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M.Pawlowski*, V.N.Pervushin

«BIG BANG» OF QUANTUM UNIVERSE

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*Soltan Institute for Nuclear Studies, Warsaw, Poland

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1. Introduction

"Big Bang" as the beginning of the Hubble evolution of a universe is described as a pure classical phenomenon on the basic of particular solutions of the Einstein equations in general relativity in the homogeneous approximation. A strange situation consists in that the highest level of the theory, i.e., the Faddeev-Popov-DeWitt generating functional for unitary S-matrix [1, 2], neglects the questions about the evolution of a universe which are in the competence of the simplest classical approximation. There is an opinion that the solution of the "Big Bang" problem in quantum theory goes beyond the scope of the unitary perturbation theory and even of general relativity. To answer these questions, we need a more general theory of the type of superstring [3].

According to another point of view, the reason of theoretical difficulties in understanding the "Big Bang" phenomenon is not the Einstein theory, but the non-invariant method of its quantization. In particular, for the Faddeev-Popov-DeWitt unitary S-matrix the non-invariant coordinate time is considered as the time of evolution, whereas an observer in a universe can observe and measure only invariants of group of diffeomorphisms of the Hamiltonian dynamics, which includes reparametrizations of the coordinate time [4, 5, 6, 7].

In the present paper we try to construct the unitary S-matrix for general relativity in a finite space-time in terms of the reparametrization-invariant evolution parameter, and to answer the questions: What do Quantum Universe and Quantum Gravity mean? What is the status of the "Big Bang" evolution in quantum theory? What does creation of Quantum Universe mean? on the level of perturbation theory, using the scheme of the time-reparametrization-invariant Hamiltonian reduction [5].

In the context of the Dirac generalized Hamiltonian theory for constrained systems [8, 9, 10, 11], this scheme means the explicit resolving of the first class constraints to determine the constraint-shell action directly in terms of invariants. In other words, we use the invariant reduction of the action instead of the generally accepted non-invariant reduction of the phase space by fixing gauges [1, 2, 12].

The example of the application of such an invariant reduction of the action is the Dirac formulation of QED [13] directly in terms of the gauge-invariant (dressed) fields as the proof of the adequateness of the Coulomb gauge to the invariant content of the classical equations. Recall that the invariant reduction of the action is the way to obtain the unconstrained Feynman integral [14] for the foundation of the intuitive Faddeev-Popov functional integral in gauges theories [12] and to reveal collective excitations of the gauge fields in the form of zero-modes of the first class constraints [15].

A constructive idea of the considered invariant Hamiltonian reduction of general relativity is to introduce the dynamic evolution parameter (i.e.,dynamic time) as the zero-mode collective excitation of metric [4, 5, 6, 7, 16, 17, 18, 19]. This dynamic time can be identified with the zero-Fourier harmonic of the space-metric determinant [5, 6] (treated in cosmology as the cosmic scale factor), whereas its conjugate momentum, i.e, the second (external) form, plays the role of the localizable Hamiltonian of evolution.

The separation of this zero-mode evolution parameter on the level of the action allows us to determine also the invariant geometrical time formed by averaging the time-like component of a metric over the space coordinates [5].

In a universe, an observer is always in the comoving frame, and he reveals the evolution of the universe (with "Big Bang") as the dependence of the dynamic time (i.e. cosmic scale factor) on the geometric one (i.e., the world proper time), in contrast with an observer of a relativistic particle in special relativity where the dynamic and geometrical times belong to different frames of reference: the rest and comoving ones, respectively.

The consistent description of the evolution of a quantum universe in terms of the proper time is based on the canonical transformations [20, 21, 22] to a new set of variables for which the total energy constraint becomes a new momentum, and its conjugate variable (i.e., a new dynamic time) coincides with the proper time.

The content of the paper is the following. In Section 2, we define the invariant Hamiltonian reduction using a relativistic particle as example. Section 3 is devoted to the reparametrization-invariant Hamiltonian reduction of general relativity to construct the generating functional for the unitary perturbation theory which includes "Big Bang" and Hubble evolution. In Section 4, "Big Bang" and Hubble evolution are reproduced in lowest order of perturbation theory. In Section 5, we research the conditions of validity of the conventional quantum field theory in the infinite space-time limit. Section 6 is devoted to the conformal generalization of general relativity.

2. Reparametrization-invariant Hamiltonian reduction

2.1. Special relativity: statement of problem

To answer the question: Why is the reparametrization-invariant Hamiltonian reduction needed?, let us consider relativistic mechanics [5] in the Hamiltonian form

$$W = \int_{\tau_1}^{\tau_2} d\tau[-P_\mu \dot{X}^\mu - \frac{N}{2m}(-P_\mu^2 + m^2)] \ . \tag{1}$$

This action is invariant with respect to reparametrizations of the coordinate time

$$\tau \to \tau' = \tau'(\tau), \qquad N'd\tau' = Nd\tau \tag{2}$$

given in the one-dimensional space with the invariant interval

$$dT := Nd\tau, \qquad T = \int_0^\tau d\bar{\tau} N(\bar{\tau}) \tag{3}$$

In special relativity, a similar invariant interval (we call this interval the geometric time) is identified with the proper time T measured by the watch of an observer in the comoving frame [5]; whereas the dynamic variable X_0 (with a negative contribution in the constraint) is dynamic time measured by the watch of an observer in the rest frame. The reparametrization- noninvariant coordinate time (τ) and the lapse-function $N(\tau)$ are not observable. In the relativistic mechanics, two invariant times (the geometric and dynamic ones) do not coincide, and they are measured by two different observers in two different frames: the comoving and rest ones, respectively.

The problem is to obtain the equivalent unconstrained theories directly in terms of the measurable times X_0 or T with the measurable Hamiltonians of evolution with respect to these times. The solution of this problem is called the dynamic (for X_0), or geometric (for T) reparametrization-invariant Hamiltonian reductions.

2.2. Dynamic reduction of the action

The dynamic reduction of the extended system (1) is the substitution, into it, of the explicit resolving of the energy constraint $(-P_\mu^2 + m^2) = 0$ with respect to the momentum P_0 with a negative contribution

$$\frac{\delta W}{\delta N} = 0 \quad \Rightarrow \quad P_0 = \pm\sqrt{m^2 + P_i^2}. \tag{4}$$

In accordance with the two signs of the solution (4), after the substitution of (4) into (1), we have two branches of the dynamic unconstrained system

$$W(\text{constraint}) = W_\pm^D = \int\limits_{X_0(\tau_1)=X_0(1)}^{X_0(\tau_2)=X_0(2)} dX_0 \left[P_i \frac{dX_i}{dX_0} \mp \sqrt{m^2 + P_i^2}\right] . \tag{5}$$

The role of the time of evolution, in this action, is played by the variable X_0 which abandons the Dirac sector of "observables" P_i, X_i, but not the sector of "measurable" quantities, as X_0 is measured by the watch of an observer in the rest frame. At the same time, its conjugate momentum P_0 converts into the corresponding Hamiltonian of evolution, values of which are the energy of a particle measured by an observer in the rest frame.

This invariant reduction of the action gives the "equivalent" unconstrained system together with definition of the invariant evolution parameter (i.e., dynamic time) measured by the watch of an observer and corresponding to a non-zero Hamiltonian. We call all invariant dynamic variables $(P_i, X_i|P_0, X_0)$ by the sector of measurable quantities including the measurable time (as one of the dynamic variables) and the measurable Hamiltonian (as the conjugate momentum of this variable). Thus, we need the reparametrization-invariant Hamiltonian reduction to determine the measurable time and its measurable Hamiltonian for reparametrization-invariant systems.

The description of the "Big Bang" in general relativity is based on the assertion that the measurable time in general relativity is one of variables of extended phase space, but not the coordinate time.

In quantum relativistic theory we got two Schrödinger equations

$$i\frac{d}{dX_0}\Psi_{(\pm)}(X|P) = \pm\sqrt{m^2 + P_i^2}\Psi_{(\pm)}(X|P) \ , \tag{6}$$

with positive and negative values of P_0 and normalized wave functions

$$\Psi_\pm(X|P) = \frac{A_P^\mp\theta(\pm P_0)}{(2\pi)^{3/2}\sqrt{2P_0}}\exp(-iP_\mu X^\mu), \qquad \left([A_P^-, A_{P'}^+] = \delta^3(P_i - P_i')\right) \ . \tag{7}$$

The coefficient A_P^+, in the secondary quantization, is treated as the operator of creation of a particle with positive energy; and the coefficient A_P^-, as the operator of annihilation of a particle also with positive energy. The physical states are formed by action of these operators on the vacuum $<0|, |0>$ in the form of out-state ($|P> = A_P^+|0>$) with positive frequencies and in-state ($<P| = <0|A_P^-$) with negative frequencies. This treatment means that positive frequencies propagate forward; and negative frequencies, rearwards, so that the negative values of energy are excluded from the spectrum, to provide the stability of the quantum system in QFT [23]. In other words, instead of changing the sign of energy, we change that of time, which leads to the causal Green function with the arrow of time

$$G^c(X) = G_+(X)\theta(X_0) + G_-(X)\theta(-X_0) = \int \frac{d^4P}{(2\pi)^4}\exp(-iPX)\frac{i}{P^2 - m^2 - i\epsilon}, \tag{8}$$

where $G_+(X) = G_-(-X)$ is the "commutative" Green function [23]

$$G_+(X) = \int \frac{d^4P}{(2\pi)^3}\exp(-iPX)\delta(P^2 - m^2)\theta(P_0) = \tag{9}$$

$$\frac{1}{2\pi}\int d^3Pd^3P' < 0|\Psi_-(X|P)\Psi_+(0|P')|0> \ .$$

These Green functions evidence the peculiarity of reparametrization-invariant theories; this peculiarity highly distinguishes them from the conventional gauge theories: in the reparametrization-invariant theories, there is a superfluous variable that is excluded by a constraint and abandons the Dirac sector of "observables", but not the sector of measurable quantities [4, 5, 7], as the dynamic time is measured by the watch of an observer in the rest frame.

2.3. The reparametrization-invariant functional integral

To obtain the reparametrization-invariant form of the functional integral adequate to the considered gauge-less reduction (5) and the causal Green function (8), we use the version of composition law for the commutative Green function with the integration over the whole measurable sector $X_{1\mu}$

$$G_+(X) = \int G(X-1)\left[d1\bar{G}_+(1)\right] \quad (1 = X_1, \ \ d1 = d^4X_1, \ \ \bar{G} = \frac{G}{2\pi\delta(0)}) \ , \qquad (10)$$

where $\delta(0) = \int dN$ is the infinite volume of the group of reparametrizations of the coordinate time. The continual limit of the multiple integral

$$G_+(X) = \int G_+(X-1)\left[\prod_{k=1}^{n} dk\bar{G}_+(k-(k+1))\right] \ , \quad (k = X_k, \quad X_{n+1} = 0) \qquad (11)$$

can be defined as the functional integral

$$G_+(X) = \int\limits_{X(\tau_1)=0}^{X(\tau_2)=X} \frac{dN(\tau_2)d^4P(\tau_2)}{(2\pi)^3} \prod_{\tau_1\leq\tau<\tau_2}\left\{d\bar{N}(\tau)\prod_{\mu}\left(\frac{dP_\mu(\tau)dX_\mu(\tau)}{2\pi}\right)\right\}\exp(iW) \qquad (12)$$

where $\bar{N} = N/2\pi\delta(0)$, and W is the initial extended action (1).

This functional integral has the form of the average over the group of reparametrization of the integral over the sector of "measurable" variables P_μ, X_μ.

2.4. Geometric reduction

The third point (besides of the introduction of the dynamic time and the reparametrization-invariant form of the functional integral) on the way of the explanation of "Big Bang" as the pure relativistic and quantum phenomenon is the fact that an observer in a universe reveals its evolution in terms of the geometric time measured in the comoving frame (an observer in a universe is always in the comoving frame). The change of a frame can be described in the form of the change of variables, in particular, there are the variables for which the dynamic time coincides with the geometric one.

A correct description of the initial conditions in the comoving frame is based on the canonical Levi-Civita - type transformation [20, 21, 22]

$$(P_\mu, X_\mu) \Rightarrow (\Pi_\mu, Q_\mu) \tag{13}$$

to the variables (Π_μ, Q_μ) for which one of equations identifies Q_0 with the proper time T. This transformation [20] converts the constraint into a new momentum

$$\Pi_0 = \frac{1}{2m}[P_0^2 - P_i^2], \quad \Pi_i = P_i, \quad Q_0 = X_0\frac{m}{P_0}, \quad Q_i = X_i - X_0\frac{P_i}{P_0} \tag{14}$$

and has the inverted form

$$P_0 = \pm\sqrt{2m\Pi_0 + \Pi_i^2}, \; P_i = \Pi_i, \quad X_0 = \pm Q_0\frac{\sqrt{2m\Pi_0 + \Pi_i^2}}{m}, \quad X_i = Q_i + Q_0\frac{\Pi_i}{m}. \tag{15}$$

After transformation (14) the action (1) takes the form

$$W = \int\limits_{\tau_1}^{\tau_2} d\tau \left[-\Pi_\mu \dot{Q}^\mu - N(-\Pi_0 + \frac{m}{2}) - \frac{d}{d\tau}S^{lc} \right], \qquad S^{lc} = (Q_0\Pi_0) \tag{16}$$

The invariant reduction is the resolving of the constraint $\Pi_0 = m/2$ which determines a new Hamiltonian of evolution with respect to the new dynamic time Q_0, whereas the equation of motion for this momentum Π_0 identifies the dynamic time Q_0 with the geometric one T $(Q_0 = T)$. The substitution of these solutions into the action (16) leads to the reduced action of a geometric unconstrained system

$$W(\text{constraint}) = W^G = \int\limits_{T_1}^{T_2} dT \left(\Pi_i \frac{dQ_i}{dT} - \frac{m}{2} - \frac{d}{dT}(S^{lc}) \right) \qquad (S^{lc} = Q_0\frac{m}{2}) \tag{17}$$

where variables Π_i, Q_i are cyclic ones and have the meaning of initial conditions in the comoving frame

$$\frac{\delta W}{\delta \Pi_i} = \frac{dQ_i}{d\tau} = 0 \Rightarrow Q_i = Q_i^{(0)}, \qquad \frac{\delta W}{\delta Q_i} = \frac{d\Pi_i}{d\tau} = 0 \Rightarrow \Pi_i = \Pi_i^{(0)} \tag{18}$$

The substitution of all geometric solutions

$$Q_0 = T, \;\; \Pi_0 = \frac{m}{2}, \;\; \Pi_i = \Pi_i^{(0)} = P_i, \;\; Q_i = Q_i^{(0)} \tag{19}$$

into the inverted Levi-Civita transformation (15) leads to the conventional relativistic solution for the rest frame

$$P_0 = \pm\sqrt{m^2 + P_i^2}, \quad P_i = \Pi_i^{(0)}, \quad X_0(T) = T\frac{P_0}{m}, \quad X_i(T) = X_i^{(0)} + T\frac{P_i}{m}. \tag{20}$$

The Schrödinger equation for the wave function

$$\frac{d}{idT}\Psi^{lc}(T,Q_i|\Pi_i) = \frac{m}{2}\Psi^{lc}(T,Q_i|\Pi_i), \tag{21}$$

$$\Psi^{lc}(T,Q_i|\Pi_i) = \exp(-iT\frac{m}{2})\exp(i\Pi_i^{(0)}Q_i)$$

contains only one eigenvalue $m/2$ degenerated with respect to the cyclic momentum Π_i. We see that there are differences between the dynamic and geometric descriptions. The dynamic time is given in the whole region $-\infty < X_0 < +\infty$, whereas the geometric one is only positive $0 < T < +\infty$, as it follows from the properties of the causal Green function (8) after the Levi-Civita transformation (14)

$$G^c(Q_\mu) = \int\limits_{-\infty}^{+\infty} d^4\Pi_\mu \frac{\exp(iQ^\mu\Pi_\mu)}{2m(\Pi_0 - m/2 - i\epsilon/2m)} = \frac{\delta^3(Q)}{2m}\theta(T), \qquad T = Q_0 \ .$$

For an observer in the comoving frame two branches of solution of the constraint (a particle and antiparticle) coincide.

3. Reparametrization-invariant reduction of GR

3.1. Action and variables

General relativity (GR) is given by the Einstein-Hilbert action with the matter fields

$$W^E(g|\mu) = \int d^4x\sqrt{-g}[-\frac{\mu^2}{6}R(g) + \mathcal{L}_f] \qquad \left(\mu^2 = M^2_{Planck}\frac{3}{8\pi}\right) \tag{22}$$

and by a measurable interval

$$(ds)^2_e = g_{\alpha\beta}dx^\alpha dx^\beta \ . \tag{23}$$

They are invariant with respect to general coordinate transformations

$$x_\mu \to x'_\mu = x'_\mu(x_0, x_1, x_2, x_3). \tag{24}$$

The Hamiltonian description of GR means the choice of the Dirac-ADM $3+1$ parametrization of metric components [24]

$$(ds)^2_e = g_{\mu\nu}dx^\mu dx^\nu = N^2dt^2 - {}^{(3)}g_{ij}\breve{dx}^i\breve{dx}^j \qquad (\breve{dx}^i = dx^i + N^i dt) \tag{25}$$

with the lapse function N, three shift vectors N^i, and six space components ${}^{(3)}g_{ij}$. This parametrization describes a set of the three-dimensional space-like hyperspaces (25) enumerated by the time-like coordinate t in the four-dimensional manifold of the world events. The Hamiltonian dynamics is defined within transformations of a kinemetric subgroup of the group of general coordinate transformations (24) [25, 5, 6, 7]

$$t \to t' = t'(t); \qquad x_i \to x_i' = x_i'(t, x_1, x_2, x_3), \tag{26}$$

which includes one global function (the time reparametrizations $t'(t)$) and three local ones ($x_i'(t,x)$). This is the group of diffeomorphisms of a set of Einstein's observers with the equivalent Hamiltonian dynamics. This continuum of "observers" with the diffeomorphism group (26) is called the kinemetric frame of reference [25].

The reparametrization invariance (26) (as we have seen in the previous Section) means that the coordinate time (t) is not measurable. To apply the reparametrization-invariant reduction discussed before, we should also answer the question : What are measurable dynamic time and the measurable Hamiltonian of evolution with respect to this time? The dynamic time is one of dynamic components of the space metric ${}^{(3)}g_{ij}$ with a negative contribution to the energy constraint. We choose this dynamic time as the zero Fourier harmonic $\varphi_0(t)$ of the space metric determinant logarithm [4, 5]. This evolution parameter can be separated by the conformal-type transformation of the metric

$$g_{\alpha\beta}(t,x) = (\frac{\varphi_0(t)}{\mu})^2 \bar{g}_{\alpha\beta}(t,x). \tag{27}$$

The transformational properties of the curvature $R(g)$ with respect to the transformations (27) lead to the action (22) in the form [5]

$$W^E(g|\mu) = W^E(\bar{g}|\varphi_0) - \int\limits_{t_1}^{t_2} dt \varphi_0 \frac{d}{dt}(\frac{\dot{\varphi}_0 V_0}{N_0}), \tag{28}$$

where N_0 is the average of the lapse function $\bar{N}$ in the metric $\bar{g}$ over the kinemetric invariant space volume

$$\frac{V_0}{N_0} = \int d^3x \frac{\sqrt{\bar{g}}}{\bar{N}}, \qquad \bar{g} = det({}^{(3)}\bar{g}) \ , \qquad V_0 = \int\limits_{V_0} d^3x \tag{29}$$

and V_0 is a free parameter which in the perturbation theory has the meaning of a finite volume of the free coordinate space. The geometric foundation of the introduction of this global variable in GR was given in [19] in the form of the theorems about the nonzero-value of the second form in the whole space. In the opposite case of the zero-value of the second form in the whole space, the positive contributions of the particle-like excitations in the energy constraint can be compensated by only by the Newton interaction term given in the class of functions with the nonzero Fourier harmonics. Therefore, the zero Fourier harmonic of the energy constraint looses all negative contributions and becomes contradictory (if we, following Dirac [9], remove the contribution of the second form imposing it to be equal to zero). Thus, the Einstein theory needs the zero-mode (27) to escape this contradiction. If this zero harmonic $\varphi_0(t)$ with its conjugate momentum P_0 are taken into account, then the action (28) takes the Dirac-Bergmann Hamiltonian form [5, 6]

$$W^E = \int\limits_{t_1}^{t_2} dt \left(\int d^3x [\sum_F P_F \dot{F} - N_q \mathcal{H} - N^i \mathcal{P}_i] - \dot{\varphi}_0 P_0 + \frac{N_0 P_0^2}{4V_0} + \frac{1}{2}\partial_t (P_0 \phi_0) \right), \tag{30}$$

where

$$\sum_F P_F \dot{F} = \sum_f p_f \dot{f} - \pi_{ij} \dot{q}^{ij} \tag{31}$$

$$\mathcal{H}(\varphi_0) = \frac{6}{\varphi_0^2} q^{ij} q^{kl} [\pi_{ik}\pi_{jl} - \pi_{ij}\pi_{kl}] + \frac{\varphi_0^2 q^{1/2}}{6} {}^{(3)}R + \mathcal{H}_f \ , \tag{32}$$

$$\mathcal{P}_i = 2[\nabla_k (q^{kl}\pi_{il}) - \nabla_i (q^{kl}\pi_{kl})] + \mathcal{P}_{if} \ , \tag{33}$$

the densities of the local excitations and $\mathcal{H}_f, \mathcal{P}_f$ are contributions of the matter fields, and we choose here the Dirac harmonical variables [8]

$$q^{ik} = \bar{g}\bar{g}^{ik}, \quad N_q = \bar{N}\bar{g}^{-1/2} \qquad \left(\frac{1}{V_0} \int\limits_{V_0} \frac{d^3x}{N_q} = \frac{1}{N_0} \right), \tag{34}$$

and the Lichnerowich conformal variables [26] for matter fields f with the conformal weight (n) fixed by the definition of the tensor field (27)

$$^{(n)}f_c = {}^{(n)}f \left(\frac{\varphi_0(t)}{\mu} \right)^n \ , \tag{35}$$

where $\bar{g} = g_c$, and $n = 2, 0, -3/2, -1$ for the tensor, vector, spinor, and scalar fields, respectively. In this case, the Planck constant μ in the Einstein equations is replaced by the dynamic time φ_0. The metric (25) takes the form

$$(ds)_e^2 = \frac{\varphi_0(t)^2}{\mu^2} q^{1/2} \left(N_q^2 dt^2 - q_{ij} \breve{dx}^i \breve{dx}^j \right), \qquad (q = det(q^{ij})). \tag{36}$$

The local part of the momentum of the space metric determinant

$$\pi(t, x) := q^{ij}\pi_{ij} = \frac{q_{ij} D_t q^{ij}}{N_q} = \frac{D_t \log q}{N_q} \tag{37}$$

is given in the class of functions with the non-zero Fourier harmonics, so that

$$\int d^3x \pi(t, x) = \int d^3x \frac{D_t \log q}{N_q} = 0 \ , \tag{38}$$

and we have the same number of variables.

There is a direct correspondence between the Einstein theory (30) with the separated zero-mode of the space metric φ_0 and a particle in special relativity (1): the unobservable coordinate times $(\tau \to t)$; the dynamic time $(X_0 \to \varphi_0)$; the Dirac reduced phase space variables

$$P_i, \;\; X_i \quad \to \quad (\; P_f, \pi_{ij}, \;\; f, q^{ij}\;) := P_F, \;\; F \tag{39}$$

and the geometric time $(dT = N d\tau \to dT = N_0 dt)$ determined by the global lapse function (29)

$$dT = N_0 dt \; . \tag{40}$$

We call the set of dynamic variables $(P_F, F|P_0, \varphi_0)$ (where P_F, F are the local excitations) the "relativistic universe", like the set of variables $(P_i, X_i|P_0, X_0)$ in SR is called a "relativistic particle".

3.2. Reparametrization-invariant reduction

The physical meaning of the dynamic time and its momentum is given the explicit resolving of the zero-Fourier harmonic of the energy constraint

$$\frac{\delta W^E}{\delta N_0(t)} = \int d^3x \mathcal{N} \frac{\delta W^E}{\delta N_q} = -\frac{P_0^2}{4V_0} + H = 0 \; , \tag{41}$$

where

$$H = \int d^3x \mathcal{N}\bar{\mathcal{H}}, \qquad \mathcal{N}(t,x) = \frac{N_q(t,x)}{N_0(t)}, \qquad \left(\frac{1}{V_0}\int\limits_{V_0} \frac{d^3x}{\mathcal{N}} = 1\right) \tag{42}$$

is the total Hamiltonian of the local excitations P_F, F (39), and the local component of the lapse function (34), respectively.

This constraint has two solutions for the global momentum P_0

$$(P_0)_\pm = \pm 2\sqrt{V_0 H} \equiv \pm H^R. \tag{43}$$

They are the generators of evolution of the Dirac sector (39) with respect to the dynamic evolution parameter φ_0 for its positive and negative values, in the correspondence with the relativistic causality, which excludes negative energies of a "relativistic universe" in the Dirac sector of "observables" (39).

The equation of motion for this global momentum P_0 takes the form

$$\frac{\delta W^E}{\delta P_0} = 0 \Rightarrow \left(\frac{d\varphi}{dT}\right)_\pm = \frac{(P_0)_\pm}{2V} = \pm\sqrt{\rho(\varphi_0)}; \quad \rho = \frac{\int d^3x \mathcal{H}}{V_0} = \frac{H}{V_0} \; . \tag{44}$$

In the Friedmann-Robertson-Walker cosmology, this type of equations is used in the homogeneous approximation to describe evolution of a universe for an observer in the comoving frame who reveals this evolution as the dependence of the geometric time (in cosmology treated as the conformal time) on the dynamic time (in cosmology treated as the cosmic scale factor φ_0/μ). The integral form of the last equation

$$T(\varphi_0) = \int\limits_0^{\varphi_0} d\varphi \rho^{-1/2}(\varphi). \tag{45}$$

is in cosmology well-known as the Friedmann-Hubble law. This equation gives the relation between the present-day value of the dynamic time $\varphi_0(T_0)$ and cosmological observations, i.e., the density of matter ρ and the Hubble parameter

$$\mathcal{H}_{hub} = \frac{\mu\varphi_0'}{\varphi_0^2} \Rightarrow \varphi_0^2(T_0) = \frac{\mu\sqrt{\rho}}{\mathcal{H}_{hub}} := \mu^2\sqrt{\Omega_0} \quad (\ 0.6 < (\Omega_0^{1/4})_{exp} < 1.2\ ). \tag{46}$$

The Hamiltonian H (42) determines the evolution of the Dirac observables (39) with respect to the geometric time T

$$F' := \frac{\partial F}{\partial T} = \sqrt{\rho}\partial_{\varphi_0} f = \{H, F\}\,. \tag{47}$$

The local part of the energy constraint

$$\frac{\delta W^E}{\delta \mathcal{N}} = 0 \ \Rightarrow \ \mathcal{N}\mathcal{H} - \frac{\rho}{\mathcal{N}} = 0 \tag{48}$$

is the projection of the energy constraint $\delta W^E/\delta N_q = 0$ onto the nonzero Fourier harmonics. In the first order of the Dirac perturbation theory [8]

$$\mathcal{N} = 1, \quad \mathcal{H}(\varphi_0) = \mathcal{H}_{fp} + \frac{\varphi_0^2}{6}\partial_i\partial_j q^{ij}, \tag{49}$$

equation (48) is the Newton law for the present-day value of the dynamic time $\varphi_0 = \mu$ (46)

The equation for shift vectors

$$\frac{\delta W^E}{\delta N^i} = \mathcal{P}_i = 0 \tag{50}$$

are considered as three local constraints.

3.3. Reparametrization-noninvariant functional integral

To give basic definitions, we consider the standard Faddeev-Popov functional integral [1] for the Einstein action given in the infinite coordinate space-time (without the separation of the dynamic time as the zero Fourier harmonic of the space metric)

$$W = \int\limits_{-\infty}^{+\infty} dt \int d^3x \left(\sum_F P_F \dot{F} - N_q \mathcal{H}(\mu) - N_i \mathcal{P}^i \right), \tag{51}$$

$$\sum_F P_F\dot{F} := \sum_f p_f\dot{f} - \pi_{ij}\dot{q}^{ij} .$$

where $\mathcal{H}(\mu)$ is defined by (32) with $\varphi_0 = \mu$.

According to the Dirac-Bergmann generalized Hamiltonian theory [8, 9, 11], the set of constraints begins with the first class primary constraints

$$P_{N_q} = 0, \qquad P_{N_i} = 0 \tag{52}$$

for the conjugate momenta P_{N_q}, P_{N_i} of time components of the metric N_q, N^i. The constraints (52) are accompanied by the gauges

$$N_q - 1 = 0, \qquad N_i = 0; \tag{53}$$

whereas the equations for the lapse function $\mathcal{H}(\mu) = 0$ and the shift vector $\mathcal{P}^i = 0$ (50) are considered as the first class secondary constraints accompanied by the Dirac gauges [8]

$$\chi^0 := \pi = 0, \qquad \chi^i := \partial_j(q^{-1/3}q^{ji}) = 0 , \tag{54}$$

respectively.

The FP-functional integral for the Dirac gauges was given by V.N.Popov in the book [10] in the form

$$G(F_1, F_2|\varphi_0 = \mu) = \int_{F_1}^{F_2} D(F, P_f)\Delta_s\Delta_t \exp\{iW_{fp}\} , \tag{55}$$

where

$$W_{fp} = \int_{-\infty}^{+\infty} dt \int d^3x \left(\sum_F P_F\dot{F} - \mathcal{H}_{fp}(\mu)\right), \qquad \mathcal{H}_{fp}(\mu) = \mathcal{H}(\mu) - \frac{\mu^2}{6}\partial_i\partial_j q^{ij} , \tag{56}$$

$$D(F, P_f) = \prod_{t,x}\left(\prod_{i<k}\frac{dq^{ik}d\pi_{ik}}{2\pi}\prod_f\frac{df\,dp_f}{2\pi}\right) \tag{57}$$

and

$$\Delta_s = \prod_{t,x,i}\delta(\mathcal{P}_i))\delta(\chi^j)detB, \qquad detB = det\{\mathcal{P}_i, \chi^j\} \tag{58}$$

$$\Delta_t = \prod_{t,x}\delta(\mathcal{H}(\mu))\delta(\pi)detA, \qquad detA = det\{\mathcal{H}, \pi\} \tag{59}$$

are the space and time parts of the FP determinant, and A and B^i_k are operators acting by the rules

$$Af = q^{ij}\nabla_i\nabla_j f + q^{1/2(3)}Rf; \tag{60}$$

$$B^i_k\eta_i = q^{-1/3}q^{lj}[\delta^i_k\partial_l\partial_j + \frac{1}{3}\delta^i_l\partial_j\partial_k]\eta_i. \tag{61}$$

The F-P integral (55) is considered as the generating functional for unitary perturbation theory in terms of S-matrix elements

$$S[-\infty|+\infty] =< out|T\exp\left\{-i\int\limits_{-\infty}^{+\infty} dtH_I\right\}|in>, \tag{62}$$

where $H_I = H - H_0$ is the Hamiltonian of interaction, and H_0 describes in- and out- states of free fields, in particular, for free gravitons H_0 takes form

$$H_0(\mu) = \int d^3x \left(\frac{6(\pi^T_{(h)})^2}{\mu^2} + \frac{\mu^2}{24}(\partial_i h^T)^2\right); \quad (h^T_{ii} = 0; \ \ \partial_j h^T_{ji} = 0), \tag{63}$$

which corresponds to the metric

$$(ds)^2 = dt^2 - (\delta_{ij} + h^T_{ij})dx^i dx^j. \tag{64}$$

We see that the reparametrization invariance of the initial Einstein theory is broken.

3.4. Reparametrization-invariant functional integral

We have seen above that the reparametrization invariance means that the coordinate time could not be the measurable time of the invariant evolution. The measurable time coincides with the global dynamic variable with a negative contribution to the energy constraint φ_0 which we call the dynamic time. The dynamic time, together with its conjugate momentum P_0 and the global component of the lapse function N_0, give the additional functional integrals for the "commutative" Green function $G_+(F_1, F_2|\varphi_1, \varphi_2)$, just as the geometric sector (P_0, X_0, N_0) in SR forms the "commutative" Green function $G_+(X_1 - X_2)$ for a relativistic quantum particle (12).

To compare the standard result (55) - (61) with the reparametrization-invariant version of the functional integral for the action (30) with the separated dynamic time φ_0, we consider this system in the class of solutions with the lapse function $\mathcal{N} = 1$ (see its normalization (42)). Then, we have the set of local constraints

$$\mathcal{H}(\varphi_0) - \rho = 0, \quad \pi = 0; \qquad\qquad \mathcal{P}_i = 0, \quad \chi^j = 0 \tag{65}$$

with their determinants

$$det\bar{A} = det\{\mathcal{H}(\varphi_0) - \rho, \pi\}; \qquad\qquad detB = det\{\mathcal{P}_i, \chi^j\}. \tag{66}$$

The reparametrization-invariant version of the functional integral (55) can be given so as to satisfy the relativistic causality in the form of the "commutative"

Green function for a relativistic quantum particle (12). Repeating the functional integral in the geometric sector (P_0, X_0, N_0) for the similar sector (P_0, φ_0, N_0) of a "relativistic quantum universe" we obtain the "commutative" Green function for a "relativistic quantum universe" in the notation of the FP-integral (55) and (66):

$$G_+(F_1, F_2|\varphi_1, \varphi_2) = \int\limits_{F_1,\varphi_1}^{F_2,\varphi_2} D(F, P_f)\Delta_s\bar{\Delta}_t \prod_t \left(\frac{d\varphi_0 dP_0 d\bar{N}_0}{2\pi}\right) \exp\left\{i\bar{W}^E\right\}, \quad (67)$$

where

$$\bar{W}^E = \int\limits_{t_1}^{t_2} dt \left\{ \int\limits_{V_0} d^3x \left(\sum_F P_F \dot{F}\right) - P_0\dot{\varphi}_0 + N_0 \left[\frac{P_0^2}{4V_0} - H(\varphi_0)\right] + \frac{1}{2}\partial_t(P_0\varphi_0)\right\}, \quad (68)$$

and

$$\bar{\Delta}_t = \prod_{t,x} \delta(\mathcal{H}(\varphi_0) - \rho)\delta(\pi) det\bar{A}\ , \qquad H(\varphi_0) = \int d^3x \mathcal{H}(\varphi_0), \quad \rho = \frac{H}{V_0}. \quad (69)$$

Comparing (55) and (67), one can see that, to get the ordinary FP-integral (55), one should to fix the dynamic time at its present-day value $\varphi_0 = \mu$ (46), remove all the zero-mode dynamics $P_0 = \dot{\varphi}_0 = 0, N_0 = 1$, and neglect the surface Newton term in the Hamiltonian. Strictly speaking, it is not a correct procedure, as it breaks the reparametrization-invariance.

4. "Big Bang" of a "free" quantum universe

Possible states of a free quantum universe are determined by the lowest order of the Dirac perturbation theory given by the well-known system of "free" conformal fields (35), (63) in a finite space-time volume [27, 7]

$$W_0^E = \int\limits_{t_1}^{t_2} dt \left(\left[\int d^3x \sum_F P_F \dot{F} \right] - P_0\dot{\varphi}_0 + N_0[\frac{P_0^2}{4V} - H_0(\varphi_0)] + \frac{1}{2}\partial_0(P_0\varphi_0) \right), \quad (70)$$

where H_0 is a sum of the Hamiltonians of "free" fields (gravitons (63), photons, massive vectors, and spinors) where all masses (including the Planck mass) are replaced by the dynamic time φ_0 [7]. The classical equations for the action (70)

$$\partial_T F = \frac{\partial H_0(\varphi_0^{\pm})}{\partial P_F}, \qquad -\partial_T P_F = \frac{\partial H_0(\varphi_0^{\pm})}{\partial F}, \qquad \frac{d\varphi_0^{\pm}}{dT} = \pm\sqrt{\rho_0(\varphi_0^{\pm})}, \qquad \left(\rho_0 = \frac{H_0}{V_0}\right) \quad (71)$$

contain two invariant times: the geometric T (measured by the watch of an observer in the universe) and the dynamic $\varphi_0^{\pm}$ connected by the geometro-dynamic (back-reaction) equation (44)

Recall that, in the reparametrization-invariant classical mechanics, the geometric and dynamic times coincide; in special relativity, the geometric time differs from the dynamic one, but both these times belong to different observers in different frames (the rest and comoving ones); in general relativity, one and the same observer detects and measures simultaneously both these times (φ_0 and T) and reveals the dependence of the geometric time T on the dynamic time φ_0 as evolution of a universe in the form of the Hubble law (46). Therefore, the reparametrization-invariant content of general relativity can be covered simultaneously by two classical unconstrained systems with the dynamic time φ_0 and the geometric one T. The dynamic system

$$W_0^E(constraint) = W_0^D = \int\limits_{\varphi(t_1)}^{\varphi(t_2)} d\varphi \left(\left[\int d^3x \sum_F P_F \partial_\varphi F \right] \mp H_0^R \pm \frac{1}{2}\partial_\varphi(\varphi H_0^R) \right), \tag{72}$$

like the rest frame in SR, has two branches for a universe with a positive energy ($P_0 > 0$), and a universe with a negative energy ($P_0 < 0$). The latter should be treated as an "antiuniverse" which propagates rearwords ($\varphi < 0$) with positive energy to provide the stability of a quantum system.

The geometric system is constructed by the Levi-Civita type transformation to the action-angle variables, so that one of them coincides with the geometric time T measured by the watch of an observer.

In the dynamic system, the content of matter in the universe, is described by the number of particles $N_{F,k}$ and their energy $\omega_F(\varphi_0, k)$ (which depends on the dynamic time φ_0 and quantum numbers k, momenta, spins, etc.). Detected particles are defined as the field variables $F = f$

$$f(x) = \sum_k \frac{C_f(\varphi_0)\exp(ik_i x_i)}{V_0^{3/2}\sqrt{2\omega_f(\varphi_0, k)}} \left(a_f^+(-k) + a_f^-(k)\right) \tag{73}$$

which diagonalize the operator of the density of matter

$$\rho_0 = \sum_{f,k} \frac{\omega_f(\varphi_0, k)}{V_0} \hat{N}_{f,k}, \qquad \hat{N}_f = \frac{1}{2}(a_f^+ a_f^- + a_f^- a_f^+) \ . \tag{74}$$

We restrict ourselves to gravitons (f=h) $C_h(\varphi_0) = \varphi_0\sqrt{12}$, $\omega_h(\varphi_0, k) = \sqrt{k^2}$ and massive vector particles (f=v) $C_v(\varphi_0) = 1$, $\omega_v(\varphi_0, k) = \sqrt{k^2 + y^2\varphi_0^2}$, where y is the mass in terms of the Planck constant.

The equations of motion (71) in terms of a^+, a^- [7] are not diagonal

$$i\chi'_{a_f} = -\hat{H}_{a_f}\chi_{a_f}, \ \chi_{a_f} = \begin{pmatrix} a_f^+ \\ a_f \end{pmatrix}; \ \hat{H}_{a_f} = \begin{vmatrix} \omega_{a_f} & , & -i\Delta_f \\ -i\Delta_f & , & -\omega_{a_f} \end{vmatrix}, \tag{75}$$

where nondiagonal terms $\Delta_{f=h,v}$ are proportional to the Hubble parameter (46)

$$\Delta_{f=h} = \frac{\varphi_0'}{\varphi_0}, \qquad \Delta_{f=v} = -\frac{\omega_v'}{2\omega_v}, \qquad \varphi_0' = \sqrt{\rho_0}. \tag{76}$$

The "geometric system" (b^+, b) is determined by the transformation to the set of variables which diagonalize equations of motion (75) and determine a set of integrals of motion of equations (75) (as conserved numbers $\{Q\}$).

To obtain integrals of motion and to choose initial conditions for the "Big Bang", we use the Bogoliubov transformations [28] of "particle" variables

$$b^+ = \cosh(r)e^{-i\theta}a^+ - i\sinh(r)e^{i\theta}a, \qquad b = \cosh(r)e^{i\theta}a + i\sinh(r)e^{-i\theta}a^+, \tag{77}$$

or

$$\chi_b = \begin{pmatrix} b^+ \\ b \end{pmatrix} = \hat{O}\chi_a,$$

which diagonalize the equations expressed in terms of "particles" (a^+, a), so that the "number of quasiparticles" is conserved

$$\frac{d(b^+b)}{dt} = 0, \qquad b = \exp(-i\int\limits_0^T d\bar{T}\bar{\omega}_b(\bar{T}))b_0, \tag{78}$$

and functions r, θ in (77) and the quasiparticle energy $\bar{\omega}_b$ in (78) are determined by the equation of diagonalization

$$i\frac{d}{dT}\chi_b = [-i\hat{O}^{-1}\frac{d}{dT}\hat{O} - \hat{O}^{-1}\hat{H}_a\hat{O}]\chi_b \equiv -\begin{pmatrix} \bar{\omega}_b, & 0 \\ 0, & -\bar{\omega}_b \end{pmatrix}\chi_b\,. \tag{79}$$

in the form obtained in [7]

$$\bar{\omega}_{fb} = (\omega_f - \theta_f')\cosh(2r_f) - (\Delta_f \cos 2\theta_f)\sinh(2r_f) \tag{80}$$

$$0 = (\omega_f - \theta_f')\sinh(2r_f) - (\Delta_f \cos 2\theta_f)\cosh(2r_f), \qquad r_f' = -\Delta_f \sin 2\theta_f.$$

Equations (76)- (80) are closed by the density of "observable particles" in terms of quasiparticles

$$\rho(\varphi) = \frac{H_0}{V} = \frac{\sum\limits_f \omega_f(\varphi)\{a_f^+ a_f\}}{V_0}; \qquad \{a^+a\} = \{b_0^+b_0\}\cosh 2r - \frac{i}{2}(b^{+2} - b^2)\sinh 2r \tag{81}$$

with

$$\bar{\omega}_{fb} = \sqrt{(\omega_f - \theta_f')^2 + (r_f')^2 - \Delta_f^2}, \;\; \cosh(2r_f) = \frac{\omega_f - \theta_f'}{\bar{\omega}_{fb}}. \tag{82}$$

The conserved numbers b_0, b_0^+ of classical theory become the set of quantum numbers in quantum theory. In other words, in the geometric frame one can quantize the initial data, like for a relativistic particle in the comoving frame considered in Subsection 2.5. This is the principle of correspondence with classical theory in the strong version.

Let us suppose that we manage to solve equations (78)– (82) with respect to the geometric time T in terms of conserved numbers b_0^+, b_0. This means that the wave function of a quantum universe can be represented in the form of a series over the conserved quantum numbers $Q = n_{f,k} =< Q|b_f^+ b_f|Q >$ of the Bogoliubov states

$$\Psi_Q(T) = \prod_{f,n_f} \exp\left\{-i\int\limits_0^T dT n_f \bar{\omega}_b(T)\right\} \frac{(b_{fo}^+)^{n_f}}{\sqrt{n_f!}}|0>_b \quad . \tag{83}$$

In this geometric system, an observer (as "geometrician" who measures the geometric time T) could not differ a universe

$$T_+(\varphi_2, \varphi_1) = \int\limits_{\varphi_1}^{\varphi_2} d\varphi \rho(\varphi)^{-1/2} > 0, \qquad \varphi_2 > \varphi_1 \tag{84}$$

and an antiuniverse

$$T_-(\varphi_2, \varphi_1) = -\int\limits_{\varphi_1}^{\varphi_2} d\varphi \rho(\varphi)^{-1/2} = \int\limits_{\varphi_2}^{\varphi_1} d\varphi \rho(\varphi)^{-1/2} > 0, \qquad \varphi_1 > \varphi_2, \tag{85}$$

like an observer of a relativistic particle in the comoving frame. However, an observer (as "physicist" who detects dynamic particles a^+, a) is simulteneously in the dynamic system (72) connected with the geometric one by the Bogoliubov transformations. These transformations restore wave functions of a universe $\varphi_2 > \varphi_1$ and a untiuniverse $\varphi_1 > \varphi_2$

$$\Psi_Q(T) = A_Q^+ \Psi_Q(\varphi_2, \varphi_1)\theta(\varphi_2 - \varphi_1) + A_Q^- \Psi_Q(\varphi_2, \varphi_1)\theta(\varphi_1 - \varphi_2) \tag{86}$$

where the first term and the second one are positive ($P_0 > 0$) and negative ($P_0 < 0$) frequency parts of the solutions with the spectrum of quasiparticles $\bar{\omega}_b$, A_Q^+ is the operator of creation of a universe with a positive "frequency" (which propagates in the positive direction of the dynamic time) and A_Q^- is the operator of annihilation of a universe (or creation of an antiuniverse) with a negative "frequency" (which propagates in the negative direction of the dynamic time). An observer as a "physicist" sees the creation of a universe and the creation of dynamic particles by the geometric vacuum ($b^+|0 >= 0$) as two different effects. The second effect disappears if we neglect gravitons and massive fields. In this case, $\rho' = 0$, and one can represent a wave function of a universe in the form of the spectral series over eigenvalues ρ_Q of the density ρ

$$\Psi(f|\varphi_2, \varphi_1) = \sum_Q \left[\Psi_Q^+ + \Psi_Q^-\right], \tag{87}$$

$$\Psi_Q^+ = \frac{A_Q^+}{\sqrt{2\rho_Q}} \exp\left\{-i(\varphi_2 - \varphi_1)\sum \frac{\bar{\omega}_f n_f}{\sqrt{\rho_Q}}\right\} < f|Q >$$

$$\Psi_Q^- = \frac{A_Q^-}{\sqrt{2\rho_Q}} \exp\left\{i(\varphi_2 - \varphi_1)\sum \frac{\bar{\omega}_f n_f}{\sqrt{\rho_Q}}\right\} < f|Q >^*$$

where $< f|Q >$ is a product of normalizable Hermite polynomials.

The equation of diagonalization (79) for the Bogoliubov coefficients (77) and the quasiparticle energy $\bar{\omega}_b$ plays the role of the equation of state of the field matter in the universe. We can show that the choice of initial conditions for the "Big Bang" in the form of the Bogoliubov (squeezed) vacuum $b|0>_b = 0$ reproduces all stages of the evolution of the Friedmann-Robertson-Walker universe in their conformal versions: anisotropic, inflation, radiation, and dust. The squeezed vacuum (i.e., the vacuum of quasiparticles) is the state of "nothing". For small φ and a large Hubble parameter, at the beginning of the Universe, the state of vacuum of quasiparticles leads to the density of matter [7]

$$_b < \rho(a^+,\ a) >_b = \rho_0 \frac{1}{2}\left(\frac{\varphi^2(0)}{\varphi^2(T)} + \frac{\varphi^2(T)}{\varphi^2(0)}\right), \qquad \theta = \frac{\pi}{4} \tag{88}$$

where $\varphi(0)$ is the initial value, and ρ_0 is the density of the Casimir energy of vacuum of "quasiparticles". The first term corresponds to the conformal version of the rigid state equation (in accordance with the classification of the standard cosmology) which describes the Kasner anisotropic stage $T_\pm(\varphi) \sim \pm\varphi^2$ (described by the Misner wave function [29]). The second term of the squeezed vacuum density (88) (for an admissible positive branch) leads to the stage with inflation of the dynamic time φ with respect to the geometric time T

$$\varphi(T)_{(+)} \simeq \varphi(0)\exp[T\sqrt{2\rho_0}/\varphi(0)].$$

It is the stage of intensive creation of "measurable particles". After the inflation, the Hubble parameter goes to zero, and gravitons convert into photon-like oscillator excitations with the conserved number of particles.

At the present-day stage, the Bogoliubov quasiparticles coincide with particles, so that the measurable density of energy of matter in the universe is a sum of relativistic energies of all particles

$$\rho_0(\varphi) = \frac{E}{V_0} = \sum_{n_f} \frac{n_f}{V_0}\sqrt{k_{fi}^2 + y_f^2\varphi^2(T)}, \tag{89}$$

where y_f is the mass of a particle in units of the Planck mass. The case of massless particles ($y = 0, \rho_0(\varphi) = constant$) correspond to the conformal version of radiation stage of the standard FRW-cosmology. And the massive particles at rest $(k = 0, \rho_0(\varphi) = \rho_{barions}\varphi/\mu)$ corresponds to the conformal version of the dust universe of the standard cosmology with the Hubble law

$$\varphi' = \pm\sqrt{\rho_0} \;\Rightarrow\; \varphi_\pm(T) = \left(\frac{\rho_{\text{barions}}}{4\mu}\right) T^2 \,. \tag{90}$$

The dynamic time is expressed through the geometric time of a quantum asymptotic state of the Universe $|out>$ and conserved quantum numbers of this state: energy E_{out} and density $\rho_0 = E_{out}/V_0$. It is well-known that E_{out} is a tremendous energy $(10^{79}GeV)$ in comparison with possible deviations of the free Hamiltonian in the laboratory processes

$$\bar{H}_0 = E_{out} + \delta H_0, \qquad < out|\delta H_0|in> << E_{out}. \tag{91}$$

We have seen that the dependence of the scale factor φ_0 on the geometric time T (or the "relation" of two classical unconstrained systems: dynamic and geometric) is the "Big Bang" and evolution of a universe. Therefore, from the point of view of an Einstein observer in special relativity, "Big Bang" is the pure relativistic effect of evolution of the geometric interval with respect to the dynamic evolution parameter which goes beyond the scope of Hamiltonian description of a single classical unconstrained system. The causal Green function

$$G_c(F_1, F_2|\varphi_1, \varphi_2) = G_+(F_1, F_2|\varphi_1, \varphi_2)\theta(\varphi_1 - \varphi_2) + G_+(F_2, F_1|\varphi_2, \varphi_1)\theta(\varphi_2 - \varphi_1), \tag{92}$$

describes the creation of a universe, but it is not sufficient to describe "Big Bang" evolution in quantum theory with the creation of matter, as an observer measures this evolution in terms of the geometric time T.

5. Infinite volume limit of Quantum Gravity

The simplest way to determine the limits of the validity of the FP-integral (55) is to use the quantum field version of the reparametrization-invariant integral (67) in the form of S-matrix elements [23]

$$S[\varphi_1, \varphi_2] =< out\ (\varphi_2)|T_\varphi \exp\left\{-i\int\limits_{\varphi_1}^{\varphi_2} d\varphi (H_I^R)\right\}|(\varphi_1)\ in > \tag{93}$$

where T_φ are symbols of ordering with respect to the dynamic time, and $< out\ (\varphi_2)|$, $|(\varphi)\ in >$ are states of quantum universes in the lowest order of the Dirac perturbation theory ($\mathcal{N} = 1$; $N^k = 0$; $q^{ij} = \delta_{ij} + h^T_{ij}$), H_I^R is the interaction Hamiltonian

$$H_I^R = H^R - H_0^R, \quad H^R = 2\sqrt{V_0 H(\varphi)}, \qquad H_0^R = 2\sqrt{V_0 H_0(\varphi)} \tag{94}$$

$H_0(\varphi)$ is the free part of the measurable Hamiltonian $H(\varphi)$.

We consider the infinite volume limit of the S-matrix element in terms of the geometric time T for the present-day stage $T = T_0, \varphi(T_0) = \mu$, and $T(\varphi_1) = T_0 - \Delta T, T(\varphi_2) = T_0 + \Delta T = T_{out}$.

One can express this matrix element in terms of the time measured by an observer of an out-state with a tremendous number of particles in the universe using equation (90) $d\varphi = dT_{out}\sqrt{\rho_{out}}$ and approximation (91) to neglect "back-reaction". In the infinite volume limit, we get

$$d\varphi[H_I^R] = d\varphi 2\left(\sqrt{V_0(H_0 + H_I)} - \sqrt{V_0 H_0}\right) = dT_{out}[\hat{F}\bar{H}_I + O(1/E_{out})] \qquad (95)$$

where H_I is the interaction Hamiltonian in GR, and

$$\hat{F} = \sqrt{\frac{E_{out}}{H_0}} = \sqrt{\frac{E_{out}}{E_{out} + \delta H_0}} \qquad (96)$$

is a multiplier which plays the role of a form factor for physical processes observed in the "laboratory" conditions when the cosmic energy E_{out} is much greater than the deviation of the free energy

$$\delta H_0 = H_0 - E_{out}; \qquad (97)$$

due to creation and annihilation of real and virtual particles in the laboratory experiments. The measurable time of the laboratory experiments $T_2 - T_1$ is much smaller than the age of the Universe T_0, but it is much greater than the reverse "laboratory" energy δ, so that the limit

$$\int\limits_{T(\varphi_1)}^{T(\varphi_2)} dT_{out} \Rightarrow \int\limits_{-\infty}^{+\infty} dT_{out}$$

is valid. If we neglect the form factor (96) that removes a set of ultraviolet divergences, we get the matrix element (62) that corresponds to the standard FP functional integral (55) where the coordinate time is replaced by the geometric (conformal) time $t \to T_{out}$.

Thus, the standard FP-integral and the unitary S-matrix for conventional quantum field theory (QFT) are not valid for the description of the early universe. On the other hand, we revealed that QFT appears as the nonrelativistic approximation of very large mass of a universe. In this approximation, QFT is expressed in terms of the conformal-invariant Lichnerowicz variables and coordinates including the conformal time as the time of evolution of these variables.

6. Conformal Relativity

The reparametrization-invariant description of the "Big Bang" evolution distinguishes conformal variables (35) and coordinates including the geometric time.

These conformal variables, coordinates, geometric time T, and the conformal Hubble parameter $\mathcal{H}^c_{hub} = \varphi'/\varphi$ can be considered as measurable quantities in the Einstein General Relativity, if the latter is treated as a scalar version of the Weyl conformal invariant theory [6]

$$W_{CUT} = -W_{PCT} + W^c_{SM}, \tag{98}$$

where

$$-W_{PCT}(\varphi, g) = \int d^4x[-\sqrt{-g}\frac{\varphi^2}{6}R(g) + \varphi\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\varphi)] \tag{99}$$

the Penrose-Chernikov-Tagirov action (with negative sign), and

$$W^c_{SM}[\varphi, V, \psi, g] = \int d^4x \left(\mathcal{L}^{SM}_{(\varphi=0)} + \sqrt{-g}[-\varphi F + \varphi^2 B]\right) \tag{100}$$

is the conformally invariant part of the SM action (i.e. the conventional SM action without the "free" part for the modulus of the Higgs $SU(2)$ doublet φ and without the Higgs mass term), B and F are the mass terms of the vector V and fermion ψ fields, respectively

$$B = V_i\hat{Y}_{ij}V_j \,;\; F = \bar{\psi}_\alpha\hat{X}_{\alpha\beta}\psi_\beta, \tag{101}$$

$\hat{Y}$, $\hat{X}$ are the ordinary matrices of vector meson and fermion mass couplings in the WS theory multiplied by a rescaling parameter [6, 30]. In terms of the Lichnerowicz conformal invariant variables, this theory locally coincides with the Einstein theory where the scalar field φ converts into the determinant of the space metric multiplied by the Planck constant μ $(g^{1/6}\mu = \varphi_c)$ [5, 6]. The action of conformal relativity (98) does not containt any dimensional parameter, except of the finite time interval and finite volume, as the universe has the beginning $T = 0$ and the end $T = T_0$, i.e., the present-day stage, where the value of the scalar field $\varphi(T = T_0) = \sqrt{\rho_{barions}}/\mathcal{H}^c_{hub} = \mu$ coincides with the coupling constant of the Newton interaction, in agreement with equations of motion and astrophysical observational data (46).

The effective Higgs potential could not be restored by the the Coleman-Weinberg perturbation theory, as the vertices with scalar field interactions are absorbed by the definition of "particles" and by the Bogoliubov transformations. Instead of the effective Higgs potential, in the exact theory, these interactions form cosmological evolution of the universe as the pure relativistic and quantum phenomenon which reproduces the conformal version of the standard Friedmann model (developed by Hoyle and Narlikar [31]).

In the conformal cosmology, the Hubble law is explained by the evolution of the masses of elementary particles, so that the photon on a star remembers the "size" of a star atom at the moment of emission, and this "size" increases during the time of traveling; as a result, we get the red shift of the star photon in comparison with a photon emitted by a standard atom on the Earth at the moment of observation. The conformal version at the dust stage (90) corresponds to the "accelerating Universe" with

$$q_c = \frac{\varphi''\varphi}{\varphi'^2} = +\frac{1}{2},$$

instead of $q_F = -1/2$ for the Friedmann version (with the measurable time $dT_F = (\varphi/\mu)dT$).

In the conformal theory (98), the Higgs mechanism of the formation of particle masses becomes superfluous and, moreover, it contradicts the equivalence principle, as, in the case of the standard Higgs mechanism, the Planck mass and masses of particles are formed by different scalar fields. The Weyl scalar field φ forms both the Planck mass (in agreement with the present-day astrophysical data) and masses of elementary particles [6] (in agreement with the principle of equivalence).

The Weyl geometrization of the modulus of the Higgs field removes the Higgs potential with its problems of tremendous vacuum energy [32], monopole creation, and the domain walls [32]. The conformal scalar field plays the role of the dynamic time and forms the Newton potential. As a consequence, the conformal version of the Higgs field looses its particle-like excitations ([30]) like the time component of the electromagnetic field. In the conformal theory (98), we obtain the σ-version of the Standard Model [5, 6, 30] without Higgs particles and with the prescription (96) which removes ultraviolet divergences from the SM sector.

7. Conclusion

The main result of the paper is the reparametrization-invariant generating functional for the unitary and causal perturbation theory in general relativity in a finite space-time. This functional contains "Big Bang", the creation of matter in a universe from "nothing", and a set of new effects, including the creation and annihilation of universes. The classical cosmology of a universe and the Faddeev-Popov-DeWitt functional correspond to different orders of decomposition of this functional over the inverse "mass" of a universe.

All these results are based on the assertion that the measurable time in any reparametrization-invariant system is not the coordinate time, but the time-like variable (φ) of an extended phase space of this system $(F_D, P_F; \varphi, P_0)$. Accordingly, the measurable Hamiltonian is a solution of the energy constraint with respect to the conjugate momentum of this variable (P_0) [5, 6]. This definition of the measurable time and Hamiltonian supposes the reduction of an action for constructing an "equivalent" unconstrained system $(F_D|\varphi)$ (but not the reduction of a phase space).

The second assertion is that such an unconstrained system cannot cover the physical content of a relativistic reparametrization-invariant system $(F_D, \varphi|T)$. This content can be covered by two "equivalent" unconstrained systems–the dynamic system $(F_D|\varphi)$ and geometric one $(F_G|Q_0 = T)$ (constructed by the Levi-Civita type canonical transformation so that a new dynamic time Q_0 coincides with the proper one T). The very notion "relativistic" means the relation between these systems. In special relativity (SR), they are the rest frame (dynamic) and the comoving frame (geometric), and the "relation" between these frames is described by the Lorentz transformation. The Lorentz transformation, in SR, realizes the "creation" of a measurable time interval by a measurable space interval (with the measure of this "creation" as the ratio of velocities of a particle and light), only light-cone intervals are free from this "creation". Similarly, in GR, the transformation between the dynamic and geometric systems realizes the creation of matter by the geometric vacuum (with the measure of this creation as the variation of masses of conformal quantum fields), only the massless light-like fields are free from this creation.

There is an essential difference between the SR and GR from the point of an Einstein's observer. In SR, the "dynamic" and "geometric" systems are alternative (we have two different observers in the rest and comoving frames), whereas, in GR, these systems are supplemented, as the same observer is simultaneously in two systems $(F_D|\varphi)$, $(F_G|T)$, and he observes the evolution of particles, i.e., the "dynamic" variables (F_D), with respect to the "geometric" time (T) measured by his watch. While, the "geometric" variables (F_G) are the Bogoliubov quasiparticles which diagonalize equations of motion and give cosmological initial conditions, and the "dynamic time" (φ) is the cosmic scale factor as the measure of the cosmic evolution of the density of matter. The creation of matter from "nothing" is the creation of "dynamic" particles from the "geometric" vacuum of the Bogoliubov quasiparticles. The coefficients of the Bogoliubov transformations, in the conventional QFT perturbation theory, determine the density of matter and reproduce all stages of the standard FRW evolution of a universe in their conformal versions. In quantum field theory, the space scale factor (as the zero Fourier harmonic of the space metric) converts into the mass scale factor, so that general relativity looks like the scalar version of the Weyl conformal theory with the measure of change of the length of a vector in its parallel transport.

We show how to construct the unitary and causal functional integral for the sector of measurable quantities which generalizes the standard Faddeev-Popov integral for infinite space-time. The stability of a quantum relativistic system determines the arrow of geometric time $T(\varphi_2, \varphi_1) \geq 0$ for the causal Green functions.

A quantum universe is defined as stable states of "free" quantum fields in the space-time with the dynamic and geometric times; and Quantum Gravity, as the theory of the unitary S-matrix between the states of the Quantum Universe.

Consistent limits of the generating functional in classical gravitation and cosmology and the conventional quantum field theory (in the form of the Faddeev-Popov generating functional for the infinite space-time) distinguish the treatment of general relativity as the scalar version of the Weyl conformal theory.

The considered quantization leads to the unification of general relativity and the Standard Model with the Weyl geometrization of the modulus of the Higss field and with the set of predictions, including the Hoyle-Narlikar cosmology (where the physical reason of red-shift is changing masses of elementary particles in the process of evolution of the Universe), the cosmic mechanism of the formation of both the masses of elementary particles and the Planck mass by the Weyl scalar field (which does not contradict the present-day astrophysical data), squeezed (geometric) vacuum inflation from "nothing" at the beginning of the Universe, accelerating conformal universe at the dust stage, and the negative result of CERN experiment on the search of Higgs particles.

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Павловски М., Первушин В.Н. E2-2000-67
«Большой взрыв» квантовой Вселенной

Дается репараметризационно-инвариантное обобщение производящего функционала Фаддеева–Попова–ДеВитта для унитарной теории возмущения в ОТО в конечном пространстве-времени.

Показано, что репараметризационно-неинвариантные (общепринятые) описания эволюции Вселенной и квантовой гравитации соответствуют разным порядкам разложения полученного инвариантного функционала по обратной «массе» Вселенной.

«Большой взрыв», эволюция Вселенной, рождение материи из «ничего» воспроизводятся на уровне репараметризационно-инвариантного производящего функционала как чисто релятивистские и квантовые эффекты.

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Pawlowski M., Pervushin V.N. E2-2000-67
«Big Bang» of Quantum Universe

The reparametrization-invariant generating functional for the unitary and causal perturbation theory in general relativity in a finite space-time is obtained. The classical cosmology of a Universe and the Faddeev–Popov–DeWitt functional correspond to different orders of decomposition of this functional over the inverse «mass» of a Universe. It is shown that the invariant content of general relativity as a constrained system can be covered by two «equivalent» unconstrained systems: the «dynamic» (with «dynamic» time as the cosmic scale factor and conformal field variables) and «geometric» (given by the Levi-Civita type canonical transformation to the action-angle variables which determine initial cosmological states with the arrow of the proper time measured by the watch of an observer in the comoving frame).

«Big Bang», the Hubble evolution, and creation of «dynamic» particles by the «geometric» vacuum are determined by «relations» between the dynamic and geometric systems as pure relativistic phenomena, like the Lorentz-type «relation» between the rest and comoving frames in special relativity.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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