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ФИЗИКА ЭЛЕМЕНТАРНЫХ ЧАСТИЦ И АТОМНОГО ЯДРА. ТЕОРИЯ

ELECTROMAGNETIC EFFECTS AND SCATTERING LENGTHS EXTRACTION FROM EXPERIMENTAL DATA ON $K \to 3\pi$ DECAYS

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The final state interactions in $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays are considered using the methods of nonrelativistic quantum mechanics. It is shown how to take into account the largest electromagnetic effect in the analysis of experimental data using the amplitudes calculated earlier. The relevant expressions for amplitude corrections valid both above and below the two charged pions production threshold $M_{\pi^0\pi^0} = 2m_{\pi^{\pm}}$, including the average effect for the threshold bin, are proposed. These formulae can be used in the procedure of pion scattering lengths measurement from $M_{\pi^0\pi^0}$ spectrum.

Взаимодействие в конечном состоянии в распадах $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ рассмотрено в рамках нерелятивистской квантовой механики. Показано, как учесть важнейшие электромагнитные эффекты при анализе экспериментальных данных, используя выражения для амплитуд, полученные ранее. Предложены соответствующие выражения для поправок к этим амплитудам, справедливые как выше, так и ниже порога $M_{\pi^0\pi^0} = 2m_{\pi^{\pm}}$. Эти формулы могут быть использованы при извлечении длин $\pi\pi$-рассеяния из измеренных $M_{\pi^0\pi^0}$-спектров.

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The experiments DIRAC and NA48/2 at CERN SPS [1, 2] are able to measure the $\pi\pi$ scattering lengths difference $a_0 - a_2$. Its value is predicted by Chiral Perturbation Theory (ChPT) with a high accuracy [3] ($a_0 - a_2 = 0.265 \pm 0.004$ in units of inverse pion mass). Thus, the extraction of a_0, a_2 from experimental data with comparable precision becomes an important task. Recently, the NA48/2 experiment has discovered an anomaly (cusp) in the invariant mass $M_{\pi^0\pi^0}$ spectrum of π^0 pairs from $K^{\pm} \to \pi^{\pm}\pi^{0}\pi^{0}$ decays. This anomaly is positioned at the charged pions production threshold $M_c = 2m_{\pi^{\pm}}$, and has been explained by N. Cabibbo [4] as a result of charge exchange reaction $\pi^{+}\pi^{-} \to \pi^{0}\pi^{0}$ in the final state. It turns out that investigation of these decays below the charged pions threshold M_c allows one to extract the scattering lengths with a precision not accessible in other current experiments.

Further development of the model has been done in the works [5–7], where the pions final state interactions were considered with the second order precision in scattering lengths terms. Approach developed in [5] provides the possibility to extract a_0, a_2 from experimental data [2] with a precision compatible to the theoretical prediction one. Nevertheless, there are two problems, requiring a special consideration that can be crucial for scattering lengths extraction from experimental data in the vicinity of the threshold.

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The first problem is the accounting for higher orders in scattering lengths terms and their impact on the value of a_0, a_2. The estimation made in [5] leads to the corresponding 5% error for $a_0 - a_2$. In comparison with the precision of ChPT prediction and with the experimental statistical error this is a rather noticeable uncertainty and its decrease is very desirable[1].

Another effect that has an impact on a_0, a_2 extraction from experimental data is the electromagnetic interaction of pions in the final state.

This issue is not a trivial task [11, 12], as Coulomb interaction between charged pions[2] below threshold M_c leads to formation of bound $\pi^+\pi^-$ states (pionic atoms $A_{2\pi^\pm}$), and their construction in the framework of perturbation expansion is a doubtful issue.

To solve this problem, we use the technique of nonrelativistic quantum mechanics[3]. Leaving the strict derivation for elsewhere, we cite here the main result for amplitude of the decay under consideration

$$T = (1 + iv_0 f_0)T_0 + 2ivf_x T_+, \quad f_0 = \frac{a_{00}}{D}, \quad f_x = \frac{a_x}{D},$$
$$D = (1 - iv_0 a_{00})(1 - 2iva_{+-}) + 2v_0 v a_x^2\,. \tag{1}$$

Here, m_0, m are the $\pi^0, \pi^\pm$ masses, respectively; $v_0 = \sqrt{M^2 - 4m_0^2}/2m_0$ and $v = \sqrt{M^2 - 4m^2}/2m$ are the neutral and charged pions velocities; T_+ and T_0 are the «unperturbed» matrix elements of $K^\pm \to \pi^\pm\pi^+\pi^-$ and $K^\pm \to \pi^\pm\pi^0\pi^0$ decays. The inelastic a_x and elastic a_{00}, a_{+-} pion–pion scattering amplitudes in the isotopic symmetry limit are $a_x = (a_0 - a_2)/3$, $a_{00} = (a_0 + 2a_2)/3$ and $a_{+-} = (2a_0 + a_2)/6$ [5, 6].

The replacement $a_i \to f_i$ has small numerical impact on results of the previous calculations done according to [5, 6], but it is crucial for inclusion of the electromagnetic interactions under threshold, where the formation of bound states ($\pi^+\pi^-$ atoms) takes place.

Expression (1) includes all successive elastic and inelastic interactions in the $\pi\pi$ system (to all orders in scattering lengths). Their interaction with a spectator pion can be taken into account at two-loop level [5, 6].

As was discussed earlier [10], to include the Coulomb interactions in the framework of considered approach, it is enough to make the simple replacement in (1):

$$v \to \tau = iv - \alpha\left[\log\left(-2ivmr_0\right) + 2\gamma + \psi(1 - i\xi)\right], \quad \xi = \frac{\alpha}{2v}, \tag{2}$$

where $\gamma = 0.5772$, $\alpha = 1/137$ are Euler and fine structure constants, whereas $\psi(\xi) = d\log\Gamma(\xi)/d\xi$ is digamma function [13]. The parameter r_0 by its meaning is the strong interaction radius, usually taken as $r_0 \sim 1/m$. Later on, we cite it in all expressions, but as we checked, its impact on the results of fitting is small. Strictly speaking, expression (2) is valid in the region $vmr_0 \leqslant 1$, however for considered kinematics it is a rather good approximation.

To go under threshold one needs to perform the common replacement $v \to i\tilde{v}$:

$$\tau = -\tilde{v} - \alpha\left[\log\left(2\tilde{v}mr_0\right) + 2\gamma + \pi\cot\left(\pi\tilde{\xi}\right) + \psi(\tilde{\xi})\right], \quad \tilde{v} = \frac{\sqrt{4m^2 - M^2}}{2m}. \tag{3}$$

[1]More pessimistic view on impact of higher order terms is given in [7].

[2]The photons radiation leads only to smearing of Coulomb contribution and will be treated elsewhere.

[3]The similar approach to $K \to 3\pi$ decays was developed by V. Gribov [8, 9].

Due to asymptotic behaviour $\psi(z) \sim \log(z)$ this expression is finite at threshold M_c. Moreover, it describes all bound states (pionium atoms) and electromagnetic interactions leading to unbound states under threshold [10].

Above the threshold expression (2) acquires the imaginary part. Using the well-known [13] relations for digamma function we obtain

$$\tau = \mathrm{Re}\,\tau + i\,\mathrm{Im}\,\tau, \quad \mathrm{Re}\,\tau = -\alpha \left[\log(2mr_0) + \gamma + \xi^2 \sum_{n=1}^{\infty} \frac{1}{n(n^2+\xi^2)}\right], \tag{4}$$

$$\mathrm{Im}\,\tau = \frac{\pi\alpha}{1 - \exp(-2\pi\xi)}\,.$$

The above expressions allow one to calculate the electromagnetic corrections in a wide kinematic region. For definiteness let us consider the fitting procedure in NA48/2. In order to extract the values of scattering lengths a_0, a_2 [2], the function

$$F(M^2) = N \int f(a_0, a_2, m^2)\Phi(M^2, m^2)dm^2 \tag{5}$$

was fitted to the experimental mass spectrum. Here, f is the theoretical distribution from [5], whereas Φ describes the experimental set-up resolution and acceptance and is obtained by Monte-Carlo simulation. Integral is replaced with a sum, and decay probability f is calculated in the middle of every $M_{\pi^0\pi^0}$ bin with a width $\delta M^2 = 0.00015$ (GeV/c)^2. The bins are centered in such a way that the threshold $4m_{\pi\pi}^2$ is placed exactly in the middle of one of them (here we will call this bin the «central» one).

This procedure is precise enough as long as one considers only strong interactions in the final state. But the electromagnetic interactions lead to the sharp peak at the threshold, thus for the central bin the averaging of theoretical predictions must be done more carefully.

So, it is convenient to consider the behaviour of the electromagnetic corrections in three separate regions. Up to the lower bound of central bin due to smallness of ξ in this region as follows from (3) we get

$$\tau = \frac{\pi\alpha}{2}\cot\left(\frac{\pi\alpha}{2\tilde{v}}\right) + \alpha\left[\log(2\tilde{v}mr_0) + \gamma - \frac{\alpha^2}{4\tilde{v}^2}\varsigma(3)\right], \quad \varsigma(3) = \sum_{1}^{\infty}\frac{1}{n^3} = 1.201. \tag{6}$$

This expression for τ has to be used there instead of the velocity formula for charged pions $\tilde{v} = \sqrt{|M^2 - 4m^2|}/2m$.

Above the charged pions production threshold beginning from the top bound of central bin the interference between direct T_0 and charge exchange T_+ terms appears (unlike in [4]). As a result, to take into account the electromagnetic effects in this region, one has to add the interference term

$$-4\alpha a_x\left[\log(2mr_0) + \gamma + \frac{\alpha^2}{4\tilde{v}^2 + \alpha^2} + \frac{1}{2}\log\left(\tilde{v}^2 + \left(\frac{\alpha}{3}\right)^2\right)\right]T_0T_+ \tag{7}$$

to the square of matrix element.

For these regions the averaging procedure (just a value in the bin's center) exploited in [2] is precise enough.

As to the central bin, where the electromagnetic corrections come from two states of $\pi^+\pi^-$ pairs (bound or unbound), the more accurate averaging is in use. Confining to the second order terms in scattering lengths, as it is done in the previous consideration [5], integrating the square of amplitude (1) in central bin, substituting the relevant velocities (3), (4) and dividing result by the square of bin width ΔM, we obtain the mean value of correction to the square of matrix element in the central bin in the form

$$\begin{gathered}\bar{T^2} = \frac{\pi\alpha^3\varsigma(3)}{2v_0v_t^2}T_+^2 - 4\alpha a_x\left[\log\left(2v_tmr_0\right) + \gamma - 0.5\right]T_+T_0,\\ v_t = v\left(4m^2 + \frac{\Delta M^2}{2}\right), \quad v_0 = v_0(4m^2).\end{gathered} \tag{8}$$

The first term describes all bound states (pionium atoms). As to the second term, it is a result of interference between the direct «unperturbed» production amplitude T_0 and electromagnetic interactions leading to unbound states. Let us note that in (8) we cite only the main electromagnetic corrections in central bin. We estimated the possible contributions from unbound states to ($\sim T_+^2$) and interference between direct amplitude T_0 and bound states ($\sim T_+T_0$) in central bin, which turns out to be small (less than 1% in comparison with main terms in (8)), and so can be safely neglected.

Numerically, the contribution of the first term (pionium atoms) from (8) to the central bin is near 2.5% of «unperturbed» decay probability that coincides with the prediction evaluated from [14]. Simple estimates show that the interference of direct amplitude T_0 with unbound Coulomb part (second term in (8)) is significant and gives approximately the contribution of the same order as the bound states one.

Now it becomes clear why the fit done in [2] with the pionium contribution as a free parameter leads to the Coulomb contribution, that is about two times larger than can be provided by solely bound states.

In conclusion, let us stress that at present we have the recipe allowing one to introduce the Coulomb effects into the expressions for the square of decay amplitude (taken, for example, from [5]), averaged correctly over every bin, that can be multiplied by phase space factor, convoluted with the experimental acceptance matrix Φ and compared with the experimental data.

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