

СООБЩЕНИЯ
ОБЪЕДИНЕННОГО
ИНСТИТУТА
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4-2000-257

A.P.Severyukhin, V.V.Voronov, D.Karadjov*

GROUND STATE CORRELATIONS
AND ANHARMONICITY OF VIBRATIONS

*Institute for Nuclear Research and Nuclear Energy, Sofia, Bulgaria

2000

1 Introduction

Many properties of the nuclear vibrational states can be described within the Random Phase Approximation (RPA), which enables one to treat some correlations in the ground state [1-5]. The low-lying nuclear vibrational states investigated with new generation of detectors [6] and the double giant dipole resonances observed in relativistic heavy ion collision [7, 8] provide an excellent test for the studies of deviations from the harmonic picture for multi-phonon excitations in real physical systems. It is well known that due to the anharmonicity of vibrations there is a coupling between one-phonon and more complex states [3, 5]. Usually such a coupling was considered for the RPA phonons only [5].

From another point of view the RPA violates the Pauli principle and many attempts have been done to improve it [9-29]. Extended RPA equations include additional corrections for the ground state correlations (GSC) [17, 18, 27, 29]. In the papers [24, 27, 29] phonons of the extended RPA containing corrections for the GSC have been used as a basis on which the quasiparticle-phonon model (QPM) equations [5, 30, 31] are generalized. The influence of the GSC on properties of nuclear vibrational states constructed by one- and two-phonon configurations was studied in [24]. Besides the GSC, the Pauli principle corrections arising in the two-phonon terms due to the fermion structure of the phonon operators were taken into account and the particle-particle channel was included too [27, 29].

However, these extended QPM equations have been derived for the case of the so called quasideagonal approximation for the Pauli principle corrections. In the present studies we don't use this approximation. The second improvement seems more important. Namely, it is found that there are additional Pauli principle corrections resulting in the anharmonicity shifts of the two-phonon configuration energies. A correspondence between the anharmonicity shifts of the QPM and Nuclear Field Theory diagrams is discussed.

2 Hamiltonian of the model

We employ the QPM hamiltonian [5, 30, 31] including an average nuclear field described as the Woods-Saxon potential, pairing interactions, the isoscalar and isovector particle-hole (p-h) and particle-particle (p-p)

residual forces in a separable form with the Bohr–Mottelson radial dependence [3]:

$$H = \sum_{\tau} \left(\sum_{jm}^{\tau} (E_j - \lambda_{\tau}) a_{jm}^{\dagger} a_{jm} - \frac{1}{4} G_{\tau}^{(0)} : P_0^{\dagger}(\tau) P_0(\tau) : - \frac{1}{2} \sum_a \sum_{\sigma=\pm 1} \sum_{\lambda\mu} \left[\left(\kappa_0^{(\lambda,a)} + \sigma \kappa_1^{(\lambda,a)} \right) : M_{\lambda\mu}^{(a)+}(\tau) M_{\lambda\mu}^{(a)}(\sigma\tau) : \right] \right), \quad (1)$$

We sum over the proton(p) and neutron(n) indexes and the notation $\{\tau = (n, p)\}$ is used and a change $\tau \leftrightarrow -\tau$ means a change $p \leftrightarrow n$; a is the channel index $a = \{ph, pp\}$. The single-particle states are specified by the quantum numbers (jm); E_j are the single-particle energies; λ_{τ} is the chemical potential; $G_{\tau}^{(0)}$ and $\kappa^{(\lambda)}$ are the strengths in the p–p and in the p–h channel, respectively. The monopole pair creation and the multipole operators entering the normal products in (1) are defined as follows:

$$P_0^+(\tau) = \sum_{jm}^{\tau} (-1)^{j-m} a_{jm}^{\dagger} a_{j-m}^{\dagger},$$

$$M_{\lambda\mu}^{(ph)+}(\tau) = \frac{1}{\sqrt{2\lambda+1}} \sum_{jj'mm'}^{\tau} (-1)^{j+m} \langle jmj' - m' | \lambda\mu \rangle f_{jj'}^{(\lambda)} a_{jm}^{\dagger} a_{j'm'}^{\dagger},$$

$$M_{\lambda\mu}^{(pp)+}(\tau) = \frac{(-1)^{\lambda-\mu}}{\sqrt{2\lambda+1}} \sum_{jj'mm'}^{\tau} \langle jmj' m' | \lambda\mu \rangle f_{jj'}^{(\lambda)} a_{jm}^{\dagger} a_{j'm'}^{\dagger},$$

where $f_{jj'}^{(\lambda)}$ are the single particle radial matrix elements of residual forces.

3 Extended RPA

In what follows we work in the quasiparticle representation, defined by the canonical Bogoliubov transformation:

$$a_{jm}^{\dagger} = u_j \alpha_{jm}^{\dagger} + (-1)^{j-m} v_j \alpha_{j-m}. \quad (2)$$

The Hamiltonian (1) can be represented in terms of bifermion quasiparticle operators (and their conjugate ones) [5, 30, 31]:

$$B(jj'; \lambda\mu) = \sum_{mm'} (-1)^{j'+m'} \langle jmj' m' | \lambda\mu \rangle \alpha_{jm}^{\dagger} \alpha_{j'-m'},$$

$$A^+(jj'; \lambda\mu) = \sum_{mm'} \langle jmj' m' | \lambda\mu \rangle \alpha_{jm}^{\dagger} \alpha_{j'm'}^{\dagger}.$$

We introduce the phonon creation operators

$$Q_{\lambda\mu i}^+ = \frac{1}{2} \sum_{jj'} (\psi_{jj'}^{\lambda i} A^+(jj'; \lambda\mu) - (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} A(jj'; \lambda - \mu)). \quad (3)$$

where the index λ denotes multipolarity and μ is its z-projection in the laboratory system. One assumes that the ground state is the ERPA phonon vacuum $|0\rangle$, i.e. $Q_{\lambda\mu i} |0\rangle = 0$. We define the excited states for this approximation by $Q_{\lambda\mu i}^+ |0\rangle$. The following relation can be proved using the exact commutators of the fermion operators:

$$\langle 0 | [Q_{\lambda\mu, i}, Q_{\lambda' \mu' i'}^+] | 0 \rangle = \frac{1}{2} \sum_{jj'} (1 - q_{jj'}) (\psi_{jj'}^{\lambda i} \psi_{jj'}^{\lambda' i'} - \varphi_{jj'}^{\lambda i} \varphi_{jj'}^{\lambda' i'}),$$

where $q_{jj'} = q_j + q_{j'}$ and q_j is the quasiparticle distribution in the ground state: $q_j = \sum_m \langle 0 | \alpha_{jm}^+ \alpha_{jm} | 0 \rangle (2j + 1)^{-1}$. The quasiparticle energies (ε_j), the energies of the ERPA excited states ($\omega_{\lambda i}$), the chemical potentials (λ_τ), the coefficients (u, v, ψ, φ) of the Bogoliubov transformations (2), (3) and the quasiparticle distributions in the ground state (q_j) are determined from the following non-linear system of equations obtained by using the general equation of motion approach [1, 11]:

$$\langle 0 | \{ \delta\alpha_{jm}, [H, \alpha_{jm}^+] \} | 0 \rangle = \varepsilon_j \langle 0 | \{ \delta\alpha_{jm}, \alpha_{jm}^+ \} | 0 \rangle, \quad (4)$$

$$\langle 0 | [\delta Q_{\lambda\mu i}, [H, Q_{\lambda\mu i}^+]] | 0 \rangle = \omega_{\lambda i} \langle 0 | [\delta Q_{\lambda\mu i}, Q_{\lambda\mu i}^+] | 0 \rangle, \quad (5)$$

with the constrained conditions:

$$\{ \alpha_{jm}, \alpha_{j'm'}^+ \} = \delta_{jm, j'm'}, \quad (6)$$

$$\langle 0 | \sum_{jm}^\tau a_{jm}^+ a_{jm} | 0 \rangle = N^{(\tau)}, \quad (7)$$

$$\langle 0 | [Q_{\lambda\mu i}, Q_{\lambda\mu i'}^+] | 0 \rangle = \delta_{i'i}, \quad (8)$$

$$q_j = \frac{1}{2} \sum_{\lambda i, j'} \left(\frac{2\lambda + 1}{2j + 1} \right) (1 - q_{jj'}) (\varphi_{jj'}^{\lambda i})^2, \quad (9)$$

where $N^{(\tau)}$ is the number of particles. The variations $\delta\alpha_{jm}$ and $\delta Q_{\lambda\mu i}$ have the following form:

$$\delta\alpha_{jm} = \delta u_j a_{jm} - (-1)^{j-m} \delta v_j a_{j-m}^+,$$

$$\delta Q_{\lambda\mu i} = \frac{1}{2} \sum_{jj'} (\delta\psi_{jj'}^{\lambda i} A(jj'; \lambda\mu) - (-1)^{\lambda-\mu} \delta\varphi_{jj'}^{\lambda i} A^+(jj'; \lambda - \mu)).$$

The system of equations (4)-(9) can be derived using various approaches, e.g. [14]. For $q_j = 0$ the system of equations (4)-(9) reduces to the usual BCS and RPA equations with the p-h and p-p channels [5, 30].

The equation (5) have been obtained under the assumptions:

$$\langle 0 | \{ \alpha_{j_1 m_1'}, [H, \alpha_{j m}] \} \alpha_{j' m'} \alpha_{j_1 m_1} | 0 \rangle \approx 0, \quad (10)$$

$$\langle 0 | [H, \alpha_{j m}^+] \alpha_{j_1 m_1} | 0 \rangle \approx \varepsilon_j q_j \delta_{j m, j_1 m_1}, \quad (11)$$

$$\langle 0 | \{ [H, \alpha_{j m}^+], \alpha_{j_1 m_1} \} \alpha_{j' m'}^+ \alpha_{j_1 m_1}' | 0 \rangle \approx \varepsilon_j q_j' \delta_{j m, j_1 m_1} \delta_{j' m', j_1 m_1}'. \quad (12)$$

If we put $q_j = 0$ then these approximations correspond to the procedure of the linearization of equations. One can derive the equation (9) using various methods [12, 16]. Using the completeness and orthogonality conditions for the phonon operators one can express bifermion operators $A^+(jj'; \lambda\mu)$ and $A(jj'; \lambda\mu)$ as

$$A^+(jj'; \lambda\mu) = (1 - q_{jj'}) \sum_i (\psi_{jj'}^{\lambda i} Q_{\lambda\mu i}^+ + (-1)^{\lambda-\mu} \varphi_{jj'}^{\lambda i} Q_{\lambda-\mu i}) \quad (13)$$

It is necessary to point out that the solutions of the system of equations (4)-(9) obey the following equalities:

$$\langle 0 | [Q_{\lambda\mu i}, [H, Q_{\lambda\mu i'}^+]] | 0 \rangle_{ERPA} = \omega_{\lambda i} \delta_{ii'}, \quad (14)$$

$$\langle 0 | H Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+ | 0 \rangle_{ERPA} = 0.$$

The proof of these statements is analogous to that in the usual RPA case [2].

As it was shown in Refs.[17, 24, 29] the ERPA calculations give a better agreement with experimental data for the characteristics of the low-lying states than the RPA ones.

4 Generalized QPM equations

The GSC affect not only the RPA, but they also should change the quasiparticle-phonon coupling (see Refs.[24, 27, 29]). To take into account such effects we follow the basic ideas of the QPM. Hereafter we

generalize the extended QPM equations [24, 27, 29]. As it was shown in [32] the pairing vibrations give a negligible contribution to q_j . On the other hand the two-phonon configurations including the pairing vibration phonons have an energy essentially higher than the configurations constructed from usual vibration phonons. That is why we do not take into account the coupling with the pairing vibrations ($\lambda = 0$) in what follows.

The initial Hamiltonian (1) can be rewritten in terms of quasiparticle and phonon operators in the following form:

$$H = h_0 + h_0^{pp} + h_{QQ} + h_{QB}, \quad (15)$$

$$h_0 + h_0^{pp} = \sum_{jm} \varepsilon_j \alpha_{jm}^+ \alpha_{jm},$$

$$h_{QQ} = h_1 + h_2 + h_3,$$

$$h_1 = -\frac{1}{4} \sum_{\lambda\mu i i' \tau} \frac{\bar{X}^{\lambda i i'}(\tau) + \bar{X}^{\lambda i' i}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda i} \mathcal{Y}_\tau^{\lambda i'}}} Q_{\lambda\mu i}^+ Q_{\lambda\mu i'},$$

$$h_2 = -\frac{1}{8} \sum_{\lambda\mu i i' \tau} \frac{\mathcal{Z}^{\lambda i i'}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda i} \mathcal{Y}_\tau^{\lambda i'}}} (-1)^{\lambda-\mu} \left(Q_{\lambda\mu i}^+ Q_{\lambda-\mu i'}^+ + Q_{\lambda-\mu i'} Q_{\lambda\mu i} \right),$$

$$h_3 = -\frac{1}{8} \sum_{\lambda\mu i i' \tau} \frac{\bar{X}_1^{\lambda i i'}(\tau) + \bar{X}_1^{\lambda i' i}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda i} \mathcal{Y}_\tau^{\lambda i'}}} \left[Q_{\lambda\mu i}, Q_{\lambda\mu i'}^+ \right],$$

$$h_{QB} = -\frac{1}{2\sqrt{2}} \sum_{\lambda\mu i \tau} \sum_{j j'}^\tau \frac{f_{j j'}^{(\lambda)}}{\sqrt{\mathcal{Y}_\tau^{\lambda i}}} \left((-1)^{\lambda-\mu} Q_{\lambda\mu i}^+ L_{j j'}^{\lambda i(+)}(\tau) + Q_{\lambda-\mu i} L_{j j'}^{\lambda i(-)}(\tau) \right) B(j j'; \lambda - \mu) + h.c.,$$

The coefficients of the hamiltonian (15) are given in Appendix A. The term h_{QB} is responsible for the mixing of the configurations. While constructing the hamiltonian (15) we have neglected the terms $\sim: B(j j'; \lambda\mu) B(j_1 j_1'; \lambda\mu) :$, which do not lead to coherent effects and the energy corrections due to these terms are small in spherical nuclei [5, 33].

The commutation relations $[[Q_1, Q_2^+], Q_3^+]$, $[[Q_1^+, Q_2^+], Q_3^+]$ and $[Q_3, [Q_1^+, Q_2^+]]$ are calculated by using the transformation (13).

$$[[Q_1, Q_2^+], Q_3^+] = \sum_4 K_1(41|23)Q_4 + K(41|23)Q_4^+ \quad (16)$$

$$[[Q_1^+, Q_2^+], Q_3^+] = \sum_4 K_3(41|23)Q_4 + K_2(41|23)Q_4^+ \quad (17)$$

$$[Q_3, [Q_1^+, Q_2^+]] = \sum_4 K_5(41|23)Q_4 + K_4(41|23)Q_4^+ \quad (18)$$

These coefficients are of fourth-order in phonon amplitudes. In what follows one needs the coefficients K , K_1 , K_2 , K_4 only. They are given in Appendix B. We obtain the anharmonic corrections taking into account these commutation relations.

Hereinafter the ground state is approximated by the ERPA phonon vacuum. Using the ERPA phonons as a basis the wave functions of the excited states of even-even nuclei can be written as:

$$\Psi_\nu(\lambda\mu) = \Omega_{\lambda\mu\nu}^+ |0\rangle, \quad (19)$$

where

$$\Omega_{\lambda\mu\nu}^+ = \sum_i R_i(\lambda\nu)Q_{\lambda\mu i}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu) (Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+)_{\lambda\mu},$$

$$(Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+)_{\lambda\mu} \equiv \sum_{\mu_1 \mu_2} C_{\mu_1 \mu_2}^{\lambda\mu} Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+.$$

The wave functions (19) have the normalization condition

$$\begin{aligned} \sum_i R_i^2(J\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} \sum_{\lambda_1' i_1' \lambda_2' i_2'} P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu) P_{\lambda_2' i_2'}^{\lambda_1' i_1'}(J\nu) \times \\ \left(\delta_{\lambda_2' i_2', \lambda_2 i_2} \delta_{\lambda_1' i_1', \lambda_1 i_1} + \delta_{\lambda_2' i_2', \lambda_1 i_1} \delta_{\lambda_1' i_1', \lambda_2 i_2} (-1)^{\lambda_1 + \lambda_2 + J} + \right. \\ \left. K^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2) \right) = 1. \end{aligned} \quad (20)$$

We use the equation of motion method to diagonalize the hamiltonian (15)

$$\langle 0 | [\delta\Omega_{\lambda\mu\nu}, [H, \Omega_{\lambda\mu\nu}^+]] | 0 \rangle = E_\nu \langle 0 | [\delta\Omega_{\lambda\mu\nu}, \Omega_{\lambda\mu\nu}^+] | 0 \rangle,$$

where the variation $\delta\Omega_{\lambda\mu\nu}$ has the following form

$$\delta\Omega_{\lambda\mu\nu} = \sum_i \delta R_i(\lambda\nu)Q_{\lambda\mu i} + \sum_{\lambda_1 i_1 \lambda_2 i_2} \delta P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu) (Q_{\lambda_2 \mu_2 i_2} Q_{\lambda_1 \mu_1 i_1})_{\lambda\mu} (-1)^{\lambda_1 + \lambda_2 + \lambda}.$$

The variational principle yields a set of linear equations for the unknown wave function coefficients $R_i(J\nu)$ and $P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)$

$$\begin{pmatrix} \mathcal{W}_1 & \mathcal{U} \\ \mathcal{U}^T & \mathcal{W}_2 \end{pmatrix} \begin{pmatrix} \mathcal{R}(\nu) \\ \mathcal{P}(\nu) \end{pmatrix} = E_\nu \begin{pmatrix} I & 0 \\ 0 & \bar{K} \end{pmatrix} \begin{pmatrix} \mathcal{R}(\nu) \\ \mathcal{P}(\nu) \end{pmatrix} \quad (21)$$

with the additional condition (20). The matrix \mathcal{U}^T is a transpose of the matrix \mathcal{U} . The number of linear equations (21) equals to the number of one- and two-phonon configurations included in the wave function (19). The notations introduced above are

$$\mathcal{W}_1^{i'}(\lambda) = \langle 0 | [Q_{\lambda\mu i'}, [H, Q_{\lambda\mu i}^+]] | 0 \rangle, \quad (22)$$

$$\begin{aligned} \mathcal{W}_2^{\lambda'_1 i'_1, \lambda'_2 i'_2}(\lambda) &= (-1)^{\lambda'_1 + \lambda'_2 + \lambda} \times \\ &\langle 0 | \left[(Q_{\lambda'_2 \mu'_2 i'_2} Q_{\lambda'_1 \mu'_1 i'_1})_{\lambda\mu}, [H, (Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+)_{\lambda\mu}] \right] | 0 \rangle, \end{aligned} \quad (23)$$

$$\mathcal{U}_{\lambda_1 i_1, \lambda_2 i_2}^{\lambda i}(\lambda) = \langle 0 | [Q_{\lambda\mu i}, [H, (Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+)_{\lambda\mu}]] | 0 \rangle, \quad (24)$$

$$I_i^{i'} = \delta_{i', i},$$

$$\begin{aligned} \overline{\mathcal{K}}_{\lambda_1 i_1, \lambda_2 i_2}^{\lambda'_1 i'_1, \lambda'_2 i'_2}(\lambda) &= \left(\delta_{\lambda'_2 i'_2, \lambda_2 i_2} \delta_{\lambda'_1 i'_1, \lambda_1 i_1} + \delta_{\lambda'_2 i'_2, \lambda_1 i_1} \delta_{\lambda'_1 i'_1, \lambda_2 i_2} (-1)^{\lambda_1 + \lambda_2 + \lambda} + \right. \\ &\left. K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 | \lambda_1 i_1, \lambda_2 i_2) \right). \end{aligned}$$

Now we calculate the matrix elements (22), (23) and as a result we get

$$\mathcal{W}_1^{i'_2}(\lambda_1) = \delta_{i'_2, i_1} \omega_{\lambda_1 i_1} - \frac{1}{8} \sum_{\lambda J' i i' \tau} \left(\frac{2J' + 1}{2\lambda_1 + 1} \right) \frac{1}{\sqrt{\mathcal{Y}_\tau^{\lambda i} \mathcal{Y}_\tau^{\lambda i'}}} \times \quad (25)$$

$$\begin{aligned} & \left(\mathcal{Z}^{\lambda i i'}(\tau) (-1)^{\lambda + \lambda_1 + J'} (K_1^{J'}(\lambda_1 i'_2, \lambda_1 i_1 | \lambda i, \lambda i') - K_2^{J'}(\lambda_1 i'_2, \lambda i | \lambda_1 i_1, \lambda i')) + \right. \\ & \left. (\overline{\mathcal{X}}_1^{\lambda i i'}(\tau) + \overline{\mathcal{X}}_1^{\lambda i' i}(\tau)) K^{J'}(\lambda_1 i'_2, \lambda i | \lambda i', \lambda_1 i_1) \right), \end{aligned}$$

$$\begin{aligned} \mathcal{W}_2^{\lambda'_1 i'_1, \lambda'_2 i'_2}(\lambda) &= \left(\delta_{i'_2, i_2} \mathcal{W}_1^{i'_1}(\lambda_1) + \delta_{i'_1, i_1} \mathcal{W}_1^{i'_2}(\lambda_2) \right) + \\ & \delta_{\lambda'_1, \lambda_2} \delta_{\lambda'_2, \lambda_1} (-1)^{\lambda_1 + \lambda_2 + \lambda} \left(\delta_{i'_2, i_1} \mathcal{W}_1^{i'_1}(\lambda_2) + \delta_{i'_1, i_2} \mathcal{W}_1^{i'_2}(\lambda_1) \right) + \\ & \Delta \overline{\omega}^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 | \lambda_1 i_1, \lambda_2 i_2). \end{aligned}$$

The matrix elements (22), (23) have been evaluated by keeping all terms containing the first power of coefficients K , K_i and products KK , only. We took into account also the terms containing the products KK_i that include

the $\psi^3\varphi$ terms. The other terms including the products KK_i are smaller and they are neglected. The terms with products K_iK_i were not taken into account for these matrix elements because they have no $\psi^4, \psi^3\varphi$ terms. It should be pointed out that the matrix element (25) differs from the phonon energy in contrast with the ERPA case (see Eq.(14)). This difference is due to the approximations (10),(11),(12). The corrections to the matrix element (14) include phonon amplitudes of the next orders. Only the terms h_2 and h_3 of the hamiltonian (15) contribute to these corrections. Then, these corrections become zero if $\varphi = 0$ (see the explicit form for \bar{X}_1, K_1, K_2). It should be mentioned that we do not use so called quasideagonal approximation for the coefficients K and K_i .

The values $\Delta\bar{\omega}^\lambda$ are anharmonic shifts of energies of the two-phonon configurations due to the Pauli principle corrections, where $\Delta\bar{\omega}^\lambda = \Delta\bar{\omega}_1^\lambda + \Delta\bar{\omega}_2^\lambda + \Delta\bar{\omega}_3^\lambda$,

$$\begin{aligned} \Delta\bar{\omega}_1^\lambda(\lambda'_2i'_2, \lambda'_1i'_1 | \lambda_1i_1, \lambda_2i_2) = & \quad (26) \\ & (\omega_{\lambda_1i_1} + \omega_{\lambda_2i_2}) K^\lambda(\lambda'_2i'_2, \lambda'_1i'_1 | \lambda_1i_1, \lambda_2i_2) - \\ & \frac{1}{4} \sum_{i\tau} \left(\frac{\bar{X}^{\lambda_1i'_1i}(\tau) + \bar{X}^{\lambda_1ii'_1}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_1i'_1} \mathcal{Y}_\tau^{\lambda_1i}}} K^\lambda(\lambda'_2i'_2, \lambda_1i | \lambda_1i_1, \lambda_2i_2) + \right. \\ & + \frac{\bar{X}^{\lambda_2i'_2i}(\tau) + \bar{X}^{\lambda_2ii'_2}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_2i'_2} \mathcal{Y}_\tau^{\lambda_2i}}} K^\lambda(\lambda_1i, \lambda_2i | \lambda_1i_1, \lambda_2i_2) (-1)^{\lambda_1+\lambda_2+\lambda} + \\ & \left. \sum_{\lambda_4\lambda_3i_3i'} \left(\frac{\bar{X}^{\lambda_4ii'}(\tau) + \bar{X}^{\lambda_4i'i}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_4i} \mathcal{Y}_\tau^{\lambda_4i'}}} \times \right. \right. \\ & \left. \left. K^\lambda(\lambda_3i_3, \lambda_4i' | \lambda_1i_1, \lambda_2i_2) K^\lambda(\lambda'_2i'_2, \lambda'_1i'_1 | \lambda_4i, \lambda_3i_3) \right) \right), \end{aligned}$$

$$\begin{aligned} \Delta\bar{\omega}_2^\lambda(\lambda'_2i'_2, \lambda'_1i'_1 | \lambda_1i_1, \lambda_2i_2) = & -\frac{1}{8} \sum_{i\tau} \left(\frac{\mathcal{Z}^{\lambda_2i_2i}(\tau) + \mathcal{Z}^{\lambda_2ii_2}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_2i_2} \mathcal{Y}_\tau^{\lambda_2i}}} \times \right. & (27) \\ & K_1^\lambda(\lambda'_2i'_2, \lambda_1i_1 | \lambda_2i, \lambda'_1i'_1) - \frac{\mathcal{Z}^{\lambda_1i'_1i}(\tau) + \mathcal{Z}^{\lambda_1ii'_1}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_1i'_1} \mathcal{Y}_\tau^{\lambda_1i}}} \times \\ & K_2^\lambda(\lambda'_2i'_2, \lambda'_1i | \lambda_1i_1, \lambda_2i_2) - \frac{\mathcal{Z}^{\lambda_2i'_2i}(\tau) + \mathcal{Z}^{\lambda_2ii'_2}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_2i'_2} \mathcal{Y}_\tau^{\lambda_2i}}} \times \end{aligned}$$

$$\begin{aligned}
& (-1)^{\lambda+\lambda'+\lambda_2} K_2^\lambda(\lambda'_1 i'_1, \lambda'_2 i \mid \lambda_1 i_1, \lambda_2 i_2) - \\
& \sum_{\lambda_4 \lambda_3 i_3 i'} \left(\frac{\mathcal{Z}^{\lambda_4 i i'}(\tau) + \mathcal{Z}^{\lambda_4 i' i}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_4 i} \mathcal{Y}_\tau^{\lambda_4 i'}}} (K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_4 i, \lambda_3 i_3) \times \right. \\
& K_2^\lambda(\lambda_3 i_3, \lambda_4 i' \mid \lambda_1 i_1, \lambda_2 i_2) - \sum_{J_1 J_2} (-1)^{\lambda_1 + \lambda'_2 + J_1 + J_2} \times \\
& \left. \left(\left\{ \begin{array}{ccc} J_1 & \lambda_3 & \lambda_1 \\ \lambda_4 & J_2 & \lambda_2 \\ \lambda'_1 & \lambda'_2 & \lambda \end{array} \right\} K^{J_1}(\lambda_3 i_3, \lambda_1 i_1 \mid \lambda_4 i, \lambda'_1 i'_1) \times \right. \right. \\
& K_1^{J_2}(\lambda'_2 i'_2, \lambda_2 i_2 \mid \lambda_4 i', \lambda_3 i_3) + (-1)^{\lambda_3 + \lambda'_1 + J_1 + J_2 + \lambda} \left. \left\{ \begin{array}{ccc} J_1 & \lambda_3 & \lambda_1 \\ \lambda_4 & J_2 & \lambda_2 \\ \lambda'_2 & \lambda'_1 & \lambda \end{array} \right\} \times \right. \\
& \left. K^{J_1}(\lambda_3 i_3, \lambda_1 i_1 \mid \lambda_4 i, \lambda'_2 i'_2) K_1^{J_2}(\lambda_3 i_3, \lambda_2 i_2 \mid \lambda_4 i', \lambda'_1 i'_1) \right) + \\
& \left. (-1)^{\lambda_2 + \lambda_3 + \lambda_4 + \lambda} \frac{(2\lambda_3 + 1) \mathcal{Z}^{\lambda_4 i i'}(\tau)}{(2\lambda_1 + 1) \sqrt{\mathcal{Y}_\tau^{\lambda_4 i} \mathcal{Y}_\tau^{\lambda_4 i'}}} \times \right. \\
& \left. \left(K_2^{\lambda_3}(\lambda_1 i_3, \lambda_4 i \mid \lambda_1 i_1, \lambda_4 i') K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_2 i_2, \lambda_1 i_3) + (-1)^\lambda \left(\frac{2\lambda_1 + 1}{2\lambda_2 + 1} \right) \times \right. \right. \\
& \left. \left. K_2^{\lambda_3}(\lambda_2 i_3, \lambda_4 i \mid \lambda_2 i_2, \lambda_4 i') K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_1 i_1, \lambda_2 i_3) \right) \right),
\end{aligned}$$

$$\Delta \bar{\omega}_3^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_1 i_1, \lambda_2 i_2) = -\frac{1}{4} \sum_{i\tau\lambda_3} \left(\frac{\bar{X}^{\lambda_2 i_2 i}(\tau) + \bar{X}^{\lambda_2 i i_2}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_2 i_2} \mathcal{Y}_\tau^{\lambda_2 i}}} \times \quad (28)$$

$$\begin{aligned}
& K_4^{\lambda_3}(\lambda'_2 i'_2, \lambda_2 i \mid \lambda_1 i_1, \lambda'_1 i'_1) (-1)^{\lambda + \lambda_3 + \lambda_2 + \lambda'_1 + 1} (2\lambda_3 + 1) \left\{ \begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda \\ \lambda'_2 & \lambda'_1 & \lambda_3 \end{array} \right\} + \\
& \frac{1}{2} \sum_{\lambda_4 i_4 i'} \frac{\bar{X}_1^{\lambda_4 i i'}(\tau) + \bar{X}_1^{\lambda_4 i' i}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_4 i} \mathcal{Y}_\tau^{\lambda_4 i'}}} \times \\
& \left(\left(\frac{2\lambda_3 + 1}{2\lambda_1 + 1} \right) K^{\lambda_3}(\lambda_1 i_4, \lambda_4 i \mid \lambda_4 i', \lambda_1 i_1) K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_1 i_4, \lambda_2 i_2) + \right. \\
& \left. \left(\frac{2\lambda_3 + 1}{2\lambda_2 + 1} \right) K^{\lambda_3}(\lambda_2 i_4, \lambda_4 i \mid \lambda_4 i', \lambda_2 i_2) K^\lambda(\lambda'_2 i'_2, \lambda'_1 i'_1 \mid \lambda_1 i_1, \lambda_2 i_4) \right).
\end{aligned}$$

One can prove that $\Delta \bar{\omega}_1^\lambda \gg \Delta \bar{\omega}_2^\lambda$ and $\Delta \bar{\omega}_1^\lambda \gg \Delta \bar{\omega}_3^\lambda$ in the case $\psi \gg \varphi$. Moreover, the shifts $\Delta \bar{\omega}_2^\lambda$ and $\Delta \bar{\omega}_3^\lambda$ become zero if $\varphi = 0$. It is necessary

to emphasize that only the terms h_2 and h_3 contribute to the shifts $\Delta\bar{\omega}_2^\lambda$ and $\Delta\bar{\omega}_3^\lambda$. The shifts $\Delta\bar{\omega}_2^\lambda$ and $\Delta\bar{\omega}_3^\lambda$ have been neglected in our previous papers. The matrix elements coupling one- and two-phonon configurations (24) are

$$U_{\lambda_1 i_1, \lambda_2 i_2}^{\lambda i}(\lambda) = \sum_{\tau} U_1^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) + U_2^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) + (-1)^{\lambda_1 + \lambda_2 + \lambda} U_2^{\lambda_2 i_2}_{\lambda_1 i_1}(\lambda i, \tau) + \bar{U}_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i, \tau),$$

where

$$U_1^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) = (-1)^{\lambda_1 + \lambda_2 + \lambda} \frac{1}{\sqrt{2}} \sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)} \sum_{j_1 j_2 j_3}^{\tau} (1 - q_{j_2 j_3}) \times$$

$$\frac{f_{j_1 j_2}^{\lambda}}{\sqrt{\mathcal{Y}_{\tau}^{\lambda i}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{Bmatrix} \left(\mathcal{L}_{j_1 j_2}^{\lambda i}(\tau) \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_3 j_1}^{\lambda_1 i_1} + \mathcal{L}_{j_2 j_1}^{\lambda i}(\tau) \psi_{j_3 j_1}^{\lambda_1 i_1} \varphi_{j_2 j_3}^{\lambda_2 i_2} \right),$$

$$\mathcal{L}_{j_1 j_2}^{\lambda i}(\tau) = \frac{1}{2} \left(L_{j_1 j_2}^{\lambda i(+)} + L_{j_2 j_1}^{\lambda i(-)} \right),$$

$$U_2^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) = (-1)^{\lambda_1 + \lambda_2 + \lambda} \frac{1}{\sqrt{2}} \sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)} \sum_{j_1 j_2 j_3}^{\tau} (1 - q_{j_2 j_3}) \times$$

$$\frac{f_{j_1 j_2}^{\lambda_2}}{\sqrt{\mathcal{Y}_{\tau}^{\lambda_2 i_2}}} \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_3 & j_2 \end{Bmatrix} \left(\mathcal{L}_{j_1 j_2}^{\lambda_2 i_2}(\tau) \varphi_{j_2 j_3}^{\lambda_1 i_1} \varphi_{j_3 j_1}^{\lambda i} + \mathcal{L}_{j_2 j_1}^{\lambda_2 i_2}(\tau) \psi_{j_3 j_1}^{\lambda i} \psi_{j_2 j_3}^{\lambda_1 i_1} \right),$$

$$\bar{U}_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i, \tau) = \sum_{\lambda' i' \lambda_3 i_3} (U_2^{\lambda' i'}_{\lambda_3 i_3}(\lambda i, \tau) K^{\lambda}(\lambda' i' \lambda_3 i_3 | \lambda_1 i_1, \lambda_2 i_2) +$$

$$U_3^{\lambda' i'}_{\lambda_3 i_3}(\lambda i, \tau) K_1^{\lambda}(\lambda_2 i_2 \lambda' i' | \lambda_3 i_3 \lambda_1 i_1,) + \sum_{J'} (2J' + 1) \sqrt{\frac{2\lambda' + 1}{2\lambda + 1}} \times \quad (29)$$

$$\left(\begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ J' & \lambda' & \lambda_3 \end{Bmatrix} \right) \left(K^{J'}(\lambda' i', \lambda i | \lambda_3 i_3, \lambda_2 i_2) U_1^{\lambda_1 i_1}_{\lambda' i'}(\lambda_3 i_3, \tau) (-1)^{\lambda + \lambda' + \lambda_2 + \lambda_3} + \right.$$

$$\left. K_1^{J'}(\lambda' i' \lambda_2 i_2 | \lambda_3 i_3, \lambda i) U_4^{\lambda_1 i_1}_{\lambda' i'}(\lambda_3 i_3, \tau) (-1)^{\lambda + \lambda' + J'} \right) +$$

$$\left\{ \begin{Bmatrix} \lambda_1 & \lambda_2 & \lambda \\ \lambda' & J' & \lambda_3 \end{Bmatrix} \right\} \left(K^{J'}(\lambda' i', \lambda i | \lambda_3 i_3, \lambda_1 i_1) U_1^{\lambda' i'}_{\lambda_2 i_2}(\lambda_3 i_3, \tau) + \right.$$

$$\left. K_1^{J'}(\lambda' i' \lambda_1 i_1 | \lambda_3 i_3, \lambda i) U_4^{\lambda' i'}_{\lambda_2 i_2}(\lambda_3 i_3, \tau) (-1)^{\lambda_1 + \lambda_3 + J'} \right)$$

$$U_3^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) = (-1)^{\lambda_1 + \lambda_2 + \lambda} \frac{1}{\sqrt{2}} \sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)} \sum_{j_1 j_2 j_3}^{\tau} (1 - q_{j_2 j_3}) \times$$

$$\frac{f_{j_1 j_2}^{\lambda_2}}{\sqrt{\mathcal{Y}_{\tau}^{\lambda_2 i_2}}} \left\{ \begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda \\ j_1 & j_3 & j_2 \end{array} \right\} \left(\mathcal{L}_{j_2 j_1}^{\lambda_2 i_2}(\tau) \varphi_{j_2 j_3}^{\lambda_1 i_1} \varphi_{j_3 j_1}^{\lambda i} + \mathcal{L}_{j_1 j_2}^{\lambda_2 i_2}(\tau) \psi_{j_3 j_1}^{\lambda i} \psi_{j_2 j_3}^{\lambda_1 i_1} \right),$$

$$U_4^{\lambda_1 i_1}_{\lambda_2 i_2}(\lambda i, \tau) = (-1)^{\lambda_1 + \lambda_2 + \lambda} \frac{1}{\sqrt{2}} \sqrt{(2\lambda_1 + 1)(2\lambda_2 + 1)} \sum_{j_1 j_2 j_3}^{\tau} (1 - q_{j_2 j_3}) \times$$

$$\frac{f_{j_1 j_2}^{\lambda}}{\sqrt{\mathcal{Y}_{\tau}^{\lambda i}}} \left\{ \begin{array}{ccc} \lambda_1 & \lambda_2 & \lambda \\ j_2 & j_1 & j_3 \end{array} \right\} \left(\mathcal{L}_{j_2 j_1}^{\lambda i}(\tau) \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_3 j_1}^{\lambda_1 i_1} + \mathcal{L}_{j_1 j_2}^{\lambda i}(\tau) \psi_{j_3 j_1}^{\lambda_1 i_1} \varphi_{j_2 j_3}^{\lambda_2 i_2} \right).$$

The terms including the coefficients K_1 in (29) were neglected in the papers [24, 27, 29].

Solving the system of equations (21) we obtain the energies (E_{ν}) of excited states and the coefficients $R_i(J\nu)$ and $P_{\lambda_2 i_2}^{\lambda_1 i_1}(J\nu)$ of the wave function (19). These equations are more general than ones derived in [24, 27, 29]. They have the same form as the basic QPM equations [5, 34]. The GSC change phonon energies $\omega_{\lambda i}$, the anharmonic shifts of two-phonon configurations and the matrix elements coupling one- and two-phonon configurations. In the case when $\psi \gg \varphi$ the system of equations (21) can be reduced to the system of equations of the extended QPM [24, 27, 29]. If we put $q_j = 0$, $\bar{X}_1 = 0$ and $K_i = 0$ we get the usual QPM equations with taking into account the Pauli principle corrections [5, 31, 34]. In the case when $q_j = 0$, $K = 0$ and $K_i = 0$ we have equations describing coupling of one- and two- RPA phonons without taking into account the Pauli principle [5].

5 Anharmonic shifts of two-phonon configurations

Now we discuss the correspondence between the QPM equations presented above and the diagrams of the Nuclear Field Theory (NFT) [3, 35]. For the sake of simplicity we compare these approaches for the hamiltonian (1) including the average nuclear field and the isoscalar particle-hole residual forces only. Besides that, we put $q_j = 0$ in the system of equations (21), i.e. conventional RPA phonons are used as the QPM basis in this case.

The NFT is a formulation of many-body perturbation theory with vibrational modes summed to all orders in RPA. Its building blocks are RPA

phonons and the single particle degrees of freedom which are described in the Hartree-Fock approximation. The coupling between them is treated diagrammatically in the perturbation theory. Diagrams illustrating first order coupling between the surface vibrations and the fermion fields are shown in Fig. 1. The wavy lines are phonon propagations, while the particles and the holes are depicted by the arrowed lines. The lowest-order anharmonic terms of NFT contributing to the energy of two-phonon states are represented by fourth-order diagrams shown in Fig. 2 [3,35-39]. These graphs are called butterfly-type (A,B), trapezoid-type (C,D), and diamond-type (E,F) diagrams. For each diagram shown in Fig. 2 there are 5 other diagrams which are obtained by changing the direction of the phonon lines.

One can rewrite the system of equations (21) in the space of two-phonon states. The diagonal approximation for the matrix element $\mathcal{W}_{1_i}^{i'}(\lambda)$ ($\mathcal{W}_{1_i}^{i'}(\lambda) = \delta_{i,i'} \mathcal{W}_{1_i}^i(\lambda)$) enables one to find the coefficient $R_i(J\nu)$ from the first equation of the system (21). Substituting it into the second equation of this system one can get the following secular equation to find energies E_ν :

$$\det \left\| \left(\delta_{i_2, i_2'} \mathcal{W}_{1_{i_1}}^{i_1'}(\lambda_1) + \delta_{i_1', i_1} \mathcal{W}_{1_{i_2}}^{i_2'}(\lambda_2) \right) \delta_{\lambda_1', \lambda_1} \delta_{\lambda_2', \lambda_2} + \right. \\ \left. \delta_{\lambda_1', \lambda_2} \delta_{\lambda_2', \lambda_1} (-1)^{\lambda_1 + \lambda_2 + \lambda} \left(\delta_{i_2', i_1} \mathcal{W}_{1_{i_2}}^{i_1'}(\lambda_2) + \delta_{i_1', i_2} \mathcal{W}_{1_{i_1}}^{i_2'}(\lambda_1) \right) + \right. \\ \left. \Delta \bar{\omega}^\lambda (\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2) - \Delta U^\lambda (\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2) - E_\nu \right\| = 0,$$

where

$$\Delta U^\lambda (\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2) = \sum_i \frac{\mathcal{U}_{\lambda_1 i_1, \lambda_2 i_2}^{\lambda i}(\lambda) \mathcal{U}_{\lambda_1' i_1', \lambda_2' i_2'}^{\lambda i}(\lambda)}{\mathcal{W}_{1_i}^i(\lambda) - E_\nu}. \quad (30)$$

It is seen that the values ΔU and $\Delta \bar{\omega}$ are anharmonic shifts of two-phonon states. The term h_{QB} does not contribute to the energy shifts of $\Delta \bar{\omega}$, while it results in the energy shifts of ΔU . Furthermore, the term h_{QB} contributes to the scattering vertices (see Fig.1 (A), (B)) only. A proof of this statement is similar to one of [38]. For example we consider terms ΔK (see (26)) and ΔU_2 (see (30)):

$$\Delta K^\lambda (\lambda_4 i_4, \lambda_3 i_3 | \lambda_1 i_1, \lambda_2 i_2) = \\ -\frac{1}{4} \sum_\tau \sum_{i_5} \frac{\bar{X}^{\lambda_3 i_3 i_5}(\tau) + \bar{X}^{\lambda_3 i_5 i_3}(\tau)}{\sqrt{\mathcal{Y}_\tau^{\lambda_3 i_3} \mathcal{Y}_\tau^{\lambda_3 i_5}}} K^\lambda (\lambda_4 i_4, \lambda_3 i_5 | \lambda_1 i_1, \lambda_2 i_2), \\ \Delta U_2^\lambda (\lambda_3 i_3, \lambda_4 i_4 | \lambda_2 i_2, \lambda_1 i_1) = \sum_\tau \sum_{i_5} \frac{U_{\lambda_1 i_1}^{\lambda_2 i_2}(\lambda i_5, \tau) U_{\lambda_3 i_3}^{\lambda_4 i_4}(\lambda i_5, \tau)}{\mathcal{W}_{1_{i_5}}^{i_5}(\lambda) - E_\nu}.$$

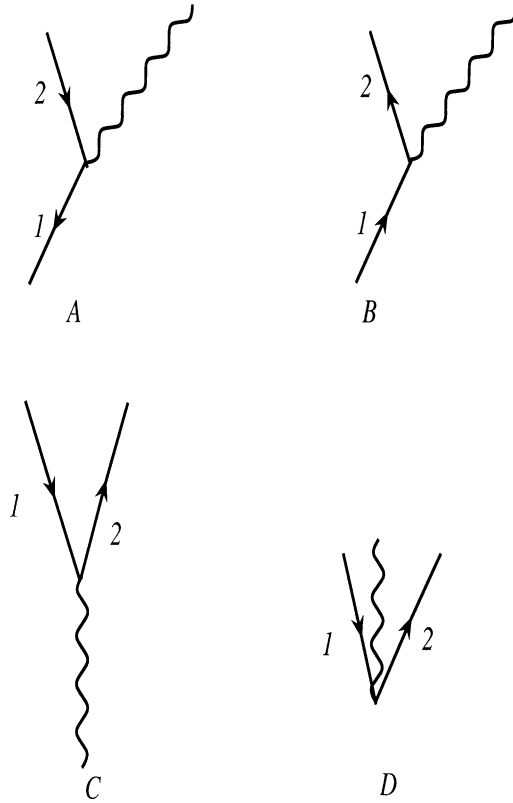


Figure 1: Diagrams illustrating the first order coupling between the surface vibrations (wavy lines) and the fermion fields (arrowed lines).

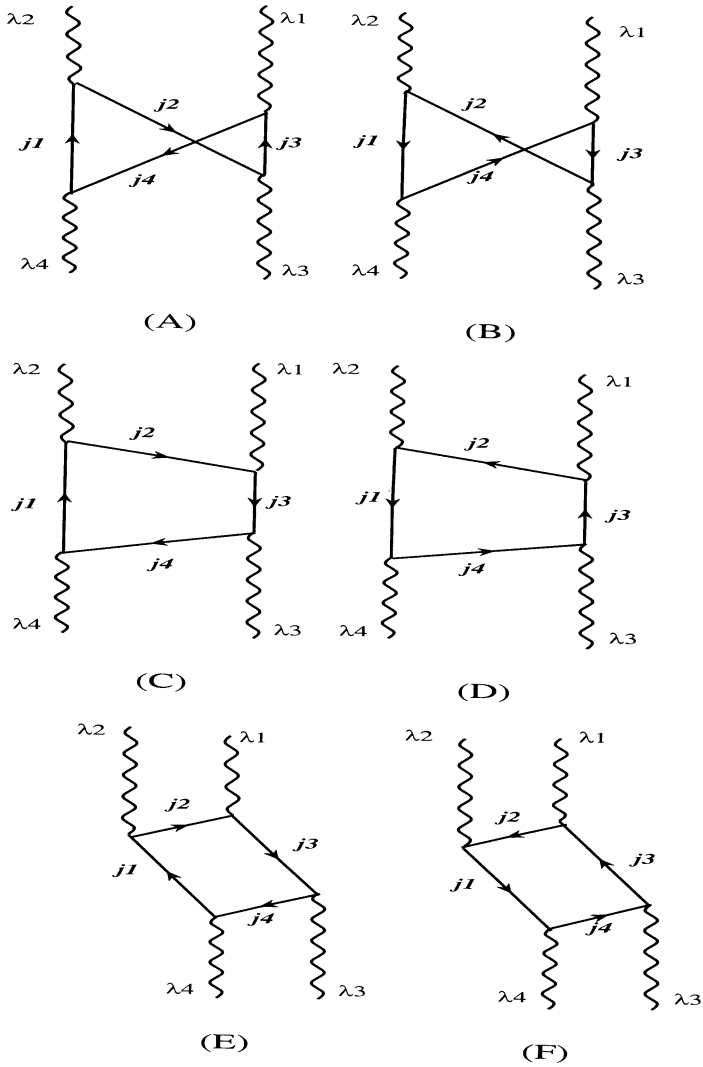


Figure 2: The NFT diagrams describing the lowest-order interaction between two phonons

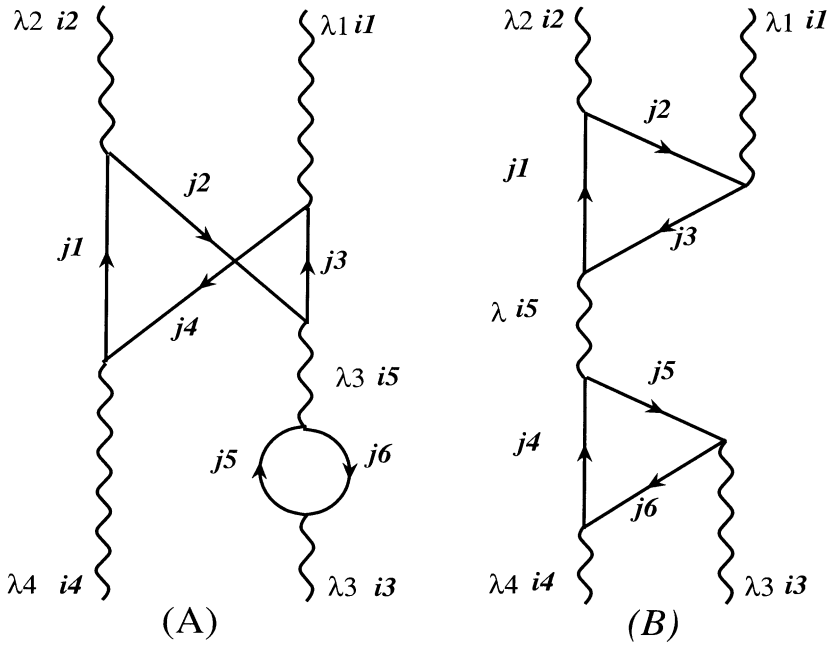


Figure 3: The QPM diagrams (A) and (B) illustrating the anharmonic shifts $\Delta K^\lambda(\lambda_4 i_4, \lambda_3 i_3 | \lambda_1 i_1, \lambda_2 i_2)$ and $\Delta U_2^\lambda(\lambda_3 i_3, \lambda_4 i_4 | \lambda_2 i_2, \lambda_1 i_1)$ of energies, respectively.

The anharmonic shifts of energies $\Delta K^\lambda(\lambda_4 i_4, \lambda_3 i_3 | \lambda_1 i_1, \lambda_2 i_2)$ and $\Delta U_2^\lambda(\lambda_3 i_3, \lambda_4 i_4 | \lambda_2 i_2, \lambda_1 i_1)$ can be illustrated by the QPM diagrams [5, 31] shown in Fig. 3 (A) and (B), respectively. If the intermediate phonon line (i_5) in these shifts are changed by the two-quasiparticle state lines (i.e. $\psi = 1$ and $\varphi = 0$), then these shifts can be represented by the fourth-order diagrams of the NFT (Fig. 2). In this case the term ΔK corresponds to the butterfly-type diagrams, while the term ΔU_2 can be presented as the diagrams of the trapezoid-type and the diamond-type. Using the same method one can prove that the shifts $\Delta\bar{\omega}$ and ΔU (see (26-28,30)) correspond to diagrams shown in Fig.2 (A), (B) and Fig. 2 (C), (D), (E), (F), respectively. The shifts $\Delta\bar{\omega}_2$ and $\Delta\bar{\omega}_3$ correspond to the butterfly-type diagrams.

6 Conclusion

A consistent treatment of the ground state correlations beyond the RPA including their influence on the pairing and the phonon-phonon coupling in nuclei is presented. A new general system of nonlinear equations for the quasiparticle phonon model is derived. It is demonstrated that this system contains as a particular case all equations derived for the QPM early. The new additional Pauli principle corrections resulting in the anharmonic shifts of energies of the two-phonon configurations are found. It is shown that the anharmonic shifts due to the Pauli principle corrections correspond with butterfly-type diagrams of the nuclear field theory, while the anharmonic shifts due to the matrix elements coupling one- and two-phonon configurations give rise to the trapezoid-type and diamond-type diagrams of the NFT.

Acknowledgments

We are grateful to Profs. W.Nawrocka, A.I.Vdovin for fruitful discussions. This work has been partially supported by the Bulgarian Plenipotentiary grant for 2000.

Appendix A

The coefficients of the hamiltonian (15) are given by the following expressions:

$$\bar{X}^{\lambda i i'}(\tau) = \sqrt{\frac{\mathcal{Y}_\tau^{\lambda i}}{2}} \left(D_0^{\lambda i}(\tau) + D_+^{\lambda i}(\tau) z_+^{\lambda i'}(\tau) + D_-^{\lambda i}(\tau) z_-^{\lambda i'}(\tau) \right)$$

$$\begin{aligned} \mathcal{Z}^{\lambda i i'}(\tau) = & \sqrt{\frac{\mathcal{Y}_\tau^{\lambda i}}{2}} \left(D_0^{\lambda i}(\tau) (1 - t_0^{\lambda i'}(\tau)) + F_0^{\lambda i}(\tau) + (D_+^{\lambda i}(\tau) + D_-^{\lambda i}(\tau)) \times \right. \\ & \left. (z_-^{\lambda i'}(\tau) - z_+^{\lambda i'}(\tau)) + F_1^{\lambda i}(\tau) t_2^{\lambda i'}(\tau) - F_2^{\lambda i}(\tau) t_1^{\lambda i'}(\tau) \right) + \\ & \sqrt{\frac{\mathcal{Y}_\tau^{\lambda i'}}{2}} \left(D_0^{\lambda i'}(\tau) (1 + t_0^{\lambda i}(\tau)) - F_0^{\lambda i'}(\tau) + (D_-^{\lambda i'}(\tau) - D_+^{\lambda i'}(\tau)) \times \right. \\ & \left. (z_-^{\lambda i}(\tau) + z_+^{\lambda i}(\tau)) + t_1^{\lambda i}(\tau) F_2^{\lambda i'}(\tau) - t_2^{\lambda i}(\tau) F_1^{\lambda i'}(\tau) \right) \end{aligned}$$

$$\begin{aligned} \bar{X}_1^{\lambda i i'}(\tau) = & \sqrt{\frac{\mathcal{Y}_\tau^{\lambda i}}{2}} \left(D_0^{\lambda i}(\tau) (2 - t_0^{\lambda i'}(\tau)) - F_0^{\lambda i}(\tau) + (D_-^{\lambda i}(\tau) - D_+^{\lambda i}(\tau)) \times \right. \\ & \left. (z_-^{\lambda i'}(\tau) - z_+^{\lambda i'}(\tau)) + t_2^{\lambda i'}(\tau) F_2^{\lambda i}(\tau) - t_1^{\lambda i'}(\tau) F_1^{\lambda i}(\tau) \right) \end{aligned}$$

$$L_{jj'}^{\lambda i(\pm)}(\tau) = v_{jj'}^{(-)} + (z_-^{\lambda i}(\tau) \pm z_+^{\lambda i}(\tau)) \left(u_{jj'}^{(-)} - u_{jj'}^{(+)} \right)$$

$$\mathcal{Y}_\tau^{\lambda i} = \frac{2(2\lambda + 1)^2}{\left(D_0^{\lambda i}(\tau) \left(\kappa_0^{(\lambda, ph)} + \kappa_1^{(\lambda, ph)} \right) + D_0^{\lambda i}(-\tau) \left(\kappa_0^{(\lambda, ph)} - \kappa_1^{(\lambda, ph)} \right) \right)^2}$$

$$z_n^{\lambda i}(\tau) = \frac{D_n^{\lambda i}(\tau) \left(\kappa_0^{(\lambda, pp)} + \kappa_1^{(\lambda, pp)} \right) + D_n^{\lambda i}(-\tau) \left(\kappa_0^{(\lambda, pp)} - \kappa_1^{(\lambda, pp)} \right)}{D_0^{\lambda i}(\tau) \left(\kappa_0^{(\lambda, ph)} + \kappa_1^{(\lambda, ph)} \right) + D_0^{\lambda i}(-\tau) \left(\kappa_0^{(\lambda, ph)} - \kappa_1^{(\lambda, ph)} \right)}$$

$$n = \{+, -\}$$

$$t_{n_1}^{\lambda i}(\tau) = \frac{F_{n_1}^{\lambda i}(\tau) \left(\kappa_0^{(\lambda, pp)} + \kappa_1^{(\lambda, pp)} \right) + F_{n_1}^{\lambda i}(-\tau) \left(\kappa_0^{(\lambda, pp)} - \kappa_1^{(\lambda, pp)} \right)}{D_0^{\lambda i}(\tau) \left(\kappa_0^{(\lambda, ph)} + \kappa_1^{(\lambda, ph)} \right) + D_0^{\lambda i}(-\tau) \left(\kappa_0^{(\lambda, ph)} - \kappa_1^{(\lambda, ph)} \right)}$$

$$n_1 = \{0, 1, 2\}$$

$$D_0^{\lambda i}(\tau) = \sum_{jj'}^{\tau} f_{jj'}^{\lambda} (1 - q_{jj'}) u_{jj'}^{(+)} (\psi_{jj'}^{\lambda i} + \varphi_{jj'}^{\lambda i})$$

$$D_{\pm}^{\lambda i}(\tau) = \sum_{jj'}^{\tau} f_{jj'}^{\lambda} (1 - q_{jj'}) u_{jj'}^{(\pm)} (\psi_{jj'}^{\lambda i} \mp \varphi_{jj'}^{\lambda i})$$

$$F_0^{\lambda i}(\tau) = \sum_{jj'}^{\tau} f_{jj'}^{\lambda} (1 - q_{jj'}) u_{jj'}^{(+)} \psi_{jj'}^{\lambda i}$$

$$F_1^{\lambda i}(\tau) = \sum_{jj'}^{\tau} f_{jj'}^{\lambda} (1 - q_{jj'}) \left(v_{jj'}^{(+)} - v_{jj'}^{(-)} \right) \psi_{jj'}^{\lambda i}$$

$$F_2^{\lambda i}(\tau) = \sum_{jj'}^{\tau} f_{jj'}^{\lambda} (1 - q_{jj'}) \left(v_{jj'}^{(+)} - v_{jj'}^{(-)} \right) \varphi_{jj'}^{\lambda i}$$

$$v_{jj'}^{(\pm)} = u_j u_{j'} \pm v_j v_{j'} \quad u_{jj'}^{(\pm)} = u_j v_{j'} \pm v_j u_{j'}$$

Appendix B

The coefficients K have the following form:

$$K^{\lambda}(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda_1 i_1, \lambda_2 i_2) = \sum_{\mu_1 \mu_2 \mu_1' \mu_2'} C_{\lambda_1 \mu_1 \lambda_2 \mu_2}^{\lambda \mu} C_{\lambda_1' \mu_1' \lambda_2' \mu_2'}^{\lambda \mu} \times \\ K(\lambda_2' \mu_2' i_2', \lambda_1' \mu_1' i_1' \mid \lambda_1 \mu_1 i_1, \lambda_2 \mu_2 i_2),$$

$$K^J(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda i, \lambda_2 i_2) = \sqrt{(2\lambda + 1)(2\lambda_2 + 1)(2\lambda' + 1)(2\lambda_2' + 1)} \times$$

$$\sum_{j_1 j_2 j_3 j_4} (1 - q_{j_1 j_2}) (-1)^{j_2 + j_4 + \lambda_2 + \lambda_2' + J} \left(\begin{array}{ccc} j_1 & j_2 & \lambda_2' \\ j_4 & j_3 & \lambda' \\ \lambda & \lambda_2 & J \end{array} \right) \times$$

$$\begin{aligned}
& \left(\psi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_3 j_4}^{\lambda' i'} \psi_{j_1 j_4}^{\lambda i} - \varphi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_3 j_4}^{\lambda' i'} \varphi_{j_1 j_4}^{\lambda i} \right) + \\
(-1)^{j_1+j_2+j_3+j_4+\lambda+\lambda_2+\lambda'+\lambda_2'+J} & \left\{ \begin{array}{ccc} \lambda & \lambda_2 & J \\ j_2 & j_4 & j_3 \end{array} \right\} \left\{ \begin{array}{ccc} \lambda' & \lambda_2' & J \\ j_2 & j_4 & j_1 \end{array} \right\} \times \\
& \left(\varphi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_1 j_4}^{\lambda' i'} \psi_{j_3 j_4}^{\lambda i} - \psi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_1 j_4}^{\lambda' i'} \varphi_{j_3 j_4}^{\lambda i} \right),
\end{aligned}$$

$$\begin{aligned}
K_1^\lambda(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda_1 i_1, \lambda_2 i_2) &= \sum_{\mu_1 \mu_2 \mu_1' \mu_2'} C_{\lambda_1' \mu_1' \lambda_1 - \mu_1}^{\lambda \mu} C_{\lambda_2 \mu_2 \lambda_2' \mu_2'}^{\lambda \mu} \times \\
(-1)^{\lambda_1 - \mu_1} K_1(\lambda_2' \mu_2' i_2', \lambda_1' \mu_1' i_1' \mid \lambda_1 \mu_1 i_1, \lambda_2 \mu_2 i_2),
\end{aligned}$$

$$\begin{aligned}
K_1^J(\lambda_2' i_2', \lambda' i' \mid \lambda i, \lambda_2 i_2) &= \sqrt{(2\lambda+1)(2\lambda_2+1)(2\lambda'+1)(2\lambda_2'+1)} \times \\
& \sum_{j_1 j_2 j_3 j_4} (1 - q_{j_3 j_4}) (-1)^{j_1+j_3+\lambda+\lambda_2'} \left\{ \begin{array}{ccc} \lambda_2 & \lambda_2' & J \\ j_4 & j_2 & j_3 \end{array} \right\} \times \\
\left((-1)^J \left\{ \begin{array}{ccc} \lambda' & \lambda & J \\ j_2 & j_4 & j_1 \end{array} \right\} \left(\psi_{j_4 j_1}^{\lambda' i'} \psi_{j_2 j_1}^{\lambda i} \varphi_{j_2 j_3}^{\lambda_2 i_2} \psi_{j_4 j_3}^{\lambda_2' i_2'} - \varphi_{j_4 j_1}^{\lambda' i'} \varphi_{j_2 j_1}^{\lambda i} \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_4 j_3}^{\lambda_2' i_2'} \right) + \right. \\
& \left. \left\{ \begin{array}{ccc} \lambda' & \lambda & J \\ j_4 & j_2 & j_1 \end{array} \right\} \left(\psi_{j_2 j_1}^{\lambda' i'} \psi_{j_4 j_1}^{\lambda i} \psi_{j_2 j_3}^{\lambda_2 i_2} \varphi_{j_4 j_3}^{\lambda_2' i_2'} - \varphi_{j_2 j_1}^{\lambda' i'} \varphi_{j_4 j_1}^{\lambda i} \varphi_{j_2 j_3}^{\lambda_2 i_2} \psi_{j_4 j_3}^{\lambda_2' i_2'} \right) \right),
\end{aligned}$$

$$\begin{aligned}
K_2^\lambda(\lambda_2' i_2', \lambda_1' i_1' \mid \lambda_1 i_1, \lambda_2 i_2) &= \sum_{\mu_1 \mu_2 \mu_1' \mu_2'} C_{\lambda_1 \mu_1 \lambda_2 \mu_2}^{\lambda \mu} C_{\lambda_1' - \mu_1' \lambda_2' \mu_2'}^{\lambda \mu} \times \\
(-1)^{\lambda_1 - \mu_1' - 1} K_2(\lambda_2' \mu_2' i_2', \lambda_1' \mu_1' i_1' \mid \lambda_1 \mu_1 i_1, \lambda_2 \mu_2 i_2),
\end{aligned}$$

$$\begin{aligned}
K_2^J(\lambda_2' i_2', \lambda' i' \mid \lambda i, \lambda_2 i_2) &= \sqrt{(2\lambda+1)(2\lambda_2+1)(2\lambda'+1)(2\lambda_2'+1)} \times \\
& \sum_{j_1 j_2 j_3 j_4} (1 - q_{j_1 j_2}) (-1)^{j_2+j_4+\lambda_2+\lambda_2'+J} \left(\left\{ \begin{array}{ccc} j_1 & j_2 & \lambda_2' \\ j_4 & j_3 & \lambda' \\ \lambda & \lambda_2 & J \end{array} \right\} \times \right. \\
& \left. \left(\psi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_3 j_4}^{\lambda' i'} \psi_{j_1 j_4}^{\lambda i} - \varphi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_3 j_4}^{\lambda' i'} \varphi_{j_1 j_4}^{\lambda i} \right) + \right. \\
(-1)^{j_1+j_2+j_3+j_4+\lambda+\lambda_2+\lambda'+\lambda_2'+J} & \left\{ \begin{array}{ccc} \lambda & \lambda_2 & J \\ j_2 & j_4 & j_3 \end{array} \right\} \left\{ \begin{array}{ccc} \lambda' & \lambda_2' & J \\ j_2 & j_4 & j_1 \end{array} \right\} \times \\
& \left. \left(\varphi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_1 j_4}^{\lambda' i'} \psi_{j_3 j_4}^{\lambda i} - \psi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_1 j_4}^{\lambda' i'} \varphi_{j_3 j_4}^{\lambda i} \right) \right),
\end{aligned}$$

$$K_4^\lambda(\lambda_2' i_2', \lambda_1' i_1' | \lambda_1 i_1, \lambda_2 i_2) = \sum_{\mu_1 \mu_2 \mu_1' \mu_2'} C_{\lambda_1 \mu_1 \lambda_2 - \mu_2}^{\lambda \mu} C_{\lambda_1' - \mu_1' \lambda_2' \mu_2'}^{\lambda \mu} \times \\ (-1)^{\lambda_1' - \mu_1' + \lambda_2 - \mu_2 - 1} K_4(\lambda_2' \mu_2' i_2', \lambda_1' \mu_1' i_1' | \lambda_1 \mu_1 i_1, \lambda_2 \mu_2 i_2),$$

$$K_4^J(\lambda_2' i_2', \lambda_1' i_1' | \lambda i, \lambda_2 i_2) = \sqrt{(2\lambda + 1)(2\lambda_2 + 1)(2\lambda' + 1)(2\lambda_2' + 1)} \times \\ \sum_{j_1 j_2 j_3 j_4} (1 - q_{j_1 j_2}) (-1)^{j_2 + j_4 + \lambda_2 + \lambda_2' + J} \left(\begin{array}{ccc} j_1 & j_2 & \lambda_2' \\ j_4 & j_3 & \lambda' \\ \lambda & \lambda_2 & J \end{array} \right) \times \\ \left(\psi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_3 j_4}^{\lambda' i_1'} \psi_{j_1 j_4}^{\lambda i} - \varphi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_3 j_4}^{\lambda' i_1'} \varphi_{j_1 j_4}^{\lambda i} \right) + \\ (-1)^{j_1 + j_2 + j_3 + j_4 + \lambda + \lambda_2 + \lambda' + \lambda_2' + J} \left\{ \begin{array}{ccc} \lambda & \lambda_2 & J \\ j_2 & j_4 & j_3 \end{array} \right\} \left\{ \begin{array}{ccc} \lambda' & \lambda_2' & J \\ j_2 & j_4 & j_1 \end{array} \right\} \times \\ \left(\varphi_{j_1 j_2}^{\lambda_2' i_2'} \psi_{j_3 j_2}^{\lambda_2 i_2} \varphi_{j_1 j_4}^{\lambda' i_1'} \psi_{j_3 j_4}^{\lambda i} - \psi_{j_1 j_2}^{\lambda_2' i_2'} \varphi_{j_3 j_2}^{\lambda_2 i_2} \psi_{j_1 j_4}^{\lambda' i_1'} \varphi_{j_3 j_4}^{\lambda i} \right).$$

References

- [1] D.J.Rowe, "Nuclear Collective Motion, Models and Theory" (Barnes and Noble, 1970).
- [2] V.G. Soloviev, "Theory of Complex Nuclei" (Pergamon Press, Oxford, 1976).
- [3] A. Bohr and B. Mottelson, "Nuclear Structure" **vol.2** (Benjamin, New York, 1975).
- [4] P.Ring and P.Schuck, "The Nuclear Many Body Problem" (Springer, Berlin, 1980).
- [5] V.G. Soloviev, "Theory of Atomic Nuclei: Quasiparticles and Phonons" (Institute of Physics, Bristol and Philadelphia, 1992).
- [6] M.Yeh et al., Phys. Rev. Lett. **76** (1996) 1208; Phys. Rev. **C57** (1998) R2085.
- [7] H. Emling, Prog. Part. Nucl. Phys. **33** (1994) 729.

- [8] T. Aumann, P.F. Bortignon and H. Emling, *Annu. Rev. Nucl. Part. Sci.* **48** (1998) 351.
- [9] K. Hara, *Progr. Theor. Phys.* **32** (1964) 88.
- [10] K. Ikeda, T. Udagawa and H. Yamaura, *Prog. Theor. Phys.* **33** (1965) 22.
- [11] D.J. Rowe, *Revs. Mod. Phys.* **40** (1968) 153.
- [12] D.J. Rowe, *Phys. Rev.* **175** (1968) 1283.
- [13] J. da Providencia, *Nucl. Phys.* **A108** (1968) 589.
- [14] R.V. Jolos and W. Rybarska-Nawrocka, *Z. Physik* **A296** (1980) 73.
- [15] W. Nawrocka-Rybarska, O. Stoyanova and Ch. Stoyanov, *Yad. Fiz.* **33** (1980) 1494.
- [16] H. Lenske and J. Wambach, *Phys. Lett.* **B249** (1990) 377.
- [17] D. Karadjov, V.V. Voronov and F. Catara, *Phys. Lett.* **B306** 197 (1993).
- [18] D. Karadjov, V.V. Voronov and F. Catara, *J.Phys.* **G20** (1994) 1431.
- [19] J. Toivanen and J. Suhonen, *Phys. Rev. Lett.* **75** (1995) 410
- [20] J. Dukelsky and P. Schuck, *Phys. Lett.* **B387** (1996) 233.
- [21] F. Catara, G. Piccitto, M. Sambataro and N. Van Giai, *Phys. Rev.* **B54** (1996) 17 536.
- [22] J.Dukelsky, G.Ropke and P.Schuck, *Nucl. Phys.* **A628** (1998) 17.
- [23] F.Krmpotic, E.J.V. de Passos, D.S.Delion, J.Dukelsky and P.Schuck, *Nucl. Phys.* **A637** (1998) 295.
- [24] D. Karadjov, V.V. Voronov, F. Catara, M. Grinberg and A.P. Severyukhin, *Nucl. Phys.* **A643** (1998) 259.
- [25] A.A. Raduta, C.M. Raduta, A.Faessler and W.A. Kaminski, *Nucl. Phys.* **A654** (1998) 497.
- [26] A.N. Storozhenko and A.I. Vdovin, *Eur. Phys. J.* **5** (1999) 263.

- [27] A.P. Severyukhin, V.V. Voronov and D. Karadjov, *Izv. RAN. ser. fiz.* **64** (2000) 2243 (Preprint JINR P4-99-121, Dubna, 1999).
- [28] F. Simkovic, A.A. Raduta, M. Veselsky and A. Faessler, *Phys. Rev.* **C61** (2000) 044319.
- [29] V.V. Voronov, D. Karadjov, F. Catara and A.P. Severyukhin *Phys. of Part. & Nucl.*, **31** (2000) 905.
- [30] A.I. Vdovin and V.G. Soloviev, *Phys. of Part. & Nucl.*, **14** (1983) 99.
- [31] V.V. Voronov and V.G. Soloviev *Phys. of Part. & Nucl.*, **14** (1983) 583.
- [32] D. Karadjov, V.V. Voronov and F. Catara, in *Proc. of 5th Int. Spring Seminar on Nuclear Structure, Ravello 1995*, World Scientific, Singapore, 1996, p. 215.
- [33] D. Karadjov, V.V. Voronov, G.Kyrchev and V.Paar, Preprint JINR E4-90-98, Dubna, 1990.
- [34] V.V. Voronov and V.G. Soloviev, *Theor. Math. Phys.* **57** (1983) 75.
- [35] P.F.Bortignon, R.A.BrogliA, D.R.Bes and R.Liotta, *Phys.Rep.* **30** (1977) 305.
- [36] D.R.Bes, G.G.Dussel, R.A.BrogliA, R.Liotta and B.R.Mottelson, *Phys. Lett.* **B52** (1974) 253.
- [37] I.Hamamoto, *Phys.Rep.* **10** (1974) 63.
- [38] D.R.Bes, R.A.BrogliA, G.G.Dussel, R.J.Liotta and H.M.Sofia, *Nucl. Phys.* **A260** (1976) 1 and 27.
- [39] I.Hamamoto, *Phys. Rev.* **C60** (1999) 054320.
- [40] G.F.Bertsch, P.F.Bortignon and K.Hagino, *Nucl. Phys.* **A657** (1999) 59.

Received by Publishing Department
on October 26, 2000.

Северюхин А.П., Воронов В.В., Караджов Д.

E4-2000-257

Корреляции в основном состоянии и ангармоничность вибраций

Представлен метод, позволяющий согласованно трактовать корреляции в основном состоянии за пределами применимости приближения случайных фаз. При этом одновременно учитывается влияние этих корреляций на спаривание и связь со сложными конфигурациями. Выведена общая система нелинейных уравнений для квазичастично-фононной модели (КФМ). Показано, что эта система уравнений содержит как частный случай уравнения КФМ, полученные ранее. Найдены новые поправки, возникающие из-за учета принципа Паули, которые приводят к сдвигам двухфотонных конфигураций. Обсуждается взаимосвязь между обобщенными уравнениями КФМ и теорией ядерных полей.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2000

Severyukhin A.P., Voronov V.V., Karadjov D.

E4-2000-257

Ground State Correlations and Anharmonicity of Vibrations

A consistent treatment of the ground state correlations beyond the random phase approximation including their influence on the pairing and phonon-phonon coupling in nuclei is presented. A new general system of nonlinear equations for the quasiparticle phonon model (QPM) is derived. It is shown that this system contains as a particular case all equations derived for the QPM early. New additional Pauli principle corrections resulting in the anharmonical shifts of energies of the two-phonon configurations are found. A correspondence between the generalized QPM equations and the nuclear field theory is discussed.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2000

Макет Т.Е.Попеко

Подписано в печать 14.11.2000
Формат 60 × 90/16. Офсетная печать. Уч.-изд. листов 2,56
Тираж 360. Заказ 52350. Цена 3 р.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области