DYNAMICAL SCHWINGER EFFECT: PROPERTIES OF THE e^+e^- PLASMA CREATED FROM VACUUM IN STRONG LASER FIELDS

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We study the dynamical Schwinger effect in the vacuum excitation of the electron–positron plasma under the action of a "laser pulse" of the simplest configuration: a linearly polarized, time-dependent and spatially homogeneous electric field. Methodical basis of this investigation is the kinetic equation which is an exact consequence of the basic equations of motion of QED in the considered field model. In the present work we investigate some features of the residual electron–positron plasma and the transient process of its formation.

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INTRODUCTION

At the present time it is well known that the electron–positron plasma (EPP) excited from vacuum passes through three stages of its evolution: the quasiparticle stage in the period of the laser pulse action, the transient period of the EPP transmutation and the final residual EPP (REPP) in the out-state (particles in this state are on their mass shell). The quasiparticle period of the EPP evolution was investigated in detail in the work [1]. In the present work we will study the transient period of the EPP evolution and its final state, the REPP. As a rule, the density of the quasiparticle EPP is larger than that of REPP. However, all the three stages give a contribution to the different observable effects as, e.g., the emission of annihilation photons from the focal spot of the colliding laser beams [2, 3]. It is rather difficult to estimate the contributions of the different stages in formation of observable effects. Therefore, the detailed investigation of each period of the EPP evolution is an important problem.

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In this work, we restrict ourselves to the consideration of the simplest model for the external ("laser") field: the linearly polarized, time-dependent and spatially homogeneous electric field pulse. It is assumed that such a field acts in the focal spot of counter-propagating laser beams in the optical or X-ray ranges.

Our investigations of the dynamical Schwinger effect in such fields are based on numerical solutions of the kinetic equation (KE) of non-Markovian type which is a nonperturbative consequence of the basic equations of motion of QED. This approach allows us to investigate the behaviour of the EPP in both cases, the tunneling and the multiphoton mechanisms of EPP excitation.

1. THE KINETIC EQUATION IN AN EXTERNAL TIME-DEPENDENT FIELD

The KE for the (quasi)particle distribution function can be derived from the Dirac equation by a canonical time-dependent Bogoliubov transformation [4]. This method is valid only in a spatially uniform time-dependent field. In the case of a linearly polarized electric field with the vector potential $A^\mu(t) = (0, 0, 0, A(t))$ (Hamiltonian gauge), we obtain a non-Markovian integro-differential collisionless KE

$$\dot{f}(\mathbf{p}, t) = \frac{1}{2}\lambda(\mathbf{p}, t) \int_{t_0}^{t} dt' \lambda(\mathbf{p}, t')[1 - 2f(\mathbf{p}, t')] \cos\theta(t, t'), \tag{1}$$

where

$$\lambda(\mathbf{p}, t) = eE(t)\varepsilon_\perp/\varepsilon^2(\mathbf{p}, t), \qquad \theta(t, t') = 2\int_{t'}^{t} d\tau\, \varepsilon(\mathbf{p}, \tau), \tag{2}$$

with the transversal energy $\varepsilon_\perp = \sqrt{m^2 + p_\perp^2}$ and the quasi-energy

$$\varepsilon(\mathbf{p}, t) = \sqrt{\varepsilon_\perp^2 + (p_\parallel + A(t))^2}. \tag{3}$$

Here λ is the amplitude of vacuum transitions and θ is a dynamical phase which describes the vacuum oscillations (Zitterbewegung) with a frequency of the energy gap $2\varepsilon(\mathbf{p}, t)$. The equation contains two characteristic time scales. The slower scale is external field period and the faster one is given by the Compton time $\tau_C = 2\pi/m$. The KE (1) is equivalent to a system of ordinary differential equations (ODEs)

$$\dot{f} = \frac{1}{2}\lambda u, \qquad \dot{u} = \lambda(1 - 2f) - 2\varepsilon v, \qquad \dot{v} = 2\varepsilon u. \tag{4}$$

The total particle number density is defined by

$$n(t) = g \int \frac{d\mathbf{p}}{(2\pi)^3} f(\mathbf{p}, t), \tag{5}$$

where $g = 4$ is the degeneracy factor due to spin and charge degrees of freedom. The system of ODEs describes transitions between the lower and the upper continuum of the energy spectrum. These processes resemble interband transitions in solid state physics. Moreover, for excitation energies smaller than the energy gap a virtual state can be created.

We solve the KE (1) numerically for the following field shapes:

$$E(t) = E_0 \cosh^{-2}\left(\frac{t}{T}\right), \qquad A(t) = T E_0 \tanh\left(\frac{t}{T}\right), \tag{6}$$

which is the Eckart field, and [5]

$$E(t) = E_0 \cos\left(\omega t + \phi\right) e^{-t^2/2\tau^2},$$

$$A(t) = -\sqrt{\frac{\pi}{8}} E_0 \tau \exp\left(-\frac{\sigma^2}{2} + i\phi\right) \mathrm{erf}\left(\frac{t}{\sqrt{2}\tau} - i\frac{\sigma}{\sqrt{2}}\right) + \mathrm{c.\,c.}, \tag{7}$$

where $\sigma = \omega\tau$. Equations (6) and (7) model standing waves created by the two counter-propagating laser beams.

2. TRANSIENT PROCESS AND REPP FORMING

The typical picture of an EPP under the influence of the smooth Eckart field (6) is presented in Fig. 1. The distribution function (panel a, $p_\perp = p_\parallel = 0$) demonstrates three stages of the EPP evolution: quasiparticle, transient and residual. For the density (panel b) the fluctuations in the transient region are lost.
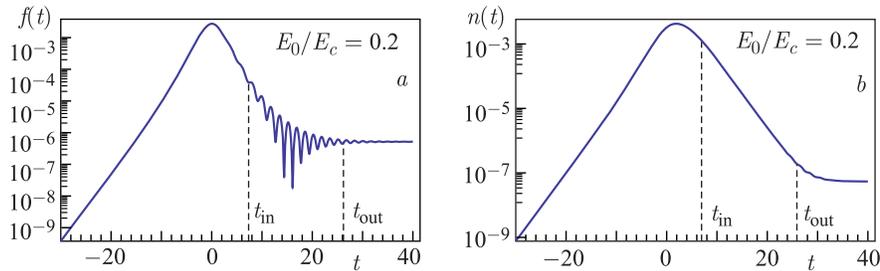

Fig. 1. Distribution function (a) and pair density (b) for the Eckart field pulse $E(t) = E_0 \cosh^{-2}(t/T)$, $T = 0.02$ nm
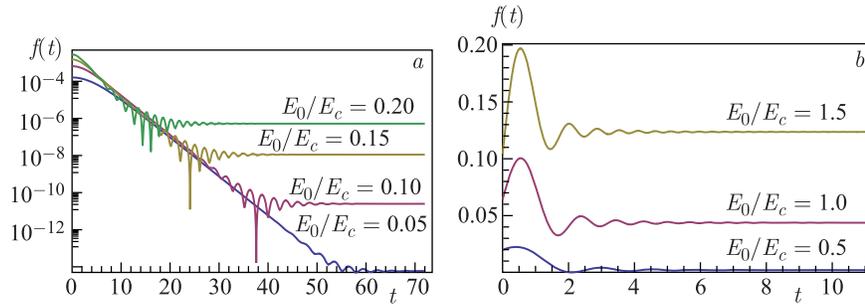
Fig. 2. Temporal behaviour of the maximum of the distribution function for different field strengths E_0. All other parameters are as in Fig. 1
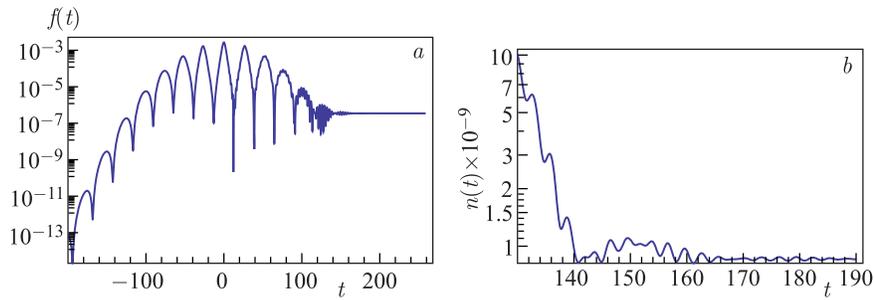

Fig. 3. The same as Fig. 1 for the Gaussian-modulated periodic field $E(t) = E_0 \cos(\omega t + \phi)\, e^{-t^2/2\tau^2}$, $\phi = 0$, $\sigma = \omega\tau = 5$
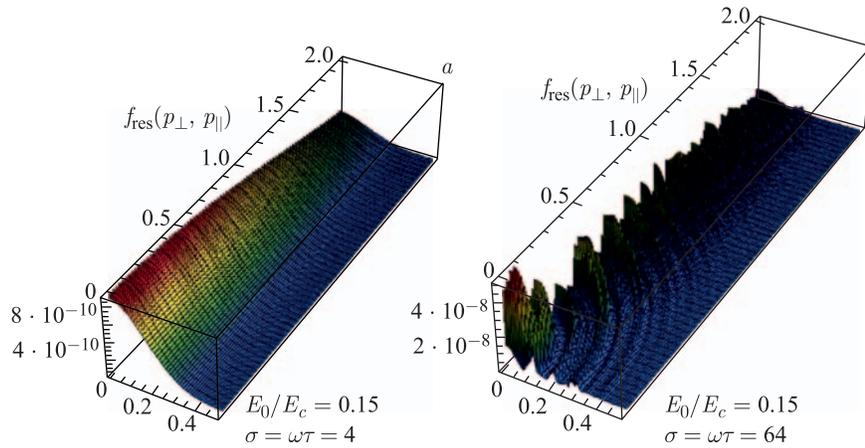

Fig. 4. Accumulation pattern of the distribution function for the Gaussian-modulated periodic field $E(t) = E_0 \cos(\omega t + \phi)\, e^{-t^2/2\tau^2}$, $\phi = 0$

The dependence of the temporal behaviour of the maximum of the distribution function on the field strength E_0 is shown in Fig. 2. The fluctuation region separates the quasiparticle state from the asymptotic out-state. Since the out-state is stationary when the external field is absent, we interpret this behaviour as a dynamical phase transition. The oscillatory behaviour of the transient process is more complicated (see Fig. 3) in the case of a Gaussian-modulated periodic field (7). In contrast to the Eckart pulse scenario, one can see oscillations of the number density. Our KE is of non-Markovian type, so one should expect to observe accumulation/memory effects. The typical picture of the accumulation case is presented in Fig. 4. For long pulses (here $\sigma = 64$) the REPP distribution function has a higher value than for short ones (here $\sigma = 4$). Moreover, the oscillation maxima form a complicated pattern which is a manifestation of multiphoton absorption. Similar resonance ring structures of $f_{\mathrm{res}}(p_\perp, p_\parallel)$ were obtained by Otto et al. [6].

CONCLUSIONS

The QED vacuum can be seen as nonlinear optical medium [7]. This gives us the chance to study its properties by optical methods. Two experimental approaches are perspective here for EPP observation: the registration of annihilation photons emitted from the region of EPP generation [2,9] and the birefringence [8]. The existing estimations are very coarse and contradictory. The reason for this is that the nonlinearity of the QED vacuum makes such studies of the distribution function strongly dependent on magnitude, shape and duration of field pulse [5]. As we have presented here, the shape of $f_{\mathrm{res}}(p_\perp, p_\parallel)$ can become complicated due to the non-Markovian and nonlinear nature of the system.

These facts, as detailed in the present work, underline that an important direction of further investigations is the search for acceptable approximate methods of the EPP description.

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