z_2) \frac{\delta^2 S}{\Delta V(z_1) \Delta V(z_2)} =$$
$$= \operatorname{tr} \int d^7 z \, v \left(\frac{1}{8} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha - i W^\alpha \mathcal{D}_\alpha + \frac{ig^2 k}{4\pi} \bar{\mathcal{D}}^\alpha \mathcal{D}_\alpha \right) v. \tag{2.11}$$

Here we do not consider the terms with vertices for quantum superfields as we restrict ourselves to one-loop computations only.

The action S_1 is responsible for the equations of motion for the background superfield and does not contribute to the effective action. The action S_2 can be rewritten in the form

$$S_2 = \operatorname{tr} \int d^7 z \, v(H_1 + H_2)v, \tag{2.12}$$

where the operators H_1 and H_2 originate from the second variations of the SYM and Chern–Simons actions, respectively,

$$H_1 = \frac{1}{4} \mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha - i W^\alpha \mathcal{D}_\alpha, \quad H_2 = \frac{ig^2 k}{4\pi} \bar{\mathcal{D}}^\alpha \mathcal{D}_\alpha. \tag{2.13}$$

Both these operators are degenerate. By fixing the gauge, we can remove the degeneracy of either of these operators. We will make them both nondegenerate since this option is more general.

Within the background field method one usually fixes the quantum gauge symmetry (2.5) keeping the invariance under the background transformations. The corresponding gauge fixing functions

$$f = i\bar{\mathcal{D}}^2 v, \quad \bar{f} = i\mathcal{D}^2 v \tag{2.14}$$

are defined with the help of the background-dependent covariant spinor derivatives (2.3). These functions are covariantly (anti)chiral and change under the quantum gauge transformations (2.5) as

$$\delta f = \frac{1}{2g} \bar{\mathcal{D}}^2 L_{gv} [\bar{\lambda} + \lambda + \coth (L_{gv})(\lambda - \bar{\lambda})]. \tag{2.15}$$

Therefore the ghost superfield action has the standard form,

$$S_{\mathrm{gh}} = \operatorname{tr} \int d^7 z \, (b + \bar{b}) L_{gv} [c + \bar{c} + \coth (L_{gv})(c - \bar{c})] = \operatorname{tr} \int d^7 z \, (\bar{b}c - b\bar{c}) + O(g). \tag{2.16}$$

The one-loop effective action is given by the following functional integral:

$$e^{i\Gamma[V]} = e^{iS[V]} \int \mathcal{D}v \, \mathcal{D}b \, \mathcal{D}c \, \delta[f - i\bar{\mathcal{D}}^2 v] \, \delta[\bar{f} - i\mathcal{D}^2 v] \, e^{iS_2[V,v] + iS_{\text{gh}}}. \quad (2.17)$$

To represent the delta-functions in Gaussian form, we average this expression with the weight

$$1 = \int \mathcal{D}f \, \mathcal{D}\varphi \, e^{iS[f] + iS[\varphi]}, \quad (2.18)$$

where

$$S[f] = \frac{1}{8\alpha} \text{tr} \int d^7 z \, f\bar{f} + \frac{i}{4\beta} \text{tr} \int d^5 z \, f^2 + \frac{i}{4\beta} \text{tr} \int d^5\bar{z} \, \bar{f}^2, \quad (2.19)$$

and the action $S[\varphi]$ coincides with (2.19), but depends on the anticommuting Nielsen–Kallosh ghost φ. Parameters α and β in (2.19) are arbitrary. For finite values of these parameters the action (2.19) describes the massive Wess–Zumino model, but one can eliminate either $f\bar{f}$ or massive terms by sending corresponding parameter to infinity. As a result, we get the gauge fixing and Nielsen–Kallosh actions in the form

$$S_{\text{gf}} = \text{tr} \int d^7 z \, v \left[-\frac{1}{16\alpha} \{\mathcal{D}^2, \bar{\mathcal{D}}^2\} + \frac{i}{\beta}\bar{\mathcal{D}}^2 + \frac{i}{\beta}\mathcal{D}^2 \right] v, \quad (2.20)$$

$$S_{\text{NK}} = -\text{tr} \int d^7 z \, \bar{\varphi}\varphi. \quad (2.21)$$

Note that the mass term for the Nielsen–Kallosh ghost vanishes because of anticommuting nature of this superfield.

One of the most simple choices for the gauge fixing parameters corresponds to $\alpha = 1$ and $\beta = 8\pi/kg^2$. Then the sum of the actions S_2 and S_{gf} reads

$$S_2 + S_{\text{gf}} = \text{tr} \int d^7 z \, v(-\Box_{\text{v}} + H)v, \quad (2.22)$$

where

$$\Box_{\text{v}} = -\frac{1}{8}\mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha + \frac{1}{16}\{\mathcal{D}^2, \bar{\mathcal{D}}^2\} + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha \mathcal{D}_\alpha =$$
$$= \mathcal{D}^m \mathcal{D}_m + G^2 + iW^\alpha \mathcal{D}_\alpha - i\bar{W}^\alpha \bar{\mathcal{D}}_\alpha, \quad (2.23)$$

$$H = \frac{ig^2 k}{8\pi}(\bar{\mathcal{D}}^\alpha \mathcal{D}_\alpha + \mathcal{D}^\alpha \bar{\mathcal{D}}_\alpha + \bar{\mathcal{D}}^2 + \mathcal{D}^2). \quad (2.24)$$

Here $\Box_{\text{v}}$ is the covariant d'Alembertian operator in the space of real superfields and H originates from the Chern–Simons part of the action.

As a result, we get the following representation for the one-loop effective action:

$$
e^{i\Gamma[V]} = e^{iS[V]} \int \mathcal{D}v \, \mathcal{D}b \, \mathcal{D}c \, \mathcal{D}\varphi \times
$$

$$
\times \exp\left[i \operatorname{tr} \int d^7 z \, v(-\Box_{\mathrm{v}} + H)v + iS_{\mathrm{gh}} + iS_{\mathrm{NK}} \right]. \quad (2.25)
$$

Schematically, it can be written as

$$
\Gamma = \Gamma_{\mathrm{v}} + \Gamma_{\mathrm{gh}}, \quad \Gamma_{\mathrm{v}} = \frac{i}{2} \operatorname{Tr}_{\mathrm{v}} \ln(\Box_{\mathrm{v}} - H), \quad \Gamma_{\mathrm{gh}} = -\frac{3i}{2} \operatorname{Tr}_{+} \ln \Box_{+}. \quad (2.26)
$$

The contribution Γ_{v} to the one-loop effective action comes from the quantum gauge superfield, while Γ_{gh} is due to ghosts. Here $\operatorname{Tr}_{\mathrm{v}}$ and Tr_{+} are the functional traces of the operators acting in the spaces of real and chiral superfields, respectively. The operator $\Box_{+}$ is the covariant d'Alembertian operator acting in the space of covariantly chiral superfields which was introduced in [65],

$$
\Box_{+} = \frac{1}{16} \bar{\mathcal{D}}^2 \mathcal{D}^2 = \mathcal{D}^m \mathcal{D}_m + G^2 + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha \mathcal{D}_\alpha. \quad (2.27)
$$

The explicit expressions for the traces of these operators can be found after one specifies the gauge group and the background gauge superfield.

3. SUPERFIELD EFFECTIVE ACTION

3.1. $\mathcal{N} = 2$ SYM. Consider pure $\mathcal{N} = 2$ SYM model with classical action (1.20). The background field method goes along the same lines as in Sec. 2, but in Eq. (2.19) we send $\beta \to \infty$ to remove the part of this action responsible for the gauge fixing in the Chern–Simons action. Then the expression for the one-loop effective action (2.26) slightly modifies,

$$
\Gamma_{\mathrm{SYM}}^{\mathcal{N}=2} = \Gamma_{\mathrm{v}} + \Gamma_{\mathrm{gh}}, \quad \Gamma_{\mathrm{v}} = \frac{i}{2} \operatorname{Tr}_{\mathrm{v}} \ln \Box_{\mathrm{v}}, \quad \Gamma_{\mathrm{gh}} = -\frac{3i}{2} \operatorname{Tr}_{+} \ln \Box_{+}, \quad (3.1)
$$

where the operators $\Box_{\mathrm{v}}$ and $\Box_{+}$ are given in (2.23) and (2.27).

We will be interested in the low-energy effective action which is a functional for the massless fields obtained by integrating out all massive ones in the functional integral. In gauge theories, the separation between massless and massive fields appears usually through the Higgs mechanism. In general, the gauge group $SU(N)$ is spontaneously broken down to its maximal Abelian subgroup, $U(1)^{N-1}$. However, in particular cases, a bigger subgroup of $SU(N)$ can be unbroken. Physically, it is interesting to consider minimal gauge symmetry

breaking, $SU(N) \rightarrow SU(N-1) \times U(1)$, because, from the point of view of D-branes, the corresponding effective action contains the potential which appears when one separates one D-brane from the stack. In this section, we will consider first the general case when the gauge group is broken down to the maximal torus and then comment on the effective action with minimal gauge symmetry breaking.

The Lie algebra $su(N)$ consists of Hermitian traceless matrices. Any element v of $su(N)$ can be represented by a decomposition over the Cartan–Weil basis in the $gl(N)$ algebra,

$$(e_{IJ})_{LK} = \delta_{IL}\delta_{JK},$$

$$v = \sum_{I<J}^{N}(v_{IJ}e_{IJ} + \bar{v}_{IJ}e_{JI}) + \sum_{I=1}^{N}v_I e_{II}, \quad \bar{v}_I = v_I, \quad \sum_{I=1}^{N}v_I = 0. \tag{3.2}$$

The background gauge superfield V belongs to the Cartan subalgebra spanned on e_{II},

$$V = \sum_{I=1}^{N}\mathbf{V}_I e_{II} = \mathrm{diag}\,(\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N), \quad \bar{\mathbf{V}}_I = \mathbf{V}_I, \quad \sum_{I=1}^{N}\mathbf{V}_I = 0. \tag{3.3}$$

In what follows, we will denote by boldface Latin letters the matrix elements of the background superfields. In particular, each matrix element $\mathbf{V}_I$ of V has superfield strength $\mathbf{G}_I = (i/2)\bar{D}^\alpha D_\alpha \mathbf{V}_I$ which is computed as in the Abelian case. We will also use the following notations:

$$\mathbf{V}_{IJ} = \mathbf{V}_I - \mathbf{V}_J, \quad \mathbf{G}_{IJ} = \mathbf{G}_I - \mathbf{G}_J, \quad \mathbf{W}_{IJ\alpha} = \mathbf{W}_{I\alpha} - \mathbf{W}_{J\alpha}. \tag{3.4}$$

Now we can do the matrix trace in the quadratic action,

$$S_2 + S_{\mathrm{gf}} = -\mathrm{tr}\int d^7 z\, v\Box_{\mathrm{v}} v = -2\sum_{I<J}^{N}\int d^7 z\, v_{IJ}\hat{\Box}_{\mathrm{v}\,IJ}\,\bar{v}_{IJ}, \tag{3.5}$$

where $\hat{\Box}_{\mathrm{v}\,IJ}$ is the Abelian version of the operator (2.23) which is constructed from the Abelian gauge superfield $\mathbf{V}_{IJ}$ and its superfield strengths,

$$\hat{\Box}_{\mathrm{v}\,IJ} = \mathcal{D}^m\mathcal{D}_m + \mathbf{G}_{IJ}^2 + i\mathbf{W}_{IJ}^\alpha\mathcal{D}_\alpha - i\bar{\mathbf{W}}_{IJ}^\alpha\bar{\mathcal{D}}_\alpha. \tag{3.6}$$

Therefore the effective action Γ_{v} can be written as

$$\Gamma_{\mathrm{v}} = i\sum_{I<J}^{N}\mathrm{Tr}_{\mathrm{v}}\ln\hat{\Box}_{\mathrm{v}\,IJ}, \tag{3.7}$$

where Tr_{v} means now only the functional trace in the space of real superfields.

In a similar way, one can analyze the contributions from the ghost superfields. Consider, for instance, the action for the Nielsen–Kallosh ghost (2.21) in which the chiral superfields are expanded over the basis (3.2) as

$$\varphi = \sum_{I \neq J}^{N} e_{IJ} \varphi_{IJ}, \quad \bar{\varphi} = \sum_{I \neq J}^{N} e_{IJ} \bar{\varphi}_{IJ}. \tag{3.8}$$

Here we omit the diagonal components because they do not interact with the background gauge superfield (3.3) and do not contribute to the effective action. Then the matrix trace in the action (2.21) is done,

$$S_{\mathrm{NK}} = -\sum_{I \neq J}^{N} \int d^7 z \, \bar{\varphi}_{IJ} \varphi_{IJ}, \tag{3.9}$$

where the superfields $\bar{\varphi}_{IJ}$ are covariantly antichiral,

$$\mathrm{e}^{-2\mathbf{V}_{IJ}} D_\alpha \, \mathrm{e}^{2\mathbf{V}_{IJ}} \bar{\varphi}_{IJ} = 0 \quad \text{for} \quad I < J,$$
$$\mathrm{e}^{2\mathbf{V}_{IJ}} D_\alpha \, \mathrm{e}^{-2\mathbf{V}_{IJ}} \bar{\varphi}_{IJ} = 0 \quad \text{for} \quad I > J. \tag{3.10}$$

We see that the chiral superfields appear in pairs with positive and negative charges with respect to the Abelian gauge superfield $\mathbf{V}_{IJ}$. This prevents the generation of the Chern–Simons term in the one-loop computations (there is no parity anomaly [75–77]). As a result, the effective action for the ghost superfields reads

$$\Gamma_{\mathrm{gh}} = -3i \sum_{I < J}^{N} \mathrm{Tr}_+ \ln \hat{\Box}_{+IJ}, \tag{3.11}$$

where $\hat{\Box}_{+IJ}$ is the Abelian version of the operator (2.27) constructed from the gauge superfield $\mathbf{V}_{IJ}$,

$$\hat{\Box}_{+IJ} = \mathcal{D}^m \mathcal{D}_m + \mathbf{G}_{IJ}^2 + \frac{i}{2}(\mathcal{D}^\alpha \mathbf{W}_{IJ\,\alpha}) + i\mathbf{W}_{IJ}^\alpha \mathcal{D}_\alpha, \tag{3.12}$$

and Tr_+ denotes the functional trace in the space of chiral superfields.

To do the explicit quantum computations of traces of logarithms in (3.7) and (3.11), we have to specify the constraints on the background Abelian superfields:

(i) The matrix components of the background gauge superfield $\mathbf{V}_{IJ}$ obey the $\mathcal{N} = 2$ supersymmetric Maxwell equations,

$$D^\alpha \mathbf{W}_{IJ\,\alpha} = \bar{D}^\alpha \bar{\mathbf{W}}_{IJ\,\alpha} = 0. \tag{3.13}$$

(ii) We study the effective action in the so-called long-wave approximation in which the space-time derivatives of the background are neglected,

$$\partial_m \mathbf{G}_{IJ} = \partial_m \mathbf{W}_{IJ\,\alpha} = \partial_m \bar{\mathbf{W}}_{IJ\,\alpha} = 0. \tag{3.14}$$

For such a background the heat kernels of the operators (3.6) and (3.12) are known, see [65]. In the present notations they read

$$K_{\mathrm{v}\,IJ}(z, z'|s) = \frac{1}{8(i\pi s)^{3/2}} \frac{s\mathbf{B}_{IJ}}{\sinh{(s\mathbf{B}_{IJ})}} \mathrm{e}^{is\mathbf{G}^2_{IJ}} \times$$

$$\times \exp\left[\frac{i}{4}(\mathbf{F}_{IJ} \coth s\mathbf{F}_{IJ})_{mn}\zeta^m(s)\zeta^n(s)\right] \zeta^2(s)\bar{\zeta}^2(s), \quad (3.15)$$

$$K_{+IJ}(z, z'|s) = -\frac{1}{4}\bar{\mathcal{D}}^2 K_{\mathrm{v}IJ}(z, z'|s), \tag{3.16}$$

where $\mathbf{B}^2_{IJ} = (1/2)D_\alpha \mathbf{W}^\beta_{IJ} D_\beta \mathbf{W}^\alpha_{IJ}$, and

$$\zeta^m = (x - x')^m - i\zeta\gamma^m\bar{\theta}' + i\theta'\gamma^m\bar{\zeta}, \quad \zeta^\alpha = \theta^\alpha - \theta'^\alpha, \quad \bar{\zeta}^\alpha = \bar{\theta}^\alpha - \bar{\theta}'^\alpha \tag{3.17}$$

are the components of supersymmetric interval. In fact, for the one-loop computations, we need these expressions only at coincident superspace points,

$$K_{\mathrm{v}\,IJ}(s) \equiv K_{\mathrm{v}\,IJ}(z, z|s) = \frac{1}{(i\pi)^{3/2}} \frac{1}{\sqrt{s}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^3_{IJ}} \times$$

$$\times \mathrm{e}^{is\mathbf{G}^2_{IJ}} \tanh\frac{s\mathbf{B}_{IJ}}{2} \sinh^2\frac{s\mathbf{B}_{IJ}}{2}, \quad (3.18)$$

$$K_{+IJ}(s) \equiv K_{+IJ}(z, z|s) = \frac{1}{8(i\pi s)^{3/2}} s^2 \mathbf{W}^2_{IJ}\, \mathrm{e}^{is\mathbf{G}^2_{IJ}} \frac{\tanh{(s\mathbf{B}_{IJ}/2)}}{s\mathbf{B}_{IJ}/2}. \tag{3.19}$$

The corresponding contributions to the effective action from these heat kernels are given by

$$\Gamma_{\mathrm{v}} = -i\sum_{I<J}^{N}\int_0^\infty \frac{ds}{s}\int d^7z\, K_{\mathrm{v}\,IJ}(s), \quad \Gamma_{\mathrm{gh}} = -3i\sum_{I<J}^{N}\int_0^\infty \frac{ds}{s}\int d^5z\, K_{+IJ}(s),$$

$$(3.20)$$

or, explicitly,

$$\Gamma_{\mathrm{v}} = -\frac{1}{\pi}\sum_{I<J}^{N}\int d^7z \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^3_{IJ}} \mathrm{e}^{is\mathbf{G}^2_{IJ}} \tanh\frac{s\mathbf{B}_{IJ}}{2} \sinh^2\frac{s\mathbf{B}_{IJ}}{2},$$

$$(3.21)$$

$$\Gamma_{\mathrm{gh}} = -\frac{3}{2\pi}\sum_{I<J}^{N}\int d^7z\left[\mathbf{G}_{IJ}\ln\mathbf{G}_{IJ}+\right.$$

$$\left. + \frac{1}{4}\int_0^\infty \frac{ds}{\sqrt{i\pi s}} \mathrm{e}^{is\mathbf{G}^2_{IJ}} \frac{\mathbf{W}^2_{IJ}\bar{\mathbf{W}}^2_{IJ}}{\mathbf{B}^2_{IJ}}\left(\frac{\tanh{(s\mathbf{B}_{IJ}/2)}}{s\mathbf{B}_{IJ}/2} - 1\right)\right], \quad (3.22)$$

where in the expression for Γ_{gh} we restored the full superspace measure. The sum of the expressions (3.21) and (3.22) gives us the resulting one-loop effective action in the pure $\mathcal{N} = 2$ SYM theory for the gauge group $SU(N)$ spontaneously broken down to $U(1)^{N-1}$. We point out that only the leading $\mathbf{G} \ln \mathbf{G}$ term in the $\mathcal{N} = 2$ SYM effective action was obtained in [78, 79] using the duality transformations while the explicit quantum computations allow us to find all higher-order F^{2n} terms encoded in the proper-time integrals (3.21) and (3.22).

Let us comment on the case of minimal gauge symmetry breaking $SU(N) \rightarrow SU(N-1) \times \mathrm{U}(1)$. In this case, it is convenient to choose the background gauge superfield in the following form:

$$V = \frac{1}{N}\mathrm{diag}\left((N-1)\mathbf{V}, \underbrace{-\mathbf{V}, \ldots, -\mathbf{V}}_{N-1}\right), \tag{3.23}$$

where $\mathbf{V}$ is the Abelian gauge superfield with the superfield strengths $\mathbf{G}$, $\mathbf{W}_\alpha$, and $\bar{\mathbf{W}}_\alpha$. One can easily repeat all the above considerations for such a background or just extract the answer from (3.21) and (3.22) by substituting the corresponding expressions for $\mathbf{V}_{IJ}$. For simplicity, we give here only two leading terms in the corresponding effective action

$$\Gamma_{\mathrm{SYM}}^{\mathcal{N}=2} = -\frac{3(N-1)}{2\pi}\int d^7 z \, \mathbf{G} \ln \mathbf{G} + \frac{9(N-1)}{128\pi}\int d^7 z \frac{\mathbf{W}^2\bar{\mathbf{W}}^2}{\mathbf{G}^5} + \ldots \tag{3.24}$$

The first term in the rhs is responsible for the $\mathcal{N} = 2$ supersymmetric (and superconformal) generalization of the Maxwell F^2 term while the second one gives F^4 among other components. The dots here stand for higher orders of the Maxwell field strength.

3.2. $\mathcal{N} = 4$ SYM. Consider the $\mathcal{N} = 4$ SYM model with the classical action (1.21). We have to extend the background field method presented in Sec. 2 with the corresponding background-quantum splitting for the chiral superfield,

$$\Phi \rightarrow \Phi + g\phi, \quad \bar{\Phi} \rightarrow \bar{\Phi} + g\bar{\phi}. \tag{3.25}$$

Here the superfields Φ, ϕ and $\bar{\Phi}$, $\bar{\phi}$ in the right-hand sides are covariantly (anti)chiral with respect to the background gauge-covariant derivatives, $\mathcal{D}_\alpha\bar{\Phi} = \mathcal{D}_\alpha\bar{\phi} = 0$, $\bar{\mathcal{D}}_\alpha\Phi = \bar{\mathcal{D}}_\alpha\phi = 0$. The quantum gauge transformations for these superfields read

$$\delta\phi = i\left[\lambda, \frac{1}{g}\Phi + \phi\right], \quad \delta\bar{\phi} = i\left[\bar{\lambda}, \frac{1}{g}\bar{\Phi} + \bar{\phi}\right], \quad \delta\Phi = \delta\bar{\Phi} = 0. \tag{3.26}$$

Upon the background-quantum splitting (2.1) and (3.25), the $\mathcal{N} = 4$ SYM action (1.21) can be expanded in a series over the quantum superfields. In

particular, for the one-loop computations we need the quadratic part of this action,

$$S_2 = -\text{tr} \int d^7 z \, v \left[-\frac{1}{8}\mathcal{D}^\alpha \bar{\mathcal{D}}^2 \mathcal{D}_\alpha + \frac{i}{2}(\mathcal{D}^\alpha W_\alpha) + iW^\alpha \mathcal{D}_\alpha + \Phi\bar{\Phi} \right] v -$$

$$- \text{tr} \int d^7 z \left(-\bar{\phi}[\Phi, v] + \phi[\bar{\Phi}, v] + \frac{1}{2}\phi\bar{\phi} \right). \quad (3.27)$$

This action is invariant under the quantum gauge transformations (2.5) and (3.26). Therefore we fix the quantum gauge symmetry by the following gauge-fixing functions:

$$f = i\bar{\mathcal{D}}^2 v - \frac{i}{2}[\Phi, \bar{\mathcal{D}}^2 \Box_-^{-1} \bar{\phi}], \quad \bar{f} = i\mathcal{D}^2 v + \frac{i}{2}[\bar{\Phi}, \mathcal{D}^2 \Box_+^{-1} \phi]. \quad (3.28)$$

In comparison with (2.14), these functions have the terms depending on the background (anti)chiral superfields Φ and $\bar{\Phi}$ which are necessary to remove the mixed terms between the quantum gauge v and (anti)chiral $\bar{\phi}$, ϕ superfields. Such a gauge fixing is usually referred to as the generalized R_ξ gauge [80–82]. The corresponding gauge-fixing action reads

$$S_{\text{gf}} = \frac{1}{8}\text{tr} \int d^7 z \, \bar{f}f = \frac{1}{8}\text{tr} \int d^7 z \left(-\frac{1}{2}v\{\mathcal{D}^2, \bar{\mathcal{D}}^2\}v - \frac{1}{2}v\bar{\mathcal{D}}^2[\bar{\Phi}, \mathcal{D}^2 \Box_+^{-1} \phi] + \right.$$

$$\left. + \frac{1}{2}v\mathcal{D}^2[\Phi, \bar{\mathcal{D}}^2 \Box_-^{-1} \bar{\phi}] + \frac{1}{4}[\Phi, \bar{\mathcal{D}}^2 \Box_-^{-1} \bar{\phi}][\bar{\Phi}, \mathcal{D}^2 \Box_+^{-1} \phi] \right). \quad (3.29)$$

It is convenient at this point to specify the constraints on the background chiral superfields Φ and $\bar{\Phi}$,

$$\mathcal{D}_\alpha \Phi = 0, \quad \bar{\mathcal{D}}_\alpha \bar{\Phi} = 0, \quad (3.30)$$

i.e., they are covariantly constant. For such a background the action (3.29) simplifies,

$$S_{\text{gf}} = \text{tr} \int d^7 z \left(-\frac{1}{16}v\{\mathcal{D}^2, \bar{\mathcal{D}}^2\}v - v[\bar{\Phi}, \phi] + v[\Phi, \bar{\phi}] + \frac{1}{2}[\Phi, \Box_-^{-1} \bar{\phi}][\bar{\Phi}, \phi] \right). \quad (3.31)$$

As a result, the quadratic part of the action for the quantum superfields becomes very simple,

$$S_2 + S_{\text{gf}} = -\text{tr} \int d^7 z \left[v(\Box_{\text{v}} + \bar{\Phi}\Phi)v + \frac{1}{2}\phi(1 + \bar{\Phi}\Phi\Box_-^{-1})\bar{\phi} \right]. \quad (3.32)$$

Here we denote $\bar{\Phi}\Phi v = [\bar{\Phi}, [\Phi, v]]$ and $\bar{\Phi}\Phi\bar{\phi} = [\bar{\Phi}, [\Phi, \bar{\phi}]]$.

The quantum gauge transformations (2.5) and (3.26) define the action for the ghost superfields,

$$
S_{\text{gh}} = \text{tr} \int d^7 z (b + \bar{b}) L_{gv} [c + \bar{c} + \coth (L_{gv})(c - \bar{c})] -
$$
$$
- \text{tr} \int d^7 z \left(b[\Phi, \Box_-^{-1}[\bar{\Phi} + g\bar{\phi}, \bar{c}]] - \bar{b}[\bar{\Phi}, \Box_+^{-1}[\Phi + g\phi, c]] \right) - \text{tr} \int d^7 z \, \bar{\varphi}\varphi.
\tag{3.33}
$$

Here b and c are standard Faddeev–Popov ghosts, while φ is the Nielsen–Kallosh ghost. All these superfields are covariantly (anti)chiral. Up to the second order in quantum superfields, the ghost superfield action is given by

$$
S_{\text{gh}} = \text{tr} \int d^7 z \left[-b(1 + \Phi\Box_-^{-1}\bar{\Phi})\bar{c} + \bar{b}(1 + \bar{\Phi}\Box_+^{-1}\Phi)c - \bar{\varphi}\varphi \right].
\tag{3.34}
$$

The functional integral for the one-loop effective action reads

$$
e^{i\Gamma_{\text{SYM}}^{\mathcal{N}=4}[V,\Phi]} = e^{iS_{\text{SYM}}^{\mathcal{N}=4}[V,\Phi]} \int \mathcal{D}v \, \mathcal{D}\phi \, \mathcal{D}b \, \mathcal{D}c \, \mathcal{D}\varphi \, e^{iS_2 + iS_{\text{gf}} + iS_{\text{gh}}}.
\tag{3.35}
$$

Schematically, the one-loop effective action can be written as

$$
\Gamma_{\text{SYM}}^{\mathcal{N}=4} = \frac{i}{2} \, \text{Tr}_{\text{v}} \, \ln \left(\Box_{\text{v}} + \bar{\Phi}\Phi \right) - i \, \text{Tr}_{+} \, \ln \left(\Box_{+} + \bar{\Phi}\Phi \right).
\tag{3.36}
$$

The first term in the rhs in this expression comes from the quantum gauge superfield, while the second one takes into account the contributions from quantum chiral superfield ϕ and ghosts.

Now let us compute the traces of the logarithms of the operators in (3.36) for the gauge group $SU(N)$ spontaneously broken down to $U(1)^{N-1}$. The background gauge superfield V is specified in (3.3). The background chiral superfield Φ has similar structure,

$$
\Phi = \text{diag} \left(\boldsymbol{\Phi}_1, \boldsymbol{\Phi}_2, \ldots, \boldsymbol{\Phi}_N \right), \quad \sum_{I=1}^{N} \boldsymbol{\Phi}_I = 0.
\tag{3.37}
$$

The quantum gauge superfield v is given by the expansion (3.2), while the quantum chiral superfield ϕ is represented by the expression similar to (3.8). It is straightforward to compute the matrix traces in (3.36),

$$
\Gamma_{\text{SYM}}^{\mathcal{N}=4} = i \sum_{I<J}^{N} \text{Tr}_{\text{v}} \ln(\hat{\Box}_{\text{v}\,IJ} + \bar{\boldsymbol{\Phi}}_{IJ}\boldsymbol{\Phi}_{IJ}) - 2i \sum_{I<J}^{N} \text{Tr}_{+} \ln(\hat{\Box}_{+IJ} + \bar{\boldsymbol{\Phi}}_{IJ}\boldsymbol{\Phi}_{IJ}),
\tag{3.38}
$$

where $\mathbf{\Phi}_{IJ} = \mathbf{\Phi}_I - \mathbf{\Phi}_J$ and the operators $\hat{\Box}_{\mathrm{v}\,IJ}$ and $\hat{\Box}_{+IJ}$ are given in (3.6) and (3.12), respectively. The traces of the logarithms of these operators are computed in a similar way as in Subsec. 3.1. As a result, we get the one-loop effective action in the $\mathcal{N}=4$ SYM theory for the gauge group $SU(N)$ broken down to $U(1)^{N-1}$,

$$
\Gamma_{\mathrm{SYM}}^{\mathcal{N}=4} = -\frac{1}{\pi} \sum_{I<J}^{N} \int d^7 z \times
$$

$$
\times \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}_{IJ}^2 \bar{\mathbf{W}}_{IJ}^2}{\mathbf{B}_{IJ}^3} \, \mathrm{e}^{is(\mathbf{G}_{IJ}^2 + \bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ})} \tanh \frac{s\mathbf{B}_{IJ}}{2} \sinh^2 \frac{s\mathbf{B}_{IJ}}{2} -
$$

$$
-\frac{2}{\pi} \sum_{I<J}^{N} \int d^7 z \left[\mathbf{G}_{IJ} \ln \left(\mathbf{G}_{IJ} + \sqrt{\mathbf{G}_{IJ}^2 + \bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}} \right) - \sqrt{\mathbf{G}_{IJ}^2 + \bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ}} + \right.
$$

$$
\left. + \frac{1}{4} \int_0^\infty \frac{ds}{\sqrt{i\pi s}} \, \mathrm{e}^{is(\mathbf{G}_{IJ}^2 + \bar{\mathbf{\Phi}}_{IJ}\mathbf{\Phi}_{IJ})} \frac{\mathbf{W}_{IJ}^2 \bar{\mathbf{W}}_{IJ}^2}{\mathbf{B}_{IJ}^2} \left(\frac{\tanh(s\mathbf{B}_{IJ}/2)}{s\mathbf{B}_{IJ}/2} - 1 \right) \right]. \quad (3.39)
$$

We point out that only the leading terms given in the second line in (3.39) were studied in [78, 79] by employing the mirror symmetry, while here we computed also all higher-order terms which are responsible in components for all higher powers of the Maxwell field strength F^{2n}, $n \geqslant 2$.

In conclusion of this section, let us briefly comment on the case of minimal gauge symmetry breaking $SU(N) \to SU(N-1) \times U(1)$. The background chiral superfield Φ is chosen similarly as the gauge one (3.23),

$$
\Phi = \frac{1}{N} \, \mathrm{diag}\left((N-1)\Phi, \underbrace{-\Phi, \dots, -\Phi}_{N-1} \right). \quad (3.40)
$$

The leading terms in the $\mathcal{N}=4$ SYM effective action in this case are given by

$$
\Gamma_{\mathrm{SYM}}^{\mathcal{N}=4} = \frac{2(N-1)}{\pi} \int d^7 z \left[\sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}} - \mathbf{G} \ln \left(\mathbf{G} + \sqrt{\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi}} \right) + \right.
$$

$$
\left. + \frac{1}{32} \frac{\mathbf{W}^2 \bar{\mathbf{W}}^2}{(\mathbf{G}^2 + \bar{\mathbf{\Phi}}\mathbf{\Phi})^{5/4}} + \dots \right]. \quad (3.41)
$$

The first two terms in the rhs of this expression are responsible for $\mathcal{N}=4$ supersymmetric (and superconformal) generalization of the Maxwell F^2 term, while the third term gives F^4 among other components and the dots stand for higher-order terms.

Finally, let us comment on the following terms in the effective action (3.41):

$$\int d^7z[\mathbf{G}\ln(\mathbf{G}+\sqrt{\mathbf{G}^2+\bar{\mathbf{\Phi}}\mathbf{\Phi}})-\sqrt{\mathbf{G}^2+\bar{\mathbf{\Phi}}\mathbf{\Phi}}], \qquad (3.42)$$

which are known as the $\mathcal{N}=2$, $d=3$ superspace action of the improved tensor multiplet [67]. Note that analogous $\mathcal{N}=1$, $d=4$ superspace action of the improved tensor multiplet was constructed in [83]. It is interesting to point out that (3.42) was obtained in [84] as a dual representation of the classical action of the Abelian Gaiotto–Witten model [85]. Hence, the classical action of the Abelian Gaiotto–Witten model in the representation (3.42) arises as the leading term in the $\mathcal{N}=4$ SYM effective action.

3.3. $\mathcal{N}=8$ SYM. Consider the $\mathcal{N}=8$ SYM model with the classical action (1.25). The background-quantum splitting of the gauge superfield (2.1) is supplemented by the following splitting of the (anti)chiral superfields:

$$\Phi_i \to \Phi_i + g\phi_i, \qquad \bar{\Phi}^i \to \bar{\Phi}^i + g\bar{\phi}^i, \qquad (3.43)$$

with the corresponding «quantum» gauge transformations

$$\delta\phi_i = i\left[\lambda, \frac{1}{g}\Phi_i + \phi_i\right], \quad \delta\bar{\phi}^i = i\left[\bar{\lambda}, \frac{1}{g}\bar{\Phi}^i + \bar{\phi}^i\right], \quad \delta\Phi_i = \delta\bar{\Phi}^i = 0. \quad (3.44)$$

The gauge fixing functions are chosen in the form similar to (3.28),

$$f = i\bar{\mathcal{D}}^2 v - \frac{i}{2}[\Phi_i, \bar{\mathcal{D}}^2\Box_-^{-1}\bar{\phi}^i], \quad \bar{f} = i\mathcal{D}^2 v + \frac{i}{2}[\bar{\Phi}^i, \mathcal{D}^2\Box_+^{-1}\phi_i]. \qquad (3.45)$$

When the background superfields are covariantly constant, $\bar{\mathcal{D}}_\alpha\bar{\Phi}^i = 0$, $\mathcal{D}_\alpha\Phi_i = 0$, the quadratic part of the action with respect to the quantum superfields takes relatively simple form,

$$S_2 + S_{\mathrm{gf}} = \mathrm{tr}\int d^7z\left[v(\Box_{\mathrm{v}}+\bar{\Phi}^i\Phi_i)v + \frac{1}{2}\phi_i(\delta^i_j+\bar{\Phi}^i\Phi_j\Box_-^{-1})\bar{\phi}^j\right] +$$
$$+ \frac{1}{4}\left(\mathrm{tr}\int d^5z\,\varepsilon^{ijk}\phi_i[\Phi_j,\phi_k]+\mathrm{c.c.}\right). \quad (3.46)$$

Here we denote $\bar{\Phi}^i\Phi_i v = [\bar{\Phi}^i[\Phi_i, v]]$. The ghost superfield action is a simple generalization of (3.34),

$$S_{\mathrm{gh}} = \mathrm{tr}\int d^7z\left[-b(1+\Phi_i\Box_-^{-1}\bar{\Phi}^i)\bar{c} + \bar{b}(1+\bar{\Phi}^i\Box_+^{-1}\Phi_i)c - \bar{\varphi}\varphi\right]. \qquad (3.47)$$

As a result, we see that the one-loop effective action is relatively simple because it is defined by only one operator,

$$\Gamma_{\mathrm{SYM}}^{\mathcal{N}=8} = \frac{i}{2}\mathrm{Tr}_{\mathrm{v}}\ln\left(\Box_{\mathrm{v}}+\bar{\Phi}^i\Phi_i\right). \qquad (3.48)$$

The contributions from ghosts and chiral superfields cancel each other at one loop for the covariantly constant background similarly as it happens for the $\mathcal{N} = 4$, $d = 4$ SYM theory.

For the gauge group $SU(N)$ spontaneously broken down to $U(1)^{N-1}$, the gauge and chiral superfields are chosen as in (3.3) and (3.37). In this case, the trace of the logarithm in (3.48) is computed by standard methods described in Subsec. 3.1,

$$\Gamma_{\text{SYM}}^{\mathcal{N}=8} = i \sum_{I<I}^{N} \text{Tr}_\text{v} \ln \left(\hat{\Box}_{\text{v}\, IJ} + \bar{\mathbf{\Phi}}_{IJ}^{i} \mathbf{\Phi}_{i\, IJ} \right) = -\frac{1}{\pi} \sum_{I<J}^{N} \int d^7 z \times$$

$$\times \int_0^\infty \frac{ds}{s\sqrt{i\pi s}} \frac{\mathbf{W}_{IJ}^2 \bar{\mathbf{W}}_{IJ}^2}{\mathbf{B}_{IJ}^3} \, e^{is(\mathbf{G}_{IJ}^2 + \bar{\mathbf{\Phi}}_{IJ}^{i} \mathbf{\Phi}_{i\, IJ})} \tanh \frac{s\mathbf{B}_{IJ}}{2} \sinh^2 \frac{s\mathbf{B}_{IJ}}{2}. \quad (3.49)$$

In the case when the gauge group $SU(N)$ is spontaneously broken down to $SU(N - 1) \times U(1)$, the background superfields should be chosen as in (3.23) and (3.40). Then, the leading term in the effective action (3.49) is given by

$$\Gamma_{\text{SYM}}^{\mathcal{N}=8} = \frac{3(N-1)}{32\pi} \int d^7 z \frac{\mathbf{W}^2 \bar{\mathbf{W}}^2}{(\mathbf{G}^2 + \bar{\mathbf{\Phi}}^i \mathbf{\Phi}_i)^{5/2}} + \ldots \sim$$

$$\sim \int d^3 x \frac{(F^{mn} F_{mn})^2}{(f^{\underline{i}} f_{\underline{i}})^{5/2}} + \ldots, \quad (3.50)$$

where $f^{\underline{i}}$, $\underline{i} = 1, 2, \ldots, 7$ are the seven scalar fields in the $\mathcal{N} = 8$ SYM theory and dots stand for the higher-order terms. In [86], it was argued that the F^4 term in the $\mathcal{N} = 8$ SYM effective action (3.50) is one-loop exact in the perturbation theory, but it receives instanton corrections.

3.4. $\mathcal{N} = 2$ Chern–Simons Model. Let us consider pure $\mathcal{N} = 2$ Chern–Simons theory with the classical action (1.27). The background field method goes along the same lines as in Sec. 2, but in Eq. (2.19), we send $\alpha \to \infty$ and $\beta = 1$ to remove the term responsible for the gauge fixing in the SYM part of the action. Then the quadratic part of the action for the quantum gauge superfields reads

$$S_2 + S_{\text{gf}} = \text{tr} \int d^7 z \, vHv, \quad (3.51)$$

with H given in (2.24).

The action for the Nielsen–Kallosh ghost vanishes because $\varphi^2 \equiv 0$ for the anticommuting superfield. Hence, only Faddeev–Popov ghosts contribute in the pure Chern–Simons theory. As a result, the structure of one-loop effective action is given by

$$\Gamma_{\text{CS}}^{\mathcal{N}=2} = \Gamma_H + \Gamma_{\text{gh}}, \quad \Gamma_H = \frac{i}{2} \text{Tr}_\text{v} \ln H, \quad \Gamma_{\text{gh}} = -i \text{Tr}_+ \ln \Box_+. \quad (3.52)$$

The operator H is first order in space-time derivatives. Therefore we need to square it,

$$\Gamma_H = \frac{i}{4} \text{Tr}_{\text{v}} \ln H^2, \tag{3.53}$$

$$H^2 = -m^2 \left[\mathcal{D}^m \mathcal{D}_m + \frac{i}{2} (W^\alpha - \bar{W}^\alpha)(\mathcal{D}_\alpha + \bar{\mathcal{D}}_\alpha) \right], \tag{3.54}$$

where the mass $m^2 = k^2 g^4/(4\pi^2)$ was introduced in (1.37). Upon such a squaring we must care about the phase of the functional determinant,

$$\frac{1}{\sqrt{\det H}} = \frac{1}{|\sqrt{\det H}|} \exp\left(\frac{i\pi}{2} \eta[V] \right), \tag{3.55}$$

where $\eta[V]$ is the so-called eta-invariant (see [36] for details in the nonsupersymmetric case). It is a background-dependent functional formally defined as

$$\eta[V] = \frac{1}{2} \lim_{s \to 0} \sum_i \text{sign}\, \lambda_i |\lambda_i|^{-s}, \tag{3.56}$$

where λ_i are the eigenvalues of the operator H. Fortunately, in [66] it was proved that

$$\eta[V] = 0. \tag{3.57}$$

Indeed, the nonvanishing value of the eta-invariant might lead only to finite shifts of the Chern–Simons coupling constant k because of quantum divergences, but it is well known that there are no such shifts in the Chern–Simons models with $\mathcal{N} > 1$ supersymmetry [87].

It is important to specify the background above which one computes the quantum corrections. Recall that in the SYM theory we used the constant field background constrained by (3.14) and (3.30). Such constraints provided us with a consistent quantum field theory as such a background was a solution of classical equations of motion. However, in the pure $\mathcal{N} = 2$ Chern–Simons theory the equations of motion have only trivial solutions with vanishing gauge superfield strengths. Therefore, in quantizing the Chern–Simons theory we do not impose any constraints on the background and compute the leading terms in the derivative expansion of the effective action. In other words, there is no Coulomb branch and we need to study the effective action in the conformal branch when all fields are massless.

3.4.1. Contributions from Quantum Vector Superfields. As the quantum vector superfields are massless, we need to introduce an effective infrared cut-off m to avoid IR divergences. Then the effective action (3.53) can be represented as

$$\Gamma_H[V] = \frac{i}{4} \text{Tr}_{\text{v}} \int_0^\infty \frac{ds}{s} e^{-m^2 s} e^{-sH^2}. \tag{3.58}$$

Recall that $\mathrm{Tr}_{\mathrm{v}} \mathcal{O}$ for a differential operator $\mathcal{O}$ acting in the space of real super-fields is computed by the rule

$$\mathrm{Tr}_{\mathrm{v}} \mathcal{O} = \int d^7 z \, \mathcal{O} \delta^7(z - z')|_{z=z'}, \tag{3.59}$$

where $\delta^7(z - z') = \delta^3(x - x') \, \delta^4(\theta - \theta')$. Hence, to get nonvanishing result, we have to accumulate exactly two derivatives $\mathcal{D}_\alpha$ and two $\bar{\mathcal{D}}_\alpha$ ones on the delta function from the expansion of e^{-sH^2}. Such a decomposition is straightforward. The expression with the minimal number of superfield strengths reads

$$\Gamma_H = -\frac{1}{256\pi m^5} \times$$

$$\times \int d^7 z \left(\mathcal{W}^{a\alpha} \mathcal{W}^b_\alpha \mathcal{W}^{c\beta} \mathcal{W}^d_\beta - \frac{1}{2} \mathcal{W}^{a\alpha} \mathcal{W}^{b\beta} \mathcal{W}^c_\alpha \mathcal{W}^d_\beta \right) \mathbf{f}_{abcd} + O(m^{-6}), \tag{3.60}$$

where $\mathcal{W}^{a\alpha} \equiv (W^{a\alpha} - \bar{W}^{a\alpha}) T_a$, $[T_a, T_b] = f_{abc} T_c$, and

$$\mathbf{f}_{a_1 a_2 a_3 a_4} = f_{b_1 a_1 b_2} f_{b_2 a_2 b_3} f_{b_3 a_3 b_4} f_{b_4 a_4 b_1}. \tag{3.61}$$

Note that these terms do not have Abelian analogs. They simply vanish in the Abelian case.

3.4.2. Contributions from Ghost Superfields. Consider one-loop contributions from the Faddeev–Popov ghost superfields,

$$\Gamma_{\mathrm{gh}} = -i \, \mathrm{Tr}_+ \ln \Box_+ = -i \int d^5 z \int_0^\infty \frac{ds}{s} \, \mathrm{e}^{-sm^2} K_+(z|s), \tag{3.62}$$

where $K_+(z|s)$ is the heat kernel for the chiral box operator (2.27) with coincident superspace points,

$$K_+(z|s) = \mathrm{tr} \, \mathrm{e}^{-s\Box_+} \delta_+(z, z')|_{z=z'} \mathbf{1}. \tag{3.63}$$

Note that for the constant superfield strengths this heat kernel has exact expression (3.19). In the present section, we need the value of this heat kernel beyond the constant field approximation. In this case only the lower-order terms in the series expansion over the parameter s can be found exactly.

Using the relation (2.27) we can restore the full superspace measure in (3.62), but for the derivative of the heat kernel,

$$\int d^5 z \frac{d}{ds} K_+(z|s) = \frac{1}{4} \mathrm{tr} \int d^7 z \, \mathcal{D}^2 \, \mathrm{e}^{-s\Box_+} \delta_+(z, z')|_{z=z'}. \tag{3.64}$$

Let us make a series decomposition

$$\operatorname{tr} \mathcal{D}^2 \, \mathrm{e}^{-s\Box_+} \, \delta_+(z, z')|_{z=z'} = \frac{1}{(4\pi s)^{3/2}} \sum s^n C_n(z), \qquad (3.65)$$

with coefficients $C_n(z)$ being superfields in full superspace. Then we have

$$\Gamma_{\mathrm{gh}} = -\frac{i}{4} \int d^7 z \int\limits_0^\infty ds \, \mathrm{e}^{-m^2 s} \sum \frac{s^n C_n(z)}{\left(n - \frac{1}{2}\right) (4\pi s)^{3/2}} =$$

$$= -\frac{i}{32\pi^{3/2}} \sum_{n=1}^\infty \frac{\Gamma(n - 1/2)}{(n - 1/2)m^{2n-1}} \int d^7 z \, C_n(z). \quad (3.66)$$

As a result, we need to compute the superfield coefficients $C_n(z)$ in the series decomposition of the lhs in (3.65). These coefficients play the role of the superfield Schwinger–DeWitt coefficients [88].

To compute the coefficients C_n in (3.65), we use the Fourier representation for the chiral delta function,

$$\delta_+(z, z') = -4 \int \frac{d^3 p}{(2\pi)^3} d^2\eta \, \mathrm{e}^{ip^m \zeta_m + \eta^\alpha \zeta_\alpha}, \qquad (3.67)$$

where ζ^m and ζ^α are the components of supersymmetric interval (3.17). The coefficient -4 here is due to our normalization of the integration measure, $\int d^2\eta \, \eta^2 = 1$. Now we act by the operators $\mathcal{D}^2$ and $\Box_+$ on $\mathrm{e}^{ip^m \zeta_m + \eta^\alpha \zeta_\alpha}$ and consider the limit of coincident superspace points,

$$\operatorname{tr} \mathcal{D}^2 \, \mathrm{e}^{-s\Box_+} \delta_+(z, z')|_{z=z'} =$$

$$= -4\operatorname{tr} \int \frac{d^3 p}{(2\pi)^3} d^2\eta \, X^\alpha X_\alpha \, \mathrm{e}^{-s(X^m X_m + iX^\alpha W_\alpha + G^2)}|_{z=z'}, \quad (3.68)$$

where

$$X_m = \mathcal{D}_m + ip_m, \quad X_\alpha = \mathcal{D}_\alpha + \eta_\alpha - p_{\alpha\beta}\bar{\zeta}^\beta. \qquad (3.69)$$

Note that the last term $p_{\alpha\beta}\bar{\zeta}^\beta$ in X_α does not contribute to the expansion of the heat kernel as it vanishes in the limit of coincident superspace points. Now we expand the exponent in a series and integrate over the momenta, e.g.,

$$\int \frac{d^3 p}{(2\pi)^3} \, \mathrm{e}^{sp^2} = \frac{i}{(4\pi s)^{3/2}}. \qquad (3.70)$$

It is clear that C_0 is independent of the background and the decomposition starts with C_1. The latter appears from $-s(\Box + G^2)$ with the factor $i/(4\pi s)^{3/2}$

which is common in the expansion. In the next order of expansion after integration over p, we have exactly $+s\square$ that cancels gauge noninvariant contribution and then we find

$$C_1 = 4i \operatorname{tr} G^2. \tag{3.71}$$

As a result, the leading contribution to the effective action due to ghost superfields has the form of Yang–Mills action in the $\mathcal{N} = 2$, $d = 3$ superspace,

$$\Gamma_{\mathrm{gh}} = \frac{1}{8\pi^2 m} \operatorname{tr} \int d^7 z \, G^2 + O(m^{-2}) =$$

$$= -\frac{1}{16\pi^2 m} \operatorname{tr} \int d^5 z \, W^\alpha W_\alpha + O(m^{-2}). \tag{3.72}$$

This demonstrates that the Yang–Mills term is generated in the effective action of pure $\mathcal{N} = 2$ supersymmetric Chern–Simons theory by the ghost superfield loop. The appearance of this term in the effective action is not surprising as we break the conformal invariance and topological nature of pure $\mathcal{N} = 2$ Chern–Simons theory. Clearly, this term vanishes on shell for $W_\alpha = 0$. Similar F^2 terms in the off-shell effective action of nonsupersymmetric Chern–Simons theory were discussed in [38, 39, 89].

The procedure developed in this section allows one to compute all other superfield coefficients in (3.65) which contain higher orders of superfield strengths, but this task is more tedious for C_n with $n > 1$.

CONCLUSIONS

We have reviewed the construction of the background field method for gauge field theories in the $\mathcal{N} = 2$, $d = 3$ superspace and demonstrated its power for calculating the low-energy effective action for $\mathcal{N} = 2, 4, 8$, $d = 3$ super Yang–Mills models and $\mathcal{N} = 2$ super Chern–Simons model. The background-field-depended operators of quadratic fluctuations, which represent the key elements of the background field formalism, are exactly found in the $\mathcal{N} = 2$ super Yang–Mills and Chern–Simons models for arbitrary gauge superfield background. The structure of one-loop effective action for these models is discussed in detail. We have developed the $\mathcal{N} = 2$, $d = 3$ superfield heat kernel technique and applied it for calculating the low-energy effective actions in a form preserving manifest gauge invariance and $\mathcal{N} = 2$ supersymmetry.

For constant gauge superfield background, the heat kernel for the operator of quadratic fluctuations in the SYM theory was exactly found. This allows us to find the one-loop effective action in the $\mathcal{N} = 2$, $\mathcal{N} = 4$ and $\mathcal{N} = 8$ SYM for such a background. However, in the pure $\mathcal{N} = 2$ Chern–Simons theory only vanishing gauge superfield background is allowed as a solution of classical

equations of motion. Therefore we compute the effective action in the $\mathcal{N} = 2$ Chern–Simons theory only in the conformal branch when all the gauge degrees of freedom are massless. In this case, we consider the arbitrary background superfield and compute the leading terms in the effective action of this model containing the lowest number of gauge superfields. We show that such an off-shell effective action contains the Yang–Mills term which appears due to ghost superfield contributions.

The methods developed in [64–66] and reviewed in this paper can be applied to a wide class of three-dimensional extended supersymmetric gauge theories. For example, it would be interesting to study the higher loop low-energy effective action in three-dimensional $\mathcal{N} = 2$ and $\mathcal{N} = 4$ SYM theories with matter and in the models containing both SYM and Chern–Simons terms together. More importantly, it is tempting to study the low-energy effective action of ABJM-like models which could correspond to the effective action of the M2 brane on the $AdS_4 \times S^7$ background. The latter can provide one more nontrivial evidence of the AdS_4/CFT_3 correspondence.

REFERENCES

1. *Kazakov D. I.* Finite $\mathcal{N} = 1$ SUSY Field Theories and Dimesional Regularization // Phys. Lett. B. 1986. V. 179. P. 352.

2. *Kazakov D. I.* Finite $\mathcal{N} = 1$ SUSY Gauge Field Theories // Mod. Phys. Lett. A. 1987. V. 2. P. 663.

3. *Ermushev A. V., Kazakov D. I., Tarasov O. V.* Finite $\mathcal{N} = 1$ Supersymmetric Grand Unified Theories // Nucl. Phys. B. 1987. V. 281. P. 72.

4. *Avdeev L. V., Grigoryev G.V., Kazakov D. I.* Renormalizations in Abelian Chern–Simons Field Theories with Matter // Nucl. Phys. B. 1992. V. 382. P. 561.

5. *Avdeev L. V., Kazakov D. I., Kondrashuk I. N.* Renormalizations in Supersymmetric and Nonsupersymmetric Non-Abelian Chern–Simons Field Theories with Matter // Nucl. Phys. B. 1993. V. 391. P. 333.

6. *Bagger J., Lambert N.* Modeling Multiple M2's // Phys. Rev. D. 2007. V. 75. P. 045020; hep-th/0611108.

7. *Bagger J., Lambert N.* Gauge Symmetry and Supersymmetry of Multiple M2 Branes // Phys. Rev. D. 2008. V. 77. P. 065008; arxiv:0711.0955.

8. *Bagger J., Lambert N.* Comments on Multiple M2 Branes // JHEP. 2008. V. 02. P. 105; arxiv:0712.3738.

9. *Bagger J., Lambert N.* Three-Algebras and $N = 6$ Chern–Simons Gauge Theories // Phys. Rev. D. 2009. V. 79. P. 025002; arxiv:0807.0163.

10. *Gustavsson A.* Algebraic Structures on Parallel M2 Branes // Nucl. Phys. B. 2009. V. 811. P. 66; arxiv:0709.1260.

11. *Gustavsson A.* Self-Dual Strings and Loop Space Nahm Equations // JHEP. 2008. V. 04. P. 083; arxiv:0802.3456.

12. *Aharony O. et al.* $N = 6$ Superconformal Chern–Simons-Matter Theories, M2 Branes and Their Gravity Duals // JHEP. 2008. V. 10. P. 091; arXiv:0806.1218.

13. *Benna M. et al.* Superconformal Chern–Simons Theories and AdS_4/CFT_3 Correspondence // JHEP. 2008. V. 09. P. 072; arxiv:0806.1519.

14. *Minahan J. A., Zarembo K.* The Bethe Ansatz for Superconformal Chern–Simons // JHEP. 2008. V. 09. P. 040; arxiv:0806.3951.

15. *Gaiotto D., Giombi S., Yin X.* Spin Chains in $N = 6$ Superconformal Chern–Simons-Matter Theory // JHEP. 2009. V. 04. P. 066; arxiv:0806.4589.

16. *Grignani G., Harmark T., Orselli M.* The $SU(2) \times SU(2)$ Sector in the String Dual of $N = 6$ Superconformal Chern–Simons Theory // Nucl. Phys. B. 2009. V. 810. P. 115; arxiv:0806.4959.

17. *Gromov N., Vieira P.* The All Loop AdS_4/CFT_3 Bethe Ansatz // JHEP. 2009. V. 01. P. 016; arxiv:0807.0777.

18. *Gromov N., Mikhaylov V.* Comment on the Scaling Function in $AdS_4 \times CP^3$ // JHEP. 2009. V. 04. P. 083; arxiv:0807.4897.

19. *Bak D., Rey S.-J.* Integrable Spin Chain in Superconformal Chern–Simons Theory // JHEP. 2008. V. 10. P. 053; arxiv:0807.2063.

20. *Minahan J. A., Schulgin W., Zarembo K.* Two-Loop Integrability for Chern–Simons Theories with $N = 6$ Supersymmetry // JHEP. 2009. V. 03. P. 057; arxiv:0901.1142.

21. *Bak D., Gang D., Rey S.-J.* Integrable Spin Chain of Superconformal $U(M) \times U(N)$ Chern–Simons Theory // JHEP. 2008. V. 10. P. 038; arxiv:0808.0170.

22. *Minahan J. A., Ohlsson Sax O., Sieg C.* Magnon Dispersion to Four Loops in the ABJM and ABJ Models // J. Phys. A. 2010. V. 43. P. 275402; arxiv:0908.2463.

23. *Minahan J. A., Ohlsson Sax O., Sieg C.* Anomalous Dimensions at Four Loops in $N = 6$ Superconformal Chern–Simons Theories // Nucl. Phys. B. 2011. V. 846. P. 542; arxiv:0912.3460.

24. *Leoni M. et al.* Superspace Calculation of the Four-Loop Spectrum in $N = 6$ Supersymmetric Chern–Simons Theories // JHEP. 2010. V. 12. P. 074; arxiv:1010.1756.

25. *Klose T.* Review of AdS/CFT Integrability, Ch. IV.3: $N = 6$ Chern–Simons and Strings on $AdS_4 \times CP^3$ // Lett. Math. Phys. 2012. V. 99. P. 401; arxiv:1012.3999.

26. *Chepelev I., Tseytlin A. A.* Long Distance Interactions of Branes: Correspondence between Supergravity and Super Yang–Mills Descriptions // Nucl. Phys. B. 1998. V. 515. P. 73; hep-th/9709087.

27. *Maldacena J. M.* Branes Probing Black Holes // Nucl. Phys. Proc. Suppl. 1998. V. 68. P. 17; hep-th/9709099.

28. *Tseytlin A. A.* Born–Infeld Action, Supersymmetry and String Theory. hep-th/9908105.

29. *Buchbinder I. L., Kuzenko S. M., Tseytlin A. A.* On Low-Energy Effective Actions in $N = 2$, $N = 4$ Superconformal Theories in Four Dimensions // Phys. Rev. D. 2000. V. 62. P. 045001; hep-th/9911221.

30. *Kuzenko S. M., McArthur I. N.* Relaxed Superself-Duality and Effective Action // Phys. Lett. B. 2004. V. 591. P. 304; hep-th/0403082.

31. *Kuzenko S. M., McArthur I. N.* Relaxed Superself-Duality and $N = 4$ SYM at Two Loops // Nucl. Phys. B. 2004. V. 697. P. 89; hep-th/0403240.

32. *Kuzenko S. M.* Self-Dual Effective Action of $N = 4$ SYM Revisited // JHEP. 2005. V. 03. P. 008; hep-th/0410128.

33. *Deser S., Jackiw R., Templeton S.* Three-Dimensional Massive Gauge Theories // Phys. Rev. Lett. 1982. V. 48. P. 975.

34. *Deser S., Jackiw R., Templeton S.* Topologically Massive Gauge Theories // Ann. Phys. 1982. V. 140. P. 372.

35. *Deser S., Kay J. H.* Topologically Massive Supergravity // Phys. Lett. B. 1983. V. 120. P. 97.

36. *Witten E.* Quantum Field Theory and the Jones Polynomial // Commun. Math. Phys. 1989. V. 121. P. 351.

37. *Pisarski R. D., Rao S.* Topologically Massive Chromodynamics in the Perturbative Regime // Phys. Rev. D. 1985. V. 32. P. 2081.

38. *Chen W., Semenoff G. W., Wu Y.-S.* Probing Topological Features in Perturbative Chern–Simons Gauge Theory // Mod. Phys. Lett. A. 1990. V. 5. P. 1833.

39. *Chen W., Semenoff G. W., Wu Y.-S.* Two-Loop Analysis of Non-Abelian Chern–Simons Theory // Phys. Rev. D. 1992. V. 46. P. 5521; hep-th/9209005.

40. *Martin C. P.* Dimensional Regularization of Chern–Simons Field Theory // Phys. Lett. B. 1990. V. 241. P. 513.

41. *Giavarini G., Martin C. P., Ruiz Ruiz F.* Chern–Simons Theory as the Large Mass Limit of Topologically Massive Yang–Mills Theory // Nucl. Phys. B. 1992. V. 381. P. 222; hep-th/9206007.

42. *Kapustin A. N., Pronin P. I.* Nonrenormalization Theorem for Gauge Coupling in $(2 + 1)$ Dimensions // Mod. Phys. Lett. A. 1994. V. 9. P. 1925; hep-th/9401053.

43. *Dunne G. V.* Aspects of Chern–Simons Theory. hep-th/9902115.

44. *Buchbinder I. L. et al.* Quantum $N = 3$, $d = 3$ Chern–Simons Matter Theories in Harmonic Superspace // JHEP. 2009. V. 10. P. 075; arXiv:0909.2970.

45. *Bianchi M. S., Penati S.* The Conformal Manifold of Chern–Simons Matter Theories // JHEP. 2011. V. 01. P. 047; arXiv:1009.6223.

46. *Bianchi M. S., Penati S., Siani M.* Infrared Stability of $N = 2$ Chern–Simons Matter Theories // JHEP. 2010. V. 05. P. 106; arXiv:0912.4282.

47. *Bianchi M. S., Penati S., Siani M.* Infrared Stability of ABJ-Like Theories // JHEP. 2010. V. 01. P. 080; arXiv:0910.5200.

48. *DeWitt B. S.* Quantum Theory of Gravity. 2. The Manifestly Covariant Theory // Phys. Rev. 1967. V. 162. P. 1195.

49. *Kallosh R.* The Renormalization in Non-Abelian Gauge Theories // Nucl. Phys. B. 1974. V. 78. P. 293.

50. *Arefeva I. Ya., Faddeev L. D., Slavnov A. A.* Generating Functional for the S-Matrix in Gauge Theories // Teor. Mat. Fiz. 1978. V. 21. P. 311 [Theor. Math. Phys. 1975. V. 21. P. 1165].

51. *Grisaru M. T., van Nieuwenhuizen P., Wu C. C.* Background Field Method versus Normal Field Theory in Explicit Examples: One-Loop Divergences in the S-Matrix and Green's Functions for Yang–Mills and Gravitational Fields // Phys. Rev. D. 1975. V. 12. P. 3203.

52. *Abbott L. F.* Introduction to the Background Field Method // Acta Physica Polonica. B. 1975. V. 13. P. 33.

53. *Vilkovisky G. A.* The Unique Effective Action in Quantum Field Theory // Nucl. Phys. B. 1984. V. 234. P. 125.

54. *Grisaru M. T., Siegel W.* Improved Methods for Supergraphs // Nucl. Phys. B. 1979. V. 159. P. 429; Supergraphity. Part 1. Background Field Formalism // Nucl. Phys. B. 1981. V. 187. P. 149.

55. *Buchbinder I. L. et al.* The Background Field Method for $\mathcal{N} = 2$ Super Yang–Mills Theories in Harmonic Superspace // Phys. Lett. B. 1998. V. 417. P. 61; hep-th/9704214.

56. *Mukhi S., Papageorgakis C.* M2 to D2 // JHEP. 2008. V. 05. P. 085; arXiv:0803.3218.

57. *Gran U., Nilsson B. E. W., Petersson C.* On Relating Multiple M2 and D2 Branes // JHEP. 2008. V. 10. P. 067; arXiv:0804.1784.

58. *Ho P.-M., Imamura Y., Matsuo Y.* M2 to D2 Revisited // JHEP. 2008. V. 07. P. 003; arXiv:0805.1202.

59. *Li T., Liu Y., Xie D.* Multiple D2 Brane Action from M2 Branes // Intern. J. Mod. Phys. A. 2009. V. 24. P. 3039; arXiv:0807.1183.

60. *Pang Y., Wang T.* From N M2's to N D2's // Phys. Rev. D. 2008. V. 78. P. 125007; arXiv:0807.1444.

61. *Ezhuthachan B., Mukhi S., Papageorgakis C.* D2 to D2 // JHEP. 2008. V. 07. P. 041; arXiv:0806.1639.

62. *Ezhuthachan B., Mukhi S., Papageorgakis C.* The Power of the Higgs Mechanism: Higher-Derivative BLG Theories // JHEP. 2009. V. 04. P. 101; arXiv:0903.0003.

63. *Buchbinder I. L. et al.* ABJM Models in $N = 3$ Harmonic Superspace // JHEP. 2003. V. 03. P. 096; arXiv: 0811.4774.

64. *Buchbinder I. L., Pletnev N. G., Samsonov I. B.* Low-Energy Effective Actions in Three-Dimensional Extended SYM Theories // JHEP. 2011. V. 01. P. 121; arXiv:1010.4967.

65. *Buchbinder I. L., Pletnev N. G., Samsonov I. B.* Effective Action of Three-Dimensional Extended Supersymmetric Matter on Gauge Superfield Background // JHEP. 2010. V. 04. P. 124; arXiv:1003.4806.

66. *Buchbinder I. L., Pletnev N. G.* The Background Field Method for $N = 2$, $d = 3$ Super Chern–Simons-Matter Theories // JHEP. 2011. V. 11. P. 085; arxiv:1108.2966.

67. *Hitchin N. J. et al.* Hyper-Kähler Metrics and Supersymmetry // Commun. Math. Phys. 1987. V. 108. P. 535.

68. *Zupnik B. M., Pak D. G.* Superfield Formulation of the Simplest Three-Dimensional Gauge Theories and Conformal Supergravities // Theor. Math. Phys. 1988. V. 77. P. 1070 [Teor. Mat. Fiz. 1988. V. 77. P. 97].

69. *Zupnik B. M., Pak D. G.* Topologically Massive Gauge Theories in Superspace // Sov. Phys. J. 1988. V. 31. P. 962.

70. *Ivanov E. A.* Chern–Simons Matter Systems with Manifest $N = 2$ Supersymmetry // Phys. Lett. B. 1991. V. 268. P. 203.

71. *Gates Jr. S. J., Nishino H.* Remarks on the $N = 2$ Supersymmetric Chern–Simons Theories // Phys. Lett. B. 1992. V. 281. P. 72.

72. *Nishino H., Gates Jr. S. J.* Chern–Simons Theories with Supersymmetries in Three Dimensions // Intern. J. Mod. Phys. A. 1993. V. 8. P. 3371.

73. *Kao H.-C., Lee K.-M.* Self-Dual Chern–Simons Systems with an $N = 3$ Extended Supersymmetry // Phys. Rev. D. 1992. V. 46. P. 4691; hep-th/9205115.

74. *Kao H.-C.* Self-Dual Yang–Mills Chern–Simons Higgs Systems with an $N = 3$ Extended Supersymmetry // Phys. Rev. D. 1994. V. 50. P. 2881.

75. *Niemi A. J., Semenoff G. W.* Axial-Anomaly-Induced Fermion Fractionization and Effective Gauge-Theory Actions in Odd-Dimensional Space-Times // Phys. Rev. Lett. 1983. V. 51. P. 2077.

76. *Redlich A. N.* Gauge Noninvariance and Parity Violation of Three-Dimensional Fermions // Phys. Rev. Lett. 1984. V. 52. P. 18.

77. *Redlich A. N.* Parity Violation and Gauge Noninvariance of the Effective Gauge-Field Action in Three Dimensions // Phys. Rev. D. 1984. V. 29. P. 2366.

78. *de Boer J., Hori K., Oz Y.* Dynamics of $N = 2$ Supersymmetric Gauge Theories in Three Dimensions // Nucl. Phys. B. 1997. V. 500. P. 163; hep-th/9703100.

79. *de Boer J. et al.* Branes and Mirror Symmetry in $N = 2$ Supersymmetric Gauge Theories in Three Dimensions // Nucl. Phys. B. 1997. V. 502. P. 107; hep-th/9702154.

80. *Banin A. T., Buchbinder I. L., Pletnev N. G.* On Low-Energy Effective Action in $N = 2$ Super Yang–Mills Theories on Non-Abelian Background // Phys. Rev. D. 2002. V. 66. P. 045021; hep-th/0205034.

81. *Ovrut B. A., Wess J.* Supersymmetric R_ξ Gauge and Radiative Symmetry Breaking // Phys. Rev. D. 1982. V. 25. P. 409.

82. *Binetruy P., Sorba P., Stora R.* Supersymmetric S-Covariant R_ξ Gauge // Phys. Lett. B. 1983. V. 129. P. 85.

83. *Lindström U., Roček M.* Scalar Tensor Duality and $N = 1, 2$ Nonlinear σ Models // Nucl. Phys. B. 1983. V. 222. P. 285.

84. *Koh E., Lee S., Lee S.* Topological Chern–Simons σ Model // JHEP. 2009. V. 09. P. 122; arXiv:0907.1641.

85. *Gaiotto D., Witten E.* Janus Configurations, Chern–Simons Couplings and the Theta-Angle in $N = 4$ Super Yang–Mills Theory // JHEP. 2010. V. 06. P. 097; arXiv:0804.2907.

86. *Dine M., Seiberg N.* Comments on Higher Derivative Operators in Some SUSY Field Theories // Phys. Lett. B. 1997. V. 409. P. 239; hep-th/9705057.

87. *Kao H.-C., Lee K.-M., Lee T.* The Chern–Simons Coefficient in Supersymmetric Yang–Mills Chern–Simons Theories // Phys. Lett. B. 1996. V. 373. P. 94; hep-th/9506170.

88. *Buchbinder I. L., Kuzenko S. M.* Ideas and Methods of Supersymmetry and Supergravity. Bristol; Philadelphia: IOP Publ., 1998.

89. *Birmingham D. et al.* Topological Field Theory // Phys. Rep. 1991. V. 209. P. 129.