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CHERN–SIMONS POTENTIAL IN MODELS OF INTERACTION OF ELECTROMAGNETIC FIELD WITH THIN FILMS

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The proposed by Symanzik approach for the modeling of interaction of a macroscopic material body with quantum fields is considered. Its application in quantum electrodynamics enables one to establish the most general form of the action functional describing the interaction of two-dimensional material surface with photon field. The models are presented that make it possible to calculate the Casimir energy, Casimir–Polder potential, characteristics of scattering processes and investigation of magneto- and electrostatic phenomena for thin films from nonideal conducting material. The specific of regularization and renormalization procedures used by calculations and the physical meaning of obtained results are discussed.

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INTRODUCTION

The models of quantum field theory describing interaction of elementary particles are considered usually in the homogeneous infinite space-time [1]. However, if one tries to construct a theory for investigation of phenomena of quantum fields interaction with macroscopic bodies, then at least its form must be presented in the model. This space-time inhomogeneity can change essentially many physical properties of vacuum and excited states of the system. The study of this problem is important both from the theoretical point of view and for possible application in nanotechnology, micromechanics and biophysics.

First quantitative results for quantum field theory model with space-time inhomogeneity were obtained by H. Casimir in 1948. He predicted [2] macroscopical attractive force between two uncharged conducting plates placed in vacuum. The force appears due to the influence of the boundary conditions on the electromagnetic quantum vacuum fluctuations. Nowadays, the Casimir effect is verified by experiments with a precision of 0.5% (see [3] for a review).

The properties of vacuum fluctuations in curved spaces, the scalar field models with various boundary conditions and their application to the description of real electromagnetic effects were actively studied throughout the last decades (see discussion and references in [4–8]). However, it was well understood from the beginning that boundary conditions must be considered just as an approximate description of a complex interaction of quantum fields with the matter. A generalization of the boundary conditions method has been proposed by Symanzik [9]. In the framework of path integral formalism he showed that the presence of material boundaries (two-dimensional defects) in the system can be modeled with a surface term added to the action functional. Such singular potentials with δ-type profile functions concentrated on the surface (defect) reproduce some simple boundary conditions (namely Dirichlet and Neumann ones) in the strong coupling limit. The additional action of the defect should not violate basic principles of the bulk model such as gauge invariance (if applicable), locality, and renormalizability.

The quantum field theory systems with δ-type potentials are mostly investigated for scalar fields, see, for instance, [10]. In [11–16], the Symanzik approach was used to describe similar problems in complete quantum electrodynamics (QED), and all δ-potentials consistent with the QED basic principles were constructed. In this paper we present typical problems which are solved for nonideal conducting materials in this approach on the cases of Casimir energy for two parallel plane films [11] and the spherical surface [14], Casimir–Polder potential for the atom near a plane [15], scattering of electromagnetic waves on an infinite plane film [16], interaction of the plane film with a parallel to it straight line current and with a point charge [11].

1. FORMULATION OF THE MODEL

The proposed by Symanzik action functional describing the interaction of the quantum field with material body has the form

$$S(\varphi) = S_V(\varphi) + S_{\rm def}(\varphi),$$

where

$$S_V(\varphi) = \int L(\varphi(x))\, d^D x, \quad S_{\rm def}(\varphi) = \int\limits_\Gamma L_{\rm def}(\varphi(x))\, d^{D'} x,$$

and Γ is a subspace of dimension $D' < D$ in D-dimensional space [9].

From the basic principles of QED — gauge invariance, locality, renormalizability — it follows that for thin film (without charges and currents) which shape is defined by equation $\Phi(x) = 0$, $x = (x_0, x_1, x_2, x_3)$, the action describing its interaction with photon field $A_\mu(x)$ reads

$$S_\Phi(A) = \frac{a}{2} \int \varepsilon^{\lambda\mu\nu\rho}\, \partial_\lambda \Phi(x)\, A_\mu(x)\, F_{\nu\rho}(x)\, \delta(\Phi(x))\, dx, \tag{1}$$

where $F_{\nu\rho}(x) = \partial_\nu A_\rho - \partial_\rho A_\nu$; $\varepsilon^{\lambda\mu\nu\rho}$ denotes totally antisymmetric tensor ($\varepsilon^{0123} = 1$); a is a constant dimensionless parameter. Expression (1) is the most general form of gauge invariant action concentrated on the defect surface being invariant in respect to reparameterization of one and not having any parameters with negative dimensions. The full action functional for electromagnetic field in the space-time with film defect including the usual free action of the photon field is written as

$$S(A,\Phi) = S_0(A) + S_\Phi(A), \quad S_0 = -\frac{1}{4}\int d^4x F^{\mu\nu}(x)F_{\mu\nu}(x). \tag{2}$$

In this paper, we consider stationary defects. In this case, $\partial_0\Phi(x) = 0$, and the action $S_\Phi(A)$ can be written as

$$S_\Phi(A) = \frac{a}{2}\int d^4x\, \delta(\Phi(x))\bigl\{2iA_0(x)\mathbf{L}_\Phi\mathbf{A}(x) + \boldsymbol{\partial}\Phi[\mathbf{A}(x)\times\partial_0\mathbf{A}(x)]\bigr\},$$

where $\mathbf{L}_\Phi \equiv i[\boldsymbol{\partial}\Phi\times\boldsymbol{\partial}]$. For the sphere with radius r_0:

$$\Phi(x) = \sqrt{x_1^2+x_2^2+x_3^2} - r_0, \quad \boldsymbol{\partial}\Phi(x) = \frac{\mathbf{x}}{|\mathbf{x}|} = \mathbf{n}(\mathbf{x}),$$

$$\mathbf{L}_\Phi = \frac{1}{|\mathbf{x}|}i[\mathbf{x}\times\boldsymbol{\partial}] = \frac{1}{|\mathbf{x}|}\mathbf{L}.$$

For the plane $x_3 = l$, the defect action reads

$$S_\Phi = \frac{a}{2}\int d^4x \epsilon^{3\mu\lambda\kappa}\delta(x_3 - l)\, A_\mu(x)\, \partial_\lambda A_\kappa(x).$$

The limit $a\to\infty$ corresponds to ideal conducting surface with conditions $n_\mu\tilde{F}^{\mu\nu}|_S = 0$.

The quantitative description of all physical phenomena caused by interaction of the photon field with film and classical charges and currents can be obtained if the generating functional of the Green functions is known. For gauge condition $\phi(A) = 0$, it is of the form

$$G(J) = C\int \mathrm{e}^{iS(A,\Phi)+iJA}\,\delta(\phi(A))\,dA, \tag{3}$$

where $S(A,\Phi)$ is given in (2), and the constant C is defined by normalization condition $G(0)|_{a=0} = 1$, i.e., in pure photodynamics without defect $\ln G(0)$ vanishes. The first term in the right-hand side is the usual action of photon field.

The full action $S(A,\Phi)$ (2) of the system can be written as $S(A,\Phi) = 1/2\, A_\mu K_\Phi^{\mu\nu} A_\nu$. The integral (3) is Gaussian and is calculated exactly:

$$G(J) = \exp\left\{\frac{1}{2}\,\mathrm{Tr}\,\ln\left(D_\Phi D^{-1}\right) - \frac{1}{2}JD_\phi J\right\},$$

where D_Φ is the propagator $D_\Phi = iK_\Phi^{-1}$ of photodynamics with defect in gauge $\phi(A) = 0$, and D is the propagator of free photon field in the same gauge. For the static defect, function $\Phi(x)$ is time-independent, and $\ln G(0)$ defines the Casimir energy.

2. REGULARIZATION AND EUCLIDEAN ROTATION

To remove the ultraviolet divergencies in $G(J)$, we introduce the Pauli–Willars regularization:

$$S_0 \to S_{0r} = -\frac{1}{4}\int d^4x F^{\mu\nu}(x)(1 + M^{-2}\partial_\lambda\partial^\lambda)F_{\mu\nu}(x),$$

$$S(A) \to S_r(A) = S_{0r} + S_{\text{def}}.$$

For performing the calculations, it is convenient to use the Euclidean version of the action S_E, which is obtained by replacement

$$x_0 \to -ix_0, \quad \partial_0 \to i\partial_0, \quad A_0 \to iA_0, \quad a \to ia.$$

In this case,

$$F^{\mu\nu}(x)F_{\mu\nu}(x) \to F_{\mu\nu}(x)F_{\mu\nu}(x), \quad d^4x \to -id^4x,$$

$$2iA_0(x)\mathbf{L}_\Phi\mathbf{A}(x) + \boldsymbol{\partial}\Phi\big[\mathbf{A}(x)\times\partial_0\mathbf{A}(x)\big] \to$$
$$\to -2A_0(x)\mathbf{L}_\Phi\mathbf{A}(x) + i\boldsymbol{\partial}\Phi\big[\mathbf{A}(x)\times\partial_0\mathbf{A}(x)\big].$$

Thus, $iS_r \to -S_{Er}$, where

$$S_{Er} = \frac{1}{4}\int d^4x\Big\{M^{-2}F_{\mu\nu}(x)(M^2 - \partial^2)F_{\mu\nu}(x) +$$
$$+ 2ia\delta(\Phi(x))\big(2A_0(x)\mathbf{L}_\Phi\mathbf{A}(x) - i\boldsymbol{\partial}\Phi\big[\mathbf{A}(x)\times\partial_0\mathbf{A}(x)\big]\big)\Big\}.$$

3. CASIMIR ENERGY

In the considered model, the Casimir energy E_{Cas} is obtained by calculation of Gaussian functional integral describing the interaction of vacuum fluctuation with defect:

$$E_{\text{Cas}} = -\frac{1}{T}\ln G(0) = -\frac{1}{T}\ln\left[C\int \mathrm{e}^{-S(A,\Phi)}DA\right].$$

It holds

$$E_{\text{Cas}} = -\frac{1}{T}Tr\ln(DD_0^{-1}),$$

where D is the propagator in the model with defect, and D_0 is the propagator for the model in homogeneous space.

For the simplest case of two plane parallel infinite films, the Casimir energy was calculated in [11]. If the defects are concentrated on planes $x_3 = 0$ and $x_3 = r$, the defect action (1) has the form

$$S_\Phi = S_{2P} = \frac{1}{2}\int (a_1\delta(x_3) + a_2\delta(x_3 - \mathrm{r}))\,\varepsilon^{3\mu\nu\rho}A_\mu(x)F_{\nu\rho}(x)\,dx.$$

For this geometry, it is convenient to use a notation like $x = (x_0, x_1, x_2, x_3) = (\mathbf{x}, x_3)$.

The defect action S_{2P} was considered in [18] in substantiation of Chern–Simons type boundary conditions chosen for studies of the Casimir effect in photodynamics. This approach based on boundary conditions is not related directly to the one we present. The defect action (1) is the main point in our model formulation, and no any boundary conditions are used.

The action S_{2P} is translationally invariant with respect to coordinates x_i, $i = 0, 1, 2$. The propagator $D_\Phi(x, y)$ is written as

$$D_{2P}(x,y) = \frac{1}{(2\pi)^3} \int D_{2P}(\mathbf{k}, x_3, y_3)\, \mathrm{e}^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}\, d\mathbf{k},$$

and $D_{2P}(\mathbf{k}, x_3, y_3)$ can be calculated exactly. Using Latin indices for the components of four-tensors with numbers 0, 1, 2 and notations

$$P^{lm}(\mathbf{k}) = \delta^{lm} - \frac{k^l k^m}{\mathbf{k}^2}, \quad L^{lm}(\mathbf{k}) = \epsilon^{lmn3} k_n$$

(δ is the Kronecker symbol), one can present the results for the Coulomb-like gauge $\partial_0 A^0 + \partial_1 A^1 + \partial_2 A^2 = 0$ as follows:

$$\begin{aligned}
D^{lm}_{2P}(\mathbf{k}, x_3, y_3) &= \frac{P^{lm}(\mathbf{k})(M_1 + M_2)}{2|\mathbf{k}|} - \\
&\quad - \frac{L^{lm}(\mathbf{k})(N_1 - N_2)}{2|\mathbf{k}|^2 \left[\left(1 + a_1 a_2 (\mathrm{e}^{2i|\mathbf{k}|\mathrm{r}} - 1)\right)^2 + (a_1 + a_2)^2\right]}, \\
D^{l3}_{2P}(\mathbf{k}, x_3, y_3) &= D^{3m}_{2P}(\mathbf{k}, x_3, y_3) = 0, \\
D^{33}_{2P}(\mathbf{k}, x_3, y_3) &= \frac{-i\delta(x_3 - y_3)}{|\mathbf{k}|^2}, \quad |\mathbf{k}| \equiv \sqrt{k_0^2 - k_1^2 - k_2^2},
\end{aligned}$$

with

$$\begin{aligned}
M_1 &= \frac{(a_1 a_2 - a_1^2 a_2^2 (1 - \mathrm{e}^{2i|\mathbf{k}|\mathrm{r}}))(\mathrm{e}^{i|\mathbf{k}|(|x_3| + |y_3 - \mathrm{r}|)} + \mathrm{e}^{i|\mathbf{k}|(|x_3 - \mathrm{r}| + |y_3|)})\, \mathrm{e}^{i|\mathbf{k}|\mathrm{r}}}{\left[(1 + a_1 a_2 (\mathrm{e}^{2i|\mathbf{k}|\mathrm{r}} - 1))^2 + (a_1 + a_2)^2\right]} - \\
&\quad - \mathrm{e}^{i|\mathbf{k}||x_3 - y_3|}, \\
M_2 &= \left[(a_1^2 + a_1^2 a_2^2 (1 - \mathrm{e}^{2i|\mathbf{k}|\mathrm{r}}))\, \mathrm{e}^{i|\mathbf{k}|(|x_3| + |y_3|)} + \right. \\
&\quad \left. + (a_2^2 + a_1^2 a_2^2 (1 - \mathrm{e}^{2i|\mathbf{k}|\mathrm{r}}))\, \mathrm{e}^{i|\mathbf{k}|(|x_3 - \mathrm{r}| + |y_3 - \mathrm{r}|)}\right] \times \\
&\quad \times \left[(1 + a_1 a_2 (\mathrm{e}^{2i|\mathbf{k}|\mathrm{r}} - 1))^2 + (a_1 + a_2)^2\right]^{-1},
\end{aligned}$$

$$\begin{aligned}
N_1 &= a_1 a_2 (a_1 + a_2)(\mathrm{e}^{i|\mathbf{k}|(|x_3| + |y_3 - \mathrm{r}|)} + \mathrm{e}^{i|\mathbf{k}|(|x_3 - \mathrm{r}| + |y_3|)})\mathrm{e}^{i|\mathbf{k}|\mathrm{r}}, \\
N_2 &= a_2 (1 + a_1 (a_1 + a_2\, \mathrm{e}^{2i|\mathbf{k}|\mathrm{r}}))\, \mathrm{e}^{i|\mathbf{k}|(|x_3 - \mathrm{r}| + |y_3 - \mathrm{r}|)} + \\
&\quad + a_1 (1 + a_2 (a_2 + a_1\, \mathrm{e}^{2i|\mathbf{k}|\mathrm{r}}))\, \mathrm{e}^{i|\mathbf{k}|(|x_3| + |y_3|)}.
\end{aligned}$$

The energy density E_{2P} of defect is defined as

$$\ln G(0) = \frac{1}{2}\,\mathrm{Tr}\,\ln\left(D_{2P}D^{-1}\right) = -iTSE_{2P},$$

where $T = \int dx_0$ is the time of defect duration, and $S = \int dx_1\,dx_2$ is the area of one. It is expressed in an explicit form in terms of polylogarithm function $\mathrm{Li}_4(x)$ in the following way:

$$E_{2P} = \sum_{j=1}^{2} E_j + E_{\mathrm{Cas}}, \quad E_j = \frac{1}{2}\int \ln\left(1 + a_j^2\right)\frac{d\mathbf{k}}{(2\pi)^3}, \quad j = 1, 2,$$

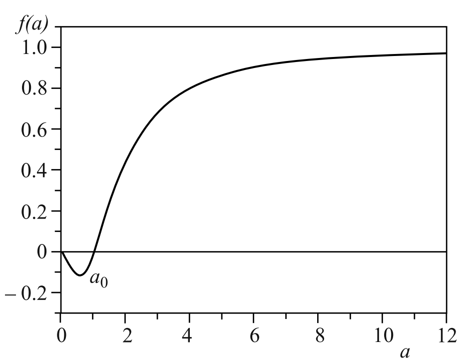
$$E_{\mathrm{Cas}} = -\frac{1}{16\pi^2 \mathrm{r}^3}\sum_{k=1}^{2}\mathrm{Li}_4\left(\frac{a_1 a_2}{a_1 a_2 + i(-1)^k(a_1 + a_2) - 1}\right).$$

Here E_j is an infinite constant, which can be interpreted as a self-energy density on the jth planes, and E_{Cas} is an energy density of their interaction. Function $\mathrm{Li}_4(x)$ is defined as

$$\mathrm{Li}_4(x) = \sum_{k=1}^{\infty}\frac{x^k}{k^4} = -\frac{1}{2}\int_0^{\infty} k^2 \ln\left(1 - x\,\mathrm{e}^{-k}\right)dk.$$

For identical defect planes ($a_1 = a_2 = a$), the force $F_{2P}(\mathrm{r}, a)$ between them is given by

$$F_{2P}(r, a) = -\frac{\partial E_{\mathrm{Cas}}(r, a)}{\partial r} = -\frac{\pi^2}{240 r^4} f(a).$$



Function $f(a)$ determining Casimir force between parallel planes

Function $f(a)$ is plotted in the Figure. It is even ($f(a) = f(-a)$) and has a minimum at $|a| = a_m \approx 0.5892$ ($f(a_m) \approx 0.11723$), $f(0) = f(a_0) = 0$ by $a_0 \approx 1.03246$, and $\lim_{a\to\infty} f(a) = 1$. For $0 < a < a_0$ ($a > a_0$), function $f(a)$ is negative (positive). Therefore the force F_{2P} is repulsive for $|a| < a_0$ and attractive for $|a| > a_0$. For large $|a|$, it is the same as the usual Casimir force between perfectly conducting planes. This model predicts that the maximal magnitude of the *repulsive* F_{2P} (about 0.1 of the Casimir force magnitude for perfectly conducting planes) is expected for $|a| \approx 0.6$.

For two infinitely thick parallel slabs the repulsive CF was predicted also in [19].

4. DIVERGENCES AND RENORMALIZATION

For the spherical defect, $E_{\rm Cas}$ diverges by $M \to \infty$. For large M, the asymptotics of the regularized Casimir energy of the spherical defect with radius r_0 has the form

$$E_{\rm Cas} = M^3 r_0^2 A(a) + MB(a) + \frac{F(a)}{r_0} + O\left(\frac{1}{M}\right),$$

with

$$F(a) = \frac{3}{64}\frac{a^2}{16+a^2} + \frac{1}{2\pi}\sum_{l=1}^{+\infty}(2l+1)\times$$
$$\times \int_0^\infty dp \left(\ln \frac{4 - a^2 \mathcal{G}_l(p)\,\mathcal{R}_l(p)}{16+a^2} + \frac{a^2(2l+1)^4}{(16+a^2)\,(4p^2+(2l+1)^2)^3}\right).$$

Here the following notations are used:

$$\mathcal{G}_l(x) = I_{l+\frac{1}{2}}(x) K_{l+\frac{1}{2}}(x),$$

$$\mathcal{R}_l(x) = \left(\frac{1}{2} I_{l+\frac{1}{2}}(x) + I'_{l+\frac{1}{2}}(x)\right)\left(\frac{1}{2} K_{l+\frac{1}{2}}(x) + K'_{l+\frac{1}{2}}(x)\right),$$

with Bessel function $I_{l+1/2}(x)$, $K_{l+1/2}(x)$.

It is finite for finite M but diverges for removing of regularization $M \to \infty$. This problem is solved by the renormalization.

For $a \to \infty$, we obtain

$$F_\infty = F(a)|_{a\to\infty} = \frac{3}{64} + \frac{1}{2\pi}\sum_{l=1}^{+\infty}(2l+1)\times$$
$$\times \int_0^\infty dp \left\{\ln[-4\mathcal{G}_l(p)\mathcal{R}_l(p)] + \frac{(2l+1)^4}{(4p^2+(2l+1)^2)^3}\right\}.$$

It is the result for ideal connecting sphere $E_{\rm Cas} = F_\infty / r_0$, coinciding with one obtained by Boyer.

For removing of the divergences of Casimir energy in the framework of usual multiplicative renormalization procedure, one needs to add to the action the terms without photon field with Lagrangian

$$L_{\rm cl}(x) = (A' r_0^2 + B')\,\delta(|\mathbf{x}| - r_0),$$

having two constant parameters A', B'. Making renormalization of them one can cancel the divergences and obtain the finite renormalized Casimir energy

$$E_{\mathrm{Cas}} = 4\pi r_0^2 \alpha + \beta + \frac{F(a)}{r_0},$$

with finite parameters α, β of dimension of surface energy density and energy. If $\alpha > 0$ and $F(\sigma) > 0$, then the function E_{Cas} has minimum with $r_0 = \sqrt[3]{F(\sigma)/8\pi\alpha}$.

5. CASIMIR–POLDER EFFECT

Casimir–Polder effect was predicted theoretically in 1948 [17]. Casimir and Polder found the energy of a neutral atom in its ground state in the presence of a perfectly conducting infinite plane. In the case of a perfectly conducting plane, one can say that the interaction of a fluctuating dipole with the electric field of its image yields the Casimir–Polder potential.

In our model, the interaction of the plane defect $x_3 = 0$ with a quantum electromagnetic field A_μ is described by the action:

$$S_{\mathrm{def}}(A) = a \int \epsilon^{\alpha\beta\gamma 3} A_\alpha(x)\, \partial_\beta A_\gamma(x)\, \delta(x_3)\, dx.$$

We will use Latin indices for the components of four-tensors with numbers 0, 1, 2 and also the following notations:

$$P^{lm}(\mathbf{k}) = g^{lm} - k^l k^m/\mathbf{k}^2,$$
$$L^{lm}(\mathbf{k}) = \epsilon^{lmn3} k_n/|\mathbf{k}|, \quad \mathbf{k}^2 = k_0^2 - k_1^2 - k_2^2,$$

where $|\mathbf{k}| = \sqrt{\mathbf{k}^2}$, and g is the metric tensor.

The atom is modeled as a localized electric dipole at the point $(x_1, x_2, x_3) = (0, 0, l)$, which is described by the current $J_\mu(x)$:

$$J_0(x) = \sum_{i=1}^{3} p_i(t)\, \partial^i \delta(x_1)\, \delta(x_2)\, \delta(x_3 - l),$$
$$J_i(x) = -\dot{p}_i(t)\, \delta(x_1)\, \delta(x_2)\, \delta(x_3 - l), \quad i = 1, 2, 3.$$

The condition of the current conservation holds:

$$\partial_\mu J^\mu = 0,$$

$p_i(t)$ is a function with a zero average and the pair correlation function

$$\langle p_j(t_1)p_k(t_2)\rangle = -i\int\limits_{-\infty}^{+\infty}\frac{\mathrm{e}^{-i\omega(t_1-t_2)}}{2\pi}\,\alpha_{jk}(\omega)\,d\omega,$$

where $\alpha_{jk}(\omega)$ for $\omega > 0$ coincides with the atomic polarizability.

The aim is to calculate the interaction energy E of the atom with a plane, and we will use the following representation for the energy:

$$E = \frac{i}{T}\left\langle\left\{\ln\int\exp\left(iS(A)+JA\right)\,DA - \ln\int\exp\left(iS(A)\right)\,DA\right\}_{(a)}\right\rangle,$$

$\{\cdots\}_{(a)}$ means that the $a = 0$ value of the a-dependent function has to be subtracted: $\{f(a)\}_{(a)} \equiv f(a) - f(0)$.

The ground state energy of a neutral atom in the presence of a plane with Chern–Simons interaction is obtained in the form [15]

$$\begin{aligned}E = -\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\Bigg(&\int\limits_0^{+\infty} d\omega \mathrm{e}^{-2\omega l}2(1+2\omega l)\alpha_{33}(i\omega)+\\ &+\int\limits_0^{+\infty} d\omega \mathrm{e}^{-2\omega l}(1+2\omega l+4\omega^2 l^2)\big(\alpha_{11}(i\omega)+\alpha_{22}(i\omega)\big)\Bigg)+\\ &+\frac{1}{64\pi^2 l^2}\frac{a}{1+a^2}\int\limits_0^{+\infty} d\omega \mathrm{e}^{-2\omega l}2\omega\big(1+2\omega l\big)\big(\alpha_{12}(i\omega)-\alpha_{21}(i\omega)\big).\end{aligned}$$

It yields the well-known Casimir–Polder potential [17] in the limit $a \to +\infty$. The part of the formula with diagonal matrix elements of matrix $\alpha_{jk}(i\omega)$ is equal to $a^2/(1+a^2)$ times the Casimir–Polder interaction of a neutral atom with a perfectly conducting plane. The last line of the formula is odd in a and contains the antisymmetric combination of the off-diagonal elements of the atomic polarizability. It is interesting to analyze the contribution in the energy E from the off-diagonal elements of the atomic polarizability to the potential in more detail. The atomic polarizability can be expressed in terms of dipole matrix elements:

$$\alpha_{jk}(\omega) = \sum_n\left(\frac{\langle 0|d_j|n\rangle\langle n|d_k|0\rangle}{\omega_{n0}-\omega-i\epsilon}+\frac{\langle 0|d_k|n\rangle\langle n|d_j|0\rangle}{\omega_{n0}+\omega-i\epsilon}\right),$$

ω_{n0} is a transition energy between the excited state $|n\rangle$ of the atom and its ground state $|0\rangle$, $\mathbf{d}$ is a dipole moment operator in the Schrödinger representation. The symmetric $\alpha^S_{jk}(\omega)$ and antisymmetric $\alpha^A_{jk}(\omega)$ parts of $\alpha_{jk}(\omega) = \alpha^S_{jk}(\omega) + \alpha^A_{jk}(\omega)$ can be written as follows:

$$\alpha^S_{jk}(\omega) = \sum_n \frac{2\omega_{n0}\,\mathrm{Re}\,M^n_{jk}}{\omega^2_{n0} - \omega^2} = \alpha^S_{kj}(\omega),$$

$$\alpha^A_{jk}(\omega) = \sum_n \frac{2i\omega\,\mathrm{Im}\,M^n_{jk}}{\omega^2_{n0} - \omega^2} = -\alpha^A_{kj}(\omega),$$

$$M^n_{jk} \equiv \langle 0|d_j|n\rangle\langle n|d_k|0\rangle.$$

Thus, the contribution of $\alpha^A_{jk}(\omega)$ to the potential is different from zero when matrix elements of a dipole moment operator have imaginary parts.

Consider the system with a nonzero $\alpha^A_{jk}(\omega)$ and assume, for simplicity, the one mode model of the atomic polarizability with a characteristic frequency ω_{10}. Then $\alpha^A_{12}(\omega) = i\omega C_2/(2(\omega^2_{10} - \omega^2))$, where C_2 is a real constant. In the limit of large separations $\omega_{10} l \gg 1$, we obtain

$$E|_{\omega_{01}l\gg 1} = -\frac{a^2}{1+a^2}\frac{\alpha_{11}(0)+\alpha_{22}(0)+\alpha_{33}(0)}{32\pi^2 l^4} - \frac{a}{1+a^2}\frac{C_2}{32\pi^2\omega^2_{10}l^5}. \tag{4}$$

At large enough separations, the first term in $E|_{\omega_{01}l\gg 1}$ always dominates. Assuming, for simplicity, $\alpha_{11}(0) = \alpha_{22}(0) = \alpha_{33}(0) = C_1/(3\,\omega_{10})$, C_1 is a positive constant, one can see from (4) that if the condition $\frac{|a|C_1}{|C_2|} < 1$ holds, then for separations $l \lesssim \frac{|C_2|}{|a|C_1\omega_{10}}$ the term with off-diagonal elements of the atomic polarizability (the second term in $E|_{\omega_{01}l\gg 1}$) dominates.

In the limit of short separations ($b \equiv \omega_{10} l \ll 1$) we obtain

$$\begin{aligned} E|_{\omega_{01}l\ll 1} = &-\frac{1}{64\pi^2 l^3}\frac{a^2}{1+a^2}\int\limits_0^{+\infty} d\omega\,\Big(\alpha_{11}(i\omega) + \alpha_{22}(i\omega) + 2\alpha_{33}(i\omega)\Big) - \\ &- \frac{C_2}{32\pi^2 l^3}\frac{a}{1+a^2}\Big(1 - \frac{\pi}{2}b + 2b^2 - \frac{\pi}{2}b^3 + \ldots\Big) \simeq \\ &\simeq -\frac{1}{32\pi^2 l^3}\Big(\frac{a^2}{1+a^2}C_1\frac{\pi}{3} + \frac{a}{1+a^2}C_2\Big) \text{ for } b \to 0. \end{aligned}$$

Hence, if the condition $\frac{|a|C_1}{|C_2|}\frac{\pi}{3} < 1$ holds, then the term with off-diagonal elements of the atomic polarizability dominates in $E|_{\omega_{01}l\ll 1}$ in the limit of short separations. Thus, if we consider the one mode model for the atomic polarizability and if the criterion $|a| \lesssim |C_2|/C_1$ holds, then the antisymmetric part of the atomic polarizability plays a dominant role in the interaction of the atom with the Chern–Simons plane.

6. MODIFICATION OF MAXWELL EQUATIONS

The interaction of electromagnetic field with the thin film changes the dynamical equation of the field. The modified homogeneous Maxwell equations are obtained as the Euler–Lagrange equations by variation of the action functional $S(A,\Phi)$ in (2). For the plane defect $\Phi(x)=x_3$, they are written as follows:

$$\frac{\delta S(A)}{\delta A_\nu}=\partial_\mu F^{\mu\nu}+a\varepsilon^{3\nu\sigma\rho}F_{\sigma\rho}\delta(x_3)=0. \tag{5}$$

Equations (5) were solved in [11]. To solve them, it is convenient to use the Fourier transform over coordinates x_0, x_1, x_2 for the vector-potential A_μ:

$$\begin{aligned} A_\mu(x)&=\frac{1}{(2\pi)^{3/2}}\int \mathrm{e}^{i\overline{px}}A_\mu(x_3,\overline{p})\,d\overline{p},\\ A_\mu(x_3,\overline{p})&=\frac{1}{(2\pi)^{3/2}}\int \mathrm{e}^{-i\overline{px}}A_\mu(x)\,d\overline{x}. \end{aligned} \tag{6}$$

Here and later we use the notation $\overline{p}$ for vector $\overline{p}=(p_0,p_1,p_2)$, $\overline{p}^2=p_0^2-p_1^2-p_2^2$, $\overline{px}=p_0x_0-p_1x_1-p_2x_2$. It follows from the second equation in (6) and reality of $A_\mu(x)$ that $A^*(x_3,\overline{p})=A(x_3,-\overline{p})$. Using this relation we can obtain an integral representation

$$\begin{aligned} A_\mu(x)&=\frac{1}{(2\pi)^{3/2}}\int \theta(p_0)\left[\mathrm{e}^{i\overline{px}}\,A_\mu(x_3,\overline{p})+\mathrm{e}^{-i\overline{px}}A^*_\mu(x_3,\overline{p})\right]d\overline{p}=\\ &=\frac{2\,\mathrm{Re}}{(2\pi)^{3/2}}\int \theta(p_0)\left[\mathrm{e}^{i\overline{px}}\,A_\mu(x_3,\overline{p})\right]d\overline{p}, \end{aligned} \tag{7}$$

where Re denotes the real part.

Action $S(A,\Phi)$ and Euler–Lagrange equations (5) are invariant in respect to the gauge transformation $A_\mu(x)\to A_\mu(x)+\partial_\mu\varphi(x)$. Thus, the solution of (5) is defined up to a gauge transformation, and we can fix it by a gauge condition. We make calculations in the temporal gauge $A_0=0$, where electric and magnetic fields $\mathbf{E}$, $\mathbf{H}$ are expressed through the vector-potential $A=(0,\mathbf{A})$ by relations $\mathbf{E}=\partial_0\mathbf{A}$, $\mathbf{H}=\boldsymbol{\partial}\times\mathbf{A}$, and we can rewrite equations (5) for $\mathbf{A}(x_3,\overline{p})$ in the form

$$\overline{p}^2A^3-\partial^3(ip_1A_1+ip_2A_2)=0, \tag{8}$$

$$p^0(ip_1A_1+ip_2A_2+\partial_3A_3)-2a(p_1A_2-p_2A_1)\delta(x_3)=0, \tag{9}$$

$$(-\overline{p}^2-\partial_3^2)A^1+ip^1(ip_1A_1+ip_2A_2+\partial_3A_3)+2aip_0A_2\delta(x_3)=0, \tag{10}$$

$$(-\overline{p}^2-\partial_3^2)A^2+ip^2(ip_1A_1+ip_2A_2+\partial_3A_3)-2aip_0A_1\delta(x_3)=0. \tag{11}$$

These linear homogeneous equations describe electromagnetic waves interacting with the material plane $x_3=0$. The general solution of (8)–(11) is constructed in [11] and the most characteristic features of scattering processes on the defect are investigated.

7. SOLUTION OF THE EULER–LAGRANGE EQUATIONS

Let us denote $A_1(0,\overline{p}) = a_1(\overline{p}),\ A_2(0,\overline{p}) = a_2(\overline{p})$. Then it follows from (9) that

$$ip_1A_1 + ip_2A_2 + \partial_3 A_3 = 2a\frac{p_1a_2 - p_2a_1}{p_0}\delta(x_3),$$

and in virtue of (10), (11), the fields A_1, A_2 satisfy the equations

$$(\overline{p}^2 + \partial_3^2)A_i + c_i\delta(x_3) = 0, \quad i = 1,2,$$

in which

$$c_1 \equiv -\frac{2ia}{p_0}\left[(p_1^2 - p_0^2)a_2 - p_1p_2a_1\right], \quad c_2 \equiv \frac{2ia}{p_0}\left[(p_2^2 - p_0^2)a_1 - p_1p_2a_2\right]. \quad (12)$$

General solution of the equation

$$\partial_t^2\psi + k^2\psi + c\delta(t) = 0$$

is

$$\psi(t) = d_1\,\mathrm{e}^{ikt} + d_2\,\mathrm{e}^{-ikt} + \frac{c\,\mathrm{e}^{-ik|t|}}{2pi},$$

where d_1, d_2 are arbitrary constants. Hence,

$$A_1(x_3,\overline{p}) = d_1^{(1)}\,\mathrm{e}^{i\rho x_3} + d_2^{(1)}\,\mathrm{e}^{-i\rho x_3} + \frac{c_1\,\mathrm{e}^{-i\rho|x_3|}}{2i\rho}, \quad (13)$$

$$A_2(x_3,\overline{p}) = d_1^{(2)}\,\mathrm{e}^{i\rho x_3} + d_2^{(2)}\,\mathrm{e}^{-i\rho x_3} + \frac{c_2\,\mathrm{e}^{-i\rho|x_3|}}{2i\rho}, \quad (14)$$

where $\rho \equiv \sqrt{\overline{p}^2}$, and $d_i^{(j)}$, $i,j = 1,2$ are functions of $\bar{p}$. We can obtain the field A_3 directly from Eq. (8):

$$A_3(x_3,\overline{p}) = d_1^{(3)}\,\mathrm{e}^{i\rho x_3} + d_2^{(3)}\,\mathrm{e}^{-i\rho x_3} + \epsilon(x_3)\frac{c_3\,\mathrm{e}^{-i\rho|x_3|}}{2i\rho}, \quad (15)$$

where

$$d_1^{(3)} = -\frac{1}{\rho}(p_1d_1^{(1)} + p_2d_1^{(2)}), \quad d_2^{(3)} = \frac{1}{\rho}(p_1d_2^{(1)} + p_2d_2^{(2)}), \quad c_3 = \frac{1}{\rho}(p_1c_1 + p_2c_2),$$

and $\epsilon(x_3) \equiv x_3/|x_3|$. We assume that the components of the vector-potential $\mathbf{A}(x_3,\overline{p})$ are limited for any value x_3. That is possible only if $\overline{p}^2 \geqslant 0$. Therefore, we consider only the case $\rho \geqslant 0$.

Putting $x_3 = 0$ in (13), (14) and denoting $D_j \equiv d_1^{(j)} + d_2^{(j)}$, $j = 1, 2$, we obtain the relations

$$a_j = d_1^{(j)} + d_2^{(j)} + \frac{c_j}{2i\rho}, \quad j = 1, 2. \tag{16}$$

In virtue of (16) and (12), $a_1,\ a_2$ satisfy the system of linear equations

$$\begin{aligned} &a_1(p_0\rho - ap_1p_2) + aa_2(p_1^2 - p_0^2) = D_1p_0, \\ &aa_1(p_2^2 - p_0^2) - a_2(p_0\rho + ap_1p_2) = -D_2p_0. \end{aligned} \tag{17}$$

Thus, it follows from (17), (12) that

$$a_1 = \frac{aD_2(p_0^2 - p_1^2) + D_1(ap_1p_2 + p_0\rho)}{p_0\rho^2(a^2+1)},$$

$$a_2 = -\frac{aD_1(p_0^2 - p_2^2) + D_2(ap_1p_2 - p_0\rho)}{p_0\rho^2(a^2+1)},$$

and

$$c_1 = -\frac{2ai[D_1(ap_0\rho - p_1p_2) - D_2(p_0^2 - p_1^2)]}{p_0\rho(a^2+1)},$$

$$c_2 = -\frac{2ai[D_2(ap_0\rho + p_1p_2)] + D_1(p_0^2 - p_2^2)]}{p_0\rho(a^2+1)}.$$

We denote $\mathbf{a} = (a_1, a_2, a_3)$, $\mathbf{c} = (c_1, c_2, c_3)/(2i\rho)$, $\mathbf{d}_j = (d_j^{(1)}, d_j^{(2)}, d_j^{(2)})$, $j = 1, 2$, and rewrite (13)–(15) in a compact form:

$$\mathbf{A}(x_3, \overline{p}) = \mathbf{d}_1(\overline{p})\,\mathrm{e}^{i\rho x_3} + \mathbf{d}_2(\overline{p})\,\mathrm{e}^{-i\rho x_3} + R(x_3)\mathbf{c}(\overline{p})\,\mathrm{e}^{-i\rho|x_3|}, \tag{18}$$

where $R(x_3)$ — diagonal matrix with elements $R_{11}(x_3) = R_{22}(x_3) = 1$, $R_{33}(x_3) = \epsilon(x_3)$.

Thus, using (7), (18), we obtain the following presentation for solution of Euler–Lagrange equations of our model:

$$\begin{aligned} \mathbf{A}(x) = \frac{\theta(x_3)2\,\mathrm{Re}}{(2\pi)^{3/2}} \int \theta(p_0)\Big\{\mathbf{d}_1(\overline{p})\,\mathrm{e}^{i(\overline{px}+\rho x_3)} + [\mathbf{d}_2(\overline{p}) + \mathbf{c}\,(\overline{p})]\,\mathrm{e}^{i(\overline{px}-\rho x_3)}\Big\}\,d\overline{p} + \\ + \frac{\theta(-x_3)2\,\mathrm{Re}}{(2\pi)^{3/2}} \int \theta(p_0)\Big\{[\mathbf{d}_1(\overline{p}) + T\mathbf{c}\,(\overline{p})]\,\mathrm{e}^{i(\overline{px}+\rho x_3)} + \mathbf{d}_2(\overline{p})\,\mathrm{e}^{i(\overline{px}-\rho x_3)}\Big\}\,d\overline{p}. \end{aligned} \tag{19}$$

Here T is a diagonal matrix with elements $T_{11} = T_{22} = -T_{33} = 1$. The first terms in the integrands in (19) describe waves moving in the negative direction of the third axis, and the second ones correspond to waves moving in the positive direction.

8. SCATTERING ON THE DEFECT

For the wave falling on the plane $x_3 = 0$ from the half space with negative coordinate x_3, we should have in a half space $x_3 > 0$ only the transmitted wave, moving from the plane $x_3 = 0$ in positive direction of the third axis. Hence, in (19) we must set $\mathbf{d}_1 = 0$. As a result, we obtain

$$\mathbf{A}(x) = \frac{\theta(x_3)2Re}{(2\pi)^{3/2}} \int \theta(p_0)\mathbf{A}_{\rm tr}(\overline{p})\, \mathrm{e}^{i(\overline{px}-\rho x_3)} d\overline{p} + \\ + \frac{\theta(-x_3)2Re}{(2\pi)^{3/2}} \int \theta(p_0) \left\{ \mathbf{A}_r\, \mathrm{e}^{i(\overline{px}+\rho x_3)} + \mathbf{A}_{\rm in}\, \mathrm{e}^{i(\overline{px}-\rho x_3)} \right\} d\overline{p},$$

where vector amplitudes $\mathbf{A}_{\rm in}(\overline{p}),\ \mathbf{A}_r(\overline{p}),\ \mathbf{A}_{\rm tr}(\overline{p})$ of the incident, transmitted, and reflected waves can be written as

$$\mathbf{A}_{\rm in}(\overline{p}) = \mathbf{d}_2(\overline{p}),\ \mathbf{A}_r(\overline{p}) = T\mathbf{c}\,(\overline{p}),\ \mathbf{A}_{\rm tr}(\overline{p}) = \mathbf{d}_2(\overline{p}) + \mathbf{c}\,(\overline{p}). \tag{20}$$

In virtue of (20), they satisfy the relation

$$\mathbf{A}_r = T(\mathbf{A}_{\rm tr} - \mathbf{A}_{\rm in}). \tag{21}$$

Thus, the vector amplitude of the reflected wave is obtained from the difference between the amplitudes of the incident and transmitted waves by changing the sign of its third component. It follows from (21) that there are only two independent vector wave amplitudes which determine the third.

9. EIGENMODES

We call eigenmodes the waves for which the amplitudes of the incident and transmitted waves are proportional to each other:

$$\mathbf{A}_{\rm tr}(\overline{p}) = \lambda \mathbf{A}_{\rm in}(\overline{p}). \tag{22}$$

For them, it follows from (21), (22) that

$$\mathbf{A}_r(\overline{p}) = (\lambda - 1)T\mathbf{A}_{\rm in}(\overline{p}), \quad a_1 = \lambda d_2^{(1)}, \quad a_2 = \lambda d_2^{(2)}.$$

Two last relations considered as a system of linear homogeneous equations for $d_2^{(1)},\ d_2^{(2)}$, have a nontrivial solution if $(a^2+1)\lambda^2 - 2\lambda + 1 = 0$. Thus, there are two eigenmodes with

$$\lambda = \frac{i}{i-a} \equiv \lambda_1, \quad \boldsymbol{\mathcal{A}}_{\rm in}^{(1)} = g_1\mathbf{V}_1; \quad \lambda = \frac{i}{i+a} \equiv \lambda_2, \quad \boldsymbol{\mathcal{A}}_{\rm in}^{(2)} = g_2\mathbf{V}_2, \tag{23}$$

where

$$\begin{aligned}\mathbf{V}_1 &\equiv (p_0^2 - p_1^2,\ -ip_0\rho - p_1p_2,\ -ip_0p_2 + p_1\rho),\\ \mathbf{V}_2 &\equiv (p_0^2 - p_1^2,\ ip_0\rho - p_1p_2,\ ip_0p_2 + p_1\rho),\end{aligned} \tag{24}$$

and $g_1,\ g_2$ are arbitrary functions of $\overline{p}$. Using the notations of (23), (24), we can write the eigenmode vector amplitudes $\boldsymbol{\mathcal{A}}_{\rm in}(\overline{p})$, $\boldsymbol{\mathcal{A}}_r(\overline{p})$, $\boldsymbol{\mathcal{A}}_{\rm tr}(\overline{p})$ of incident, reflected, and transmitted waves in the form

$$\boldsymbol{\mathcal{A}}_{\rm in}^{(j)} = g_j(\overline{p})\mathbf{V}_j(\overline{p}),\quad \boldsymbol{\mathcal{A}}_r^{(j)} = g_j(\overline{p})K_r^{(j)}T\mathbf{V}_j(\overline{p}),$$
$$\boldsymbol{\mathcal{A}}_{\rm tr}^{(j)} = g_j(\overline{p})K_{\rm tr}^{(j)}\mathbf{V}_j(\overline{p}),\quad j = 1, 2.$$

Here we used the notations

$$K_r^{(1)} = \frac{ia + a^2}{1 + a^2},\quad K_r^{(2)} = \frac{-ia + a^2}{1 + a^2},\quad K_{\rm tr}^{(1)} = \frac{1 - ia}{1 + a^2},\quad K_{\rm tr}^{(2)} = \frac{1 + ia}{1 + a^2}.$$

The obtained characteristics of eigenmodes satisfy the following relations: $\lambda_2 = \lambda_1^*$, $\mathbf{V}_2 = \mathbf{V}_1^*$, $\mathbf{V}_1\mathbf{V}_2^* = 0$, $|\mathbf{V}_1|^2 = |\mathbf{V}_2|^2 = 2p_0^2(p_0^2 - p_1^2)$, $K_{\rm tr}^{(2)} = K_{\rm tr}^{(1)*}$, $K_r^{(2)} = K_r^{(1)*}$.

10. PLANE WAVES

Choosing the functions $g_1(\bar{p})$, $g_2(\bar{p})$ in (23), we can represent in general case the amplitude of incident plane wave as a linear combination of eigenmodes

$$\boldsymbol{\mathcal{A}}_{\rm in} = \boldsymbol{\mathcal{A}}_{\rm in}^{(1)} + \boldsymbol{\mathcal{A}}_{\rm in}^{(2)} = g_1\mathbf{V}_1 + g_2\mathbf{V}_1^* = f_1\mathbf{U}_1 + if_2\mathbf{U}_2.$$

We used the notations $f_1 = g_1 + g_2$, $f_2 = g_1 - g_2$,

$$\mathbf{U}_1 = \Re V_1 = (p_0^2 - p_1^2,\ -p_1p_2,\ p_1\rho),\quad \mathbf{U}_2 = \Im V_1 = (0,\ -p_0\rho,\ -p_0p_2).$$

It is easy to see that $|\mathbf{U}_1| = |\mathbf{U}_2| = |\mathbf{V}_1|/2$, $\mathbf{U}_1\mathbf{U}_2 = 0$. If we denote $\mathbf{Y}_1 \equiv a\mathbf{U}_1 - \mathbf{U}_2$, $\mathbf{Y}_2 \equiv a\mathbf{U}_2 + \mathbf{U}_1$, then for the amplitudes of reflected and incident waves, we have

$$\boldsymbol{\mathcal{A}}_r = \frac{a}{1 + a^2}(f_1T\mathbf{Y}_1 + if_2T\mathbf{Y}_2),\quad \boldsymbol{\mathcal{A}}_{\rm tr} = \frac{1}{1 + a^2}(f_1\mathbf{Y}_2 - if_2\mathbf{Y}_1).$$

The plane wave is characterized by its propagation direction $\mathbf{n}$ and frequency ω, which are expressed through the components of the momentum $p = (p_0, \mathbf{p})$: $p_0 = \omega, \mathbf{p} = \omega\mathbf{n}$. In virtue of (7), the vector potentials $\mathbf{A}_{\rm in}(p_{\rm in}; x)$, $\mathbf{A}_r(p_r; x)$, $\mathbf{A}_{\rm tr}(p_{\rm tr}; x)$ of incident reflected and transmitted waves have the form

$$\mathbf{A}_{\rm in}(p_{\rm in}; x) = \alpha_{\rm in}\mathbf{U}_1 - \beta_{\rm in}\mathbf{U}_2,\quad \mathbf{A}_r(p_r; , x) = \alpha_rT\mathbf{Y}_1 - \beta_rT\mathbf{Y}_2,$$
$$\mathbf{A}_{\rm tr}(p_{\rm tr}; x) = \alpha_{\rm tr}\mathbf{Y}_2 + \beta_{\rm tr}\mathbf{Y}_1,$$

where $p_{\rm in} = p_{\rm tr} = (p_0, p_1, p_2, \rho)$, $p_r = (p_0, p_1, p_2, -\rho)$,

$$\alpha_{\rm in} = \frac{2|f_1|}{(2\pi)^{3/2}}\cos(p_{\rm in}x+\phi_1), \quad \alpha_r = \frac{a}{1+a^2}\frac{2|f_1|}{(2\pi)^{3/2}}\cos(p_r x+\phi_1),$$
$$\alpha_{\rm tr} = \frac{1}{1+a^2}\alpha_{\rm in},$$
$$\beta_{\rm in} = \frac{2|f_2|}{(2\pi)^{3/2}}\sin(p_{\rm in}x+\phi_2), \quad \beta_r = \frac{a}{1+a^2}\frac{2|f_2|}{(2\pi)^{3/2}}\sin(p_r x+\phi_2),$$
$$\beta_{\rm tr} = \frac{1}{1+a^2}\beta_{\rm in},$$

and $\phi_i = -i\ln f_i/|f_i|$, $i = 1,2$.

In the gauge $A_0 = 0$, the electric field $\mathbf{E}$ is the derivative over x_0 of the vector potential $\mathbf{A}$

$$\mathbf{E}_{\rm in} = -p_0\left(\frac{\beta_{\rm in}|f_1|}{|f_2|}\mathbf{U}_1 + \frac{\alpha_{\rm in}|f_2|}{|f_1|}\mathbf{U}_2\right),$$
$$\mathbf{E}_r = -p_0\left(\frac{\beta_r|f_1|}{|f_2|}T\mathbf{Y}_1 + \frac{\alpha_r|f_2|}{|f_1|}T\mathbf{Y}_2\right),$$
$$\mathbf{E}_{\rm tr} = -p_0\left(\frac{\beta_{\rm tr}|f_1|}{|f_2|}\mathbf{Y}_2 - \frac{\alpha_{\rm tr}|f_2|}{|f_1|}\mathbf{Y}_1\right).$$

The magnetic field is calculated by the formula $\mathbf{H} = [\boldsymbol{\partial}\times\mathbf{A}]$, from which we obtain immediately the following result:

$$\mathbf{H}_{\rm in} = -\frac{[\mathbf{p}_{\rm in}\times\mathbf{E}_{\rm in}]}{p_0}, \quad \mathbf{H}_r = -\frac{[\mathbf{p}_r\times\mathbf{E}_r]}{p_0}, \quad \mathbf{H}_{\rm tr} = -\frac{[\mathbf{p}_{\rm tr}\times\mathbf{E}_{\rm tr}]}{p_0}.$$

For the intensities $I_{\rm in}$, I_r, $I_{\rm tr}$ of the incident, reflected, and transmitted waves we have

$$I_{\rm in} \equiv \frac{|\boldsymbol{\mathcal{A}}_{\rm in}(\overline{p})\,{\rm e}^{i(\overline{px}-\rho x_3)}|^2}{2\pi^3} = \frac{|\boldsymbol{\mathcal{A}}_{\rm in}(\overline{p})|^2}{2\pi^3}, \quad \mathbf{I}_r = \frac{|\boldsymbol{\mathcal{A}}_r(\overline{p}))|^2}{2\pi^3}, \quad \mathbf{I}_{\rm tr} = \frac{|\boldsymbol{\mathcal{A}}_{\rm tr}(\overline{p}))|^2}{2\pi^3}.$$

Therefore,

$$I_r = \frac{a^2}{1+a^2}I_{\rm in}, \quad I_{\rm tr} = \frac{1}{1+a^2}I_{\rm in}.$$

Hence, the reflection $K_r \equiv I_r/I_{\rm in}$ and transmission coefficients $K_{\rm tr} \equiv I_{\rm tr}/I_{\rm in}$ for flat waves scattering on the plane do not depend on the frequency and incidence angle and can be expressed through the characterizing the scattering material coupling constant a:

$$K_r = \frac{a^2}{1+a^2}, \quad K_{\rm tr} = \frac{1}{1+a^2}.$$

Let us consider the movement of waves along the axis x_3. In this case, $p_1 = p_2 = 0$, $\rho = p_0$,

$$\mathbf{E}_{\rm in} = p_0^3(-\beta_{\rm in},\, \alpha_{\rm in}, 0), \quad \mathbf{E}_{\rm tr} = \frac{1}{1+a^2}\mathbf{E}_{\rm in} + \frac{a}{1+a^2}\mathbf{Q},$$

$$\mathbf{Q} \equiv p_0^3(\alpha_{\rm in},\, \beta_{\rm in},\, 0), \quad \mathbf{E}_r = \frac{ap_0^3}{1+a^2}(-\beta_r a - \alpha_r,\, \alpha_r a - \beta_r,\, -0),$$

and replacing in $\mathbf{E}_r$ the sign of x_3 on the opposite one, we obtain

$$T\mathbf{E}_r = \frac{a^2}{1+a^2}\mathbf{E}_{\rm in} - \frac{a}{1+a^2}\mathbf{Q}.$$

We see that by the scattering of waves moving perpendicular to the plane, apart from the usual for process of scattering waves, there are waves with electric field rotated by an angle $\pi/2$ ($\mathbf{E}_{\rm in}\mathbf{Q} = 0$).

11. EXTERNAL CLASSICAL CHARGE AND CURRENT

The classical charge and the wire with current near defect plane are modeled by appropriately chosen 4-current J in (3). The mean vector potential $\mathcal{A}_\mu$ generated by J and the plane $x_3 = 0$, with $a_1 = a$ can be calculated as

$$\mathcal{A}^\mu = -i\frac{\delta G(J)}{\delta J_\mu}\bigg|_{a_1=a,a_2=0} = iD_{2P}^{\mu\nu}J_\nu|_{a_1=a,a_2=0}. \tag{25}$$

Using notations $\mathcal{F}_{ik} = \partial_i\mathcal{A}_k - \partial_k\mathcal{A}_i$, one can present electric and magnetic fields as $\vec{E} = (\mathcal{F}_{01}, \mathcal{F}_{02}, \mathcal{F}_{03})$, $\mathbf{H} = (\mathcal{F}_{23}, \mathcal{F}_{31}, \mathcal{F}_{12})$. For charge e at the point $(x_1, x_2, x_3) = (0, 0, l)$, $l > 0$, the corresponding classical 4-current is

$$J_\mu(x) = 4\pi e\delta(x_1)\,\delta(x_2)\,\delta(x_3 - l)\,\delta_{0\mu}.$$

In virtue of (25), the electric field in considered system is the same as one generated in usual classical electrostatics by charge e placed on distance l from infinitely thick slab with dielectric constant $\epsilon = 2a^2 + 1$. The defect plane induces also a magnetic field $\mathbf{H} = (H_1, H_2, H_3)$:

$$H_1 = \frac{eax_1}{(a^2+1)\rho^3}, \quad H_2 = \frac{eax_2}{(a^2+1)\rho^3}, \quad H_3 = \frac{ea(|x_3|+l)}{(a^2+1)\rho^3},$$

where $\rho = (x_1^2 + x_2^2 + (|x_3| + l)^2)^{1/2}$. It is an anomalous field which does not arise in classical electrostatics. Its direction depends on sign of a. A current with density j flowing in the wire along the x_1 axis is modeled by

$$J_\mu(x) = 4\pi j\delta(x_3 - l)\,\delta(x_2)\,\delta_{\mu 1}.$$

For magnetic field from (25) we obtain usual results of classical electrodynamics for the rectilinear current parallel to infinitely thick slab with permeability $\mu = (2a^2+1)^{-1}$. There is also an anomalous electric field $\mathbf{E} = (0, E_2, E_3)$:

$$E_2 = \frac{2ja}{a^2+1}\frac{x_2}{\tau^2}, \quad E_3 = \frac{2ja}{a^2+1}, \frac{|x_3|+l}{\tau^2},$$

where $\tau = (x_2^2 + (|x_3|+l)^2)^{1/2}$. Comparing both formulae for parameter a, we obtain the relation $\epsilon\,\mu = 1$. It holds for material of thick slab interaction which with point charge and current in classical electrodynamics was compared with results for thin film of our model. The speed of light in this hypothetical material is equal to one in the vacuum. From the physical point of view, it could be expected, because interaction of film with photon field is a surface effect which cannot generate the bulk phenomena like decreasing the speed of light in the considered slab.

CONCLUSION

Most essential features of presented results are the following. The considered approach enables one to investigate, in the framework of one model with small number of parameter, many physical phenomena. In this model, the interaction of thin film with the electromagnetic fields is described by the defect action (1) obtained by most general assumptions consistent with locality, gauge invariance and renormalizability. Thus, basic principles of quantum electrodynamics are used essentially for investigation of interaction effects between quantum and classical degrees of freedom in considered systems.

For plane films, it was demonstrated that the Casimir force is not universal and depends on properties of the material presented by the interaction constant a. For $a \to \infty$, one obtains the usual force for ideal conducting planes. In this case, the model coincides with photodynamics considered in [20] with boundary condition $\epsilon^{ijk3}F_{jk} = 0$ $(i = 0, 1, 2)$ on orthogonal to the x_3-axis planes. For sufficiently small a, the Casimir force appears to be repulsive. Interaction of plane films with charges and currents generates anomalous magnetic and electric fields which do not arise in classical electrodynamics.

The Casimir energy was calculated for spherical films interacting with quantum electromagnetic field. The result obtained in the framework of multiplicative renormalization procedure depends on three parameters. One of them is dimensionless and is a coupling constant of the sphere with photon field. If it is given, the $1/r_0$-contribution to Casimir energy is calculated exactly. The renormalization procedure requires the presence in the model of two complementary parameters, which make two additional terms to Casimit energy: one independent of the radius r_0 and one proportional to r_0^2.

Thus, the Casimir energy appears to be nonuniversal and dependent on the properties of material. The presented approach can by applied for the problem of stability of fullerenes and nanotubules.

In the framework of quantum electrodynamics with the Chern–Simons potential describing the interaction, a two-dimensional plane with neutral atom (molecule), the energy of the system as a function of the atom distance from the plane, was calculated. In the limit $a \to +\infty$ for coupling constant, the result coincides with the Casimir–Polder potential [17] for the energy of interaction of a neutral atom with a perfectly conducting plane. The essential feature of the result is the term depending on the antisymmetric part of a dipole correlation function for finite values of the parameter a. The criterion of its dominance in terms of imaginary and real parts of dipole matrix elements of the atom and the parameter a of the Chern–Simons ponential was presented.

The scattering processes in the model of the Chern–Simons interaction with the coupling constant a between electromagnetic field and the material plane were investigated. The Euler–Lagrange equations of the model are the modified Maxwell equations including the parameter a characterizing the material of the scattering plane. They were solved in the temporal gauge $A_0 = 0$. For the spatial part of the vector potential, the result is represented as a linear combination of two orthogonal eigenmodes of the scattering problem. For the case of a monochromatic plane wave with arbitrary polarization, the vectors of electric and magnetic fields of the reflected and transmitted waves, and also the transmission and reflection coefficients are obtained in an explicit form. The transmission and reflection coefficients $K_{\rm tr}$, K_r are expressed through coupling constant a: $K_{\rm tr} = (1+a^2)^{-1}$, $K_r = a^2(1+a^2)^{-1}$. They do not depend on the wave frequency and propagation direction. For waves propagating in the orthogonal to the plane direction by small a, the electric field vector of the reflected wave is rotated on the close to $\pi/2$ angle with respect to its direction for the case of a perfectly conducting plane ($a \to \infty$). Electric field vector of the transmitted wave, vanishing for large a, turned toward his direction in the incident wave on an angle close to $\pi/2$.

The presented effects may be used for experimental determination of the parameter a of material of thin films. The measurement of the Casimir force and Casimir–Polder potential for thin material films, studies of scattering on them of electromagnetic waves, investigation of magneto- and electrostatic properties of films would give the possibility to verify the correctness of the proposed in [11] approach for the theoretical investigation of the nanophysical phenomena.

One can expect that quantum Hall effect systems, graphene, fullerene two-dimensional magnetoelectric materials [21], sharp boundaries of material bodies [22] are the most promising from known materials for this aim. The measurements of the antisymmetric part of the atomic polarizability by means of the Casimir–Polder effect can be an independent possibility for the study of antisymmetric parts of atomic polarizabilities in various atomic and molecular systems.

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