COLLINS AND SIVERS EFFECTS IN $p^\uparrow p \to$ jet πX: UNIVERSALITY AND PROCESS DEPENDENCE

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In this paper, we briefly review the transverse-momentum-dependent generalized parton model and its application to the study of azimuthal asymmetries in the distribution of leading hadrons (mainly pions) inside large transverse momentum jets inclusively produced in polarized proton–proton collisions. We put particular emphasis on the phenomenological interest of these observables, in combination with similar asymmetries measured in semi-inclusive deeply inelastic scattering, Drell–Yan processes and e^+e^- collisions, for the study of the universality properties of the transverse-momentum-dependent parton distribution and fragmentation functions. We present results for RHIC kinematics at center-of-mass energies $\sqrt{s} = 200$ and 500 GeV, for central and mainly forward jet rapidities, in particular for the Sivers distribution and the Collins fragmentation function, that are believed to be responsible for many of the largest asymmetries measured in the last years. We also briefly discuss the case of inclusive jet production and recent phenomenological applications of other theoretical approaches, like the colour gauge-invariant generalized parton model and the collinear twist-three approach, aiming at clarifying the issues of the universality and process dependence of transverse-momentum-dependent functions.

Представлен обзор обобщенной партонной модели с зависимостью от поперечного импульса и ее применение для изучения азимутальных асимметрий в распределении ведущих адронов (главным образом пионов) внутри струй с большими поперечными импульсами, образующимися инклюзивно в столкновениях поляризованных протонов. Особое внимание уделяется феноменологическому смыслу рассматриваемых наблюдаемых с учетом похожих асимметрий, измеряемых в полуинклюзивном глубоконеупругом рассеянии, процессах Дрелла–Яна и столкновениях e^+e^-, при изучении свойств универсальности, зависящих от поперечного импульса распределений партонов и функций фрагментации. Результаты представлены для кинематики RHIC при энергиях в системе центра масс $\sqrt{s} = 200$ и 500 ГэВ для центральных быстрот и движущихся вперед струй, в частности, рассматриваются распределение Сиверса и функция фрагментации Коллинза,

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которые, как полагают, описывают многие из наиболее заметных асимметрий, измеренных в последнее время. Также коротко обсуждаются случай инклюзивного рождения струи и недавние феноменологические описания, сделанные с помощью других теоретических приближений, таких как цветная калибровочно-инвариантная обобщенная партонная модель и коллинеарное приближение с тремя поворотами. Цель этого обсуждения — прояснить природу универсальности и зависимости от процесса функций, зависящих от поперечного импульса.

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INTRODUCTION

In the last years the study and knowledge of the full three-dimensional dynamical nucleon structure in polarized high-energy collisions have witnessed impressive progress (see, e.g., [1, 2] for recent reviews). Motivated by several experimental results on spin and azimuthal asymmetries, a class of partonic, transverse-momentum-dependent, distribution and fragmentation functions (nowadays largely known as TMDs for short) have been introduced and analyzed. In high-energy hadronic processes where two energy scales play a role (a large perturbative scale and a small transverse momentum scale) the usual leading-twist QCD collinear factorization schemes, making use of the corresponding collinear parton distribution (PDFs) and fragmentation (FFs) functions, often fail to describe several puzzling experimental measurements on spin asymmetries. When a small transverse scale is involved, one needs to take care more accurately of the intrinsic motion of constituent partons inside parent hadrons. Typical examples are: the low transverse momentum distribution of dilepton pairs in Drell–Yan (DY) processes and the corresponding asymmetries in the azimuthal distribution of the observed pair [3–5]; the low transverse momentum spectrum of hadrons produced in the current region in semi-inclusive deeply inelastic scattering (SIDIS) [6–8]; the azimuthal asymmetries in the correlations of two leading hadrons (typically pions) observed in opposite jets produced in e^+e^- collisions [9, 10]. Despite the theoretical and experimental difficulties associated with the study of these reactions, they offer a unique opportunity to learn about the hadron structure in the transverse directions (with respect to the usual light-cone one).

From a historical perspective, the first sizable single-spin asymmetries were observed in single inclusive hadron production at large values of the Feynman variable, $x_F = p_L/p_L^{\max} \simeq 2p_L/\sqrt{s}$, and moderately large transverse momentum in polarized hadronic collisions. However, the theoretical study of this process is made difficult by the fact that there is no small transverse momentum scale. Intrinsic transverse momenta of partons are integrated out in the observable. This complicates the treatment of such an (higher twist) asymmetry, since several

possible effects are mixed up and it is not obvious how to disentangle them. Moreover, a TMD factorization scheme similar to that developed for the reactions discussed above (DY, SIDIS, e^+e^- annihilations) has never been proved and, at least for double inclusive jet and/or hadron production processes, like the one studied here, there are clear indications that factorization may be broken (for a recent discussion, see, e.g., [11] and references therein).

Quite recently, it has been suggested to study azimuthal asymmetries in hadronic collisions by looking at the azimuthal distribution of leading hadrons (pions or kaons) inside a large transverse momentum jet inclusively produced in polarized proton–proton collisions [12, 13]. Although also in this case a proof of TMD factorization is not available (if one takes into account intrinsic motion in the initial colliding hadrons), the observables considered are rather similar to those measured in SIDIS. In particular, leading-twist asymmetries appear and different contributions (like the Sivers or Collins effects) can be disentangled by taking appropriate moments of the azimuthal distributions, much in the same way adopted in the SIDIS or DY cases.

The detailed analysis of this process can be of crucial relevance, when compared with analogous studies in the DY and SIDIS cases, for the theoretical and phenomenological understanding of the process dependence and the universality properties of the Sivers distribution and the validation of the expected universality of the Collins fragmentation function. Other TMDs can also be tested in the same way.

In this review, we summarize recent results concerning the study of the Sivers and Collins azimuthal asymmetries in the distribution of leading pions inside a jet in $p^\uparrow p \to \mathrm{jet}\,\pi X$ processes. After a short description of the TMD theoretical approach adopted, the so-called generalized parton model (GPM) [14, 15], we present a selection of interesting results involving the Sivers and Collins effects, that are expected to be the dominant contributions to the single-spin asymmetries considered here.

We will also discuss in some detail an extension of the GPM [16], named colour gauge invariant (CGI) GPM, including colour gauge factors in the approach, and its application to the study of the process dependence of the Sivers distribution [17]. This is indeed expected in perturbative QCD, due to the essential role played by initial- and final-state interactions among active partons and parent hadrons for the nonvanishing of these single-polarized observables.

Finally, we will shortly summarize other recent attempts to study the process dependence of the Sivers distribution (and other TMD PDFs and FFs) in different processes and adopting different theoretical approaches. Hopefully, the combined phenomenological analysis of several reactions and observables will help in clarifying essential theoretical issues crucial for a full understanding of these interesting phenomena in the realm of QCD.

1. KINEMATICS

We consider the process

$$A(p_A; S) + B(p_B) \to \text{jet}\,(p_j) + \pi(p_\pi) + X, \tag{1}$$

where A and B are two spin-1/2 hadrons carrying momenta p_A and p_B, respectively. One of the two hadrons, A, is in a pure transverse spin state described by the four-vector S ($S^2 = -1$ and $p_A \cdot S = 0$), while B is unpolarized. We work mainly in the center-of-mass (c.m.) frame of A and B, where $s = (p_A + p_B)^2$ is the total energy squared, and, as depicted in Fig. 1, A moves along the positive direction of the $\hat{\mathbf{Z}}_{\text{cm}}$ axis. The production plane containing the colliding beams and the observed jet is taken as the $(XZ)_{\text{cm}}$ plane, with $(\mathbf{p}_j)_{X_{\text{cm}}} > 0$. In this frame the four-momenta of the particles and the spin vector S are given by

$$p_A = \frac{\sqrt{s}}{2}(1,0,0,1), \quad S = S_T = (0, \cos\phi_S, \sin\phi_S, 0),$$

$$p_B = \frac{\sqrt{s}}{2}(1,0,0,-1),$$

$$p_j = (E_j, p_{jT}, 0, p_{jL}) = E_j(1, \sin\theta_j, 0, \cos\theta_j) = p_{jT}(\cosh\eta_j, 1, 0, \sinh\eta_j), \tag{2}$$

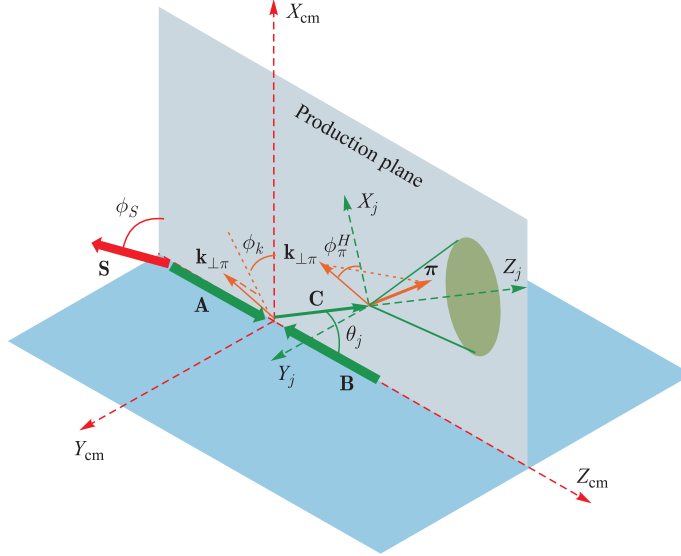$$p_\pi = E_\pi(1, \sin\theta_\pi \cos\phi_\pi, \sin\theta_\pi \sin\phi_\pi, \cos\theta_\pi),$$

Fig. 1. Color online. Kinematics for the process $A(p_A; S) + B(p_B) \to \text{jet}\,(p_j) + \pi(p_\pi) + X$ in the center-of-mass frame of the two incoming hadrons, A and B

where all masses have been neglected and η_j denotes the jet (pseudo)rapidity, $\eta_j = -\log\left[\tan\left(\theta_j/2\right)\right]$.

At leading order in perturbative QCD, the reaction proceeds via the partonic hard scattering subprocesses $ab \rightarrow cd$, where the outgoing parton c fragments into the observed hadronic jet. For the partonic momenta in the hadronic c.m. frame, one has

$$p_a = \left(x_a\frac{\sqrt{s}}{2} + \frac{k_{\perp a}^2}{2x_a\sqrt{s}}, k_{\perp a}\cos\phi_a, k_{\perp a}\sin\phi_a, x_a\frac{\sqrt{s}}{2} - \frac{k_{\perp a}^2}{2x_a\sqrt{s}}\right),$$

$$p_b = \left(x_b\frac{\sqrt{s}}{2} + \frac{k_{\perp b}^2}{2x_b\sqrt{s}}, k_{\perp b}\cos\phi_b, k_{\perp b}\sin\phi_b, -x_b\frac{\sqrt{s}}{2} + \frac{k_{\perp b}^2}{2x_b\sqrt{s}}\right), \quad (3)$$

$$p_c \equiv p_j,$$

where $k_{\perp a,b} = |\mathbf{k}_{\perp a,b}|$. Here we have introduced the variables $x_{a,b}$ and $\mathbf{k}_{\perp a,b}$, which are, respectively, the light-cone momentum fractions and the intrinsic transverse momenta of the incoming partons a and b. From Eqs. (2) and (3) one can calculate the partonic Mandelstam variables

$$\hat{s} = (p_a + p_b)^2 = x_a x_b s\left[1 - 2\left(\frac{k_{\perp a}k_{\perp b}}{x_a x_b s}\right)\cos(\phi_a - \phi_b) + \left(\frac{k_{\perp a}k_{\perp b}}{x_a x_b s}\right)^2\right], \quad (4)$$

$$\hat{t} = (p_a - p_c)^2 = -x_a E_j\sqrt{s}\left[1 - \cos\theta_j - \right.$$

$$\left. - 2\left(\frac{k_{\perp a}}{x_a\sqrt{s}}\right)\sin\theta_j\cos\phi_a + \left(\frac{k_{\perp a}}{x_a\sqrt{s}}\right)^2(1 + \cos\theta_j)\right] =$$

$$= -x_a p_{jT}\sqrt{s}\left[e^{-\eta_j} - 2\left(\frac{k_{\perp a}}{x_a\sqrt{s}}\right)\cos\phi_a + \left(\frac{k_{\perp a}}{x_a\sqrt{s}}\right)^2 e^{\eta_j}\right], \quad (5)$$

$$\hat{u} = (p_b - p_c)^2 = -x_b E_j\sqrt{s}\left[1 + \cos\theta_j - \right.$$

$$\left. - 2\left(\frac{k_{\perp b}}{x_b\sqrt{s}}\right)\sin\theta_j\cos\phi_b + \left(\frac{k_{\perp b}}{x_b\sqrt{s}}\right)^2(1 - \cos\theta_j)\right] =$$

$$= -x_b p_{jT}\sqrt{s}\left[e^{\eta_j} - 2\left(\frac{k_{\perp b}}{x_b\sqrt{s}}\right)\cos\phi_b + \left(\frac{k_{\perp b}}{x_b\sqrt{s}}\right)^2 e^{-\eta_j}\right], \quad (6)$$

with the condition $\hat{s} + \hat{t} + \hat{u} = 0$ giving an additional constraint.

The helicity frame of the fragmenting parton c has axes denoted by $\hat{\mathbf{x}}_j$, $\hat{\mathbf{y}}_j$, $\hat{\mathbf{z}}_j$, with $\hat{\mathbf{z}}_j$ along the direction of motion of c. It can be reached from the hadronic

c.m. frame by performing a simple rotation by the angle θ_j around $\hat{\mathbf{Y}}_{\text{cm}} \equiv \hat{\mathbf{y}}_j$, as can be seen from Fig. 1. Hence, in this frame,

$$\tilde{p}_c = \tilde{p}_j = E_j(1, 0, 0, 1),$$

$$\tilde{p}_\pi = \left(E_\pi, \mathbf{k}_{\perp\pi}, \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2} \right) =$$

$$= \left(E_\pi, k_{\perp\pi} \cos \phi_\pi^H, k_{\perp\pi} \sin \phi_\pi^H, \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2} \right), \tag{7}$$

with ϕ_π^H being the azimuthal angle of the pion three-momentum around the jet axis, as measured in the fragmenting parton helicity frame. From Eq. (7), one can obtain the expression for the light-cone momentum fraction of the pion,

$$z = \frac{\tilde{p}_\pi^+}{\tilde{p}_c^+} \equiv \frac{\tilde{p}_\pi^+}{\tilde{p}_j^+} = \frac{\tilde{p}_\pi^0 + \tilde{p}_\pi^3}{\tilde{p}_j^0 + \tilde{p}_j^3} = \frac{E_\pi + \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2}}{2 E_j}. \tag{8}$$

By writing down explicitly the three-momentum of the pion $\mathbf{p}_\pi$ in the parton c helicity frame and in the hadronic c.m. frame, respectively,

$$\mathbf{p}_\pi = k_{\perp\pi} \cos \phi_\pi^H \hat{\mathbf{x}}_j + k_{\perp\pi} \sin \phi_\pi^H \hat{\mathbf{y}}_j + \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2} \hat{\mathbf{z}}_j =$$

$$= \left[k_{\perp\pi} \cos \phi_\pi^H \cos \theta_j + \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2} \sin \theta_j \right] \hat{\mathbf{X}}_{\text{cm}} +$$

$$+ k_{\perp\pi} \sin \phi_\pi^H \hat{\mathbf{Y}}_{\text{cm}} + \left[-k_{\perp\pi} \cos \phi_\pi^H \sin \theta_j + \sqrt{E_\pi^2 - \mathbf{k}_{\perp\pi}^2} \cos \theta_j \right] \hat{\mathbf{Z}}_{\text{cm}}, \tag{9}$$

one finds that the intrinsic transverse momentum of the pion in the hadronic c.m. frame can be written as

$$\mathbf{k}_{\perp\pi} = k_{\perp\pi} \cos \phi_\pi^H \cos \theta_j \hat{\mathbf{X}}_{\text{cm}} + k_{\perp\pi} \sin \phi_\pi^H \hat{\mathbf{Y}}_{\text{cm}} - k_{\perp\pi} \cos \phi_\pi^H \sin \theta_j \hat{\mathbf{Z}}_{\text{cm}}. \tag{10}$$

Therefore, denoting by ϕ_k the azimuthal angle of $\mathbf{k}_{\perp\pi}$, *as measured in the hadronic c.m. frame*, one obtains

$$\tan \phi_k = \frac{\tan \phi_\pi^H}{\cos \theta_j} . \tag{11}$$

In [12], where only forward jet production was considered, azimuthal asymmetries were given in terms of ϕ_k (named ϕ_h there). In this kinematical configuration, $\cos \theta_j \to 1$ and the angles ϕ_π^H and ϕ_k become practically identical. On the other hand, for central-rapidity jets ($\theta_j = \pi/2$), $\phi_k = \pi/2$, implying that azimuthal asymmetries expressed as a function of ϕ_k, would be artificially suppressed. For this reason, ϕ_π^H has to be considered as the physically relevant angle in the present analysis.

2. THE GENERALIZED PARTON MODEL

The single transversely polarized cross section for the process $p(S) + p \to$ jet $+ \pi + X$ has been calculated in the GPM framework, using the helicity formalism, in [13], to which we refer for further details. Its final expression has the following general structure:

$$
\begin{aligned}
2d\sigma(\phi_S, \phi_\pi^H) \sim &\, d\sigma_0 + d\Delta\sigma_0 \sin\phi_S + d\sigma_1 \cos\phi_\pi^H + \\
&+ d\sigma_2 \cos 2\phi_\pi^H + d\Delta\sigma_1^- \sin(\phi_S - \phi_\pi^H) + d\Delta\sigma_1^+ \sin(\phi_S + \phi_\pi^H) + \\
&+ d\Delta\sigma_2^- \sin(\phi_S - 2\phi_\pi^H) + d\Delta\sigma_2^+ \sin(\phi_S + 2\phi_\pi^H), \quad (12)
\end{aligned}
$$

where, as discussed in Sec. 1, ϕ_π^H is the azimuthal angle of the pion three-momentum around the jet axis, and ϕ_S is the azimuthal angle of the spin polarization vector S of the polarized proton, as measured in the hadronic c.m. frame. The numerator of the related single-spin asymmetry is given by

$$
\begin{aligned}
d\sigma(\phi_S, \phi_\pi^H) - d\sigma(\phi_S + \pi, \phi_\pi^H) \sim &\, d\Delta\sigma_0 \sin\phi_S + \\
&+ d\Delta\sigma_1^- \sin(\phi_S - \phi_\pi^H) + d\Delta\sigma_1^+ \sin(\phi_S + \phi_\pi^H) + \\
&+ d\Delta\sigma_2^- \sin(\phi_S - 2\phi_\pi^H) + d\Delta\sigma_2^+ \sin(\phi_S + 2\phi_\pi^H), \quad (13)
\end{aligned}
$$

while for the denominator we have

$$
\begin{aligned}
d\sigma(\phi_S, \phi_\pi^H) + d\sigma(\phi_S + \pi, \phi_\pi^H) \equiv & \\
\equiv 2d\sigma^{\mathrm{unp}}(\phi_\pi^H) \sim &\, d\sigma_0 + d\sigma_1 \cos\phi_\pi^H + d\sigma_2 \cos 2\phi_\pi^H. \quad (14)
\end{aligned}
$$

The various terms contributing to the cross section in Eq. (12) are explicitly given by convolutions of different TMD parton distribution and fragmentation functions with hard scattering (polarized) cross sections. For example, if we keep only the leading contributions after integrating over the intrinsic transverse momenta of the initial partons, the symmetric term in Eq. (14) is given by

$$
\begin{aligned}
d\sigma_0 \equiv E_j \frac{d\sigma_0}{d^3\mathbf{p}_j \, dz \, d^2\mathbf{k}_{\perp\pi}} = & \\
= \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a}{x_a} d^2\mathbf{k}_{\perp a} \int \frac{dx_b}{x_b} d^2\mathbf{k}_{\perp b} \delta(\hat{s} + \hat{t} + \hat{u}) H_{ab \to cd}^U(\hat{s}, \hat{t}, \hat{u}) \times & \\
\times f_{a/A}(x_a, \mathbf{k}_{\perp a}^2) f_{b/B}(x_b, \mathbf{k}_{\perp b}^2) \, D_1^c(z, \mathbf{k}_{\perp\pi}^2), \quad (15)
\end{aligned}
$$

where $H_{ab \to cd}^U(\hat{s}, \hat{t}, \hat{u})$ is the unpolarized squared hard scattering amplitude for the partonic process $a\,b \to c\,d$, related to the elementary cross section as follows:

$$
\frac{d\hat{\sigma}_{ab \to cd}}{d\hat{t}} = \frac{\pi\alpha_s^2}{\hat{s}^2} H_{ab \to cd}^U. \quad (16)
$$

By $f_{a/A}(x_a, \mathbf{k}^2_{\perp a})$ and $f_{b/B}(x_b, \mathbf{k}^2_{\perp b})$ we denote the unpolarized TMD distributions for parton a inside hadron A and for parton b inside hadron B, respectively, while $D_1^c(z, \mathbf{k}^2_{\perp \pi})$ is the unintegrated fragmentation function for the unpolarized parton c that fragments into a pion. The term containing the $\sin \phi_S$ modulation in Eq. (13) is related to the Sivers effect,

$$d\Delta\sigma_0 \sin \phi_S \equiv E_j \frac{d\Delta\sigma^{(\text{Sivers})}}{d^3\mathbf{p}_j \, dz \, d^2\mathbf{k}_{\perp\pi}} =$$

$$= \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a}{x_a} d^2\mathbf{k}_{\perp a} \int \frac{dx_b}{x_b} d^2\mathbf{k}_{\perp b} \delta(\hat{s} + \hat{t} + \hat{u}) H^U_{ab \to cd}(\hat{s}, \hat{t}, \hat{u}) \times$$

$$\times \left(-\frac{k_{\perp a}}{M} \right) f_{1T}^{\perp a}(x_a, \mathbf{k}^2_{\perp a}) \cos \phi_a f_{b/B}(x_b, \mathbf{k}^2_{\perp b}) D_1^c(z, \mathbf{k}^2_{\perp \pi}) \sin \phi_S, \quad (17)$$

where M is the proton mass and $f_{1T}^{\perp a}(x_a, \mathbf{k}^2_{\perp a})$ is the Sivers function, also denoted as $\Delta^N f_{a/p^\uparrow} = -2(k_\perp/M) f_{1T}^{\perp a}$ [18]. Notice that, for a direct comparison with the CGI GPM approach, in this review, we adopt the so-called Amsterdam notation [6,7] instead of the usual GPM notation [1,15].

The term containing the $\sin(\phi_S - \phi_\pi^H)$ modulation in Eq. (13) corresponds to the Collins effect,

$$d\Delta\sigma_1^- \sin(\phi_S - \phi_\pi^H) \equiv E_j \frac{d\Delta\sigma^{(\text{Collins})}}{d^3\mathbf{p}_j \, dz \, d^2\mathbf{k}_{\perp\pi}} =$$

$$= \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a}{x_a} d^2\mathbf{k}_{\perp a} \int \frac{dx_b}{x_b} d^2\mathbf{k}_{\perp b} \delta(\hat{s} + \hat{t} + \hat{u}) H^U_{ab \to cd}(\hat{s}, \hat{t}, \hat{u}) \times$$

$$\times h_1^a(x_a, \mathbf{k}^2_{\perp a}) \cos(\phi_a - \psi) f_{b/B}(x_b, \mathbf{k}^2_{\perp b}) \times$$

$$\times \frac{k_{\perp \pi}}{z M_\pi} H_1^{\perp c}(z, \mathbf{k}^2_{\perp \pi}) d_{NN}(\hat{s}, \hat{t}, \hat{u}) \sin(\phi_S - \phi_\pi^H), \quad (18)$$

where the Collins fragmentation function of the struck quark c, $H_1^{\perp c}(z, \mathbf{k}^2_{\perp \pi})$ (or $\Delta^N D_{\pi/c^\uparrow} = 2(k_{\perp \pi}/z M_\pi) H_1^{\perp c}$), is convoluted with the unintegrated transversity distribution, $h_1^a(x_a, \mathbf{k}^2_{\perp a})$, that is the distribution of transversely polarized quarks in a transversely polarized hadron. In Eq. (18), M_π is the pion mass, d_{NN} is the spin transfer asymmetry for the partonic process $a^\uparrow b \to c^\uparrow d$,

$$d_{NN} = \frac{\sigma^{a^\uparrow b \to c^\uparrow d} - \sigma^{a^\uparrow b \to c^\downarrow d}}{\sigma^{a^\uparrow b \to c^\uparrow d} + \sigma^{a^\uparrow b \to c^\downarrow d}}, \quad (19)$$

and ψ is the corresponding azimuthal phase [13].

In order to single out the different contributions to the polarized cross section, we introduce the following average values of the circular functions of ϕ_S and ϕ_π^H

appearing in Eq. (12):

$$\langle W(\phi_S, \phi_\pi^H)\rangle(\mathbf{p}_j, z, k_{\perp\pi}) = \frac{\int d\phi_S\, d\phi_\pi^H\, W(\phi_S, \phi_\pi^H)\, d\sigma(\phi_S, \phi_\pi^H)}{\int d\phi_S\, d\phi_\pi^H\, d\sigma(\phi_S, \phi_\pi^H)}. \qquad (20)$$

For single-spin asymmetries one can preferably define azimuthal moments, similarly to the SIDIS case,

$$A_N^{W(\phi_S, \phi_\pi^H)}(\mathbf{p}_j, z, k_{\perp\pi}) =$$
$$= 2\frac{\int d\phi_S\, d\phi_\pi^H\, W(\phi_S, \phi_\pi^H)[d\sigma(\phi_S, \phi_\pi^H) - d\sigma(\phi_S + \pi, \phi_\pi^H)]}{\int d\phi_S\, d\phi_\pi^H\, [d\sigma(\phi_S, \phi_\pi^H) + d\sigma(\phi_S + \pi, \phi_\pi^H)]}, \qquad (21)$$

with $W(\phi_S, \phi_\pi^H)$ now being one of the angular modulations in Eq. (13). In the following, we will focus mainly on the two observables that are the most relevant from the phenomenological point of view: the Collins and the Sivers contributions to A_N, namely, $A_N^{\sin(\phi_S - \phi_\pi^H)}$ and $A_N^{\sin\phi_S}$.

3. PHENOMENOLOGICAL RESULTS

Here, as well as in the following sections, we review some phenomenological implications of the TMD generalized parton model approach for the $p^\uparrow p \to \mathrm{jet}\,\pi X$ and $p^\uparrow p \to \mathrm{jet}\,X$ processes in kinematical configurations accessible at RHIC by the STAR and PHENIX experiments. We consider both central ($\eta_j = 0$) and forward ($\eta_j = 3.3$) (pseudo)rapidity configurations, at c.m. energies $\sqrt{s} = 200$ and 500 GeV. A more detailed account and additional phenomenological results are given in [13].

The preliminary STAR results at $\sqrt{s} = 200$ GeV for the Collins azimuthal asymmetry in the process $p^\uparrow p \to \mathrm{jet}\,\pi^\pm X$ in the mid-rapidity region [19] and for the Collins and Sivers azimuthal asymmetries in $p^\uparrow p \to \mathrm{jet}\,\pi^0 X$ at forward rapidities [20] are also available. A phenomenological analysis of these results in the GPM approach, with proper account of all jet kinematical cuts, is in progress and will be presented elsewhere [21].

In the sequel TMD parton distribution and fragmentation functions are parameterized with a simplified functional dependence on the parton light-cone momentum fraction and on the transverse motion, which are completely factorized. Notice, however, that kinematical constraints due to usual parton model requirements (implemented in numerical calculations) effectively lead to correlations between the light-cone momentum fraction and the transverse momentum, particularly at very small and very large ($\to 1$) momentum fractions (for more details, see, e.g., Appendix A of [14]). Moreover, we assume a Gaussian-like flavour-independent shape for the transverse momentum component. Preliminary lattice QCD calculations seem to support the validity of this assumption, see, e.g., [22].

Concerning the parameterizations of the quark transversity and Sivers distributions, and of the quark Collins functions, we will consider two sets: SIDIS 1 [10, 23] and SIDIS 2 [24, 25].

The set SIDIS 1 includes the u, d quark Sivers functions of [23], the u, d quark transversity distributions, and the favoured and disfavoured Collins FFs of [10]. The Kretzer set [26] for collinear pion FFs was used.

Instead, the set SIDIS 2 includes the u, d and sea-quark Sivers functions of [24], and the updated set of the u, d quark transversity distributions and of the favoured and disfavoured Collins FFs of [25]. In this case, the DSS set [27] for collinear pion and kaon FFs was adopted.

In both cases, for the usual collinear parton distributions, the LO unpolarized set GRV98 [28] and the corresponding longitudinally polarized set GRSV2000 [29] (needed in order to implement the Soffer bound [30] for the transversity distribution) were adopted.

Notice that, quite recently, updated parameterizations of the transversity distribution and of the Collins function within the GPM approach have been released [31]. Since they are qualitatively similar to those adopted in [13], for ease of comparison they will not be used in the following.

Since the jet transverse momentum (the hard scale in the process) covers a significant range, one should properly take into account the QCD evolution of all TMDs.

This could indeed play a role both in the size and shape of the Sivers and Collins (see the next section) contributions to the spin asymmetries, as well as for the relevant issue of the process dependence of TMDs (see Sec. 4).

On the other hand, a formal proof of TMD factorization for such processes is still missing and the study of TMD evolution is at present in its earlier stage. Therefore, we tentatively take into account proper evolution with scale, at leading order, for the usual collinear PDFs and FFs, while keeping the transverse momentum component of all TMDs fixed.

The study of the formal aspects and the related phenomenology of the correct QCD evolution with scale of TMD PDFs and FFs has received a lot of attention quite recently. Several papers have investigated proper TMD evolution equations for the Sivers function and their phenomenological implications, see, e.g., [32–42]. The TMD evolution of the helicity and transversity parton distributions has been considered, e.g., in [43]. No information is available yet on the TMD evolution of the Collins fragmentation functions.

Some progress has been recently made in comparing the different approaches to the definition of TMDs, their proper QCD evolution and process dependence, even if some controversy still persists. These formal aspects will then likely find a valuable support from detailed phenomenological analyses. In this respect, the present study can be considered complementary to those, more direct, dedicated to the DY and SIDIS processes, where the TMD factorization has been proved.

In all cases considered, $\mathbf{k}_{\perp\pi}$ is integrated out and, since we are interested in leading particles inside the jet, we present results obtained integrating over the light-cone momentum fraction of the observed hadron, z, in the range $z \geqslant 0.3$. Different choices, according to the kinematical cuts of interest in specific experiments, can be easily implemented in the numerical calculations.

We have considered first, for the π^+ production only, an extreme scenario in which the effects of all TMD functions are overmaximized. By this we mean that all TMDs are maximized in size by imposing natural positivity bounds. The transversity distribution has been fixed at the initial scale by saturating the Soffer bound [30] and then we let it evolve. Moreover, the relative signs of all active partonic contributions are chosen so that they sum up additively. In this way, we set an upper bound on the absolute value of any of the effects playing a potential role in the azimuthal asymmetries. Therefore, all effects that are negligible or even marginal in this scenario may be directly discarded in subsequent refined phenomenological analyses. See [13] for a more detailed discussion.

As a second step in our study we consider, for both neutral and charged pions, only the dominant contributions, that is, the Collins and the Sivers effects, involving the TMD functions for which parameterizations are available from independent fits to other spin and azimuthal asymmetries data in the SIDIS and e^+e^- processes (the SIDIS 1 and SIDIS 2 sets discussed above).

3.1. The Collins Asymmetries.

The Collins fragmentation function contributes to two of the azimuthal moments defined in Eq. (21), namely, $A_N^{\sin(\phi_S+\phi_\pi^H)}$ and $A_N^{\sin(\phi_S-\phi_\pi^H)}$. In $A_N^{\sin(\phi_S+\phi_\pi^H)}$ it is convoluted with two different terms:

$$A_N^{\sin(\phi_S+\phi_\pi^H)} \sim \left[h_{1T}^{\perp q}(x_a, \mathbf{k}_{\perp a}^2) \otimes f_1(x_b, \mathbf{k}_{\perp b}^2) + \right.$$
$$\left. + f_{1T}^\perp(x_a, \mathbf{k}_{\perp a}^2) \otimes h_1^{\perp q}(x_b, \mathbf{k}_{\perp b}^2) \right] \otimes H_1^{\perp q}(z, \mathbf{k}_{\perp\pi}^2). \quad (22)$$

The first term is related to the so-called pretzelosity distribution $h_{1T}^{\perp q}$, while the second one, which enters also in the expression for $A_N^{\sin(\phi_S-\phi_\pi^H)}$, involves in the convolution the Sivers and Boer–Mulders ($h_1^{\perp q}$) functions. As described above, and in more detail in [13], it has been checked that the upper bound of this asymmetry is always negligible, hence it will not be considered again in the following. Same conclusions hold for the Collins-like azimuthal moment $A_N^{\sin(\phi_S+2\phi_\pi^H)}$, originating from the fragmentation of linearly polarized gluons, which has a structure similar to Eq. (22), with quarks replaced by gluons.

The azimuthal asymmetry $A_N^{\sin(\phi_S-\phi_\pi^H)}$ is dominated by a convolution of the transversity distribution and the Collins fragmentation function,

$$A_N^{\sin(\phi_S-\phi_\pi^H)} \sim h_1^q(x_a, \mathbf{k}_{\perp a}^2) \otimes f_1(x_b, \mathbf{k}_{\perp b}^2) \otimes H_1^{\perp q}(z, \mathbf{k}_{\perp\pi}^2), \quad (23)$$

see Eq. (18). A similar expression holds for its gluonic counterpart $A_N^{\sin(\phi_S - 2\phi_\pi^H)}$. Their upper bounds turn out to be sizeable, at least in some kinematic domains [13].

In Figs. 2 and 3, we show our estimates for $A_N^{\sin(\phi_S - \phi_\pi^H)}$ at the RHIC energies $\sqrt{s} = 200$ and 500 GeV, respectively, as a function of the transverse momentum
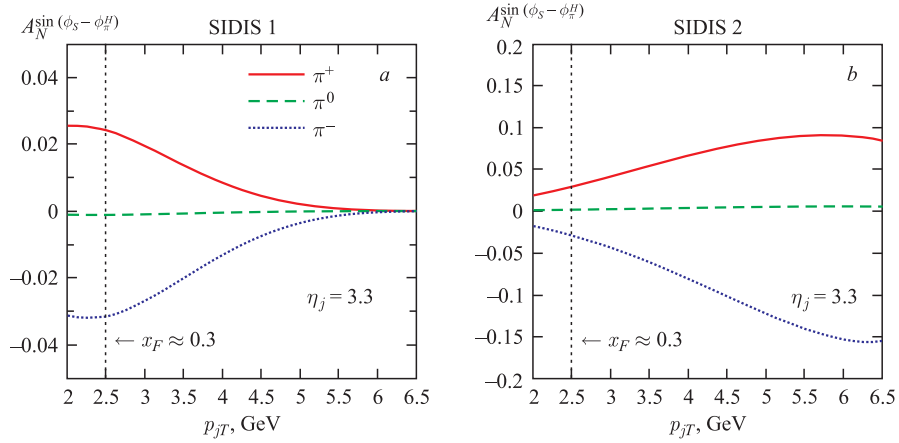

Fig. 2. Color online. The Collins asymmetry $A_N^{\sin(\phi_S - \phi_\pi^H)}$ for the process $p^\uparrow p \to \text{jet } \pi X$, as a function of p_{jT}, at fixed value of the rapidity η_j and c.m. energy $\sqrt{s} = 200$ GeV. Estimates are obtained by adopting the SIDIS 1 (*a*) and SIDIS 2 (*b*) parameterizations. The dotted vertical line delimits the region $x_F \approx 0.3$, beyond which the currently available parameterizations for the quark transversity distributions, extracted from the SIDIS data, are affected by large uncertainties
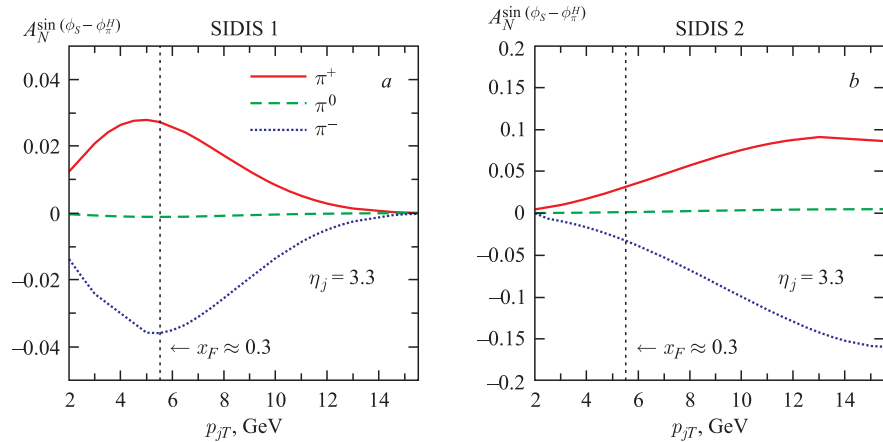

Fig. 3. Color online. The same as in Fig. 2, but at c.m. energy $\sqrt{s} = 500$ GeV

of the jet, p_{jT}, and at fixed jet rapidity ($\eta_j = 3.3$). These results have been obtained by adopting the SIDIS 1 and SIDIS 2 parameterizations. Notice that, while the results of Fig. 2 are taken from [13], those of Fig. 3 are presented here for the first time. Our prediction of an almost vanishing asymmetry for neutral pions, confirmed very recently by preliminary data at $\sqrt{s} = 200$ GeV from the STAR Collaboration [20], is a consequence of the comparable size and the opposite sign, in both parameterizations, of the favoured (e.g., $u \to \pi^+$) and disfavoured (e.g., $d \to \pi^+$) Collins fragmentation functions. In fact, because of isospin invariance, the Collins function for neutral pions is given by half the sum of the fragmentation functions for charged pions, hence turning out to be very small. In addition, further cancellations among quark contributions are due to the relative opposite sign of the transversity distribution for the u and d flavours. Concerning charged pions, the two parameterizations give comparable results only in the kinematic domain, where the Feynman variable $x_F = 2p_{jL}/\sqrt{s}$ is equal to or smaller than the value $x_F \approx 0.3$, denoted by the dotted vertical lines in Figs. 2 and 3 (notice the different scales used in the two panels). This corresponds to the Bjorken x region covered by the SIDIS data that have been used to determine the available parameterizations for the transversity distributions. Extrapolation beyond $x_F \approx 0.3$, where transversity is not constrained, leads to completely different estimates at large p_{jT}, as shown in the figures.

Based on these considerations, in a recent paper [44] (to which we refer for more details) a different and complementary analysis (denoted as «scan procedure») has been performed. The large x behaviour of the quark transversity distribution is mainly controlled by the parameters β_q ($q = u, d$) in the factor $(1 - x)^{\beta_q}$ of the parameterization [13], which are basically unconstrained by the SIDIS data. Therefore, starting from a reference fit (with a given total χ^2, χ_0^2) to the updated SIDIS and e^+e^- data (hence, although using the same collinear PDFs and FFs, slightly different from the SIDIS 1 set), the following procedure has been implemented: first, we fix $\beta_{u,d}$ within the range $[0, 4]$ by discrete steps of 0.5, for a total of 81 different $\{\beta_u, \beta_d\}$ configurations; second, for each of these $\{\beta_u, \beta_d\}$ pairs, we perform a new fit of the other parameters and evaluate its corresponding total χ^2. Only those configurations with $\Delta\chi^2 = \chi^2 - \chi_0^2$ less than a statistically significant reference value (see [44] for further details) have been kept. In practice, in this case all 81 configurations fulfill the selection criterion, reinforcing the conclusion that the presently available SIDIS data do not constrain the large x behaviour of the TMD transversity distribution.

For a given process of interest and the related azimuthal asymmetries, like, e.g., the inclusive particle production in polarized pp collisions studied in this review (in particular, in the large x_F region), the final step of the scan procedure consists in taking the full envelope of the values of the asymmetry generated by considering all the selected configuration sets. This envelope gives an estimate of
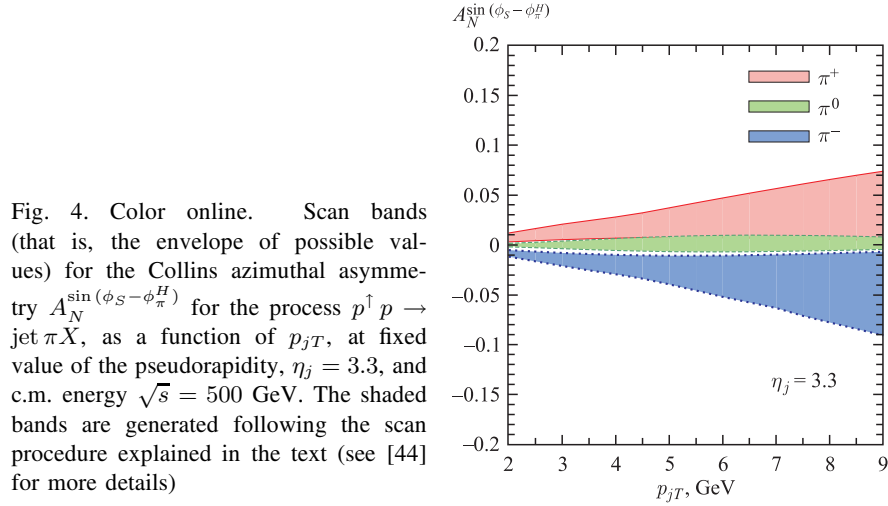
$A_N^{\sin(\phi_S - \phi_\pi^H)}$

Fig. 4. Color online.   Scan bands (that is, the envelope of possible values) for the Collins azimuthal asymmetry $A_N^{\sin(\phi_S - \phi_\pi^H)}$ for the process $p^\uparrow p \to \text{jet}\,\pi X$, as a function of p_{jT}, at fixed value of the pseudorapidity, $\eta_j = 3.3$, and c.m. energy $\sqrt{s} = 500$ GeV. The shaded bands are generated following the scan procedure explained in the text (see [44] for more details)

the uncertainty in the asymmetry calculation due to the limited x_B range covered by the SIDIS data and the consequent indeterminacy in the large x behaviour of the quark transversity distribution.

As an example, in Fig. 4 we show the resulting scan bands for the Collins azimuthal asymmetry $A_N^{\sin(\phi_S - \phi_\pi^H)}$ for neutral and charged pions at the RHIC c.m. energy $\sqrt{s} = 500$ GeV, as a function of the jet transverse momentum and fixed jet pseudorapidity, $\eta_j = 3.3$ (that is, the same kinematical configuration of Fig. 3).

It is clear from this plot how the uncertainty on the asymmetry grows as p_{jT} (and consequently x_F) increases. This information is complementary and integrates the indications obtained comparing the results of the specific SIDIS 1 and SIDIS 2 sets in Figs. 2 and 3.

It is also clear that future measurements of the Collins asymmetries for charged pions in the processes $p^\uparrow p \to \text{jet}\,\pi X$ would be very helpful in delineating the large x behaviour of the quark transversity distributions. We point out that in the central rapidity region these asymmetries are much smaller. Nevertheless, they are currently under active investigation by the STAR Collaboration [19, 20].

Finally, analogous estimates for the azimuthal moment $A_N^{\sin(\phi_S - 2\phi_\pi^H)}$ cannot be provided, since the underlying TMD gluon distribution and fragmentation functions are still completely unknown.

3.2. The Sivers Asymmetries. In analogy to Eqs. (22) and (23), the azimuthal moment $A_N^{\sin\phi_S}$ can be written schematically as

$$A_N^{\sin\phi_S} \sim f_{1T}^\perp(x_a, \mathbf{k}_{\perp a}^2) \otimes f_1(x_b, \mathbf{k}_{\perp b}^2) \otimes D_1(z, \mathbf{k}_{\perp\pi}^2), \qquad (24)$$

i.e., as a convolution of the Sivers function for the parton inside the transversely polarized proton with the unpolarized TMD distribution and fragmentation functions of the two other active partons in the hard scattering. The explicit expression for the numerator of the asymmetry is given in Eq. (17). Both the quark and gluon Sivers functions contribute to this observable, and in principle these contributions cannot be separated. Nevertheless, it should be possible to select between the two
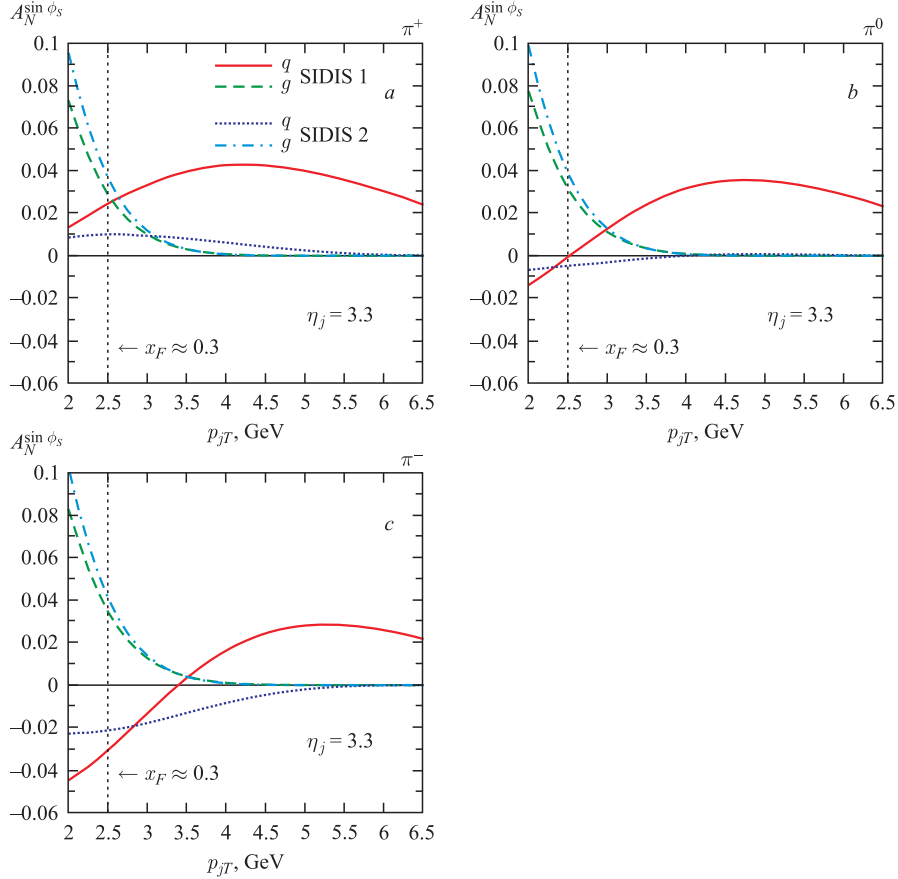

Fig. 5. Color online. The Sivers asymmetry $A_N^{\sin \phi_S}$ for the process $p^\uparrow p \to \text{jet} \, \pi X$, as a function of p_{jT}, at fixed value of the rapidity η_j and c.m. energy $\sqrt{s} = 200$ GeV. Estimates for the quark contribution are obtained by adopting the SIDIS 1 and SIDIS 2 parameterization sets. The gluon Sivers function is assumed to be positive and to saturate an updated version of the bound in [45]. The dotted vertical line delimits the region $x_F \approx 0.3$, beyond which the currently available parameterizations for the quark Sivers function, extracted from the SIDIS data, are affected by large uncertainties

terms by looking at particular kinematic domains, in which only one of them is expected to be sizeable and dominates the asymmetry [13].

In Fig. 5, $A_N^{\sin\phi_S}$ is presented, for both neutral and charged pions, at the c.m. energy $\sqrt{s} = 200$ GeV and in the forward rapidity region ($\eta_j = 3.3$), as a function of p_{jT}. The quark Sivers contribution is estimated adopting the SIDIS 1 and SIDIS 2 parameterizations, which give comparable results only in the p_{jT} region, where they are constrained by the SIDIS data (see, as for the case of the Collins asymmetry, the dotted vertical line). The almost unknown gluon Sivers function is tentatively taken positive and saturates an updated version of the bound calculated in [45] by analyzing the PHENIX data for transverse single-spin asymmetries for the process $p^\uparrow p \to \pi^0 X$, with the neutral pion being produced in the central rapidity region.

Clearly, the measurement of $A_N^{\sin\phi_S}$ at large p_{jT}, where the role of the gluon Sivers function becomes negligible, could be quite helpful in discriminating between the SIDIS 1 and SIDIS 2 parameterizations and constraining the large x behaviour of the u, d quark Sivers functions.

The present analysis can be extended to the transverse single-spin asymmetry $A_N^{\sin\phi_S}$ for inclusive jet production in $p^\uparrow p \to \mathrm{jet}\,X$, by simply integrating the results for the process $p^\uparrow p \to \mathrm{jet}\,\pi X$ over the pion phase space. In this case, in the general structure of the asymmetry in Eq. (13), only the $\sin\phi_S$ modulation will be present, since all the mechanisms related to the fragmentation process cannot play a role. The numerator of $A_N^{\sin\phi_S}$ will be given by Eq. (17), in which the fragmentation function $D_1^c(z, \mathbf{k}_{\perp\pi}^2)$ is replaced by $\delta(z-1)\,\delta^2(\mathbf{k}_{\perp\pi})$. As already done for pion-jet production, we have checked explicitly that, for the kinematic configurations under study, all other possible contributions rather than the Sivers one are numerically irrelevant, and therefore can be safely neglected.

In Fig. 6, we present our results for $A_N^{\sin\phi_S}$ for inclusive jet production at the c.m. energy $\sqrt{s} = 200$ GeV, as a function of p_{jT} and fixed rapidities $\eta_j = 0$ (plot a) and $\eta_j = 3.3$ (plot b). As before, they have been obtained utilizing the SIDIS 1 and SIDIS 2 parameterizations for the quark Sivers functions and an updated version of the bound presented in [45] for the gluon Sivers function (taken to be positive). Predictions in the forward rapidity region are very similar to those for neutral pion-jet production shown in Fig. 5, b, where the gluon component dominates only at very low values of p_{jT} and decreases quickly as p_{jT} increases. On the other hand, in the central rapidity region, the gluon component is always larger than the quark one, the latter being practically negligible. A measurement of $A_N^{\sin\phi_S}$ in this kinematic domain would therefore be ideal to probe the gluon Sivers function [13, 46]. Results for the RHIC kinematics at $\sqrt{s} = 500$ GeV will be discussed in the next section.

Notice that, the scan procedure discussed in Subsec. 3.1 and in [44] for the Collins effect in the process $p^\uparrow p \to hX$, and for the large x behaviour of the transversity distribution, can also be applied, for the same process, to the Sivers
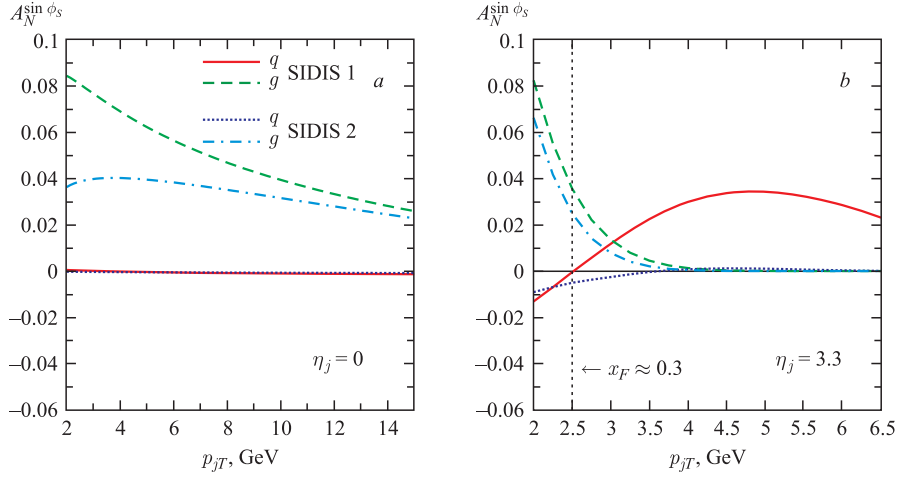
Fig. 6. Color online. The Sivers asymmetry $A_N^{\sin \phi_S}$ for the process $p^\uparrow p \to$ jet X, as a function of p_{jT}, at fixed value of the rapidity η_j and c.m. energy $\sqrt{s} = 200$ GeV. Estimates for the quark contribution are obtained by adopting the SIDIS 1 and SIDIS 2 parameterization sets. The gluon Sivers function is assumed to be positive and to saturate an updated version of the bound in [45]. The dotted vertical line delimits the region $x_F \approx 0.3$, beyond which the currently available parameterizations for the quark Sivers function, extracted from the SIDIS data, are affected by large uncertainties

asymmetry and, in this case, the large x behaviour of the Sivers distribution, see [47]. We will present some new results obtained utilizing the scan procedure for the Sivers azimuthal asymmetry $A_N^{\sin \phi_S}$ in the processes $p^\uparrow p \to$ jet πX and $p^\uparrow p \to$ jet X in the next section.

4. A STUDY OF THE PROCESS DEPENDENCE OF THE SIVERS FUNCTION

In the GPM approach adopted so far, TMD distribution and fragmentation functions are assumed to be universal. In particular, the Sivers function in Eq. (17) is taken to be the same as the one extracted from SIDIS [13, 48],

$$f_{1T}^{\perp a}(x_a, \mathbf{k}_{\perp a}^2) \equiv f_{1T}^{\perp a, \text{SIDIS}}(x_a, \mathbf{k}_{\perp a}^2). \qquad (25)$$

There is at present a large consensus on the universality of the Collins fragmentation function (which, however, must be verified phenomenologically), at least for the processes where the QCD factorization has been proven. On the contrary, several naively time-reversal odd (T-odd) TMD distributions crucially

depend on initial and/or final-state interactions (embedded via gauge links) among struck partons and soft remnants in the process.

Recently, the azimuthal asymmetries for the distribution of leading pions inside jets have been studied allowing for the process dependence of the quark Sivers function [17] within the framework of the so-called colour gauge invariant GPM [16]. In the CGI GPM the existence of a nonzero Sivers function in a transversely polarized hadron is due to the effects of initial- (ISIs) and final- (FSIs) state interactions between the struck parton and the spectator remnants from the polarized proton. These interactions depend on the particular process considered and make the Sivers function non-universal. The typical example is provided by the predicted opposite sign of the quark Sivers functions in SIDIS, where only FSIs are present, and in the DY process, in which only ISIs can be active.

The colour factor structure of the Sivers function for the reaction under study, involving hadrons in both the initial and the final states, is more complicated because both ISIs and FSIs contribute. Equation (17) has then to be replaced by

$$
E_j \frac{d\Delta\sigma^{(\text{Sivers})}}{d^3\mathbf{p}_j\, dz\, d^2\mathbf{k}_{\perp\pi}} =
$$
$$
= \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a}{x_a}\, d^2\mathbf{k}_{\perp a} \int \frac{dx_b}{x_b}\, d^2\mathbf{k}_{\perp b}\, \delta(\hat{s}+\hat{t}+\hat{u})\, H^U_{ab\to cd}(\hat{s},\hat{t},\hat{u}) \times
$$
$$
\times \left(-\frac{k_{\perp a}}{M}\right) f_{1T}^{\perp a, ab\to cd}(x_a, \mathbf{k}_{\perp a}^2) \cos\phi_a\, f_{b/B}(x_b, \mathbf{k}_{\perp b}^2)\, D_1^c(z, \mathbf{k}_{\perp\pi}^2) \sin\phi_S, \quad (26)
$$

in which a *process-dependent* Sivers function denoted as $f_{1T}^{\perp a, ab\to cd}$ is used. The resulting colour factors, C_I (C_{F_c}), for initial- (final-) state interactions determine the proper Sivers function to be used for each of the different partonic scattering processes $ab \to cd$. They are the same as the ones calculated in [16] for single inclusive hadron production using a one-gluon exchange approximation. Finally, the process dependence of the Sivers function can be absorbed into the squared hard partonic scattering amplitude $H^U_{ab\to cd}$, that is

$$
f_{1T}^{\perp a, ab\to cd}\, H^U_{ab\to cd} \equiv f_{1T}^{\perp a, \text{SIDIS}}\, H^{\text{Inc}}_{ab\to cd}, \quad (27)
$$

where the new hard function $H^{\text{Inc}}_{ab\to cd}$ has been introduced. Details on the connection between the CGI GPM and the twist-three collinear formalism [49, 50], suggested by Eq. (27), can be found in [16].

Since our aim is to study the process dependence of the quark Sivers function, we analyze pion-jet production in the forward rapidity region, where possible contributions from the sea-quark and gluon Sivers functions are expected to be negligible. This assumption is supported by studies of SSAs in SIDIS [24] and
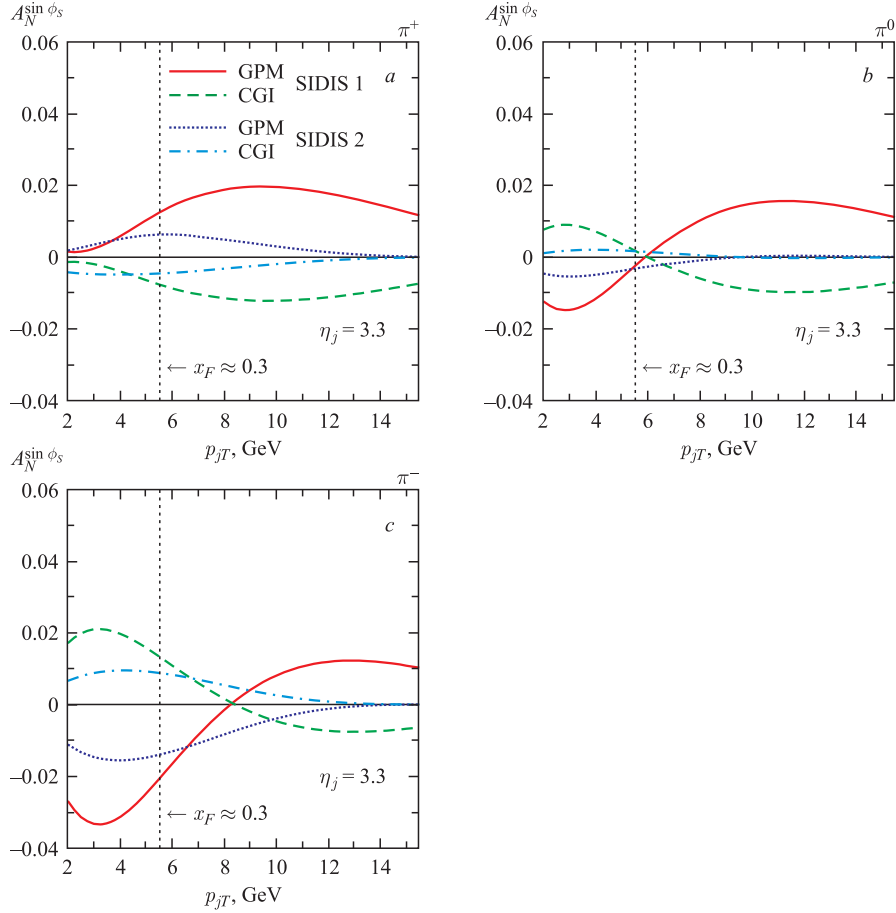
Fig. 7. Color online. The quark contribution to the Sivers asymmetry $A_N^{\sin\phi_S}$ in the GPM and CGI GPM approaches for the process $p^\uparrow p \to \mathrm{jet}\,\pi X$, as a function of p_{jT}, at fixed value of the rapidity η_j and c.m. energy $\sqrt{s} = 500$ GeV. Estimates are obtained by adopting the SIDIS 1 and SIDIS 2 parameterization sets. The dotted vertical line delimits the region $x_F \approx 0.3$, beyond which the currently available parameterizations for the quark Sivers function, extracted from the SIDIS data, are affected by large uncertainties

in the processes $pp \to \pi X$ at central rapidities [45, 51, 52], and by the analysis performed in [53]. Our results are shown in Fig. 7, where $A_N^{\sin\phi_S}$, integrated over $\mathbf{k}_{\perp\pi}$ and z ($z \geqslant 0.3$), is plotted as a function of the jet transverse momentum p_{jT} at fixed jet rapidity $\eta_j = 3.3$, for the RHIC energy $\sqrt{s} = 500$ GeV. The solid and dotted lines represent our predictions in the GPM formalism using the two available sets, SIDIS 1 and SIDIS 2, respectively, for the quark Sivers

function, while the dashed and dash-dotted lines describe the analogous predictions in the CGI GPM formalism. As one can easily see, the results obtained with and without inclusion of colour gauge factors are comparable in size but have *opposite signs* [17], in close analogy to the DY case. The reason is that, at forward rapidity, the dominant channel is $qg \to qg$, where the final quark is identified with the observed jet, for which the effects of ISIs/FSIs lead to

$$H^{\text{Inc}}_{qg \to qg} \sim -\frac{N_c^2 + 2}{N_c^2 - 1}\frac{\hat{s}^2}{\hat{t}^2} \tag{28}$$

in the CGI GPM, while

$$H^{U}_{qg \to qg} \sim \frac{2\hat{s}^2}{\hat{t}^2} \tag{29}$$

in the GPM. Moreover, as already pointed out in the previous section, our estimates obtained adopting the two different parameterizations SIDIS 1 and SIDIS 2 are similar only in the region $x_F \leqslant 0.3$, corresponding to $p_{jT} \leqslant 5.5$ GeV at $\sqrt{s} = 500$ GeV. Therefore, this is the optimal kinematic region to test directly the process dependence of the Sivers function: the measurement of a sizeable asymmetry for $p_{jT} \leqslant 5.5$ GeV could easily discriminate between the two different approaches and probe the universality properties of the Sivers function. At the c.m. energy $\sqrt{s} = 200$ GeV our predictions would be qualitatively similar to the ones presented in Fig. 7, becoming almost twice as large. However, the range of p_{jT} covered would now be narrower, $p_{jT} \leqslant 6.5$ GeV, and $x_F \leqslant 0.3$ would correspond to $p_{jT} \leqslant 2.5$ GeV.

As already discussed in the previous section for the Collins azimuthal asymmetry, the scan procedure introduced in [44, 47] offers a different and more complete information. In fact, it gives the envelope of all possible values of $A_N^{\sin \phi_S}$ coming from parameterizations of the Sivers function leading to good fits of the SIDIS data on the analogous asymmetry. Therefore, in Fig. 8 we present the analogous of Fig. 7, obtained using new results of the Sivers scan procedure. These plots confirm the conclusions drawn from Fig. 7: the low-intermediate p_{jT} region is the most interesting for a discrimination between the GPM and CGI GPM approaches. As soon as p_{jT} grows beyond 4–6 GeV, the two scan bands start overlapping and we loose predictive power. For this reason, we cut our plots at $p_{jT} = 9$ GeV, although the kinematical limit is larger (see Fig. 7). Clearly, the most favourable situation seems to be that of the π^-, for which the asymmetry is larger and the scan bands for the GPM and CGI GPM cases are well separated up to $p_{jT} \simeq 5$ GeV.

Finally, we consider also single inclusive jet production in proton–proton scattering. Data for this observable are now available and have been presented in [54, 55]. The results obtained for $A_N^{\sin \phi_S}$ are plotted in Fig. 9. In panel *a*, we show $A_N^{\sin \phi_S}$ for the process $p^\uparrow p \to$ jet X, as a function of p_{jT} and fixed
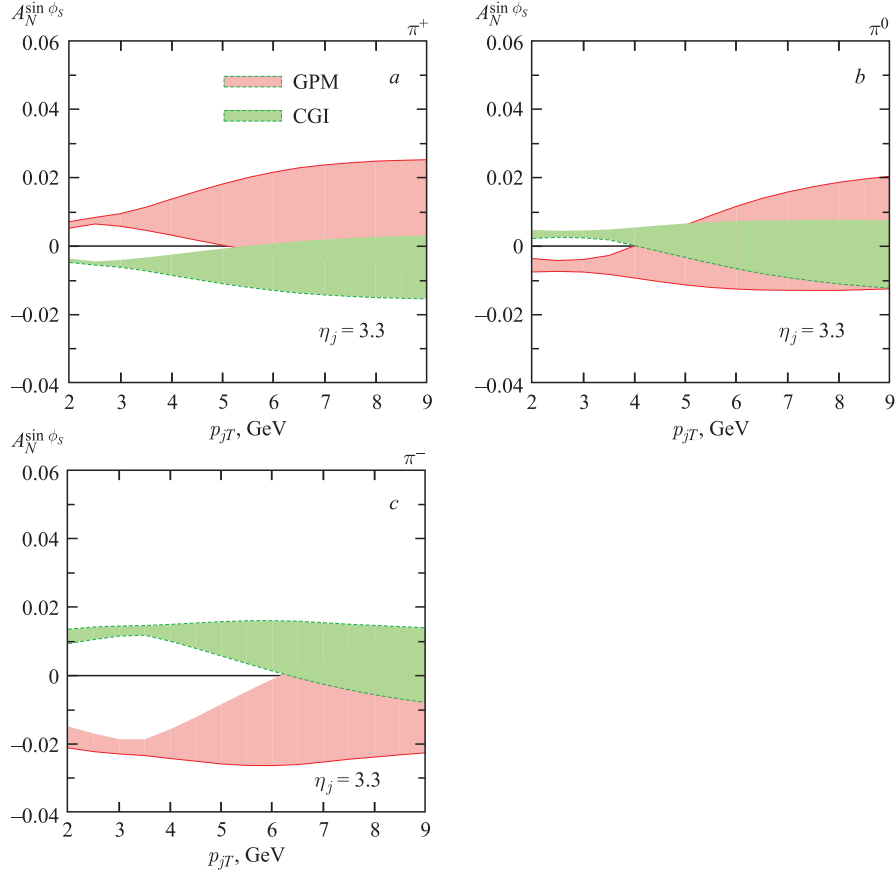
Fig. 8. Color online. Scan bands (that is, the envelope of possible values) for the quark contribution to the Sivers asymmetry $A_N^{\sin\phi_S}$ in the GPM and CGI GPM approaches, for the process $p^\uparrow p \to \mathrm{jet}\,\pi X$, as a function of p_{jT}, at fixed value of the rapidity η_j and c.m. energy $\sqrt{s} = 500$ GeV. The shaded bands are generated following the scan procedure explained in the text (see [44, 47] for more details)

pseudorapidity, $\eta_j = 3.3$, at RHIC c.m. energy $\sqrt{s} = 500$ GeV. The results look very similar to those for the case of neutral pion-jet production, shown in Fig. 7, b. In Fig. 9, b, we compare the GPM and CGI GPM scan bands for the Sivers asymmetry $A_N^{\sin\phi_S}$ with recent results by the A_NDY Collaboration [54, 55], shown as a function of x_F, at fixed pseudorapidity $\eta_j = 3.25$ and $\sqrt{s} = 500$ GeV. As expected, beyond $x_F \sim 0.3$, since the u, d quark Sivers functions are poorly constrained by the present SIDIS data, the scan bands become larger and overlap almost completely. Therefore, at this stage we cannot draw any conclusion by
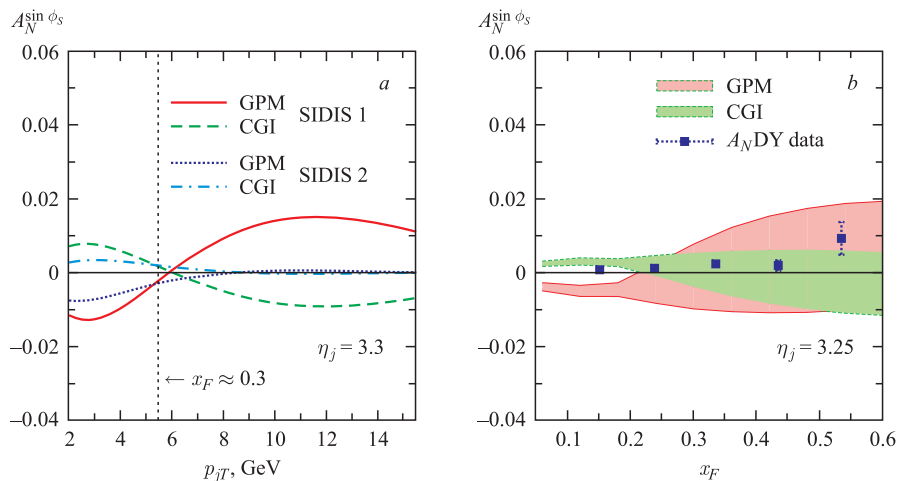
Fig. 9. Color online. *a*) The quark contribution to the Sivers asymmetry $A_N^{\sin\phi_S}$ in the GPM and CGI GPM approaches for the process $p^\uparrow p \to \mathrm{jet}\,X$, as a function of p_{jT}, at fixed value of the rapidity $\eta_j = 3.3$ and c.m. energy $\sqrt{s} = 500$ GeV. Estimates are obtained by adopting the SIDIS 1 and SIDIS 2 parameterization sets. The dotted vertical line delimits the region $x_F \approx 0.3$, beyond which the currently available parameterizations for the quark Sivers function, extracted from the SIDIS data, are affected by large uncertainties. *b*) Scan bands (that is, the envelope of possible values) for the quark contribution to the Sivers asymmetry $A_N^{\sin\phi_S}$ in the GPM and CGI GPM approaches, for the process $p^\uparrow p \to \mathrm{jet}\,X$, as a function of x_F, at fixed value of the rapidity $\eta_j = 3.25$ and c.m. energy $\sqrt{s} = 500$ GeV. The shaded bands are generated following the scan procedure explained in the text (see [44, 47] for more details)

looking solely at these results. Only the first and the last A_NDY data points seem to favour, respectively, the CGI GPM and GPM approaches, but much more work is needed. See also [56] for a similar study comparing the GPM and collinear twist-three results.

5. OTHER TESTS OF THE PROCESS DEPENDENCE OF THE TMD FUNCTIONS

In this section, we present a short overview of other possible tests of the process dependence of the TMD parton distribution and fragmentation functions proposed in the literature. Due to the lack of space, we will not cover thoroughly all aspects of the subject, limiting ourselves to a discussion of the more interesting phenomenological tests. A detailed treatment may be found in the original papers quoted in the bibliography.

Basically, all these phenomenological studies try to compare predictions for spin asymmetries coming from different formalisms (like the collinear twist-three, the GPM and CGI GPM approaches) in kinematical situations where typically only one of the many possible effects dominates. If the predictions of the various approaches are very different (in particular, in sign), then interesting phenomenological investigations can be performed.

In [57], it was proposed to study a weighted asymmetry in the azimuthal distribution of photon-jet pairs in the polarized process $p^\uparrow p \to \gamma\,\mathrm{jet}\,X$. It was shown that for specific kinematical configurations reachable at RHIC, the asymmetry is dominated by the quark Sivers effect, making its interpretation much more clear. Moreover, predictions coming from gluonic-pole cross sections [58], directly related to the Wilson lines preserving colour gauge invariance and leading to process-dependent effects, are almost opposite to those of the generalized parton model. Therefore, experimental tests of these results offer an interesting alternative way to investigate the process dependence of the Sivers function and the predicted relative sign difference in the SIDIS and Drell–Yan processes.

As we have already discussed in the previous section, in [16] Gamberg and Kang have discussed a modified version of the generalized parton model, the colour gauge invariant GPM. Assuming, as in the GPM, the validity of factorization for single inclusive particle production in hadronic collisions, this approach includes the process dependence of TMDs by taking into account initial- and final-state interactions between the struck parton and the parent hadron remnants. Once more, these interactions come out from appropriate, process-dependent colour gauge links. It was also shown that the CGI GPM is in close connection with the collinear twist-three approach. The phenomenological implications of the CGI GPM for the process dependence of the Sivers effect in the reactions $p^\uparrow p \to \pi^0, \gamma + X$ were investigated. Once again, the main result is that the transverse single-spin asymmetry due to the quark Sivers contribution has a similar size but opposite sign with respect to the original GPM that assumes the universality of TMDs. Applications of the approach to pion-jet production were discussed in the previous section and in more detail in [17].

The study of the universality and process dependence of the Sivers function is of relevance also in the context of the so-called «sign mismatch» issue for the collinear twist-three approach [59]. Since in this formalism factorization has been proven for both SIDIS processes and single inclusive particle production in hadronic collisions at large-energy scales, the multiparton soft correlation functions involved are universal and process-independent. On the other hand, factorization holds also for the TMD approach in SIDIS, for large Q^2 and small transverse momentum of the final hadron. It has been shown that there is a common region of validity of these two approaches, and this allows one to find a relation among the twist-three quark–gluon correlation function and the first $\mathbf{k}_\perp$ moment of the TMD Sivers function. However, if one uses this relation from

the SIDIS processes for the calculation in the twist-three approach of A_N in the processes $p^\uparrow p \to \pi^0, \gamma + X$, one finds results opposite in sign with respect to those obtained by directly fitting, in the same approach, the RHIC data for $p^\uparrow p \to \pi^0 X$.

In [60], the authors have explored the possibility of escaping this sign-mismatch problem for the twist-three approach by accounting for nodes of the quark Sivers function (either in its x or $\mathbf{k}_\perp$ dependence). They found that by allowing for a single node in the quark Sivers function, one is not able to «cure» the sign-mismatch problem and explain both the STAR and BRAHMS A_N data for the reactions $p^\uparrow p \to \pi X$. However, one must not forget that the Sivers effect is not the only possible contribution to A_N. In fact, it may be that the Sivers effect gives a subdominant contribution, and the asymmetry is mainly due to the Collins effect in the fragmentation sector. To investigate this eventuality, it is crucial to collect experimental information for processes like, e.g., $p^\uparrow p \to \gamma X$ and $p^\uparrow p \to \text{jet}\,X$, where fragmentation in the final state is absent.

As we have seen, quite recently the A_NDY Collaboration at RHIC [54,55] has presented preliminary results for $A_N(p^\uparrow p \to \text{jet}\,X)$ at forward rapidity and c.m. energy $\sqrt{s} = 500$ GeV. Gamberg, Kang and Prokudin [56] have performed a new fit of the Sivers function using the HERMES and COMPASS data on the $A_N^{\sin(\phi_h - \phi_S)}$ asymmetry. Then, using this information and the relation among the twist-three quark–gluon correlation function and the first $\mathbf{k}_\perp$ moment of the Sivers function discussed above, they have estimated the spin asymmetry for $p^\uparrow p \to \text{jet}\,X$ in the collinear twist-three approach, comparing it with the A_NDY data. They found, taking into account that the large x behaviour of the Sivers function is poorly constrained by the present SIDIS data, that their estimate is consistent with experimental data and there is in fact no strong sign-mismatch problem, contrary to the case of pion single-spin asymmetries discussed above.

Kang and Qiu [61] have proposed to probe the (modified) universality of the quark and gluon Sivers functions, that is, the change of sign between the Sivers functions in the the SIDIS and DY processes, by studying the transverse single-spin asymmetry A_N for W production and inclusive lepton production from W decays in polarized proton–proton collisions at the RHIC energies. Although the lepton asymmetry is diluted by W decays, its size can reach several percents over a large range of lepton rapidity at RHIC. Therefore, this process can offer an additional phenomenological test of the predicted sign change of the Sivers function. Moreover, because of the weak interaction, it can provide unique information, with respect to the DY case, on the flavour dependence and the functional form of the Sivers function.

Let us finally add some comments on the process dependence of the T-odd TMD fragmentation functions, like the Collins function and the so-called «polarizing» fragmentation function [6,7,62]. They have been shown to be universal by several authors, see, e.g., [12,63–66]. Testing phenomenologically universality in

the fragmentation sector is as important as the tests for the modified universality of the Sivers functions discussed above. However, for the Collins function, the study of its universality is made difficult by its chiral-odd nature. In any physical observable it will always appear coupled to another chiral-odd object, either in the distribution or in the fragmentation sector. As well-known examples, the Collins function couples to the TMD transversity distribution in SIDIS and in the pion-jet production process considered in detail here. It couples to another Collins FF in the reactions $e^+e^- \to h_1 h_2 X$. Therefore, relative signs among these coupled chiral-odd functions are difficult to determine and require the study of different observables involving additional chiral-odd functions.

Based on these considerations, the authors of [67] have suggested to study the universality of the polarizing fragmentation functions and to test factorization by looking at the transverse polarization of Λ hyperons in the SIDIS processes and e^+e^- annihilations. They found that, despite the large uncertainties in these functions, definite signs for the hyperon polarization in different processes can be obtained, possibly allowing for a robust test of universality in this sector.

CONCLUSIONS

In the last years, impressive progress has been made in the theoretical understanding of the origin of the sizeable azimuthal and spin asymmetries measured by several experiments in polarized hadronic processes at large-energy scales. The crucial role of colour gauge invariance, and of the proper account of gauge links (Wilson lines) also in the transverse plane with respect to the usual light-cone direction, has been emphasized and investigated in depth. Several processes and polarized observables, for which factorization may not hold and universality can be broken, have been recognized. However, it is always difficult to assess, for the ongoing experiments, as well as for the ones which are going to be performed in the near future, the real relevance and size of process-dependent terms and factorization-breaking effects. Clearly, theoretical, more formal, developments must be complemented by corresponding detailed phenomenological analyses. These can be of great help and valuable guidance for further theoretical progress in this field.

In this review, we have discussed, in the framework of the so-called generalized parton model, the phenomenological relevance and usefulness of the reaction $p^\uparrow p \to \mathrm{jet}\,\pi X$ for the study of the process dependence of the TMD PDFs and FFs, in particular for the Sivers distribution and the Collins fragmentation function. We have shown how the study of this process can well complement information coming from the SIDIS, Drell–Yan and e^+e^- annihilations, particularly for the knowledge of the large x behaviour of the TMD quark transversity distributions and of the quark Sivers functions. We have also summarized additional phe-

nomenological tests, formulated within various theoretical approaches, recently suggested in the literature for the study of the universality properties and the process dependence of TMDs.

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