# ELASTIC $e D$ SCATTERING IN THE BETHE-SALPETER APPROACH FOR THE DEUTERON WITH THE POSITIVE- AND NEGATIVE-ENERGY STATES 

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#### Abstract

Recent results obtained by the application of the Bethe-Salpeter approach to the analysis of elastic electron-deuteron scattering with the separable $N N$ kernel are presented. We analyze the impact of the $P$ waves (negative-energy components) on the electromagnetic properties of the deuteron and compare it with experimental data. It was shown that the contribution of the $P$ waves must be taken into account to explain tensor polarization and charge form factor of the deuteron.


Представлены результаты расчетов упругого электрон-дейтронного рассеяния в подходе БетеСолпитера с сепарабельным ядром нуклон-нуклонного взаимодействия. Анализируется роль $P$-состояний (компонентов дейтронной амплитуды Бете-Солпитера с отрицательной энергией) при описании электромагнитных свойств дейтрона и проводится сравнение с экспериментальными данными. Показано, что учет вкладов $P$-состояний значительно улучшает согласие зарядового формфактора и компонентов тензора поляризации дейтрона с экспериментальными данными.

## INTRODUCTION

The study of electromagnetic properties of the deuteron helps us to construct the theory of strong interactions and, in particular, the nucleon-nucleon interaction (see, for example, [1]). Theoretical research in this field is of topical interest, which is reflected in recent review articles [2-8]. A large amount of available experimental data stimulate a further development of theoretical methods, which are often restricted to qualitative predictions. The forthcoming experiments are expected to provide high-precision data, which will allow us to explore the region of large-momentum transfer in elastic, inelastic and deep inelastic (DIS) electronnucleus reactions.

The fact that nuclei consist of bound nucleons introduces a major problem for theoretical description of relativistic $l-A$ interactions. The deuteron is naturally the first object in the row of many other nuclei, and has received a vast number of treatments within many different approaches. One also finds that nonrelativistic schemes of calculations are widely employed in the analysis, which can be justified for a few particular cases. On the other hand, the consistent consideration of the relativistic bound states is offered within the Bethe-Salpeter (BS) formalism (see, for example, review [8]), which allows a qualitatively new interpretation

[^0]of the physics of the relativistic bound state and should not be regarded as an alternative scheme only.

We emphasize the covariant description of the BS formalism by taking the separable interaction, which is still at a stage of infancy. In particular, the role of the abnormal parity states has not yet been confronted with experimental data, though the necessity is demonstrated in this paper.

## 1. BASIC FORMALISM OF THE BETHE-SALPETER APPROACH

We start with the Bethe-Salpeter Equation (BSE) for $N N T$ matrix:

$$
\begin{align*}
T_{\alpha \beta, \delta \gamma}\left(P, p^{\prime}, p\right) & =V_{\alpha \beta, \delta \gamma}\left(P, p^{\prime}, p\right)+ \\
+ & i \int \frac{d^{4} k}{(2 \pi)^{4}} V_{\alpha \beta, \epsilon \lambda}\left(P, p^{\prime}, k\right) S_{\epsilon \eta}(P / 2+k) S_{\lambda \rho}(P / 2-k) T_{\eta \rho, \delta \gamma}(P, k, p) \tag{1}
\end{align*}
$$

where $P$ is the total momentum; $p$ and $p^{\prime}$ are the relative 4 -momenta of the two nucleons before and after the interaction. They are connected with 4 -momenta of the first $\left(q_{1}\right)$ and second $\left(q_{2}\right)$ particles: $P=q_{1}+q_{2}, p=\left(q_{1}-q_{2}\right) / 2, q_{1}=P / 2+p, q_{2}=P / 2-p . S_{\alpha \beta}(k)$ is the one-particle Green function: $S_{\alpha \beta}(k)=[1 /(k \cdot \gamma-m+i \epsilon)]_{\alpha \beta}$.

The bound state corresponds to a pole in $T$ matrix at $P^{2}=M_{B}^{2}\left(M_{B}\right.$ is the mass of bound state) and takes the form:

$$
\begin{equation*}
T_{\alpha \beta, \delta \gamma}\left(P, p^{\prime}, p\right)=\underbrace{\frac{\Gamma_{\alpha \beta}\left(P, p^{\prime}\right) \bar{\Gamma}_{\delta \gamma}(P, p)}{P^{2}-M_{B}^{2}}}_{\text {bound state }\left(\text { mass }=M_{B}\right)}+\underbrace{R_{\alpha \beta, \delta \gamma}\left(P, p^{\prime}, p\right)}_{\text {other states }} \tag{2}
\end{equation*}
$$

where $\Gamma_{\alpha \beta}$ is the vertex function of BSE , and $R_{\alpha \beta, \delta \gamma}$ is regular at $P^{2}=M_{B}^{2}$.
We can express the BS amplitude by the vertex function as

$$
\begin{equation*}
\Phi_{\alpha \beta}(P, p)=S_{\alpha \gamma}\left(\frac{P}{2}+p\right) S_{\beta \delta}\left(\frac{P}{2}-p\right) \Gamma_{\gamma \delta}(P, p) \tag{3}
\end{equation*}
$$

and we obtain the equation for the BS amplitude from Eqs. (1)-(3):

$$
\begin{equation*}
\Phi_{\alpha \beta}(P, p)=i S_{\alpha \eta}\left(\frac{P}{2}+p\right) S_{\beta \rho}\left(\frac{P}{2}-p\right) \int \frac{d^{4} k}{(2 \pi)^{4}} V_{\eta \rho ; \gamma \delta}(P, p, k) \Phi_{\gamma \delta}(P, k) \tag{4}
\end{equation*}
$$

## 2. PARTIAL-WAVE DECOMPOSITION OF THE BS AMPLITUDE

We determine two-particle spinor basis in c.m. frame as $U_{\mu_{1}}^{\rho_{1}(1)}(\mathbf{p}) \otimes U_{\mu_{2}}^{\rho_{2}(2) T}(-\mathbf{p})$, where $\mu$ is the spin projection, $\rho_{1,2}$ is the so-called $\rho$ spin, which distinguishes the positive- and negative-energy states. Both of them are necessary to prepare the complete set for the twoparticle bound state. The spinors $U_{\mu_{1}}^{\rho_{1}}(\mathbf{p})$ are connected with the Dirac free spinors $u_{\mu}(\mathbf{p})$ and $v_{\mu}(\mathbf{p})$ as

$$
U_{\mu}^{\rho}(\mathbf{p})= \begin{cases}u_{\mu}(\mathbf{p}), & \rho=+  \tag{5}\\ v_{-\mu}(-\mathbf{p}), & \rho=-\end{cases}
$$

The connections between the propagators and the spinors can be written as

$$
\begin{aligned}
& {[S(P / 2+p)]^{-1} U_{\mu}^{\rho(1)}(\mathbf{p})=\rho S_{\rho}^{(1)-1} U_{\mu}^{\rho(1)}(-\mathbf{p})} \\
& {[S(P / 2-p)]^{-1} U_{\mu}^{\rho(2)}(-\mathbf{p})=\rho S_{\rho}^{(2)-1} U_{\mu}^{\rho(2)}(\mathbf{p})}
\end{aligned}
$$

where $E_{\mathbf{p}}=\sqrt{\mathbf{p}^{2}+m^{2}}$,

$$
\begin{equation*}
S_{ \pm}^{(1)}=1 /\left(\sqrt{s} / 2+p_{0} \mp E_{\mathbf{p}}\right), \quad S_{ \pm}^{(2)}=1 /\left(\sqrt{s} / 2-p_{0} \mp E_{\mathbf{p}}\right) . \tag{6}
\end{equation*}
$$

Here, we can write the partial wave expansion of the BS amplitude as

$$
\begin{align*}
\Phi_{\alpha \beta}^{J M}(P, p)=S_{\alpha \gamma}(P / 2+p) \Gamma_{\gamma \delta}^{J M} & (P, p) S_{\delta \beta}^{T}(P / 2-p)= \\
& =\sum_{L S \rho_{1} \rho_{2}} S_{\rho_{1}}^{(1)} S_{\rho_{2}}^{(2)} g_{J L S \rho_{1} \rho_{2}}\left(p_{0},|\mathbf{p}|\right) \Gamma^{J L S \rho_{1} \rho_{2}}(\mathbf{p}) U_{c} \tag{7}
\end{align*}
$$

where $U_{c}=i \gamma_{2} \gamma_{0}$, and $\Gamma^{J L S \rho_{1} \rho_{2}}(\mathbf{p})$ is the spin-angular function defined as

$$
\begin{align*}
\Gamma^{J L S \rho_{1} \rho_{2}}(\mathbf{p}) U_{c}=i^{L} \sum_{\mu_{1} \mu_{2} m_{L} m_{S}}\left(L m_{L} S m_{S} \mid J \mathcal{M}\right) & \left(\left.\frac{1}{2} \mu_{1} \frac{1}{2} \mu_{2} \right\rvert\, S m_{S}\right) \times \\
& \times Y_{L m_{L}}(\hat{\mathbf{p}}) U_{\mu_{1}}^{\rho_{1}(1)}(\mathbf{p}) \otimes U_{\mu_{2}}^{\rho_{2}(2) T}(-\mathbf{p}) \tag{8}
\end{align*}
$$

We introduce the symmetrical notation of $\rho$ spin for convenience, the radial part of the BS amplitude can be written as

$$
\begin{equation*}
\phi_{J L S \rho}\left(p_{0},|\mathbf{p}|\right)=\sum_{\rho^{\prime}} S_{\rho \rho^{\prime}}\left(p_{0},|\mathbf{p}|\right) g_{J L S \rho}\left(p_{0},|\mathbf{p}|\right) \tag{9}
\end{equation*}
$$

where $S_{\rho \rho^{\prime}}$ is

$$
\begin{align*}
& S_{+}=S_{++}=\left(\sqrt{s} / 2+p_{0}-E_{\mathbf{p}}\right)^{-1}\left(\sqrt{s} / 2-p_{0}-E_{\mathbf{p}}\right)^{-1} \\
& S_{-}=S_{--}=\left(\sqrt{s} / 2+p_{0}+E_{\mathbf{p}}\right)^{-1}\left(\sqrt{s} / 2-p_{0}+E_{\mathbf{p}}\right)^{-1} \\
& S_{e}=S_{e e}=S_{o o}=\left(s / 4-p_{0}^{2}-E_{\mathbf{p}}^{2}\right)\left(\left(s / 4-p_{0}^{2}-E_{\mathbf{p}}^{2}\right)^{2}-4 p_{0}^{2} E_{\mathbf{p}}^{2}\right)^{-1}  \tag{10}\\
& S_{o}=S_{e o}=S_{o e}=\left(2 p_{0} E_{\mathbf{p}}\right)\left(\left(s / 4-p_{0}^{2}-E_{\mathbf{p}}^{2}\right)^{2}-4 p_{0}^{2} E_{\mathbf{p}}^{2}\right)^{-1}, \quad \text { others }=0
\end{align*}
$$

The BS amplitude for the deuteron has 8 states: ${ }^{3} S_{1}^{+},{ }^{3} D_{1}^{+},{ }^{1} P_{1}^{e},{ }^{1} P_{1}^{o},{ }^{3} P_{1}^{e},{ }^{3} P_{1}^{o},{ }^{3} S_{1}^{-}$, ${ }^{3} D_{1}^{-} \cdot{ }^{3} S_{1}^{+},{ }^{3} D_{1}^{+}$are positive-energy states and the others include negative-energy states.

## 3. SOLUTION OF THE BSE

After the partial wave decomposition, the BS equation for $T$ matrix has the following form:

$$
\begin{align*}
& T_{\alpha \beta}\left(p_{0}^{\prime},\left|\mathbf{p}^{\prime}\right|, p_{0},|\mathbf{p}| ; s\right)=V_{\alpha \beta}\left(p_{0}^{\prime},\left|\mathbf{p}^{\prime}\right|, p_{0},|\mathbf{p}| ; s\right)+ \\
& +\frac{i}{2 \pi^{2}} \int d k_{0} \mathbf{k}^{2} d|\mathbf{k}| \sum_{\gamma \delta} V_{\alpha \gamma}\left(p_{0}^{\prime},\left|\mathbf{p}^{\prime}\right|, k_{0},|\mathbf{k}| ; s\right) S_{\gamma \gamma}\left(k_{0},|\mathbf{k}| ; s\right) T_{\gamma \beta}\left(k_{0}^{\prime},|\mathbf{k}|, p_{0},|\mathbf{p}| ; s\right), \tag{11}
\end{align*}
$$

here the indexes of Greek character correspond to the partial states ( $\alpha: J L S \rho$ ).

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We introduce separable ansatz to transform the BSE to a system of the linear equation in the following manner:

$$
\begin{equation*}
V_{\alpha \beta}\left(p_{0}^{\prime},\left|\mathbf{p}^{\prime}\right|, p_{0},|\mathbf{p}| ; s\right)=\sum_{i, j=1}^{N} \lambda_{i j} g_{i}^{(\alpha)}\left(p_{0}^{\prime},\left|\mathbf{p}^{\prime}\right|\right) g_{j}^{(\beta)}\left(p_{0},|\mathbf{p}|\right), \quad \lambda_{i j}=\lambda_{j i} \tag{12}
\end{equation*}
$$

Then, the solution for the radial part of the BS amplitude can be written as

$$
\begin{equation*}
\phi_{J L S \rho}\left(p_{0},|\mathbf{p}|\right)=\sum_{\rho^{\prime}} \sum_{i, j=1}^{N} S_{\rho \rho^{\prime}}\left(p_{0},|\mathbf{p}| ; s\right) \lambda_{i j} g_{i}^{\left(J L S \rho^{\prime}\right)}\left(p_{0},|\mathbf{p}|\right) c_{j}(s) \tag{13}
\end{equation*}
$$

where $c_{i}(s)$ satisfy the following system of equations:

$$
\begin{gather*}
c_{i}(s)-\sum_{k, j=1}^{N} H_{i k}(s) \lambda_{k j} c_{j}(s)=0  \tag{14}\\
H_{i j}(s)=\frac{i}{2 \pi^{2}} \sum_{L S \rho \rho^{\prime}} \int d k_{0} \mathbf{k}^{2} d|\mathbf{k}| S_{\rho \rho^{\prime}}\left(k_{0},|\mathbf{k}| ; s\right) g_{i}^{(J L S \rho)}\left(k_{0},|\mathbf{k}|\right) g_{j}^{\left(J L S \rho^{\prime}\right)}\left(k_{0},|\mathbf{k}|\right) . \tag{15}
\end{gather*}
$$

## 4. COVARIANT GRAZ-II INTERACTION $+P$ WAVES

To calculate various electromagnetic observables, we use the kernel which added $p$-wave parts based on covariant Graz-II interaction. In covariant Graz-II interaction (only positiveenergy states are taken into account: ${ }^{3} S_{1}^{+},{ }^{3} D_{1}^{+}$), the functions $g_{i}$ have the following form [9]:

$$
\begin{align*}
g_{1}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right) & =\frac{1-\gamma_{1}\left(p_{0}^{2}-\mathbf{p}^{2}\right)}{\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{11}^{2}\right)^{2}} \\
g_{2}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right) & =-\frac{\left(p_{0}^{2}-\mathbf{p}^{2}\right)}{\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{12}^{2}\right)^{2}},  \tag{16}\\
g_{3}^{3} D_{1}^{+}\left(p_{0},|\mathbf{p}|\right) & =\frac{\left(p_{0}^{2}-\mathbf{p}^{2}\right)\left(1-\gamma_{2}\left(p_{0}^{2}-\mathbf{p}^{2}\right)\right)}{\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{21}^{2}\right)\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{22}^{2}\right)^{2}} \\
g_{1}^{3} D_{1}^{+}\left(p_{0},|\mathbf{p}|\right) & =g_{2}^{{ }^{3} D_{1}^{+}}\left(p_{0},|\mathbf{p}|\right)=g_{3}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right) \equiv 0
\end{align*}
$$

Parameters of covariant Graz-II are given in the Table.
In addition, we take into account the negative-energy states: ${ }^{1} P_{1}^{e}$ and ${ }^{1} P_{1}^{o}$. We take $g_{i}$ of $P$ waves as follows:

$$
\begin{align*}
& g_{4}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right)=g_{4}^{3} D_{1}^{+}\left(p_{0},|\mathbf{p}|\right)=g_{1,2,3}^{{ }^{1} P_{1}^{e, o}}\left(p_{0},|\mathbf{p}|\right) \equiv 0 \\
& g_{4}^{1} P_{1}^{e}  \tag{17}\\
&\left(p_{0},|\mathbf{p}|\right)=\frac{|\mathbf{p}|}{\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{3}^{2}\right)^{2}} \\
& g_{4}^{1} P_{1}^{o}\left(p_{0},|\mathbf{p}|\right)=\gamma_{3} \frac{p_{0}}{m} \frac{|\mathbf{p}|}{\left(p_{0}^{2}-\mathbf{p}^{2}-\beta_{3}^{2}\right)^{2}}
\end{align*}
$$

## Parameters of covariant Graz-II interaction

| $\gamma_{1}$ | $28.69550 \mathrm{GeV}^{-2}$ | $\lambda_{11}$ | $2.718930 \cdot 10^{-4} \mathrm{GeV}^{6}$ |
| :---: | :---: | :---: | :---: |
| $\gamma_{2}$ | $64.9803 \mathrm{GeV}^{-2}$ | $\lambda_{12}$ | $-7.16735 \cdot 10^{-2} \mathrm{GeV}^{4}$ |
| $\beta_{11}$ | $2.31384 \cdot 10^{-1} \mathrm{GeV}$ | $\lambda_{13}$ | $-1.51744 \cdot 10^{-3} \mathrm{GeV}^{6}$ |
| $\beta_{12}$ | $5.21705 \cdot 10^{-1} \mathrm{GeV}$ | $\lambda_{22}$ | $16.52393 \mathrm{GeV}^{2}$ |
| $\beta_{21}$ | $7.94907 \cdot 10^{-1} \mathrm{GeV}$ | $\lambda_{23}$ | $0.28606 \mathrm{GeV}^{4}$ |
| $\beta_{22}$ | $1.57512 \cdot 10^{-1} \mathrm{GeV}$ | $\lambda_{33}$ | $3.48589 \cdot 10^{-3} \mathrm{GeV}^{6}$ |

The solution of the BSE can be written as

$$
\begin{align*}
& \phi_{3 S_{1}^{+}}\left(p_{0},|\mathbf{p}|\right)=\left(c_{1} \lambda_{11}+c_{2} \lambda_{12}+c_{3} \lambda_{13}+c_{4} \lambda_{14}\right) S_{+} g_{1}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right)+ \\
& +\left(c_{1} \lambda_{12}+c_{2} \lambda_{22}+c_{3} \lambda_{23}+c_{4} \lambda_{24}\right) S_{+} g_{2}^{3} S_{1}^{+}\left(p_{0},|\mathbf{p}|\right),  \tag{18}\\
& \phi_{3_{1}^{+}}\left(p_{0},|\mathbf{p}|\right)=\left(c_{1} \lambda_{13}+c_{2} \lambda_{23}+c_{3} \lambda_{33}+c_{4} \lambda_{34}\right) S_{+} g_{3}^{{ }^{3} D_{1}^{+}}\left(p_{0},|\mathbf{p}|\right), \\
& \phi_{1 P_{1}^{e}}\left(p_{0},|\mathbf{p}|\right)=\left(c_{1} \lambda_{14}+c_{2} \lambda_{24}+c_{3} \lambda_{34}+c_{4} \lambda_{44}\right)\left(S_{e} g_{4}^{1} P_{1}^{e}\left(p_{0},|\mathbf{p}|\right)+S_{o} g_{4}^{1} P_{1}^{o}\left(p_{0},|\mathbf{p}|\right)\right), \\
& \phi_{P_{1}^{o}}\left(p_{0},|\mathbf{p}|\right)=\left(c_{1} \lambda_{14}+c_{2} \lambda_{24}+c_{3} \lambda_{34}+c_{4} \lambda_{44}\right)\left(S_{e} g_{4}^{1} P_{1}^{o}\left(p_{0},|\mathbf{p}|\right)+S_{o} g_{4}^{1} P_{1}^{e}\left(p_{0},|\mathbf{p}|\right)\right) \text {. } \tag{19}
\end{align*}
$$

$\phi_{1} P_{1}^{e}$ is even and $\phi_{1} P_{1}^{\circ}$ is odd under $p_{0} \rightarrow-p_{0}$, which are decided by Eq. (7).

## 5. ELASTIC ELECTRON-DEUTERON SCATTERING

In the relativistic impulse approximation, the deuteron current matrix element can be written as

$$
\begin{gather*}
\left\langle D^{\prime} \mathcal{M}^{\prime}\right| J_{\mu}|D \mathcal{M}\rangle=i \mathrm{e} \int \frac{d^{4} p}{(2 \pi)^{4}} \operatorname{Tr}\left[\bar{\Phi}_{\mathcal{M}^{\prime}}\left(P^{\prime}, p^{\prime}\right) \Gamma_{\mu}^{(p+n)}(q) \Phi_{\mathcal{M}}(P, p)\left(S^{(2) T}\left(q_{2}\right)\right)^{-1}\right]  \tag{20}\\
\Gamma_{\mu}^{(S)}(q)=\gamma_{\mu} F_{1}^{(S)}\left(q^{2}\right)-\frac{\gamma_{\mu} \hat{q}-\hat{q} \gamma_{\mu}}{4 m} F_{2}^{(S)}\left(q^{2}\right) \tag{21}
\end{gather*}
$$

where $\Phi_{\mathcal{M}}(P, p)$ is BS amplitude of the deuteron; $P^{\prime}=P+q$ and $p^{\prime}=p+q / 2 . q$ is the momentum transfer and $\eta=-q^{2} / 4 M^{2}=Q^{2} / 4 M^{2}$, where $M$ is the deuteron mass. The vertex of $\gamma N N$ interaction $\Gamma_{\mu}^{(S)}(q)$ is of on-mass-shell form. The isoscalar form factors of the nucleon $F_{1,2}^{(S)}$ are the summation of two nucleons. To calculate the deuteron form factors, one should know at least three matrix elements with different total angular momentum projections and current component, for example, $\langle 0| J_{0}|0\rangle,\langle 1| J_{0}|1\rangle$ and $\langle 1| J_{1}|0\rangle$. The electric $F_{C}\left(q^{2}\right)$, quadrupole $F_{Q}\left(q^{2}\right)$ and magnetic $F_{M}\left(q^{2}\right)$ form factors are normalized as $F_{C}(0)=1$, $F_{Q}(0)=M^{2} Q_{D}, F_{M}(0)=\mu_{D} M / m$, where $m$ is the nucleon mass, $Q_{D}$ and $\mu_{D}$ are quadrupole and magnetic moments of the deuteron, respectively. The tensor polarization
components of the final deuteron are expressed through the deuteron form factors as follows:

$$
\begin{align*}
& T_{20}\left[A+B \tan ^{2} \frac{\theta_{e}}{2}\right]=-\frac{1}{\sqrt{2}}\left[\frac{8}{3} \eta F_{C} F_{Q}+\frac{8}{9} \eta^{2} F_{Q}^{2}+\frac{1}{3} \eta\left(1+2(1+\eta) \tan ^{2} \frac{\theta_{e}}{2}\right) F_{M}^{2}\right] \\
& T_{21}\left[A+B \tan ^{2} \frac{\theta_{e}}{2}\right]=\frac{2}{\sqrt{3}} \eta\left(\eta+\eta^{2} \sin ^{2} \frac{\theta_{e}}{2}\right)^{1 / 2} F_{M} F_{Q} \sec \frac{\theta_{e}}{2}  \tag{22}\\
& T_{22}\left[A+B \tan ^{2} \frac{\theta_{e}}{2}\right]=-\frac{1}{2 \sqrt{3}} \eta F_{M}^{2}
\end{align*}
$$

where $A$ and $B$ are the deuteron structure functions.

## 6. CALCULATIONS AND RESULTS

To see the contribution of $P$ waves, we fix the Graz-II parameters in the Table. And we introduce the conditions to limit the freedom of the parameters for $P$ waves:

$$
\begin{align*}
& \lambda_{14}=-\sqrt{\lambda_{11}} u_{4}, \lambda_{24}=\sqrt{\lambda_{22}} u_{4}, \lambda_{34}=\sqrt{\lambda_{33}} u_{4}, \lambda_{44}=u_{4}^{2},  \tag{23}\\
& \left.H_{44}\right|_{s=M^{2}}=\left.\frac{i}{2 \pi^{2}} \int d k_{0} \mathbf{k}^{2} d|\mathbf{k}|\left[S_{e}\left(g_{4}{ }^{1} P_{1}^{e} 2+g_{4}{ }^{1} P_{1}^{o} 2\right)+S_{o}\left(g_{4}{ }^{1} P_{1}^{e} g_{4}^{1} P_{1}^{o}\right)\right]\right|_{s=M^{2}}=0 . \tag{24}
\end{align*}
$$

The deuteron binding energy $E d$ can be fixed under the condition of Eq. (24). Now we have two free parameters for $P$ waves: $u_{4}, \gamma_{3}$. For example, to fit the $F_{C}$ node, we calculate the changing point of the sign, at $\gamma_{3}=-15$. Then the parameter is decided as $u_{4} \simeq-10$ or 9.75. The results of calculations using the set of parameters: $\gamma_{3}=-15, u_{4}=-10$, $\beta_{3}=0.4819 \mathrm{GeV}$, are given in Figs. 1-5. In Figs. 1-3 the experimental data are taken from [12]; in Figs. 4, 5 the experimental data are taken from [13] and [14], respectively. Curve 4 denotes calculation for the covariant Graz-II interaction with only positive-energy states: ${ }^{3} S_{1}^{+},{ }^{3} D_{1}^{+}[8]$ and with the dipole-type nucleon form factors. Curves 1,2 , and 3 represent calculations with the dipole-type, Vector Meson Dominance Model (VMDM) [10] and Relativistic Harmonic Oscillator Model (RHOM) [11] nucleon form factors, respectively.


Fig. 1. Charge form factor $F_{C}\left(q^{2}\right)$


Fig. 2. Quadrupole form factor $F_{Q}\left(q^{2}\right)$


Fig. 3. Tensor polarization $T_{20}\left(q^{2}\right)$ calculated at $\theta_{e}=70^{\circ}$


Fig. 4. Tensor polarization $T_{21}\left(q^{2}\right)$ calculated at $\theta_{e}=70^{\circ}$


Fig. 5. Tensor polarization $T_{22}\left(q^{2}\right)$ calculated at $\theta_{e}=70^{\circ}$

## CONCLUSION

We made an attempt to extract information about $P$ waves by analyzing the charge form factor of the deuteron $F_{C}$. Why the $F_{C}$ is appropriate characteristic of the deuteron? To receive answer to this question, we can remember that in nonrelativistic approach, to explain properties of the $F_{C}$, one must take into account mesonic exchange currents especially the so-called pair currents, which have the direct connections with negative-energy states in the deuteron [8]. We could see the contribution of the negative-energy states $\left({ }^{1} P_{1}^{e},{ }^{1} P_{1}^{o}\right)$ by fitting $F_{C}$, at a certain set of parameters. We reproduced the $F_{C}, F_{Q}$, and $T_{20}$ at over $Q^{2}=2(\mathrm{GeV} / c)^{2}$ and $T_{21}$ at $0<Q^{2}<0.7(\mathrm{GeV} / c)^{2}$. Furthermore, we can calculate the form factors to fit $F_{M}$ or $B$. Of course, this consideration has qualitative character only and the further investigation in this direction must be done.

Acknowledgements. This work was partly supported by Sasakawa Scientific Research Grants from the Japan Science Society and Osaka University supporters' association and by Russian Foundation for Basic Research, Grant No. 05-02-17698.

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