ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА
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PROBLEMS RELATED TO GAUGE INVARIANCE AND MOMENTUM, SPIN DECOMPOSITION IN NUCLEON STRUCTURE STUDY

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How do the quark and gluon share the nucleon momentum? How does the nucleon spin distribute among its constituents? What do the quark and gluon momentum, spin and orbital angular momentum mean? These problems are analyzed and a solution is proposed based on *gauge invariance principle, canonical quantization rule* and *Poincaré covariance*.

Как кварки и глюоны делят импульс в нуклоне? Как распределен спин нуклона среди его составляющих? Что такое импульсы кварков и глюонов, спины и орбитальные угловые моменты? Эти проблемы анализируются, и решение основывается на *калибровочно-инвариантном принципе, канонических правилах квантования* и *ковариантности Пуанкаре*.

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INTRODUCTION

Nucleon is the dominant component of visible mass in the universe. It is a fundamental laboratory for the study of the microscopic structure of matter controlled by strong interaction and the low-energy scale properties of strong interaction theory, the nonperturbative QCD. The quark model proposed in 1964 suggested a naive picture of the nucleon internal structure. It is assumed to consist of three valence quarks, all of them occupying the lowest s-wave orbit. Therefore, the nucleon spin is solely due to quark spin. This picture explains the anomalous nucleon magnetic moments, discovered in 1933, quite well. The lepton–nucleon deep inelastic scattering (DIS) in 1960s–1970s confirmed that there are really colored, spin one-half, fractional charged particles within the nucleon. However,

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the polarized μ–p deep inelastic scattering (DIS) measurement in 1987 showed that the quark spin contribution is only a small amount of nucleon spin and this led to the so-called «proton spin crisis». After the experimental effort of a quarter of century, it is confirmed that the quark spin contribution is about $0.25(\hbar/2)$. How the nucleon spin distributes among its constituents is still a controversial issue. Even how the quark and gluon share the nucleon momentum is also under debate. We found that there have been confusions about what are the quark and gluon momentum, spin and orbital angular momentum which complicated the nucleon structure study. In this report we will analyze these problems and propose a solution based on *gauge-invariance principle, canonical quantization rule* and *Poincaré covariance*.

1. THE FIRST PROTON SPIN CRISIS AND THE QUARK SPIN CONFUSION

M. Gell-Mann and G. Zweig proposed the quark model to describe the nucleon internal structure in 1964. In this model the nucleon is assumed to consist of three valence quarks which are all located in the lowest s-wave orbital state. The spin-flavor part is assumed to be in an $SU^{f\sigma}(6) \supset SU^{f}(3) \times SU^{\sigma}(2)$ symmetric state. Based on this picture, the nucleon spin is solely due to quark spin contribution. The quark orbital angular momentum (OAM) does not contribute. The gluon spin and OAM do not contribute, either. For proton, the quark spin contributions are

$$\begin{gathered} \Delta u = \frac{4}{3}, \quad \Delta d = -\frac{1}{3}, \quad \Delta s = 0, \\ L_q = L_G = \Delta G = 0, \end{gathered} \tag{1}$$

where $\Delta u, \Delta d, \Delta s, \Delta G$ stand for flavor u, d, s quark and gluon polarizations; L_q, L_G are the quark and gluon OAM contributions. In 1988, the European Muon Collaboration (EMC) published their polarized μ–p DIS result, which shows a surprisingly small quark spin contribution [1],

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \sim 0. \tag{2}$$

There had been a long-time debate on what this result means [2]. Experimentalists spent 25 years to improve the measurement continuously. The present overall fit result is [3]

$$\begin{aligned} &\Delta\Sigma = 0.817 - 0.453 - 0.055 = 0.255 \; (Q^2 = 1 \text{ GeV}^2, \; \Delta G = -0.118), \\ &\Delta\Sigma = 0.814 - 0.456 - 0.056 = 0.245 \; (Q^2 = 4 \text{ GeV}^2, \; \Delta G = -0.096), \\ &\Delta\Sigma = 0.813 - 0.458 - 0.057 = 0.242 \; (Q^2 = 10 \text{ GeV}^2, \; \Delta G = -0.084), \end{aligned} \tag{3}$$

where the numbers in the above equations are $\Delta u, \Delta d, \Delta s$, respectively, and ΔG is gluon spin polarization. Theoretically, the gluon anomaly contribution to the matrix element (ME) of the flavor singlet axial charge can be absorbed into the quark spin contribution in the gauge-invariant factorization scheme. Therefore, the popular idea is that the «proton spin crisis» is standing and the quark model picture of nucleon structure is invalidated by the polarized lepton–nucleon DIS measurements.

We did an analysis of the nucleon spin puzzle in 1998 [4], where we pointed out: (1) the polarized DIS measured «quark spin» contribution is the ME of the flavor singlet quark axial vector current operator for a longitudinal polarized proton. Even though this axial vector current operator is the relativistic field-theoretical extension of the nonrelativistic Pauli spin operator, it is different from the latter which is calculated in quark model to obtain the quark spin contribution. One should calculate the ME of the axial vector current operator which includes the relativistic correction and $q\bar{q}$ creation (annihilation) terms in addition to the Pauli spin. (2) The pure valence quark q^3 configuration is only the dominant component even for a ground-state proton, and there is sea-quark excitation or the QCD vacuum polarization component in the ground-state proton. One should take these sea-quark components into account in a refined quark model calculation.

Based on this, we did a Fock space expansion quark model calculation of the ME of the axial vector current operator and found that this model predicts the quark spin content correctly with almost the same model parameters as used in the standard ones. Moreover, this model reproduces all of the masses and magnetic moments of the ground-state octet and decuplet baryons. In this model, the relativistic correction shifts ~ 0.32 quark spin to quark OAM. The $q^3 \rightleftharpoons q^3 q\bar{q}$ crossing ME due to the $q\bar{q}$ creation (annihilation) shifts ~ 0.44 quark spin to quark OAM. The u-, d-, s-quark magnetic moment modifications almost cancel each other and leave the nonrelativistic quark model predicting the correct magnetic moments of the ground-state baryons. It also gives the correct $g_A = \Delta u - \Delta d = 1.23$. The numerical results of the u-, d-, s-quark polarizations are term-by-term consistent with the lattice results [5]. The physical mechanism is also the same; our model and lattice calculation both show that it is the disconnected and connected diagrams which shift quark spin to OAM. We had done a transformation to transform our model u-, d-, s-quark momentum distributions in the rest frame to the infinite momentum frame by the method developed in [6] and found that qualitatively the model parton distributions are similar to the measured ones for low Q^2, especially in the valence quark kinematic region.

Then where does the nucleon get its spin? Our model answer is that the relativistic field-theoretical quark OAM will compensate the quark spin losing. The relativistic quark spin $\mathbf{S}_q$ (the axial vector current operator) and OAM operator $\mathbf{L}_q$ can be expanded as follows through the decomposition of the Dirac spinor into its large and small components,

$$
\begin{aligned}
\mathbf{S}_q = \int d^3x\, \psi^\dagger \frac{\mathbf{\Sigma}}{2}\psi = \\
= \frac{1}{2}\int d^3x\, \bar{\psi}\boldsymbol{\gamma}\gamma^5\psi = \\
= \sum_{i,\lambda,\lambda'} \int d^3k\, \chi^\dagger_\lambda \frac{\sigma}{2}\chi_{\lambda'}(a^\dagger_{i,\mathbf{k},\lambda}a_{i,\mathbf{k},\lambda'} - b^\dagger_{i,\mathbf{k},\lambda'}b_{i,\mathbf{k},\lambda}) - \\
- \frac{1}{2}\sum_{i,\lambda,\lambda'} \int d^3k\, \chi^\dagger_\lambda \frac{\sigma\cdot\mathbf{k}}{k_0(k_0+m_i)} i\sigma\times\mathbf{k}\chi_{\lambda'}(\mathbf{a}^\dagger_{\mathbf{i},\mathbf{k},\lambda}\mathbf{a}_{\mathbf{i},\mathbf{k},\lambda'} - \mathbf{b}^\dagger_{\mathbf{i},\mathbf{k},\lambda'}\mathbf{b}_{\mathbf{i},\mathbf{k},\lambda}) + \\
+ \sum_{i,\lambda,\lambda'} \int d^3k\, \chi^\dagger_\lambda \frac{i\sigma\times\mathbf{k}}{2k_0}\chi_{\lambda'} a^\dagger_{i,\mathbf{k},\lambda} b^\dagger_{i,-\mathbf{k},\lambda'} + \text{h.c.}
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\mathbf{L}_q = \int d^3x\, \psi^\dagger \mathbf{x}\times\frac{\nabla}{\mathbf{i}}\psi = \\
= \sum_{i,\lambda} \int d^3k\, (a^\dagger_{i,\mathbf{k},\lambda} i\nabla_\mathbf{k}\times\mathbf{k}\mathbf{a}_{\mathbf{i},\mathbf{k},\lambda} + \mathbf{b}^\dagger_{\mathbf{i},\mathbf{k},\lambda}\mathbf{i}\nabla_\mathbf{k}\times\mathbf{k}\mathbf{b}_{\mathbf{i},\mathbf{k},\lambda}) + \\
+ \frac{1}{2}\sum_{i,\lambda,\lambda'} \int d^3k\, \chi^\dagger_\lambda \frac{\sigma\cdot\mathbf{k}}{k_0(k_0+m_i)} i\sigma\times\mathbf{k}\chi_{\lambda'}(\mathbf{a}^\dagger_{\mathbf{i},\mathbf{k},\lambda}\mathbf{a}_{\mathbf{i},\mathbf{k},\lambda'} - \mathbf{b}^\dagger_{\mathbf{i},\mathbf{k},\lambda'}\mathbf{b}_{\mathbf{i},\mathbf{k},\lambda}) - \\
- \sum_{i,\lambda,\lambda'} \int d^3k\, \chi^\dagger_\lambda \frac{i\sigma\times\mathbf{k}}{2k_0}\chi_{\lambda'} a^\dagger_{i,\mathbf{k},\lambda} b^\dagger_{i,-\mathbf{k},\lambda'} + \text{h.c.}
\end{aligned} \tag{5}
$$

Equation (4) shows what we already mentioned above: that the relativistic quark spin includes three terms, the third line is the usual nonrelativistic Pauli spin which is what one calculated in the usual nonrelativistic quark model to obtain the quark spin contribution expressed in Eq. (1), the fourth line is the relativistic correction which shifts quark spin to quark OAM, the fifth line is the $q\bar{q}$ creation (annihilation) term which will couple the dominant valence q^3 component to the minor valence quark core and meson cloud components $q^3q\bar{q}$ and shift further the quark spin to the OAM. The relativistic reduction of quark spin had been calculated in many relativistic quark models. However, the $q\bar{q}$ creation (annihilation) term contribution has almost never been calculated in the vast quark model calculations even after the «proton spin crisis» and this is the most important mechanism (the disconnected and connected diagrams) to shift the quark spin to OAM [7].

Comparing Eqs. (4) and (5), one can find that the relativistic correction and $q\bar{q}$ creation (annihilation) terms in the relativistic field-theoretical-quark spin and OAM cancel each other exactly and so we have

$$\mathbf{S}_q^{\mathrm{nr}} + \mathbf{L}_q^{\mathrm{nr}} = \mathbf{S}_q^r + \mathbf{L}_q^r. \tag{6}$$

This means that the relativistic quark OAM contribution will compensate the quark spin losing exactly. If we assume the static gluon field does not contribute to nucleon spin and the quantum fluctuation of the background gluon field is small, then the nucleon spin can be viewed as mainly due to the nonrelativistic quark spin $\mathbf{S}_q^{\rm nr}$ and a very small amount of nonrelativistic quark OAM $\mathbf{L}_q^{\rm nr}$ because of the small sea-quark component or a small amount of the relativistic quark spin $\mathbf{S}_q^r$ and a large amount of the relativistic quark OAM $\mathbf{L}_q^r$ contributions. Equations (4), (5) and (6) also show that the nonrelativistic spin and OAM operators used for a long time in the nonrelativistic quantum mechanics, if added together, are correct. Of course, the relativistic field-theoretical ones are more realistic.

The nonlinear realization of chiral symmetry spontaneous breaking

$$\tilde{\psi}(x) = \mathrm{e}^{i\gamma^5 \xi_a(x)\lambda_a}\, \psi(x) \tag{7}$$

transforms the current quark to the constituent quark and this transformation keeps the quark vector and axial vector current operators invariant [8]. This serves as the nonperturbative QCD basis of our model numerical calculation.

Based on these analyses, we conclude that there is no «proton spin crisis» and the quark model picture of the nucleon structure is qualitatively correct. But there is «quark spin confusion» in misidentifying the relativistic field-theoretical spin to the nonrelativistic Pauli spin!

2. THE SECOND PROTON SPIN CRISIS AND THE QUARK ORBITAL ANGULAR MOMENTUM CONFUSION

R. L. Jaffe talked about the second proton spin crisis at the 1st International Symposium on Science at J-PARK, Mito, Ibaraki, Japan, 2008, where he compared the quark model OAM with the lattice calculated and the «measured» quark OAM through the generalized parton distribution (GPD). A. W. Thomas made a quantitative comparison of their quark model OAM contribution and the lattice calculated and GPD «measured» one [7]. Such kind of comparisons are popular in the literature. However, these are in fact quite different quantities. The quark model calculated one is the ME of the nonrelativistic *canonical* quark OAM $\mathbf{L}_q^{\rm nr}$ or the relativistic *canonical* one $\mathbf{L}_q^r$. The existing lattice calculated one or the «measured» one through GPD is the ME of the following operator:

$$\mathbf{L}_{qk} = \int d^3x\, \psi^\dagger \mathbf{x} \times \frac{\mathbf{D}}{\mathbf{i}} \psi, \quad \mathbf{D} = \nabla - ig\mathbf{A}. \tag{8}$$

This «OAM» $\mathbf{L}_{qk}$ includes the gauge potential $\mathbf{A}(\mathbf{x})$ of the studied physical system. For example, within the nucleon or meson the $\mathbf{A}(\mathbf{x})$ is different and so the $\mathbf{L}_{qk}$ for nucleon and meson will have different meaning. Moreover, this operator does not satisfy the angular momentum algebra,

$$\mathbf{L}_{qk} \times \mathbf{L}_{qk} \neq \mathbf{i}\mathbf{L}_{qk}. \tag{9}$$

So the present lattice calculated and the measured one through GPD are not the canonical OAM and to compare them with what had been calculated in quark models is nonsense. Even such an «OAM» is measured completely many years later, it is useless for checking our qualitative picture about the nucleon internal spin distribution obtained from quark models. If we misidentify this «measured quark OAM» to the quark model calculated one, it will lead to the second «proton spin crisis» as already happened in the literature.

3. WHAT IS THE GLUON SPIN?

It is widely believed that one can measure the gluon spin contribution to nucleon spin through the measurement of the gluon parton helicity distribution. It is also widely believed that there is no gauge-invariant local operator corresponding to the first moment of gluon helicity parton distribution. Even more generally, there is no gauge-invariant local operator corresponding to the spin of a massless particle including gluon and photon. On the other hand, the optical community believed that they had measured the photon spin. In all of the microscopic structure study, from atom to hadron, the multipole radiation analysis is crucial where the spin and OAM of photon is unavoidable. All of these call for a spin operator for the massless particles, the photon and gluon.

4. DECOMPOSITION OF THE TOTAL MOMENTUM AND ANGULAR MOMENTUM OF A GAUGE FIELD SYSTEM INTO ITS CONSTITUENTS

In the study of the nucleon structure, it is essential to understand how the mass, the momentum, the angular momentum or the spin distribute among its constituents. The above discussions show that one has to do a critical analysis on what are the momentum, spin and orbital angular momentum of quark and gluon first. Atom is a QED system and nucleon is a QCD system, they are both gauge field systems and therefore the above problems are common to atom and nucleon. We will choose the simple QED system to analyze the above problems and propose a solution based on *gauge invariance principle, canonical quantization rule* and *Poincaré covariance* first. The results are then extended to QCD case.

R. L. Jaffe and A. Manohar first obtain a decomposition of the total angular momentum of the QCD gauge system into quark and gluon spin and OAM parts in 1990 [9],

$$\mathbf{J} = \int \mathbf{d}^3\mathbf{x}\,\psi^\dagger \frac{1}{2}\mathbf{\Sigma}\psi + \int \mathbf{d}^3\mathbf{x}\,\psi^\dagger \mathbf{x} \times \frac{1}{\mathbf{i}}\nabla\psi + \int \mathbf{d}^3\mathbf{x}\,\mathbf{E}\times\mathbf{A} + \int \mathbf{d}^3\mathbf{x}\,\mathbf{E}^\mathbf{i}\mathbf{x}\times\nabla\mathbf{A}^\mathbf{i}. \tag{10}$$

This expression can be applied both to QED and to QCD, while for QCD case, the color indices are omitted. The first term is the electron (quark) spin, the second term is the electron (quark) OAM, the third term is the photon (gluon) spin, and the fourth term is the photon (gluon) OAM. The advantage of this decomposition is that each term satisfies the canonical angular momentum algebra and so is qualified to be called electron (quark) spin, OAM and photon (gluon) spin, OAM. The disadvantage of this decomposition is that only the first term is gauge-invariant and all the other three terms are gauge-dependent and so not measurable.

To remedy this drawback, our group and X. D. Ji obtained another decomposition in 1997 [10]

$$\mathbf{J} = \int \mathbf{d}^3\mathbf{x}\,\psi^\dagger \frac{1}{2}\mathbf{\Sigma}\psi + \int \mathbf{d}^3\mathbf{x}\,\psi^\dagger \mathbf{x} \times \frac{1}{\mathbf{i}}\mathbf{D}\psi + \int \mathbf{d}^3\mathbf{x}\,\mathbf{x}\times(\mathbf{E}\times\mathbf{B}). \tag{11}$$

This expression can be applied both to QED and to QCD and the color indices are omitted again. The advantage of this decomposition is that each term is gauge-invariant. The disadvantage is that the individual term does not satisfy the canonical angular momentum algebra except the first electron (quark) spin term. So to call them electron (quark) OAM $\mathbf{L}_{qk}$ and total angular momentum of photon (gluon) at least will cause confusion. This really happened as we discussed in the second section. Moreover, such kind of OAM is a mixing of electron and electro-magnetic potential in QED case and a mixing of quark and gluon potential in QCD case. For different systems, different atom or molecule in QED, different baryon or meson in QCD, such an OAM will include different gauge potential contribution. The $\mathbf{L}^2_{qk}$, $\mathbf{L}_{z,qk}$ do not commute and so cannot be diagonalized simultaneously. Another disadvantage is that the total photon (gluon) «angular momentum», i.e., the third term, has not been further decomposed into photon (gluon) spin and OAM. For a long time it is widely believed that it is impossible to further decompose the total angular momentum of a massless particle into gauge-invariant spin and OAM. This conflicts with the photon spin measurements in optics and the ongoing gluon spin measurement. It is also inconsistent with the widely used multipole radiation analysis where both photon spin and OAM are inevitable. Despite these serious drawbacks, this decomposition has been widely accepted as a benchmark for the study of nucleon spin structure. However, we ourselves never suppose it is a good decomposition for the angular momentum of a gauge field system!

Instead, we had turned to another direction to develop A. S. Wightman's idea: what quantum measurement measures is the ME of an operator. Gauge-dependent operator might have gauge-invariant ME for physical states [11]. If this can be proved, then one can use the Jaffe–Manohar gauge-dependent decomposition to study the nucleon spin structure. In order to go beyond the Lorentz gauge, we use path-integral formalism to study this possibility [10], because Wightman's argument is limited to the Lorentz gauge. Unfortunately, in the course of this study we found that the path-integral formalism does not always give reliable result, mainly because it involves divergent integrals [12]. Therefore, whether a gauge-dependent operator could have gauge-invariant ME for physical states is still an open question. Then we turn back to search for a decomposition, in which each term is individually *gauge-invariant*, satisfies *canonical quantization rule* and satisfies *Poincaré covariance* as much as possible. This is possible both for QED and for QCD. Let us discuss the simpler QED case first. The Abelian $U(1)$ gauge potential in QED case can be decomposed as follows:

$$\begin{gathered} A_\mu(x) = A_{\mu,\mathrm{phys}}(x) + A_{\mu,\mathrm{pure}}(x), \quad A_{\mu,\mathrm{phys}}(x) = \frac{1}{\partial^2}\partial_i F_{i\mu}(x), \\ A_{\mu,\mathrm{pure}}(x) = A_\mu(x) - A_{\mu,\mathrm{phys}}(x). \end{gathered} \tag{12}$$

Under suitable boundary condition the solution of Eq. (12) is unique. It is easy to check

$$F_{\mu\nu} = \partial_\mu A_{\nu,\mathrm{phys}} - \partial_\nu A_{\mu,\mathrm{phys}}, \quad F_{\mu\nu,\mathrm{pure}} = \partial_\mu A_{\nu,\mathrm{pure}} - \partial_\nu A_{\mu,\mathrm{pure}} = 0, \tag{13}$$

$$\partial_i A_{i,\mathrm{phys}} = \partial_i \frac{1}{\partial^2}\partial_k F_{ki} = 0, \quad \partial_i A_{i,\mathrm{pure}} = \partial_i A_i. \tag{14}$$

The spatial part of $A_{\mu,\mathrm{phys}}$ and $A_{\mu,\mathrm{pure}}$ are nothing else but the transverse and longitudinal parts of the well-known Helmholtz decomposition. Under a local gauge transformation,

$$\begin{gathered} \psi'(x) = \exp\left(-ie\omega(x)\right)\psi(x), \\ A'_\mu(x) = A_\mu(x) + \partial_\mu \omega(x), \end{gathered} \tag{15}$$

the $A_{\mu,\mathrm{phys}}$ and $A_{\mu,\mathrm{pure}}$ will transform as follows:

$$A'_{\mu,\mathrm{phys}}(x) = A_{\mu,\mathrm{phys}}, \quad A'_{\mu,\mathrm{pure}} = A_{\mu,\mathrm{pure}} + \partial_\mu \omega(x). \tag{16}$$

Please, note the $A_{\mu,\mathrm{phys}}$ is gauge-invariant! Based on these properties of $A_{\mu,\mathrm{phys}}$ and $A_{\mu,\mathrm{pure}}$, we obtain the following decomposition:

$$\begin{aligned} \mathbf{J} = \int d^3x\, \psi^\dagger \frac{1}{2}\mathbf{\Sigma}\psi + \int \mathbf{d^3x}\, \psi^\dagger \mathbf{x} \times \frac{1}{\mathbf{i}}\mathbf{D}_{\mathrm{pure}}\psi + \\ + \int d^3x\, \mathbf{E} \times \mathbf{A}_{\mathrm{phys}} + \int \mathbf{d^3x}\, \mathbf{E^i}\mathbf{x} \times \nabla \mathbf{A^i_{\mathrm{phys}}}. \end{aligned} \tag{17}$$

Here $\mathbf{D}_{\rm pure}$ is

$$\mathbf{D}_{\rm pure} = \nabla - \mathbf{ie}\mathbf{A}_{\rm pure}. \tag{18}$$

The $\mathbf{D}_{\rm pure}/i$ is the gauge-invariant extension of the canonical momentum which reduces to the canonical momentum in the $\mathbf{A}_{\rm pure} = 0$ gauge, the Coulomb gauge. The three components of $\mathbf{D}_{\rm pure}/i$ commute with each other, the same as the canonical momentum. The commutator between this operator and the OAM, the second term in the above equation, is the same as those of canonical momentum and OAM in the Poincaré algebra. Due to these properties we call them the «physical momentum». In fact, we can obtain a corresponding decomposition of the total momentum of the QED system

$$\mathbf{P} = \int \mathbf{d}^3\mathbf{x}\, \psi^\dagger \frac{\mathbf{D}_{\rm pure}}{\mathbf{i}} \psi + \int \mathbf{d}^3\mathbf{x}\, \mathbf{E}^\mathbf{i} \nabla \mathbf{A}^\mathbf{i}_{\rm phys}. \tag{19}$$

It is not hard to check that the individual term in these two decompositions satisfies gauge invariance and the canonical momentum and angular momentum quantization rule. This is our proposed decomposition of the momentum and angular momentum for the QED system. It is the first one to decompose the total angular momentum of a massless particle into a gauge-invariant spin (the third term) and OAM (the fourth term) operator and provides the gauge-field theoretical basis for the optical measurements of photon spin and OAM and the widely used multipole radiation analysis from atomic to hadron spectroscopy. In general this decomposition includes nonlocal operator if one uses Eq. (12) to calculate the $A_{\mu,\rm phys}$. However, in Coulomb gauge, where $A_{\mu,\rm pure} = 0$ and $A_{\mu,\rm phys} = A_\mu$, one no longer needs to use Eq. (12) to calculate the $A_{\mu,\rm phys}$, this decomposition will include local operator only. In fact, in Coulomb gauge our decomposition reduces to Jaffe–Manohar decomposition Eq. (10). Therefore, one can use the simple Jaffe–Manohar definition of electron (quark) spin, OAM and photon (gluon) spin, OAM to do calculations in Coulomb gauge. In this case, all of these operators have their familiar canonical forms used in quantum mechanics. This also explains why we get the correct results in the vast atomic, molecular, optical, nuclear, hadronic (including our Fock space extension quark model) calculations even though superficially gauge-dependent operators have been used. It is also a warning: to use these gauge-dependent operators to do calculations beyond Coulomb gauge is dangerous! The time-dependent gauge transformation dependence of the eigenvalue calculation of the hydrogen Hamiltonian is a typical example [13].

This decomposition can be extended to QCD, the $SU(3)$ non-Abelian gauge field system. To simplify the expressions, we will omit the color index and Gell-Mann $SU(3)$ color matrix $\lambda^a/2$ except a few necessary cases. The gauge potential $A_\mu(x) = A^a_\mu(x)(\lambda^a/2)$ is decomposed in the same way as in Eq. (12):

$$A_\mu(x) = A_{\mu,\rm phys}(x) + A_{\mu,\rm pure}(x). \tag{20}$$

The physical condition for the separation of the $SU(3)$ non-Abelian gauge potential is different from the Abelian $U(1)$ case due to the complication of nonlinearity:

$$F_{\mu\nu,\mathrm{pure}} = \partial_\mu A_{\nu,\mathrm{pure}} - \partial_\nu A_{\mu,\mathrm{pure}} + ig[A_{\mu,\mathrm{pure}}, A_{\nu,\mathrm{pure}}] = 0, \tag{21}$$

$$D_i^{\mathrm{adj}} A_{i,\mathrm{phys}} = \partial_i A_{i,\mathrm{phys}} + ig[A_{i,\mathrm{pure}}, A_{i,\mathrm{phys}}] = 0, \tag{22}$$

$$\partial_i A_{0,\mathrm{phys}} = \partial_i A_0 - \partial_0 A_{i,\mathrm{pure}} + ig[A_{i,\mathrm{pure}}, A_{0,\mathrm{pure}}]. \tag{23}$$

Here the bracket $[\ldots,\ldots]$ is the commutator of the color $SU(3)$ matrix. We cannot get the concise solution as in the QED case, but we have found perturbative solution [14]. Under local gauge transformation

$$\begin{aligned} \psi'(x) &= U(x)\psi(x), \quad U(x) = \exp\left(-ig\omega^a(x)\frac{\lambda^a}{2}\right), \\ A'_\mu(x) &= U(x)\, A_\mu(x)\, U^\dagger(x) - \frac{i}{g} U(x)\, \partial_\mu U^\dagger(x), \end{aligned} \tag{24}$$

the $A_{\mu,\mathrm{phys}}$ and $A_{\mu,\mathrm{pure}}$ transform as follows:

$$\begin{aligned} A'_{\mu,\mathrm{phys}}(x) &= U(x) A_{\mu,\mathrm{phys}}(x) U^\dagger(x), \\ A'_{\mu,\mathrm{pure}}(x) &= U(x) A_{\mu,\mathrm{pure}} U^\dagger(x) - \frac{i}{g} U(x)\partial_\mu U^\dagger(x). \end{aligned} \tag{25}$$

Based on these properties of $A_{\mu,\mathrm{phys}}$ and $A_{\mu,\mathrm{pure}}$, we obtain a decomposition of the total angular momentum $\mathbf{J}$ of the QCD system. The expression is the same as Eq. (17) if the color indices are omitted but the last term, where the ∇ should be replaced by the $\mathbf{D}^{\mathrm{adj}}$ in Eq. (22):

$$\begin{aligned} \mathbf{J} = \int d^3x\, \psi^\dagger \frac{\mathbf{\Sigma}}{2}\psi + \int d^3x\, \psi^\dagger \mathbf{x} \times \frac{\mathbf{D}_{\mathrm{pure}}}{\mathbf{i}}\psi + \\ + \int d^3x\, \mathbf{E} \times \mathbf{A}_{\mathrm{phys}} + \int \mathbf{d}^3\mathbf{x}\, \mathbf{E}^{\mathbf{i}} \mathbf{x} \times \mathbf{D}^{\mathrm{adj}} \mathbf{A}^{\mathbf{i}}_{\mathrm{phys}}. \end{aligned} \tag{26}$$

The total momentum of QCD system can be decomposed as follows:

$$\mathbf{P} = \int \mathbf{d}^3\mathbf{x}\, \psi^\dagger \frac{\mathbf{D}_{\mathrm{pure}}}{\mathbf{i}}\psi + \int \mathbf{d}^3\mathbf{x}\, \mathbf{E}^{\mathbf{i}} \mathbf{D}^{\mathrm{adj}} \mathbf{A}^{\mathbf{i}}_{\mathrm{phys}}. \tag{27}$$

This momentum decomposition is consistent with the angular momentum decomposition given in the above equation. In these momentum and angular momentum decompositions, each term is gauge-invariant and satisfies part of the Poincaré algebra for the three momentum and OAM components.

Let us digress a little bit but still a related problem. There is also a long-standing problem in nonrelativistic and relativistic quantum mechanics that the fundamental operators, the momentum, OAM and Hamiltonian of a charged particle moving in electromagnetic field do not have gauge-invariant ME. We proposed to use the following operators:

$$p^{\mu} - eA^{\mu}_{\rm pure}, \quad \mathbf{x} \times (\mathbf{p} - \mathbf{eA}_{\rm pure}), \tag{28}$$

as the four-momentum operator and OAM. This proposal does not change the Dirac equation for the electron or quark and the Maxwell equation for the photon or gluon field. However, the interpretation of the Dirac equation is modified a little bit:

$$(i\gamma^{\mu}((\partial_{\mu} + ieA_{\mu,\rm pure}) + ieA_{\mu,\rm phys}) - m)\psi = 0, \tag{29}$$

we call $(\partial_{\mu} + ieA_{\mu,\rm pure})/i$ as four «physical momentum», the interaction term only includes the physical part of gauge potential $A_{\mu,\rm phys}$. The advantage of this proposal is that the four «physical momenta» now all have gauge-invariant ME and so are measurable. One can choose the $A_{\mu,\rm pure}$ to transform as the same as the ∂_{μ} under a Lorentz transformation, then the «physical momentum» will transform as a normal four vector. The three spatial components of momentum and OAM still satisfy the canonical commutation relations. The disadvantage is that they are no longer the space-time translation generator of the electron or quark field. Only in a special gauge, the $A_{\mu,\rm pure} = 0$ Coulomb gauge, they reappear as the translation generator. It is impossible to have full Poincaré algebra for the electron (or quark) operators among themselves, especially the three momentum do not commute with the Hamiltonian. On the other hand, if we take the $P^{\mu} - eA^{\mu}$ as four momentum (they are called kinetic momentum in the literature), as in the other decomposition depicted in Eq. (11),

$$\mathbf{P} = \int \mathbf{d}^3\mathbf{x}\,\psi^{\dagger}\frac{\mathbf{D}}{\mathbf{i}}\psi + \int \mathbf{d}^3\mathbf{x}\,\mathbf{E}\times\mathbf{B}. \tag{30}$$

Even though each term is gauge-invariant, no one satisfies the canonical momentum algebra. They are no longer the space translation generators, do not satisfy the full Poincaré algebra, either. Especially, because the three components of the quark kinetic momentum do not commute, they cannot be diagonalized simultaneously and so cannot form a complete momentum operator set to describe the three-dimensional quark parton momentum distribution. The Dirac equation appears to be a free particle equation and indeed it appears as a free particle momentum in the light-cone gauge. This has caused confusions to identify this kinetic momentum as canonical momentum. The other weak point is the Poynting vector $\mathbf{E}\times\mathbf{B}$, even it had been identified as the momentum density of photon for a long time, it is in fact not the right momentum density because the $\mathbf{x}\times(\mathbf{E}\times\mathbf{B})$

is not the OAM but the total angular momentum of photon which includes both photon spin and OAM.

Equations (17), (19) and (26), (27) are our proposed solution of the momentum and angular momentum decomposition [15]. If we keep the gauge-invariance requirement, the Jaffe–Manohar decomposition depicted in Eq. (10) is excluded. If we require to keep the canonical quantization rule further, the Chen–Wang and Ji decomposition depicted in Eqs. (11) and (30) will be excluded, too. A gauge-invariant spin operator of massless particle is obtained in our proposed decomposition. M. Wakamatsu proved the total angular momentum of gluon (photon) in the Chen–Wang and Ji decomposition can be decomposed further into spin and OAM and the obtained gauge-invariant spin operator is the same as the one given in Eqs. (17) and (26) [16]. So the remaining problem is which momentum and OAM should be chosen? Different choices will give different physical picture of the quark gluon contribution to nucleon (also different electron photon contribution to atom) observables [17].

From atomic structure to hadron spectroscopy, one always uses the momentum and angular momentum expressed in Eq. (10) and their gauge-invariant version Eqs. (17) and (20). The popular idea, however, is that the kinetic momentum and angular momentum expressed in Eqs. (11) and (30) are the right choice to describe the quark, gluon momentum and angular momentum measured in DIS. They are derived from the symmetric, gauge-invariant Belinfante energy-momentum tensor. It is also widely believed that the density of the energy momentum tensor (DEMT) is fixed only up to a surface term and one has the freedom to choose either the canonical asymmetric or Belinfante symmetric one. How large freedom do we really have? The DEMT of the classical scalar field of the hydrodynamics is measurable and it is symmetric. The DEMT of the classical electromagnetic field and the Dirac electron field should be measurable too. In the Belinfante version, $T^{\mu\nu}_{\rm el}$ (electron part) and $T^{\mu\nu}_{\rm ph}$ (photon part) are both gauge-invariant and in principle measurable. In the usual canonical DEMT, both the electron and photon parts are gauge-dependent and so not measurable. However, we have derived a new version:

$$\begin{aligned} T^{\mu\nu} &= T^{\mu\nu}_{\rm el} + T^{\mu\nu}_{\rm ph}, \\ T^{\mu\nu}_{\rm el} &= \frac{i}{2}\bar{\psi}\gamma^{\mu}(\partial^{\nu} + ieA^{\nu}_{\rm pure})\psi + {\rm h.c.}, \\ T^{\mu\nu}_{\rm ph} &= -F^{\mu\rho}\partial^{\nu}A_{\rho,{\rm phys}} - g^{\mu\nu}\mathcal{L}_{\rm ph}. \end{aligned} \tag{31}$$

This is the $T^{\mu\nu}$ which we used to derive our momentum and angular momentum decomposition Eqs. (17) and (19). Both the electron and photon parts are gauge-invariant and in principle measurable. We therefore suggest to measure the DEMT of the classical electron and photon field which might be helpful in making the choice discussed above.

Let us discuss an ideal experiment to illustrate this point. Suppose, we have a spin polarized and orbital unpolarized electron beam propagating along the z direction. To do a proper discussion, one should use a wave packet. But to simplify the discussion, we still use plane wave approximation. We measure the total J_z carried by this beam in a fixed volume. It should be the total number N of electrons in this volume multiplied by the spin S_z of electron:

$$\int dV J_z(x) = NS_z. \tag{32}$$

If we start from the Belinfante symmetric DEMT, the above equation can be reexpressed as

$$\int dV J_z(x) = \int dV(\mathbf{x} \times \mathbf{p}(\mathbf{x})_{\mathrm{el}})_z. \tag{33}$$

Due to the symmetric property of the Belinfante DEMT, the momentum density in the above equation is equal to the energy flow density. The energy flow density divided by the energy ε of individual electron, which should be the same for a plane wave, should equal the electron number density flow $n(x)$. This electron number density flow should equal the electric current density divided by the charge e of electron. Taking these relations into account, we can reexpress the above equation further as

$$\int dV(\mathbf{x} \times \mathbf{p}(x)_{\mathrm{el}})_z = \frac{\varepsilon}{e}\int dV(\mathbf{x} \times \mathbf{j})_z. \tag{34}$$

The last integral should equal the electron number N in this volume multiplied by the spin magnetic moment μ_z of electron but with a factor 2, which equals $2(e/\varepsilon)S_z$ and finally we have

$$\int dV(\mathbf{x} \times \mathbf{p}(\mathbf{x})_{\mathrm{el}})_z = 2NS_z. \tag{35}$$

This obviously contradicts the result shown in Eq. (32) and it originates from the symmetry property of the Belinfante DEMT:

$$T^{0i}_{\mathrm{el},B} = T^{i0}_{\mathrm{el},B} = \frac{T^{0i}_{\mathrm{el},C} + T^{i0}_{\mathrm{el},C}}{2}. \tag{36}$$

Here the subscript B and C mean Belinfante and canonical DEMT. The Belinfante momentum (energy flow) density is a mixing of the physical canonical momentum and energy flow density. In the canonical version, the $\mathbf{x}\times\mathbf{p}$ is related to the OAM density only and will not cause such a contradiction because the gyromagnetic ratio is 1 for orbital magnetic moment to OAM not 2 as in the spin case. The fundamental spin can never be related to the orbital motion. The Belinfante

DEMT expresses the spin and OAM superficially both as $\mathbf{x}\times\mathbf{p}$ which is physically misleading.

Such an unphysical feature also appears in photon case. In the Belinfante version, the total angular momentum of photon is expressed as $\mathbf{x} \times (\mathbf{E} \times \mathbf{B})$. Our optical colleagues already showed experimentally that the spin and OAM of photon are different. D. P. Ghai, S. Senthikumaran and R. S. Sirohi showed a diffraction pattern of orbital polarized light beam with $L_z = \pm 1$. The pattern is distorted due to the transverse orbital motion [18]. Our group measured a diffraction pattern of spin polarized light beam with $S_z = \pm 1$. There is no distortion because there is no orbital motion related to photon spin [19]. However, in the Belinfante description, they are both $J_z = \pm 1$ light beam!

All of these results show that the Belinfante symmetric DEMT might be not physical and the canonical asymmetric one might be physical. The surface term manipulation is unavoidable in decomposing the momentum and angular momentum into contributions of constituents for a gauge field system. The question is to what extent such a manipulation still gives physical result. We propose to measure the DEMT of gauge field system to test its symmetry property. One example is to use the existing longitudinal spin polarized electron beam to realize the above ideal experiment.

5. DISCUSSIONS

Since our proposal of the decomposition of momentum and angular momentum of gauge field systems there have been very intense studies and hot debates on this topic. E. Leader and C. Lorcé (L. L.) gave an in-time critical review in arXiv: 1309.4235[hep-ph]. Many problems have been clarified there. We feel the following problems need to be studied further:

(1) L. L. give a new definition of gauge invariance and gauge independence which are different from what had been given by Wightman [11]. We suggest it is better not to do so if it is not absolutely necessary because it will cause confusion in an already full of confusions field. Up to now we cannot prove rigorously that a gauge-dependent operator might have gauge-invariant ME for some physical states, we'd better keep the requirement that an operator corresponding to an observable should be gauge-invariant under the assumption that the physical states are gauge-invariant as in the QCD case.

(2) For QED there are only two photon helicity states observed in Compton scattering and in Coulomb gauge only these two components of the electromagnetic potential are kept. In the other popular Lorentz or light-cone gauge, the unphysical components remain. In this sense, Coulomb gauge is special, but certainly this situation does not contradict the spirit of gauge invariance. On the contrary, gauge invariance will not change the fact that there are only two

physical components for the gauge potential. The Aharonov–Bohm effect is also induced by these transverse components of electromagnetic potential nonlocally produced by the electric current within the local solenoid [20]. The well-known gauge principle introduces pure and physical gauge potential together, but one can have a gauge-invariant formalism without physical interaction:

$$(i\gamma^{\mu}(\partial_{\mu} + ieA_{\mu,\mathrm{pure}}) - m)\psi = 0. \tag{37}$$

Here the pure gauge potential is introduced solely for the Dirac equation to be gauge-invariant under a local phase change of the Dirac field. This pure gauge potential can be eliminated by a local phase change of the Dirac field. On the other hand, the transverse components, the physical part of the gauge potential is not changed under such a local gauge transformation and cannot be eliminated by any gauge transformation. Our condition to separate the gauge potential into physical and pure gauge parts is based on this physics. The Helmholtz separation just meets this requirement. There are infinite possibilities to separate the gauge potential into gauge-invariant and variant parts, but no one is able to separate the physical and unphysical parts so clear-cut. In this sense, the physical and pure gauge separation is unique. For QCD case we extend the transverse condition to meet the gauge covariance also taking into account the fact that there should be only two helicity components for gluon. But our physical condition is reduced to transverse condition only in $A^{\mu}_{\mathrm{pure}} = 0$ case and the gauge-covariance transformation of A^{μ}_{phys} will mix the unphysical part into physical one in addition to Gribov ambiguity. So it should be studied further [21].

(3) L. L. already explain the general Lorentz covariance in their review paper. Because there is misunderstanding of the Lorentz covariance in the literature, we suppose it is worth discussing this problem here a little bit. The Lorentz transformation of four coordinates, four momentum, electromagnetic field tensor are determined by measurements and it is the well-known Lorentz transformation law (LTL). Hereafter we call it homogeneous LTL. However, the transformation law of the gauge potential is not measurable because of the gauge degree of freedom. J. D. Bjorken and S. D. Drell, and S. Weinberg already used the more general LTL for the Coulomb gauge potential [22]. Hereafter we call it inhomogeneous LTL. The popular misunderstanding is to require the gauge potential to follow the homogeneous LTL no matter which gauge is, otherwise one supposes that it violates the Lorentz covariance or says it is not manifestly Lorentz covariant. A typical example is in X. Ji's comment; he criticized that the physical condition $\nabla \cdot \mathbf{A}(\mathbf{x})_{\mathrm{phys}} = 0$ violates the Lorentz covariance [23]. In fact, it is impossible to prove that gauge potential should transform with homogeneous LTL in general. Even for the gauge potential in Lorentz gauge it is still possible to transform inhomogeneously if the residual gauge degree of freedom is taken into account. Of course, usually one prefers to assume that it transforms with homogeneous LTL. For $\mathbf{A}_{\mathrm{phys}}$, if one assumes it to transform with homogeneous LTL, the unphysical

components will mix in and so one needs to do an additional gauge transformation to eliminate the unphysical components:

$$A'^{\mu}_{\rm phys}(x') = \Lambda^{\mu}_{\nu}(A^{\nu}_{\rm phys}(x) + \partial^{\nu}\Omega(\Lambda)). \tag{38}$$

This new $A'^{\mu}_{\rm phys}(x')$ will be still physical, i.e.,

$$\nabla' \cdot \mathbf{A}'_{\rm phys}(\mathbf{x}') = 0. \tag{39}$$

The inhomogeneous term $\partial^{\nu}\Omega(\Lambda)$ is not hard to fix and certainly one doesn't need to do Lorentz boosting! In general, the gauge potential transforms with inhomogeneous LTL.

(4) We suppose what means «measurable» might need further study. First, we have to distinguish classical measurable and quantum measurable. X. Ji and M. Wakamatsu argued that the kinetic or mechanical momentum $m\mathbf{v}$ is measurable. We think it is classically measurable but quantum mechanically is questionable, the three components of $m\mathbf{v} = \mathbf{p} - \mathbf{A}$ do not commute, how can they be measured simultaneously? Second, we have to distinguish what is in principle not measurable and what is not measurable at present. M. Wakamatsu argued that the quark canonical OAM in nucleon might be not measurable. Yes, up to now we don't know how to measure it. But we suppose the same canonical OAM of electron in atom is measurable. Usually we believe the multipole radiation analysis gives us the information about the electron canonical OAM in atom, even the quark canonical OAM in nucleon. In this sense we don't think the relevant decompositions of nucleon spin listed in L. L.'s review paper is complete. The momentum, spin and OAM proposed by us are in principle measurable and we'd better not close the door to explore the measuring method of these observables.

Certainly, there are more problems which we have not discussed here. We also apologize that we have not discussed many interesting developments contributed by other authors in this field due to space limitation.

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