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PROPOSAL OF THE EXPERIMENT TESTING THE FINE STRUCTURE OF THE VAVILOV–CHERENKOV RADIATION

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It is shown that the combined experimental and theoretical study of the unfocused Cherenkov rings gives possibility to obtain information on the physical processes accompanying the Vavilov–Cherenkov radiation in the finite spatial interval (bremsstrahlung, transition of the light velocity barrier, etc.).

Показано, что совместное экспериментальное и теоретическое исследование тонкой структуры несфокусированных черенковских колец дает возможность получить информацию о физических процессах, сопровождающих излучение Вавилова–Черенкова на конечном пространственном интервале (тормозное излучение, прохождение барьера скорости света в веществе и т. д.).

INTRODUCTION

The classical Tamm–Frank theory [1] explaining the main properties of the Vavilov–Cherenkov (VC) effect [2, 3] grounds on the assertion that a charge uniformly moving in medium with the velocity v greater than the velocity of light c_n in medium radiates spherical waves from each point of its trajectory [4]. The envelope to these spherical waves propagating with the velocity c_n is the Cherenkov cone with its apex attached to a moving charge and with its normal inclined at the angle θ_c towards the motion axis. Here $\cos\theta_c = 1/\beta_n$, $\beta_n = \beta n$, $\beta = v/c$, $c_n = c/n$ (c is the velocity of light in vacuum and n is the medium refractive index).

The radiation of a charge moving uniformly in a finite medium space interval is usually studied in the framework of the so-called Tamm problem [5]. In this problem, a point charge is at rest at a spatial point up to an instant when it exhibits an instantaneous acceleration acquiring the velocity greater or smaller than c_n. With this velocity a charge moves over some time interval at the end of which it exhibits an instantaneous deceleration coming to the permanent state of rest. Tamm obtained the remarkably simple approximate formula, which is frequently used by experimentalists to identify the charge velocity [6, 7].

When analyzing the angular spectrum of the radiation arising in the Tamm problem, Ruzicka and Zrelov [8] came to the paradoxical result that this spectrum can be interpreted as an interference of two bremsstrahlung (BS) shock waves arising at the beginning and at the end of the charge motion. There was no room for the VC radiation in their analysis based on the use of the Tamm approximate formula. Tamm himself thought that his formula describes both the VC radiation and BS.

To resolve this controversy, the exact solution of the Tamm problem was obtained and investigated in [9]. It was shown there that, side by side with BS shock waves, the Cherenkov shock wave (CSW) exists. The results obtained in [9] remove the above-mentioned inconsistency between [5] and [8] in the following way: Although the Tamm problem describes both the VC and BS, its approximate solution (i. e., the Tamm formula) does not describe the CSW properly.

We see that, due to the approximations involved, an important physics has dropped out from the consideration. It is the goal of this report to analyze the experimental and theoretical aspects of this new physics. For this we obtain the exact (numerical) and approximate (analytical) theoretical radiation intensities describing a charge motion in finite spatial interval and compare them with existing experimental data. Theoretical intensities predict the existence of the CSW of finite extension manifesting itself as a plateau in the radiation intensity and of the BS shock wave manifesting itself as the intensity bursts at the ends of this plateau. It turns out that the theoretical (numerical and analytical) and experimental intensities are in satisfactory agreement with each other, but disagree sharply with the Tamm formula. The reasons for this are given in Sec. 2.

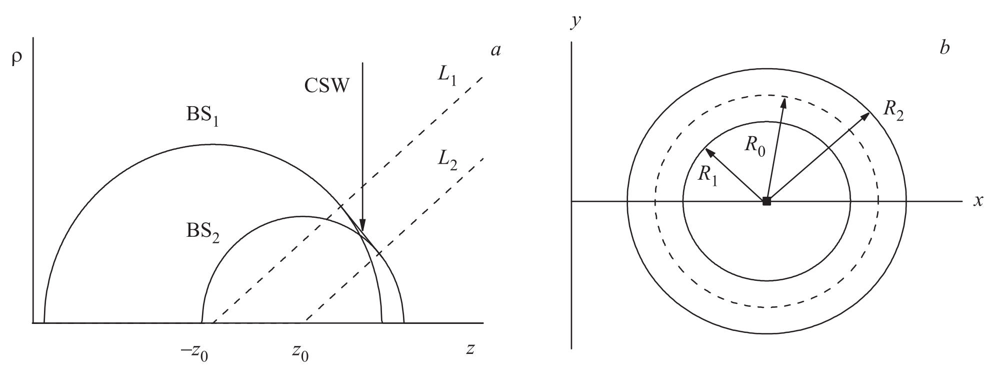

Fig. 1. *a*) The position of the Cherenkov shock wave (CSW) and the BS ones arising at the beginning (BS_1) and at the end (BS_2) of the charge motion at the fixed instant of time. The CSW is enclosed between L_1 and L_2 straight lines originating from the points corresponding to the boundaries of the motion interval and inclined at the angle θ_c towards the motion axis. *b*) The CSW in the $z = \text{const}$ plane, cuts off the ring with internal and external radii R_1 and R_2, respectively. The width $R_2 - R_1$ of the Cherenkov ring and the energy released in it do not depend on the position z of the observational plane

According to [9], when a charge moves in the interval $(-z_0, z_0)$ of the medium, the CSW is enclosed between the moving charge and the L_1 straight line originating from the $-z_0$ point corresponding to the beginning of motion and inclined at the angle θ_c towards the motion axis. For an arbitrary instant of time $t > t_0$, the CSW is enclosed between L_1 and the L_2 straight line originating from the z_0 point corresponding to the end of motion and parallel to L_1. The CSW is perpendicular to L_1 and L_2 and tangential to BS_1 and BS_2 shock waves. The positions of BS_1 and BS_2 shock waves and the CSW at the fixed instant of time are shown in Fig. 1, *a*. The length of CSW (coinciding with the distance between L_1 and L_2) is $L/\beta_n\gamma_n$, where $L = 2z_0$ is the motion interval and $\gamma_n = 1/\sqrt{|1-\beta_n^2|}$. As time goes on, the CSW propagates between L_1 and L_2 with the velocity c_n. Let the measurements of the radiation intensity be made in the plane perpendicular to the motion axis z. Then, the intersection of the CSW with $z = \mathrm{const}$ plane looks like a ring with minor and major radii equal to $R_1 = R_0 - L/2\gamma_n$ and $R_2 = R_0 + L/2\gamma_n$, respectively (Fig. 1, *b*). Here $R_0 = z/\gamma_n$ is the middle radius of the ring.

This qualitative consideration implies only the possible existence of the Cherenkov ring of the finite width. To find the distribution of the radiation intensity within and outside it, the numerical calculations are needed. When the ratio of the motion interval to the observed wavelength is very large (this is a usual thing in the Cherenkov-like experiments), the Tamm formula has a sharp δ-type peak within the Cherenkov ring. Due to this, it cannot describe a rather uniform distribution of the radiation intensity inside the Cherenkov ring.

The observation of the above shock waves encounters certain difficulties when the focusing devices are used which collect radiation from the part of the charge trajectory lying inside the radiator into the sole ring, thus projecting the VC radiation and BS into the same place.

To see how the VC radiation and BS are distributed in space, we turn to experiments in which the VC radiation was observed without using the focusing devices. These successful (although qualitative) experiments were performed by V. P. Zrelov (unpublished) in 1962 when preparing illustrations to monograph [6] devoted to the VC radiation and its applications. In this paper we processed these experimental data. The results are presented in the next section.

1. SIMPLE EXPERIMENT WITH 657-MeV PROTONS

The 657-MeV ($\beta = 0.80875$) proton beam of the phasotron of the JINR Laboratory of Nuclear Problems was used. The experimental setup is shown in Fig. 2, *a*. The collimated

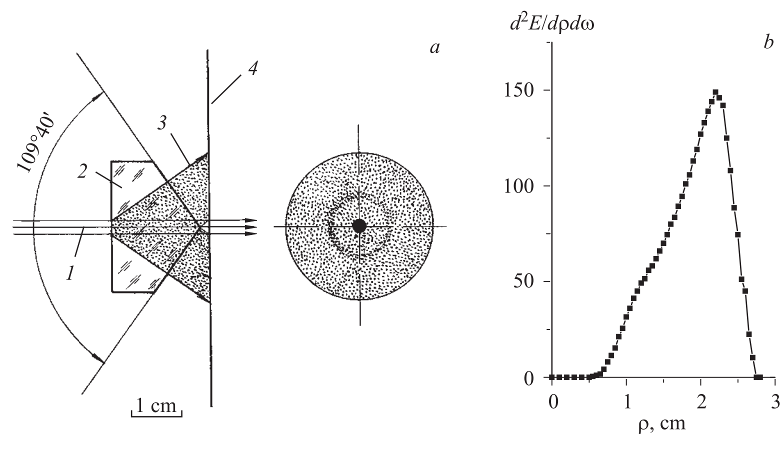

Fig. 2. *a*) The experimental setup of the discussed experiment. The proton beam (*1*) passing through the conical plexiglass radiator (*2*) induces the VC radiation (*3*, shaded region) propagating in the direction perpendicular to the cone surface. The radiation was detected by the plane color photofilm (*4*) placed perpendicularly to the motion axis. *b*) The photometric curve corresponding to the part *a*. One observes the increment of the radiation intensity at $\rho \approx 2.25$ cm which corresponds to the Cherenkov ray emitted from the point where the proton beam enters the radiator

proton beam (*1*) with diameter 0.5 cm was directed to the conic polishing plexiglass radiator (*2*) ($n = 1.505$ for $\lambda = 4 \cdot 10^{-5}$ cm). The apex angle $109.7°$ of the cone enabled the VC radiation (*3*) to go out from the radiator in the direction perpendicular to the cone surface. The radiation was detected by the plane color photofilm placed perpendicularly to the beam at a distance of 0.3 cm from the cone apex. Nearly 10^{12} protons passed through the conical radiator. The corresponding photometric curve (from which the beam background was subtracted) is shown in Fig. 2, *b*. The photometric curve describes the distribution $d\mathcal{E}(\rho)/d\rho$ of the energy released inside the ring of the finite width. More accurately, $d\rho \cdot d\mathcal{E}(\rho)/d\rho$ is the energy released in the elementary ring with minor and major radii ρ and $\rho + d\rho$, respectively. It is seen from this figure that the increment of the radiation intensity takes place at the radius $\rho = 2.25$ cm corresponding to the radiation emitted at the Cherenkov angle θ_c from the boundary point where the charge enters the radiator.

Theoretical consideration [9] and numerical calculations presented below show that the just mentioned radiation intensity maxima should indeed take place and they are due to the discontinuities at the beginning and at the end of the charge motion.

2. NUMERICAL RESULTS AND DISCUSSION

In the past, the finite width of the Cherenkov rings on the observational sphere S of the finite radius r was studied analytically and numerically in [9] in the framework of the Tamm problem. It was shown there that the angular region to which the Cherenkov ring is confined is large for small r and diminishes with increase of r. However, the width of the band on the observational sphere corresponding to the Cherenkov ring remains finite even for infinite values of r. Unfortunately, the authors of [9] were unaware of Zrelov's unpublished experiments discussed above. Since the measurements in these experiments were made in the plane perpendicular to the motion axis (which we identify with the z axis), we adjusted formulae obtained in [9] to the case treated.

In Fig. 3, the radiation intensities are presented for various distances δz of the observational plane (δz is the distance from the point corresponding to the termination of motion). We observe the qualitative agreement of the exact radiation intensity with the analytic Fresnel one and its sharp disagreement with the Tamm radiation intensity. Both of them sharply disagree with the Tamm intensity which does not contain the CSW responsible for the appearance of plateau. Fig. 3, *c* demonstrates that at large observational distances ($\delta z = 100$ cm) the Tamm radiation intensity approaches the exact one outside the Cherenkov ring.

In Introduction it was mentioned about the special optical devices focusing the rays directed at the Cherenkov angle into one ring. In the case treated, it is the plateau shown in Figs. 3 and the BS peaks at its ends that are focused into this ring. The remaining part of BS will form the tails of the focused total radiation intensity. Probably, for such a compressed radiation distribution the Tamm formula has a greater range of applicability.

We evaluated also the radiation intensities in the quasi-classical approximation which is unique in the sense that contributions of the VC radiation and the BS are clearly separated in electromagnetic field strengths and, therefore, in radiation intensities. In Fig. 4, *b*, we present the quasi-classical intensity for $\delta z = 0.3$ cm. We observe perfect agreement between it and the exact one, shown in Fig. 4, *a*, everywhere except for the boundaries of the region to which the VC radiation is confined. In accordance with quasi-classical predictions, one

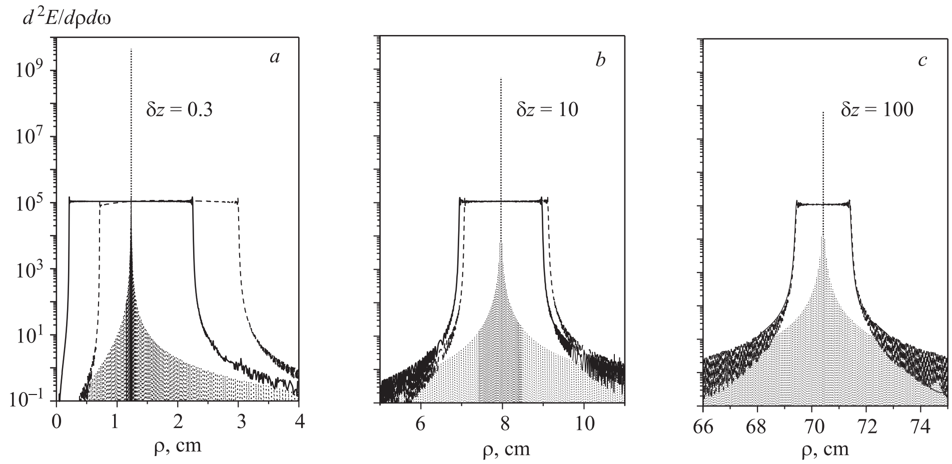

Fig. 3. Theoretical radiation intensities in a number of planes perpendicular to the motion axis for the experimental setup shown in Fig. 2, *a*; δz means the distance (in cm) from the cone vertex to the observational plane. The solid, dashed, and dotted curves refer to the exact and analytic (Fresnel and Tamm) intensities. In this figure and the following ones, the theoretical radiation intensities are in e^2/cz_0 units

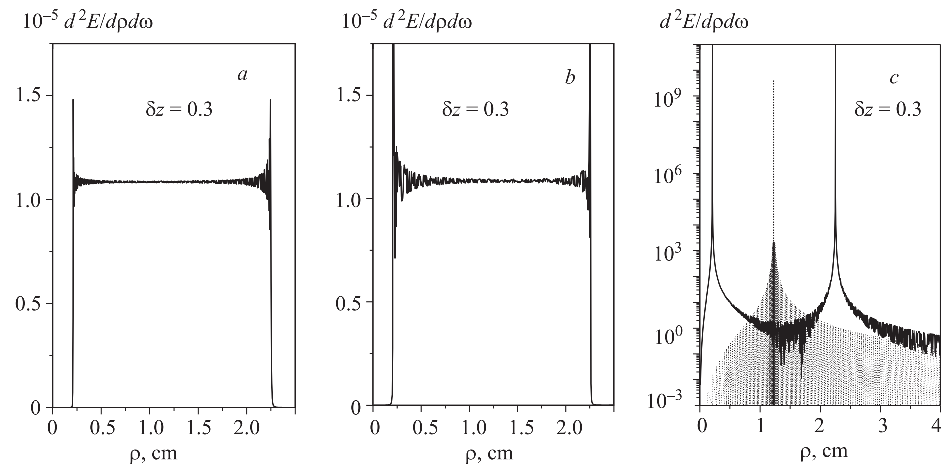

Fig. 4. The exact (*a*) and quasi-classical (*b*) radiation intensity in the $\delta z = 0.3$ cm plane. *c*) The quasi-classical bremsstrahlung intensity (solid curve) and the Tamm one (dotted curve) in the $\delta z = 0.3$ cm plane

sees the maxima at the ends of the $(z - z_0)/\gamma_n < \rho < (z + z_0)/\gamma_n$ interval. To see the contribution of the BS, we omit the conribution of the CSW in the quasi-classical radiation intensities. The resulting intensity describing BS is shown in Fig. 4, *c*. It sharply disagrees with the Tamm intensity. From the smallness of the BS intensity everywhere except for the boundaries of the Cherenkov ring it follows that oscillations of the total radiation intensity inside the Cherenkov ring are due to the interference of the VC radiation and the BS.

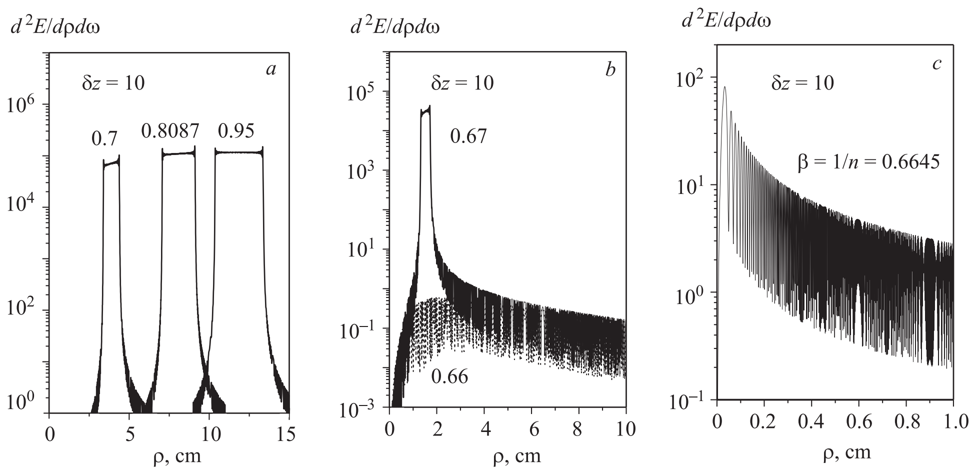

Fig. 5. *a*) Radiation intensities for a number of charge velocities above the Cherenkov threshold in the $\delta z = 10$ cm plane. As the charge velocity approaches the light velocity in medium, the position of the Cherenkov ring approaches the motion axis while its width diminishes. *b*) Radiation intensities for the charge velocity slightly above and below the Cherenkov threshold in the $\delta z = 10$ cm plane. *c*) Radiation intensity at the Cherenkov threshold in the $\delta z = 10$ cm plane. In accordance with theoretical predictions it is much smaller than that above the threshold

Figure 5, a demonstrates that the position of the radiation intensity maximum approaches the motion axis, while its width diminishes as the charge velocity approaches the Cherenkov threshold ($\beta = 1/n \approx 0.665$). The radiation intensities presented in Fig. 5, b show their behavior just above ($\beta = 0.67$) and below ($\beta = 0.66$) the Cherenkov threshold. It is seen that the maxima of the underthreshold and the overthreshold intensities differ by 10^5 times. Far from the maximum position, they approach each other. The radiation intensity at the Cherenkov threshold, shown in Fig. 5, c, is three orders smaller than the one corresponding to $\beta = 0.67$.

Strictly speaking, the formulae obtained above and describing the fine structure of the Cherenkov rings are valid if the observations are made in the same medium where a charge moves. Because of this, the plateau of the radiation intensity and its bursts at the ends of this plateau cannot be associated with the transition radiation which appears when a charge intersects the boundary between two media. Turning to the comparison with experiment, we observe that it corresponds to the charge moving subsequently in air, in medium and, finally, again in air. The transition radiation arising at the boundary of medium with air is approximately 100 times smaller than the VC radiation. Since the uniformly moving charge does not radiate in air where $\beta n < 1$ and radiates in medium where $\beta n > 1$, the observer inside the medium associates the radiation with instantaneous appearance and disappearance of a charge at the medium boundaries and with its uniform motion inside medium. We quote, e. g., Jelly ([7, p. 59]): «A situation alternative to that of a particle of constant velocity traversing a finite slab may arise in the following way; suppose instead that we have an infinite medium and that a charged particle, initially at rest at a point A, is rapidly accelerated up to a constant velocity (above the Cherenkov threshold) which it maintains until, at a point B, it is brought abruptly to rest. If, as in the first case, the distance $AB = d$, the output of Cherenkov radiation will be the same as before. In this case, there will be radiation at the two points A and B; this will be now identified as a form of acceleration radiation. This and transition radiation are essentially the same; the intensities work out the same in both cases and it is only convention which decides which term shall be used». This justifies the applicability of the Tamm problem for the description of the discussed experiments.

Comparing theoretical intensities with the experimental ones, we see that:

i) Theoretical intensities have a plateau (Figs. 3–5), while the experimental ones have a triangle form (Fig. 2, *b*). Such a form of the observed radiation intensities may be due to the smooth change of the charge velocity inside the dielectric. For such a motion, the radiation intensities obtained in [9] had indeed a triangle form. We estimate now the energy losses for the experiment treated. For the protons with an energy of 657 MeV, the energy ionization losses in plexiglass with density $\rho = 1.2$ g/cm^3 are $\Delta E/\Delta z = 2.91$ MeV/cm. This gives $\Delta E = 8.58$ MeV for a radiator length of 2.95 cm. The corresponding proton velocity change is $\Delta\beta = 2.3 \cdot 10^{-3}$. Alternatively, it can be associated with a smooth change of the refractive index at the border of vacuum and dielectric.

ii) The observed radiation peaks at the boundaries of the Cherenkov rings are not so pronounced as the predicted ones. This can be understood taking into account that the analyzed experiment was performed with a relatively broad proton beam (0.5 cm in diameter). This leads to the smoothing of the boundary peaks after averaging over the proton beam diameter.

CONCLUSION

For the uniform charge motion in unbounded medium, a photoplate placed perpendicularly to the motion axis will be darkened with the intensity proportional to $1/\rho$ (ρ is the distance from the motion axis) without any maximum at the Cherenkov angle. Despite its increase for small ρ, the energy emitted in a particular ring with the width $d\rho$ is independent of ρ. The surface of the cylinder coaxial with the motion axis will be uniformly darkened. The Cherenkov ring can be observed only for the finite motion interval. In the $z = \mathrm{const}$ plane, the ring width is proportional to the charge motion interval L: $\Delta R = L/\gamma_n$ ($\gamma_n = 1/\sqrt{|1-\beta_n^2|}$, $\beta_n = \beta n$). It does not depend on the position z of the observation plane. The frequency dependence enters only through the refractive index n. The radiation emitted into a particular ring does not depend on z. For the fixed observation plane, the radiation intensity oscillates within the Cherenkov ring. These oscillations are due to the interference of bremsstrahlung and the Vavilov–Cherenkov radiation. The large characteristic peaks at the ends of the Cherenkov ring are due to the bremsstrahlung shock waves, which include shock waves originating from the jumps of velocity, acceleration, other higher velocity time derivatives and from the transition of the medium light velocity barrier. The finite width of the Cherenkov ring in the $z = \mathrm{const}$ plane is due to the Cherenkov shock wave. Inside the Cherenkov ring ($R_1 < \rho < R_2$), the Tamm formula does not describes the radiation intensity at any position of the observation plane (see Fig. 3). Outside the Cherenkov ring ($\rho < R_1$ and $\rho > R_2$), the exact radiation intensity and the one given by the Tamm formula are rather small. In this angular region they approach each other at large distances satisfying $kz_0^2/r \ll 1$. For the experiments treated in the text, the l. h. s. of this inequality equals unity at the distance $r \approx 1$ km.

We conclude that the experiments performed with a relatively broad 657-MeV proton beam passing through various radiators point to the existence of diffused radiation peaks at the boundary of the broad Cherenkov rings. This supports predictions on the existence of the shock waves arising when the charge motion begins and when the charge velocity coincides with the light velocity in medium.

It is desirable to repeat similar experiments with the charged particle beam of a smaller diameter ($\approx$ 0.1 cm), with a thick dielectric sample, without using the focusing devices and for various observation distances. This should result in appearance of well pronounced radiation peaks.

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