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EXISTENCE OF THE σ-MESON BELOW 1 GeV AND CHIRAL SYMMETRY

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1 Introduction

A problem of scalar mesons is most troublesome and long-lived in the light meson spectroscopy. A main difficulty in understanding the scalar-isoscalar sector seems to be related with the possibly-considerable influence of the vacuum and such effects as the instanton contributions that are difficult to take into account. But there is another difficulty related to a strong model-dependence of information on multichannel states obtained in analyses based on the specific dynamic models or using an insufficiently-flexible representation of states (*e.g.*, the standard Breit – Wigner form). Especially, this concerns scalar mesons due to the most weak kinematic diminution of their widths. It was observed that the scalar mesons are either very large or, if narrow, lie near the channel thresholds. Earlier, we have shown [1] that an inadequate description of multichannel states gives not only their distorted parameters when analyzing data but also can cause the fictitious states when one neglects important (even energetic-closed) channels. In this paper we are going, conversely, to demostrate that the large background (*e.g.*, that happens in analyzing $\pi\pi$ scattering), can hide low-lying states (even such important for theory as a σ-meson [2]). With this object, a very interesting and instructive history is related. A majority of analyses rejected this meson by resolving the known "up-down ambiguity" in the 700-900-MeV region in the solutions of the $\pi\pi$ phase-shift analyses for δ_0^0 in favour of the "down" one, because to the "up" solution, one related the $\epsilon(800)$ resonance of width $\sim$ 150-200 MeV. However, some of theorists continued to insist on the existence of that state because it is required by most of the models (like the linear σ-models or the Nambu – Jona-Lasinio models [3]) for spontaneous breaking of chiral symmetry. Since all the analyses of the s-wave $\pi\pi$ scattering gave a large $\pi\pi$-background, it was said that this state (if exists) is "unobservably"-wide. Recently, new analyses of the old and new experimental data have been performed which give a very wide scalar-isoscalar state in the energy region 500-850 MeV [4]-[8]. However, these analyses use either the Breit – Wigner form (even if modified) or specific forms of interactions in a quark model, unitarized by taking the relevant process thresholds into account, or in a multichannel approach to the considered processes; therefore, there one cannot talk about a model independence of results. Besides, in these analyses, a large $\pi\pi$-background is obtained. We are going to show that a proper detailing of the background (as allowance for the left-hand branch-point) permits us to extract from the latter a very wide (but observable) state below 1 GeV even in the "down" solution for the $\pi\pi$ phase-shift, and, therefore, it is highly important in studying lightest states.

An adequate consideration of multichannel states and a model-independent information on them can be obtained on the basis of the first principles (analyticity, unitarity and Lorentz invariance) immediately applied to analyzing experimental data. The way of realization is a consistent allowance for the nearest singularities on all sheets of the Riemann surface of the S-matrix. The Riemann-surface structure is taken into account by a proper choice of the uniformizing variable. Earlier, we have proposed this method for 2- and 3-channel resonances and developed the concept of standard clusters (poles on the Riemann surface) as a qualitative characteristic of a state and a sufficient condition of its existence as well as a criterion of a quantitative description of the coupled-process amplitudes when all the complifications of the analytic structure due to a finite width of resonances and crossing channels and high-energy "tails" are accumulated in quite a smooth background [1, 9, 10]. Let us stress that for a wide state, the pole position (the pole cluster one for multichannel states) is a more stable characteristic than the mass and width which are strongly dependent on a model. The cluster kind is determined from the analysis of experimental data and is related to the state nature. At all events, we can, in a model-independent manner, discriminate between bound states of particles and the ones of quarks and gluons, qualitatively predetermine the relative strength of coupling of a state with the considered channels, and obtain an indication on its gluonium nature.

Since, in this work, a main stress is laid on studying lowest states, it is sufficient to restrict itself to a two-channel approach when considering simultaneously the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$. Furthermore, in the uniformizing variable, one must take into account, besides the branch-points corresponding to the thresholds of the processes $\pi\pi \to \pi\pi, K\overline{K}$, also the left-hand branch-point at $s = 0$, related to the background in which the crossing-channel contributions are contained.

The layout of the paper is as follows. In Section 2, we outline the two-coupled-channel formalism, determine the pole clusters on the Riemann surface as characteristics of multichannel states, and introduce a new uniformizing variable, allowing for the branch-points of the right-hand (unitary) and left-hand cuts of the $\pi\pi$-scattering amplitude. In Section 2, we analyze simultaneously experimental data on the processes $\pi\pi \to \pi\pi, K\overline{K}$ in the isoscalar s-wave on the basis of the presented approach. In the Conclusion, the obtained results are discussed.

2 Two-Coupled-Channel Formalism

Considering the multichannel problem (here the 2-channel one), we pursue two aims: to obtain a model-independent information about the multichannel resonances and an indication about their QCD nature, and to describe the experimental data on the coupled processes. The first purpose is achieved through the account of the nearest (to the physical region of interest) singularities of the S-matrix. Herewith it is important to analyze simultaneously experimental data on the coupled processes.

Here we consider the coupled processes of $\pi\pi$ and $K\overline{K}$ scattering and $\pi\pi \to K\overline{K}$. Therefore, we have the two-channel S-matrix determined on the 4-sheeted Riemann surface. The S-matrix elements $S_{\alpha\beta}$, where $\alpha, \beta = 1(\pi\pi), 2(K\overline{K})$, have the right-hand (unitary) cuts along the real axis of the s-variable complex plane, starting at the points $4m_\pi^2$ and $4m_K^2$ and extending to ∞, and the left-hand cuts, which are related to the crossing-channel contributions and extend along the real axis towards $-\infty$ and begin at $s = 0$ for S_{11} and at $4(m_K^2 - m_\pi^2)$ for S_{22} and S_{12}. We number the Riemann-surface sheets according to the signs of analytic continuations of the channel momenta

$$k_1 = (s/4 - m_\pi^2)^{1/2}, \qquad k_2 = (s/4 - m_K^2)^{1/2} \tag{1}$$

as follows: signs $(\mathrm{Im}k_1, \mathrm{Im}k_2) = ++, -+, --, +-$ correspond to the sheets I,II,III,IV.

Then, for instance, from the physical region on sheet I we pass across the cut below the $K\overline{K}$ threshold to sheet II; above the $K\overline{K}$ threshold, to sheet III.

To elucidate the resonance representation on the Riemann surface, we express analytic continuations of the matrix elements to the unphysical sheets $S^L_{\alpha\beta}$ $(L = II, III, IV)$ in terms of them on the physical sheet $S^I_{\alpha\beta}$. Those expressions are convenient for our purpose because, on sheet I (the physical sheet), the matrix elements $S^I_{\alpha\beta}$ can have only zeros beyond the real axis. Using the reality property of the analytic functions and the 2-channel unitarity, one can obtain

$$\begin{array}{lll} S_{11}^{II} = \dfrac{1}{S_{11}^I}, & S_{11}^{III} = \dfrac{S_{22}^I}{\det S^I}, & S_{11}^{IV} = \dfrac{\det S^I}{S_{22}^I}, \\ S_{22}^{II} = \dfrac{\det S^I}{S_{11}^I}, & S_{22}^{III} = \dfrac{S_{11}^I}{\det S^I}, & S_{22}^{IV} = \dfrac{1}{S_{22}^I}, \\ S_{12}^{II} = \dfrac{iS_{12}^I}{S_{11}^I}, & S_{12}^{III} = \dfrac{-S_{12}^I}{\det S^I}, & S_{12}^{IV} = \dfrac{iS_{12}^I}{S_{22}^I}, \end{array} \qquad (2)$$

Here $\det S^I = S^I_{11}S^I_{22} - (S^I_{12})^2$. Provided a resonance has the only decay mode (1-channel case), in the matrix element, the resonance (in the limit of its narrow width) is represented by a pair of complex conjugate poles on the IInd sheet and by a pair of conjugate zeros on the physical sheet at the same points of complex energy. This model-independent statement about the poles as the nearest singularities holds also when taking account of the finite width of a resonance. In the case of two coupled channels, formulae (2) immediately give the resonance representation by poles and zeros on the 4-sheeted Riemann surface. Here one must discriminate between three types of resonances – which are described (**a**) by a pair of complex conjugate poles on sheet II and, therefore, by a pair of complex conjugate zeros on the Ist sheet in S_{11}; (**b**) by a pair of conjugate poles on sheet IV and, therefore, by a pair of complex conjugate zeros on sheet I in S_{22}; (**c**) by one pair of conjugate poles on each of sheets II and IV, that is by one pair of conjugate zeros on the physical sheet in each of matrix elements S_{11} and S_{22}.

As is seen from (2), to the resonances of types (**a**) and (**b**) one has to make correspond a pair of complex conjugate poles on sheet III which are shifted relative to a pair of poles on sheet II and IV , respectively (if the coupling among channels were absent, i.e. $S_{12} = 0$, the poles on sheet III would lay exactly (**a**) under the poles on the IInd sheet, (**b**) above the poles on the IVth sheet). To the states of type (**c**) one must make correspond two pairs of conjugate poles on sheet III which are reasonably expected to be a pair of the complex conjugate compact formations of poles. Thus, we arrive at the notion of three standard pole-clusters which represent two-channel bound states of quarks and gluons. It is convenient to discriminate between those clusters according to the presence of zeros, corresponding to the state, on the physical sheet in matrix element S_{11} (**a**), S_{22} (**b**) or in both (**c**).

Note that this resonance division into types is not formal. In paricular, the resonance, coupled strongly with the first $(\pi\pi)$ channel, is described by the pole cluster of type (**a**); if the resonance is coupled strongly with the $K\overline{K}$ and weakly with $\pi\pi$ channel (say, if it has a dominant $s\bar{s}$ component), then it is represented by the cluster of type (**b**); finally, since a most noticeable property of a glueball is the flavour-singlet structure of its wave function and, therefore, (except the factor $\sqrt{2}$ for a channel with neutral particles) practically equal coupling with all the members of the nonet, then a glueball must be represented by the pole cluster of type (**c**) as a necessary condition.

Just as in the 1-channel case, the existence of a particle bound-state means the presence of a pole on the real axis under the threshold on the physical sheet, so in the 2-channel case, the existence of a bound state in channel 2 ($K\overline{K}$ molecule), which, however, can decay into channel 1 ($\pi\pi$ decay), would imply the presence of a pair of complex conjugate poles on sheet II under the threshold of the second channel without an accompaniment of the corresponding shifted pair of poles on sheet III. Namely, according to this test, earlier an interpretation of the $f_0(980)$ state as $K\overline{K}$ molecule has been rejected [1, 9, 11].

Generally, formulae (2) are a solution of the 2-channel problem in the sense of giving a chance to predict (on the basis of the data on one process) the coupled-process amplitudes under a certain conjecture about the background. We made this earlier in the 2-channel approach [9]. It was a success to describe ($\chi^2/ndf \approx 1.06$) the experimental isoscalar s-wave of $\pi\pi$ scattering from the threshold to 1.9 GeV, to predict satisfactorily (on the basis of data on $\pi\pi$ scattering) the behaviour of the s-wave of $\pi\pi \to K\overline{K}$ process approximately up to 1.25 GeV. To take account of the proper right-hand branch-points, the corresponding uniformizing variable has been used. However, for the simultaneous analysis of experimental data on the coupled processes it is more convenient to use the Le Couteur-Newton relations [12] representing compactly all features given by formulae (2) and expressing the S-matrix elements of all coupled processes in terms of the Jost matrix determinant $d(k_1, k_2) \equiv d(s)$, the real analytic function with the only square-root branch-points at the process thresholds $k_i = 0$ [13]. Earlier, this was done by us in the 2-channel consideration [9] with the uniformizing variable

$$z = \frac{k_1 + k_2}{\sqrt{m_K^2 - m_\pi^2}}, \tag{3}$$

which was proposed in Ref. [13] and maps the 4-sheeted Riemann surface with two unitary cuts, starting at the points $4m_\pi^2$ and $4m_K^2$, onto the plane. (Note that other authors have been also applied the parametrizations with using the Jost functions at analyzing the s-wave $\pi\pi$ scattering in the one-channel approach [14] and in the two-channel one [11]. In latter work, the uniformizing variable k_2 has been used, therefore, their approach cannot be emploied near by the $\pi\pi$ threshold.)

When analyzing the processes $\pi\pi \to \pi\pi, K\overline{K}$ by the above methods in the 2-channel approach, two resonances ($f_0(975)$ and $f_0(1500)$) are found to be sufficient for a satisfactory description ($\chi^2/\text{ndf} \approx 1.00$). However, in this case, the large $\pi\pi$-background has been obtained. A character of the representation of the background (the pole of second order on the imaginary axis on sheet II and the corresponding zero on sheet I) suggests that a wide light state is possible to be hidden in the background. To check this, one must work out the background in some detail.

Now we will take, in the uniformizing variable, into account also the left-hand branch-point at $s = 0$. We use the uniformizing variable

$$v = \frac{m_K\sqrt{s - 4m_\pi^2} + m_\pi\sqrt{s - 4m_K^2}}{\sqrt{s(m_K^2 - m_\pi^2)}}, \tag{4}$$

which maps the 4-sheeted Riemann surface, having (in addition to two above-indicated unitary cuts) also the left-hand cut starting at the point $s = 0$, onto the v-plane. (Note that the analogous uniformizing variable has been used, *e.g.*, in Ref. [15] at studying the forward elastic $p\bar{p}$ scattering amplitude.) It is convenient to write also the function $s(v)$

$$s = -\frac{16m_K^2 m_\pi^2 v^2}{(m_K^2 - m_\pi^2)(v-b)(v+b)(v-b^{-1})(v+b^{-1})}, \tag{5}$$

where $b = \sqrt{(m_K + m_\pi)/(m_K - m_\pi)}$ is the point into which $s = \infty$ is mapped on the v-plane, and the symmetry properties of this function

$$s(v) = s(-v) = s(v^{-1}) = s(-v^{-1}) = s^*(v^*) \tag{6}$$

demostrate which points on the v-plane correspond to the same point on the s-plane. In Fig.1, the plane of the uniformizing variable v for the $\pi\pi$-scattering amplitude is

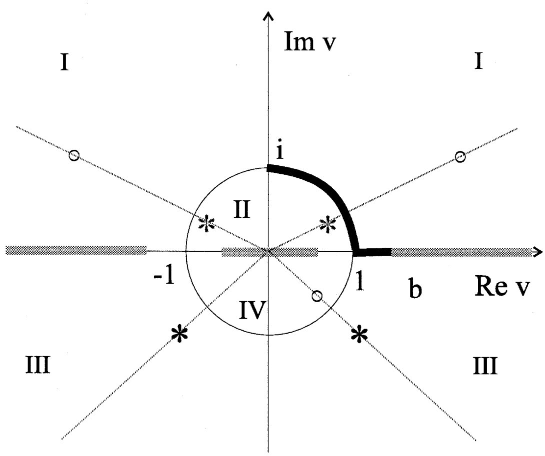

Figure 1: Uniformization plane for the $\pi\pi$-scattering amplitude. The Roman numerals (I,...,IV) denote the images of the corresponding sheets of the Riemann surface; the thick line represents the physical region (the points i, 1 and b correspond to the $\pi\pi, K\overline{K}$ thresholds and $s = \infty$), respectively); the shaded lines are the images of the corresponding edges of the left-hand cut. The depicted positions of poles ($*$) and of zeros ($\circ$) give the representation of the type (**a**) resonance in S_{11}.

depicted. The Roman numerals (I,...,IV) denote the images of the corresponding sheets of the Riemann surface; the thick line represents the physical region; the points i, 1 and b correspond to the $\pi\pi, K\overline{K}$ thresholds and $s = \infty$, respectively; the shaded intervals $(-\infty, -b]$, $[-b^{-1}, b^{-1}]$, $[b, \infty)$ are the images of the corresponding edges of the left-hand cut. The depicted positions of poles ($*$) and of zeros ($\circ$) give the representation of the type (**a**) resonance in S_{11}. In Fig.1, a very symmetric picture is shown which ensures the known fact that the $\pi\pi$ interaction is practically elastic up to the $K\overline{K}$ threshold (the contribution of the multiparticle states $(4\pi, 6\pi)$ is negligible within the up-to-date experiment accuracy). This property of the $\pi\pi$ interaction is satisfied since the poles and zeros are symmetric to each other with respect to the unit circle. If the $\pi\pi$ scattering were elastic also above the $K\overline{K}$ threshold, there would be the symmetry of the poles and zeros with respect to the real axis. The symmetry of the whole picture relative to the imaginary axis ensures the property of the real analyticity.

On v-plane the Le Couteur-Newton relations are [9, 13]

$$S_{11} = \frac{d(-v^{-1})}{d(v)}, \quad S_{22} = \frac{d(v^{-1})}{d(v)}, \quad S_{11}S_{22} - S_{12}^2 = \frac{d(-v)}{d(v)}. \tag{7}$$

Then, the condition of the real analyticity implies

$$d(-v^*) = d^*(v) \tag{8}$$

for all v, and the unitarity needs the following relations to hold true for the physical v-values:

$$|d(-v^{-1})| \leq |d(v)|, \quad |d(v^{-1})| \leq |d(v)|, \quad |d(-v)| = |d(v)|. \tag{9}$$

The d-function that on the v-plane already does not possess branch-points is taken as

$$d = d_B d_{res}, \tag{10}$$

where $d_B = B_\pi B_K$; B_π contains the possible remaining $\pi\pi$-background contribution, related to exchanges in crossing channels; B_K is that part of the $K\overline{K}$ background which does not contribute to the $\pi\pi$-scattering amplitude. The most considerable part of the background of the considered coupled processes related to the influence of the left-hand branch-point at $s = 0$ is taken already in the uniformizing variable v (4) into account. The function $d_{res}(v)$ represents the contribution of resonances, described by one of three types of the pole-zero clusters, *i.e.*, except for the point $v = 0$, it consists of zeros of clusters:

$$d_{res} = v^{-M} \prod_{n=1}^{M} (1 - v_n^* v)(1 + v_n v), \tag{11}$$

where n runs over the independent zeros; therefore, for resonances of the types (**a**) and (**b**), n has two values, for the type (**c**), four values; M is the number of pairs of the conjugate zeros.

3 Analysis of experimental data

Using the described 2-channel approach, we analyze simultaneously the available experimental data on the $\pi\pi$-scattering [16] and the process $\pi\pi \to K\overline{K}$ [17] in the channel with $I^G J^{PC} = 0^+0^{++}$. As data, we use the results of phase analyses which are given for phase shifts of the amplitudes (δ_1 and δ_{12}) and for moduli of the S-matrix elements η_1 (the elasticity parameter) and ξ:

$$S_a = \eta_a e^{2i\delta_a} \quad (a = 1, 2), \qquad S_{12} = i\xi e^{i\delta_{12}}. \tag{12}$$

(Remember that "1" denotes here the $\pi\pi$ channel, "2" – $K\overline{K}$). The 2-channel unitarity condition gives

$$\eta_1 = \eta_2 = \eta, \qquad \xi = (1 - \eta^2)^{1/2}, \qquad \delta_{12} = \delta_1 + \delta_2. \tag{13}$$

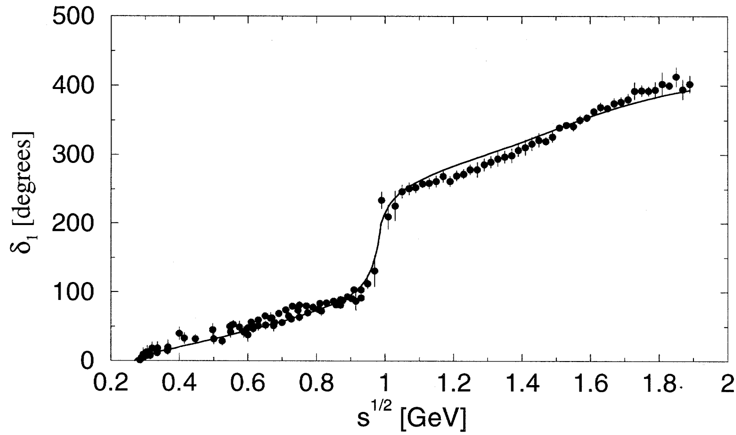

Figure 2: The energy dependence of the phase shift (δ_1) of the $\pi\pi$-scattering amplitude obtained on the basis of a simultaneous analysis of the experimental data on the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$. The data on the $\pi\pi$ scattering are taken from Refs.[16].

We have taken the data on the $\pi\pi$ scattering from the threshold up to 1.89 GeV. Then, comparing experimental data for ξ with values of ξ, calculated by eq.(13) with using the experimental points for the elasticity parameter η, one can see that the 2-channel unitarity takes place approximately to 1.4 GeV. To obtain the satisfactory description of the s-wave $\pi\pi$ scattering from the threshold to 1.89 GeV (Fig.2 and Fig.3), we have taken $B_\pi = 1$ in eq.(10), and three multichannel resonances turned out to be sufficient: the two ones of the type (**a**) ($f_0(665)$ and $f_0(980)$) and $f_0(1500)$ of the type (**c**). Therefore, in eq.(11) $M = 8$ and the following zero positions on the v-plane, corresponding to these resonances, have been established in this situation with the parameterless description of the background:

$$
\begin{array}{lll}
\text{for } f_0(665): & v_1 = 1.36964 + 0.208632i, & v_2 = 0.921962 - 0.25348i, \\
\text{for } f_0(980): & v_3 = 1.04834 + 0.0478652i, & v_4 = 0.858452 - 0.0925771i, \\
\text{for } f_0(1500): & v_5 = 1.2587 + 0.0398893i, & v_6 = 1.2323 - 0.0323298i, \\
 & v_7 = 0.809818 - 0.019354i, & v_8 = 0.793914 - 0.0266319i.
\end{array}
$$

Here for the phase shift δ_1 and the elasticity parameter η, 113 and 50 experimental points [16], respectively, are used; when rejecting the points at energies 0.61, 0.65, and 0.73 GeV for δ_1 and at 0.99, 1.65, and 1.85 GeV for η, which give an anomalously large contribution to χ^2, we obtain for χ^2/ndf the values 2.7 and 0.72, respectively; the total χ^2/ndf in the case of the $\pi\pi$ scattering is 1.96.

With the presented picture, the satisfactory description for the modulus (ξ) of the $\pi\pi \to K\overline{K}$ matrix element is given from the threshold to $\sim$ 1.4 GeV (Fig.4). Here

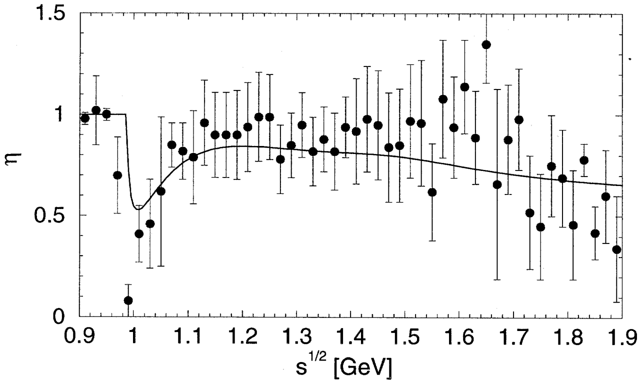

Figure 3: The same as in Fig.2 but for the elasticity parameter η.

35 experimental points [17] are used; $\chi^2/\mathrm{ndf} \approx 1.11$ when eliminating the points at energies 1.002, 1.265, and 1.287 GeV (with especially large contribution to χ^2). However, for the phase shift $\delta_{12}(s)$, slightly excessive curve is obtained. Therefore, keeping the *parameterless* description of the $\pi\pi$ background, one must take into account the part of the $K\overline{K}$ background that does not contribute to the $\pi\pi$-scattering amplitude. Furthermore, this contribution is to be elastic. Note that the variable v is uniformizing for the $\pi\pi$-scattering amplitude, *i.e.*, on the v-plane, S_{11} has no cuts, however, the amplitudes of the $K\overline{K}$ scattering and $\pi\pi \to K\overline{K}$ process do have the cuts on the v-plane, which arise from the left-hand cut on the s-plane, starting at the point $s = 4(m_K^2 - m_\pi^2)$. Under the $s \to v$ conformal mapping (4), this left-hand cut is mapped into cuts which begin at the points

$$v = \frac{m_K\sqrt{m_K^2 - 2m_\pi^2} \pm i m_\pi}{m_K^2 - m_\pi^2}$$

on the unit circle on the v-plane, go along it up to the imaginary axis, and occupy the latter. This left-hand cut will be neglected in the Riemann-surface structure, and the contribution on the cut will be taken into account in the $K\overline{K}$ background as a pole on the real s-axis on the physical sheet in the sub-$K\overline{K}$-threshold region; on the v-plane, this pole gives two poles on the unit circle in the upper half-plane, symmetric to each other with respect to the imaginary axis, and two zeros, symmetric to the poles with respect to the real axis, *i.e.* at describing the process $\pi\pi \to K\overline{K}$, one additional parameter is introduced, say, a position p of the zero on the unit circle. Therefore, for B_K in eq. (10) we take the form

$$B_K = v^{-4}(1 - pv)^4(1 + p^*v)^4. \tag{14}$$

Fourth power in (14) is stipulated by the following. First, a pole on the real s-axis on the physical sheet in S_{22} is accompanied by a pole in sheet II at at the same s-value (as

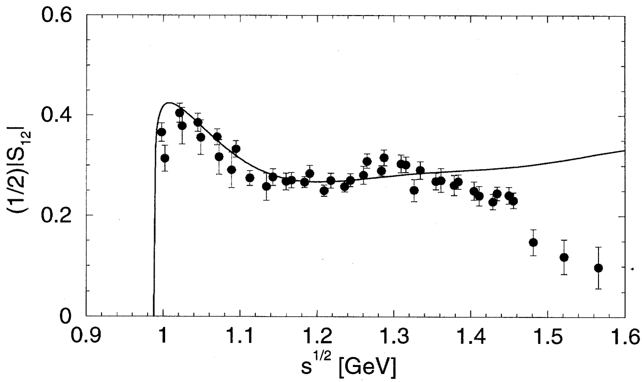

Figure 4: The energy dependence of the ($|S_{12}|$) obtained on the basis of a simultaneous analysis of the experimental data on the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ in the channel with $I^G J^{PC} = 0^+0^{++}$. The data on the process $\pi\pi \to K\overline{K}$ are taken from Ref.[17].

it is seen from eqs. (2)); on the v-plane, this implies the pole of second order (and also zero of the same order, symmetric to the pole with respect to the real axis). Second, for the s-channel process $\pi\pi \to K\overline{K}$, the crossimg u- and t-channels are the $\pi - K$ and $\overline{\pi} - K$ scattering (exchanges in these channels give contributions on the left-hand cut); this results in the additional doubling of the multiplicity of the indicated pole on the v-plane. Zeros of the fourth order in B_K (and, correspondingly, poles of the fourth order in the $K\overline{K}$-amplitude) provide the better description of the $K\overline{K}$ background than the ones of the first order in our recent work [18]. One can verify that the expression (14) does not contribute to S_{11}, *i.e.* the parameterless description of the $\pi\pi$ background is kept. A satisfactory description of the phase shift $\delta_{12}(\sqrt{s})$ (Fig.5) is obtained approximately to 1.52 GeV with the value of the parameter $p = 0.948201 + 0.31767i$ (this corresponds to the position of the pole on the s-plane at $s = 0.434\text{GeV}^2$). Here 59 experimental points [17] are considered; $\chi^2/\text{ndf} \approx 3.05$ when eliminating the points at energies 1.117, 1.247, and 1.27 GeV (with especially large contribution to χ^2). The total χ^2/ndf for four analyzed quantities to describe the coupled processes $\pi\pi \to \pi\pi, K\overline{K}$ is 2.12; the number of adjusted parameters is 17, where they all (except a single relating to the $K\overline{K}$ background) are positions of poles describing resonances.

In Table 1, the obtained parameter values of poles on the corresponding sheets of the Riemann surface are cited on the complex energy plane ($\sqrt{s_r} = \mathrm{E}_r - i\Gamma_r$). We stress that these are not masses and widths of resonances. Since, for wide resonances, values of masses and widths are very model-dependent, it is reasonable to report characteristics of pole clusters which must be rather stable for various models.

Now we can calculate the constants of the obtained-state couplings with the $\pi\pi$ – "1" and $K\overline{K}$ – "2" systems through the residues of amplitudes at the pole on sheet II.

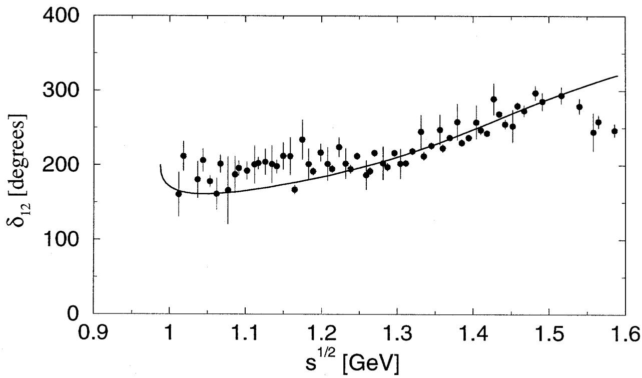

Figure 5: The same as in Fig.4 but for the phase shift (δ_{12}).

Table 1:

Sheet	$f_0(665)$ E, MeV	$f_0(665)$ Γ, MeV	$f_0(980)$ E, MeV	$f_0(980)$ Γ, MeV	$f_0(1500)$ E, MeV	$f_0(1500)$ Γ, MeV
II	610±14	620±26	988±5	27±8	1530±25	390±30
III	720±15	55±9	984±16	210±22	1430±35	200±30
					1510±22	400±34
IV					1410±24	210±38

Expressing the T-matrix via the S-matrix as

$$S_{ii} = 1 + 2i\rho_i T_{ii}, \qquad S_{12} = 2i\sqrt{\rho_1\rho_2}T_{12}, \tag{15}$$

where $\rho_i = \sqrt{(s-4m_i^2)/s}$, and taking the resonance part of the amplitude in the form

$$T_{ij}^{res} = \sum_r g_{ir}g_{rj}D_r^{-1}(s), \tag{16}$$

where $D_r(s)$ is an inverse propagator ($D_r(s) \propto s - s_r$), we define the coupling constants as

$$g_i g_j = \frac{16m_K^2 m_\pi^2}{3(m_K^2 - m_\pi^2)}\left|\frac{v_r^{*2} - v_r^{*-2}}{(v_r^{*2}-b^2)(v_r^{*2}-b^{-2})(v_r^{*-2}-b^2)(v_r^{*-2}-b^{-2})}\lim_{v\to v_r^{*-1}}(1-v_r^* v)\frac{S_{ij}(v)}{\sqrt{\rho_i\rho_j}}\right|. \tag{17}$$

Here we denote the coupling constants with the $\pi\pi$ and $K\overline{K}$ systems through g_1 and g_2, respectively. The obtained values of the coupling constants of the observed states are given in Table 2.

Table 2

	$f_0(665)$	$f_0(980)$	$f_0(1500)$
g_1, GeV	0.7477 ± 0.095	0.1615 ± 0.03	0.899 ± 0.093
g_2, GeV	0.834 ± 0.1	0.438 ± 0.028	

In this 2-channel approach, there is no point in calculating the coupling constant of the $f_0(1500)$ state with the $K\overline{K}$ system, because the 2-channel unitarity is valid only to 1.4 GeV, and, above this energy, there is a considerable disagreement between the calculation of the amplitude modulus S_{12} and the experimental data.

Let us indicate also scattering lengths calculated in our approach. For the $K\overline{K}$ scattering, we obtain

$$a_0^0(K\overline{K}) = -1.188 \pm 0.13 + (0.648 \pm 0.09)i, \; m_{\pi^+}^{-1}.$$

A presence of the imaginary part in $a_0^0(K\overline{K})$ reflects the fact, that already at the threshold of the $K\overline{K}$ scattering, other channels ($2\pi, 4\pi$ etc.) are opened. In Table 3, we have presented our result for the $\pi\pi$ scattering length a_0^0 and its comparison with results of some other works both theoretical and experimental.

Table 3

a_0^0, $m_{\pi^+}^{-1}$	References	Remarks
0.27 ± 0.06	our paper	model-independent approach
0.26 ± 0.05	L. Rosselet et al.[16]	analysis of the decay $K \to \pi\pi e\nu$ with using Roy's model
0.24 ± 0.09	A.A. Bel'kov et al.[16]	analysis of the process $\pi^- p \to \pi^+\pi^- n$ with using the effective range formula
0.23	S. Ishida et al.[6]	modified analysis of $\pi\pi$ scattering with using Breit-Wigner forms
0.16	S. Weinberg [19]	current algebra (non-linear σ-model)
0.20	J. Gasser, H. Leutwyler [20]	chiral theory with one-loop corrections, non-linear realization of chiral symmetry
0.217	J. Bijnens at al.[21]	chiral theory with two-loop corrections, non-linear realization of chiral symmetry
0.26	M.K. Volkov [22]	chiral theory, linear realization of chiral symmetry

We have here presented model-independent results: the pole positions, coupling constants and scattering lengths. The formers can be used further for calculating masses and widths of these states in various models.

If we suppose, that the obtained state $f_0(665)$ is the σ-meson, then from the known relation of the σ-model between the coupling constant of the σ with the $\pi\pi$-system and masses

$$g_{\sigma\pi\pi} = \frac{m_\sigma^2 - m_\pi^2}{\sqrt{2} f_{\pi^0}}$$

(here f_{π^0} is the constant of the weak decay of the π^0: $f_{\pi^0} = 93.1$ MeV), we obtain $m_\sigma \approx 342$ MeV. That small value of the σ-mass can be a result of the mixing with the $f_0(980)$ state [23].

4 Conclusions

In the present work, in the model-independent approach consisting in the immediate application to the analysis of experimental data of first principles (analyticity-causality and unitarity), a satisfactory simultaneous description of the isoscalar s-wave channel of the processes $\pi\pi \to \pi\pi, K\overline{K}$ from the thresholds to the energy values, where the 2-channel unitarity is valid, is obtained. A parameterless description of the $\pi\pi$ background is first given by allowance for the left-hand branch-point in the proper uniformizing variable. It is shown that the large $\pi\pi$-background, usually obtained, combines in reality the influence of the left-hand branch-point and the contribution of a very wide resonance at ~ 665 MeV. Thus, a model-independent confirmation of the state, already discovered in other works [5]-[8] (or pretending to this discovery) and denoted in the PDG issues by $f_0(400-1200)$ [2], is obtained. This is the σ-meson required by majority of models for spontaneous breaking of chiral symmetry. Three states ($f_0(665) - \sigma$-meson, $f_0(980)$ and $f_0(1500)$) are sufficient to describe the analyzed data.

The discovery of the $f_0(665)$ state solves one important mystery of the scalar-meson family that is related to the Higgs boson of the hadronic sector. This is a result of principle, because the schemes of the nonlinear realization of the chiral symmetry have been considered which do without the Higgs mesons. One can think that a linear realization of the chiral symmetry (at least, for the lightest states and related phenomena) is valid. First, this is a simple and beautiful mechanism that works also in other fields of physics, for example, in superconductivity. Second, the effective Lagrangians obtained on the basis of this mechanism (the Nambu – Jona-Lasinio and other models) describe perfectly the ground states and related phenomena. The only weak link of this approach was the absence of the σ-meson below 1 GeV.

Note also that the $f_0(665)$ changes but does not solve the problem of unusual properties of the scalar mesons that prevent the scalar nonet to be made up.

Let us also notice that the character of the $f_0(665)$ pole-cluster (namely, a considerable shift of the pole on sheet III towards the imaginary axis) can point to the unconsidered channel with which this state is, possibly, coupled strongly, and the threshold of which is situated below 600 MeV. In this energy region, only one channel is opened: this is the 4π channel. It is interesting to verify this assumption, because it concerns such an important state.

This analysis does not reveal the $f_0(1370)$ resonance; therefore, if this meson exists, it must be weakly coupled with the $\pi\pi$ channel, *i.e.* be described by the pole cluster of the type (**b**) (this would testify to the dominant $s\bar{s}$ component in this state; as to that assignment of the $f_0(1370)$ resonance, we agree, *e.g.*, with the work [24]).

The $f_0(1500)$ state is represented by the pole cluster on the Riemann surface of the S-matrix of the type (**c**) which corresponds to a glueball. This type of cluster (*i.e.* the presence of the zeros, corresponding to the state, on the physical sheet of both $\pi\pi$ and $K\overline{K}$ scattering) reflects the flavour-singlet structure of the glueball wave-function and is only a necessary condition of the glueball nature of the $f_0(1500)$ state. Let us also pay attention to the strong coupling of the $f_0(1500)$ state with the $\pi\pi$ system, and to that in the model-independent approach, one can obtain a qualitative indication – how much is the admixture of other states ($q\bar{q}, q\bar{q}g$, etc.)? To this end, one must consider the $\pi\pi \to K\overline{K}$ process in our 3-channel approach [1] and determine the coupling constants of the $f_0(1500)$ with the other members of the pseudoscalar nonet.

We emphasize that the obtained results are model-independent, since they are based on the first principles and on the mathematical fact that a local behaviour of analytic functions, determined on the Riemann surface, is governed by the nearest singularities on all sheets.

We think that multichannel states are most adequately represented by clusters, *i.e.* by the pole positions on all corresponding sheets. The pole positions are rather stable characteristics for various models, whereas masses and widths are very model-dependent for wide resonances.

Finally, note that in the model-independent approach, there are many adjusted parameters (although, *e.g.* for the $\pi\pi$ scattering, they all are positions of poles describing resonances). The number of these parameters can be diminished by some dynamic assumptions, but this is another approach and of other value.

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References

[1] D. Krupa, V.A. Meshcheryakov, Yu.S. Surovtsev, Nuovo Cimento **109 A**, 281 (1996).

[2] Review of Particle Physics, Europ. Phys. J. **C3**, 1 (1998).

[3] Y. Nambu, G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961); M.K. Volkov, Ann. Phys. **157**, 282 (1984); T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 223 (1994); R. Delbourgo, M.D. Scadron, Mod. Phys. Lett. **A10**, 251 (1995).

[4] B.S. Zou, D.V. Bugg, Phys. Rev. **D48**, R3948 (1993); **D50**, 591 (1994).

[5] M. Svec, Phys. Rev. **D53**, 2343 (1996).

[6] S. Ishida et al., Progr. Theor. Phys. **95**, 745 (1996); **98**, 621 (1997).

[7] N. Törnqvist, Phys. Rev. Lett. **76**, 1575 (1996).

[8] R. Kamiński, L. Leśniak, B. Loiseau, Eur. Phys. J. C 9 (1999) 141.

[9] D. Krupa, V.A. Meshcheryakov, Yu.S. Surovtsev, Yad. Fiz. **43**, 231 (1986); Czech. J. Phys. **B38**, 1129 (1988).

[10] D. Krupa, V.A. Meshcheryakov, Yu.S. Surovtsev, *Pole-Cluster Representations as Tests of Multichannel Resonances and Coupled Processes in Scalar Sector.* Proc. Intern. Conf."HADRON STRUCTURE'96" (Stará Lesná, Slovak Republic, 12-16 February 1996), ed. L.Martinovič and P.Striženec, Dubna, 1996, p.86.

[11] D. Morgan, M.R. Pennington, Phys. Rev. **D48**,1185 (1993).

[12] K.J. Le Couteur, Proc. Roy. Soc. **A256**, 115 (1960); R.G. Newton, J. Math. Phys. **2**, 188 (1961).

[13] M. Kato, Ann. Phys. **31**, 130 (1965).

[14] J. Bohacik, H. Kühnelt, Phys. Rev. **D21**, 1342 (1980).

[15] B.V. Bykovsky, V.A. Meshcheryakov, D.V. Meshcheryakov, Yad. Fiz. **53**, 257 (1990).

[16] B. Hyams et al., Nucl. Phys. **B64**, 134 (1973); ibid., **B100**, 205 (1975). A. Zylbersztejn et al., Phys. Lett. **B38**, 457 (1972). P. Sonderegger, P. Bonamy, Proc. 5th Intern. Conf. on Elementary Particles, Lund, 1969, paper 372. J.R. Bensinger et al., Phys. Lett. **B36**, 134 (1971). J.P. Baton et al., Phys. Lett. **B33**, 525, 528 (1970). P. Baillon et al., Phys. Lett. **B38**, 555 (1972). L. Rosselet et al., Phys. Rev. **D15**, 574 (1977). A.A. Kartamyshev et al., Pis'ma v Zh. Eksp. Teor. Fiz. **25**, 68 (1977). A.A. Bel'kov et al., Pis'ma v Zh. Eksp. Teor. Fiz. **29**, 652 (1979).

[17] A.B. Wicklund et al., Phys. Rev. Lett. **45**, 1469 (1980). D. Cohen et al., Phys. Rev. **D22**, 2595 (1980). A. Etkin et al., Phys. Rev. **D25**, 1786 (1982).

[18] Yu.S. Surovtsev, D. Krupa, M.Nagy, *e-Print Archive*: hep-ph/0005090 (2000).

[19] S. Weinberg, Phys. Rev. Lett. **17**, 616 (1966). B.W. Lee, H.T. Nieh, Phys. Rev. **166**, 1507 (1968).

[20] J. Gasser, H. Leutwyler, Ann. Phys. **158**, 142 (1984).

[21] J. Bijnens et al., Phys. Lett. **B374**, 210 (1996).

[22] M.K. Volkov, Physics of Elementary Particles and Atomic Nuclei, v.17, part 3, 433 (1986).

[23] M.K. Volkov, V.L. Yudichev, M. Nagy, Nuovo Cimento **112 A**, 225 (1999).

[24] L.S. Celenza, Huangsheng Wang, C.M. Shakin, *Preprint of Brooklyn College of the City Univ. of New York*, BCCNT: 00/041/289 (2000).

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Крупа Д., Надь М., Суровцев Ю.С. E2-2000-214
Существование σ-мезона ниже 1 ГэВ и киральная симметрия

В модельно-независимом подходе, заключающемся в непосредственном применении к анализу экспериментальных данных таких общих принципов, как аналитичность и унитарность, на основе одновременного описания изоскалярных s-волновых каналов $\pi\pi$-рассеяния (от порога до 1,9 ГэВ) и процесса $\pi\pi \to K\overline{K}$ (от порога до ~1,4 ГэВ, где выполняется 2-канальная унитарность) получены подтверждение σ-мезона при ~ 665 МэВ и указание для глобальной природы состояния f_0 (1500). Впервые дано беспараметрическое описание $\pi\pi$-фона посредством учета левой точки ветвления в надлежащей униформизирующей переменной. Показано, что обычно получаемый в анализах большой $\pi\pi$-фон в действительности объединяет влияние левой точки ветвления и вклад широкого резонанса при ~ 665 МэВ. Получены константы связи наблюденных состояний с $\pi\pi$- и $K\overline{K}$-системами и длины $\pi\pi$- и $K\overline{K}$-рассеяний. Существование состояния f_0 (665) и полученная длина $\pi\pi$-рассеяния ($\alpha_0^0 = 0{,}27 \pm 0{,}06$, $m_{\pi^+}^{-1}$), по-видимому, свидетельствуют о линейной реализации киральной симметрии.

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Krupa D., Nagy M., Surovtsev Yu.S. E2-2000-214
Existence of the σ-Meson Below 1 GeV and Chiral Symmetry

In the model-independent approach consisting in the immediate application to the analysis of experimental data of such general principles as analyticity and unitarity, a confirmation of the σ-meson at ~ 665 MeV and an indication for the glueball nature of the f_0 (1500) state are obtained on the basis of a simultaneous description of the isoscalar s-wave channel of the $\pi\pi$ scattering (from the threshold up to 1.9 GeV) and of the $\pi\pi \to K\overline{K}$ process (from the threshold to ~1.4 GeV where the 2-channel unitarity is valid). A parameterless description of the $\pi\pi$ background is first given by allowance for the left-hand branch-point in the proper uniformizing variable. It is shown that the large $\pi\pi$-background, usually obtained, combines, in reality, the influence of the left-hand branch-point and the contribution of a wide resonance at ~ 665 MeV. The coupling constants of the observed states with the $\pi\pi$ and $K\overline{K}$ scattering are obtained. Existence of the f_0 (665) state and the obtained $\pi\pi$-scattering length ($\alpha_0^0 = 0.27 \pm 0.06$, $m_{\pi^+}^{-1}$) seems to suggest the linear realization of chiral symmetry.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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