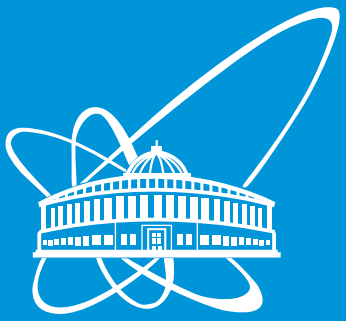

ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4-2000-267

E.B.Balbutsev

LARGE AMPLITUDE MOTION IN THE SUZUKI MODEL

Submitted to «Ядерная физика»

2000

1 Introduction

One-dimensional harmonic oscillator model with a monopole-monopole residual interaction was suggested by Suzuki [1] in 1973 year. Using a mean field approach and a small amplitude approximation he gave the qualitative explanation of the nature of the recently discovered giant monopole resonance. However, the main interest in this model is connected with the fact that it has the exact solution. It is known that in a Hartree approximation the model is reduced to a time dependent Schrödinger equation with a harmonic oscillator Hamiltonian, the frequency of which is time dependent. Its analytical solution has been found by Popov and Perelomov [2]. So we have here the rare example of the nonlinear problem having the analytical solution. That is why the Suzuki model is widely used as a test for different approximate methods to describe the large amplitude nuclear collective motion [3, 4, 5]. The first attempt to treat this model without a small amplitude approximation has been made by Kirson [6] who gave its algebraic analysis and found numerically its "exact" solution. The interest to investigate a collective motion going beyond the usual RPA (small amplitude approach) has been spurred after the experimental discovery of high-energy structures in heavy-ion grazing collisions and their interpretation in terms of multiphonon excitations of giant quadrupole resonances [7, 8, 9]. We consider this model with the aim to verify the possibilities of the method of the Wigner Function Moments (WFM) in studying the large amplitude motion and nonlinear effects accompanying it. The simplicity of the model allows one to observe the appearance of the anharmonicity in the collective spectra and clarify some problems of quantization of classical equations of motion for the collective variables describing giant resonances. Besides, using the WFM method one is able to perform more extensive analysis of the collective aspects of the Suzuki model.

2 Formulation of the method

The basis of our method for the description of collective nuclear dynamics is the equation of motion for the one-body density matrix $\rho(\mathbf{r}_1, \mathbf{r}_2, t) = \langle \mathbf{r}_1 | \hat{\rho}(t) | \mathbf{r}_2 \rangle$:

$$i\hbar \frac{\partial \hat{\rho}}{\partial t} = \left[\hat{H}, \hat{\rho} \right], \tag{1}$$

where $\hat{H}$ is the self-consistent one-body Hamiltonian depending implicitly on the density matrix.

This equation is modified by applying the Wigner transform of the density matrix [10]

$$f(\mathbf{r}, \mathbf{p}, t) = \int d^3s \, \exp(-i\mathbf{p} \cdot \mathbf{s}/\hbar) \rho(\mathbf{r} + \frac{\mathbf{s}}{2}, \mathbf{r} - \frac{\mathbf{s}}{2}, t) \tag{2}$$

and of the Hamiltonian

$$H^W(\mathbf{r}, \mathbf{p}) = \int d^3s \, \exp(-i\mathbf{p} \cdot \mathbf{s}/\hbar) \langle \mathbf{r} + \frac{\mathbf{s}}{2} \left| \hat{H} \right| \mathbf{r} - \frac{\mathbf{s}}{2} \rangle. \tag{3}$$

Using (2,3) one arrives at [11, 12]

$$\frac{\partial f}{\partial t} = \frac{2}{\hbar} \sin \left(\frac{\hbar}{2} (\nabla_{\mathbf{r}}^H \cdot \nabla_{\mathbf{p}}^f - \nabla_{\mathbf{p}}^H \cdot \nabla_{\mathbf{r}}^f) \right) H^W f, \tag{4}$$

where the upper index on the nabla operator stands for the function on which this operator acts. The right-hand side is just the brief notation of the infinite series corresponding to the expansion of the sine-function. When one takes into account only the first term of this expansion, one obtains the equation equivalent to the Vlasov equation for the distribution function. If the Hamiltonian is a sum of a kinetic term and a local potential $V(\mathbf{r})$, its Wigner transform is just the classical version of the same Hamiltonian

$$H^W = p^2/2m + V(\mathbf{r}). \tag{5}$$

Then, equation (4) becomes

$$\frac{\partial f}{\partial t} + \frac{1}{m} \mathbf{p} \cdot \nabla_{\mathbf{r}} f = \frac{2}{\hbar} \sin \left(\frac{\hbar}{2} \nabla_{\mathbf{r}}^V \cdot \nabla_{\mathbf{p}}^f \right) V f. \tag{6}$$

Now we apply the WFM method to derive a closed system of dynamical equations for Cartesian tensors of second rank. This method has been suggested in [13] and is described in detail in ref. [14]. Its idea is based on the virial theorems of Chandrasekhar and Lebovitz [15]. So without going into details we integrate equation (6) over the phase space $\{\mathbf{p},\mathbf{r}\}$ with the weights x_ix_j, p_ix_j, p_ip_j to get the following system:

$$\begin{aligned}
\frac{d}{dt}J_{ij}(t) - \frac{1}{m}(L_{i,j}+L_{j,i}) &= 0,\\
\frac{1}{2}\frac{d}{dt}L_{i,j}(t) + \frac{1}{2}\int d\{\mathbf{p},\mathbf{r}\}\, x_i\frac{\partial V}{\partial x_j}f(\mathbf{r},\mathbf{p},t) - \Pi_{ij}(t) &= 0,\\
\frac{d}{dt}\Pi_{ij}(t) + \frac{1}{2m}\int d\{\mathbf{p},\mathbf{r}\}\, \left[p_i\frac{\partial V}{\partial x_j}\right]_{ij} f(\mathbf{r},\mathbf{p},t) &= 0, \qquad (7)
\end{aligned}$$

where $\int d\{\mathbf{p},\mathbf{r}\} \equiv 4/(2\pi\hbar)^3 \int d^3p \int d^3r$ and $[\ldots]_{ij}$ means symmetrization with respect to the indices i and j ($[a_ib_j]_{ij} = a_ib_j + a_jb_i$). We have introduced the notation

$$J_{ij}(t) = \int d\{\mathbf{p},\mathbf{r}\}\, x_ix_jf(\mathbf{r},\mathbf{p},t)$$

for an inertia tensor,

$$L_{i,j}(t) = \int d\{\mathbf{p},\mathbf{r}\}\, x_ip_jf(\mathbf{r},\mathbf{p},t)$$

for a mixed momentum-position tensor and

$$\Pi_{ij}(t) = \frac{1}{2m}\int d\{\mathbf{p},\mathbf{r}\}\, p_ip_jf(\mathbf{r},\mathbf{p},t)$$

for the integral kinetic energy tensor.

We thus have derived a system of three dynamic equations for three collective variables $J_{ij}(t)$, $L_{i,j}(t)$ and $\Pi_{ij}(t)$. It is necessary to stress that these equations are exact because up to this moment we have not made any approximations. To close this system of equations, one needs to represent the integrals involving derivatives of the potential $V(\mathbf{r})$ in terms of the three variables mentioned. This problem can be solved rigorously in the case of V with quadratic coordinate dependence (which is the subject of this paper).

3 Suzuki model

The microscopic Hamiltonian of the Suzuki model [1] is

$$H = \sum_{i=1}^{A}(\frac{p_i^2}{2m} + \frac{1}{2}m\omega^2 x_i^2) + \frac{1}{2}\kappa\sum_{i\neq j}^{A}(x_i^2 - x_0^2/A)(x_j^2 - x_0^2/A), \tag{8}$$

where x_0^2 is the value of the tensor J_{11} for an oscillator ground state. Usually, it is studied in a Hartree approximation. The time dependent mean field of the model is

$$V(x,t) = \frac{m}{2}\omega^2 x^2 + \kappa(J - x_0^2)(x^2 - x_0^2/A), \tag{9}$$

where $J = J_{11}(t)$. It looks like the harmonic oscillator potential with the time dependent frequency $\omega^2(t) = \omega^2 + \frac{2}{m}\kappa(J - x_0^2)$. The exact solution of the Schrödinger equation with such a potential has been found by Popov and Perelomov [2]. Roughly speaking, it is the usual oscillator wave function whose arguments are modified by the linear independent solutions Z_1 and Z_2 of the classical equation

$$\ddot{Z} + \omega^2(t)Z = 0. \tag{10}$$

One can write Z_1 and Z_2 in the form

$$Z_1 = r(t)e^{i\gamma(t)}, \quad Z_2 = Z_1^*,$$

with $r(t)$ and $\gamma(t)$ obeying the differential equations

$$\begin{aligned} \ddot{r} - \frac{W^2}{r^3} + \omega^2(t)r &= 0, \\ \dot{\gamma} - W/r^2 &= 0. \end{aligned} \tag{11}$$

The constant W, being proportional to the wronskian of eq. (10)

$$2iW = \dot{Z}_1 Z_2 - \dot{Z}_2 Z_1,$$

is determined by the Initial Conditions (IC).

Suzuki estimated the energy of the giant monopole resonance in a small amplitude approximation, neglecting the nonlinear effects of the model. They were

considered in the papers [3, 4, 5] where this model was used to test the different approaches to the investigation of the large amplitude collective motion. We pursue just the same goal with our method.

3.1 Equations of motion

For the potential (9) equations (7) become

$$\begin{aligned} \frac{m}{4}\ddot{J} + J\left[\frac{m}{2}\omega^2 + \kappa(J - J_0)\right] - \Pi &= 0, \\ \dot{\Pi} + \dot{J}\left[\frac{m}{2}\omega^2 + \kappa(J - J_0)\right] &= 0 \end{aligned} \tag{12}$$

with $J_0 \equiv x_0^2$, $\Pi = \Pi_{11}(t)$. The time dependence of tensors is omitted for the sake of simplicity. The second equation of this system is reduced to the integral of motion

$$\Pi + \frac{m}{2}\omega^2 J + \frac{\kappa}{2}(J - J_0)^2 = E, \tag{13}$$

whose physical meaning is that the total energy of the system is a conserved quantity. It is indeed easy to see that it is equal to the Hartree-Fock average of the microscopic Hamiltonian (8), i.e. $E = \langle \Psi \mid H \mid \Psi \rangle$. Another integral can be found by multiplying the second equation of (12) by J and subtracting it from the first equation multiplied by $\dot{J}$

$$J(t)\Pi(t) - \frac{m}{8}\dot{J}(t)^2 = c, \tag{14}$$

where the constant c is determined by IC. With the help of equation (14) one is able to reduce the system (12) to the single equation

$$\frac{m}{4}\ddot{J} + \frac{m}{2}\omega^2 J + \kappa(J - J_0)J - \frac{c}{J} - \frac{m}{8J}\dot{J}^2 = 0. \tag{15}$$

Changing the variable $J = J_0 r^2$ this equation can be written in the form

$$\ddot{r} + \omega^2(t)r - \omega^2\frac{c'}{r^3} = 0, \tag{16}$$

where $\omega^2(t) = \omega^2[1 + 2\bar{\kappa}(r^2 - 1)]$, $\bar{\kappa} = \kappa\dfrac{J_0}{m\omega^2}$ and $c' = 2c/(m\omega^2 J_0^2)$. One recognizes immediately in this equation one of the above written equations (11), the constants

W and c' being linked by the relation $W^2 = \omega^2 c'$. If one supposes here $c' = 1$ this equation becomes identical to the corresponding equations of refs. [3, 4]. Such a choice of c' was dictated by their choice of the initial condition $\omega(t = -\infty) = \omega$ which is not accidental - just this initial condition has been used in [2] to find the exact solution of the problem. The analysis of the c'-dependence of equation (16) allows one to find out new properties of the model.

Solving equations (13) and (14) with respect to Π, one can rewrite the energy in a more traditional form, as a sum of kinetic and potential energies

$$E = \frac{m}{8J}\dot{J}^2 + \frac{c}{J} + \frac{m}{2}\omega^2 J + \frac{\kappa}{2}(J - J_0)^2 \tag{17}$$

or in terms of r

$$E = \frac{m}{2}J_0\{\dot{r}^2 + \omega^2[r^2 + c'/r^2 + \bar{\kappa}(r^2 - 1)^2]\}. \tag{18}$$

The r-dependence of the potential

$$V(r) = \frac{m}{2}\omega^2 J_0[r^2 + c'/r^2 + \bar{\kappa}(r^2 - 1)^2] \tag{19}$$

for various values of $\bar{\kappa}$ is schematically illustrated in Fig.1.

3.2 Equilibrium state and small amplitude approximation

By definition, at equilibrium the kinetic energy is equal to zero and the potential energy is at its minimum. The equation determining the extremums of $V(r)$ is

$$g(r) - c' = 0, \tag{20}$$

where $g(r) = r^4[1 + 2\bar{\kappa}(r^2 - 1)]$. The function g(r) is sketched in Fig. 2.

It is seen that in the case of $\bar{\kappa} > 0$ the polynom (20) has only one positive root for $c' > 0$ which corresponds to the minimum of the potential (see fig.1). It describes the stable equilibrium state which is more compressed ($J_{eq} < J_0$) than that of the harmonic oscillator when $c' < 1$ and less compressed ($J_{eq} > J_0$) when $c' > 1$. Using an analogy with equilibrium deformation, one can say that the system has positive

static compression for $c' < 1$ and negative static compression (dilatation) for $c' > 1$ if one assumes that the oscillator ground state has zero static compression.

There is no necessity to analyse the situation with $c' < 0$ (see however the next section) because this integral of motion cannot be negative in the state of equilibrium. Really, substituting $\dot{J} = 0$ into (14) we find

$$c_{eq} = J_{eq}\Pi_{eq}, \tag{21}$$

J and Π being positive by definition; hence c_{eq} and c'_{eq} are positive definite.

In the case of $\bar{\kappa} < 0$ the polynom (20) has two positive roots if $0 \leq c' < (1 - 2\bar{\kappa})^3/(27\bar{\kappa}^2)$. The smaller root corresponds to the minimum of the potential well and the bigger one corresponds to the maximum of the barrier. The latter equilibrium state is metastable due to the finite value of the barrier height. For $\bar{\kappa} \leq -1$ the equilibrium state has positive static compression independent of the c' value. For $\bar{\kappa} > -1$ the equilibrium state has positive static compression when $c' < 1$ and negative one when $c' > 1$. The potential has no extrema, when $c' \geq (1 - 2\bar{\kappa})^3/(27\bar{\kappa}^2)$, possessing only an inflection point at $r^2 = (-c'/\bar{\kappa})^{1/3}$.

To find the energy of small vibrations around the equilibrium state, we apply the linearization procedure. Writing equation (16) in terms of the new variable $y = r - r_{eq}$ and neglecting y^2 terms we find

$$\ddot{y} + y\omega^2 \left[1 + 3c'/r_{eq}^4 + 2\bar{\kappa}(3r_{eq}^2 - 1)\right] + \omega^2 \left[r_{eq} - c'/r_{eq}^3 + 2\bar{\kappa}(r_{eq}^3 - r_{eq})\right] = 0. \tag{22}$$

This equation is transformed into

$$\ddot{y} + 4\omega^2[1 + \bar{\kappa}(3r_{eq}^2 - 2)]y = 0 \tag{23}$$

after taking into account eq. (20) satisfied by r_{eq}. The corresponding eigenfrequency is

$$\tilde{\Omega} = 2\omega\sqrt{1 + \bar{\kappa}(3r_{eq}^2 - 2)}. \tag{24}$$

Assuming here $J_0 = 0$, we reproduce the result of the paper [5]

$$\tilde{\Omega} = 2\omega\sqrt{1 + \frac{3\kappa}{m\omega^2}J_{eq}}.$$

Equation (20) is solved elementary when $c' = 1$. One positive extremum lies at $r^2 = 1$. It corresponds to the maximum of the barrier for $\bar{\kappa} < -1$ and to the minimum of the potential for $\bar{\kappa} > -1$. Only this minimum was analysed in [3] and [4]. From formula (24) one gets the corresponding expression for the RPA frequency

$$\tilde{\Omega} = 2\omega\sqrt{1+\bar{\kappa}}. \tag{25}$$

Another positive extremum lies at $r^2 = -(1+\sqrt{1-8\bar{\kappa}})/(4\bar{\kappa})$. It corresponds to the maximum of the barrier for $0 > \bar{\kappa} > -1$ and to the minimum of the potential for $\bar{\kappa} < -1$. The corresponding expression for the RPA frequency is

$$\tilde{\Omega}^2 = \omega^2(1 - 8\bar{\kappa} - 3\sqrt{1-8\bar{\kappa}}). \tag{26}$$

The strength constant $\bar{\kappa} = -1$ is the critical one. With this $\bar{\kappa}$ the potential has neither a minimum nor a maximum and the point $r^2 = 1$ turns out to be its inflection point.

3.3 Analysis of the exact solution

To find an exact expression for the function $J(t)$, it is convenient to use equation (17). Its solution can be expressed in terms of the Jacobian elliptic function [16]

$$J(t) = \eta_1 + (\eta_2 - \eta_1)\mathrm{sn}^2(\bar{\omega}t + \chi). \tag{27}$$

Here $\bar{\omega} = \omega\sqrt{(\eta_1 - \eta_3)\kappa/m}$ and η_i are the real roots of the polynom

$$P(J) = -J^3 + a_2J^2 + a_1J + a_0 \tag{28}$$

with $a_2 = 2J_0 - m\omega^2/\kappa$, $a_1 = 2E/\kappa - J_0^2$, $a_0 = -2c/\kappa$. The roots satisfy the condition $\eta_1 > \eta_2 > \eta_3$ for $\kappa > 0$ and $\eta_1 < \eta_2 < \eta_3$ for $\kappa < 0$. The phase χ is determined by IC. The function $\mathrm{sn}(\phi)$ is periodic with the period $\Delta\phi = 4\mathbf{K}$ where

$$\mathbf{K}(k) = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - k^2\sin^2\phi}} \tag{29}$$

is a complete elliptic integral of the first kind with $k^2 = \frac{\eta_2 - \eta_1}{\eta_3 - \eta_1}$. There exists an analytical expression for the Fourier expansion of this function [17]

$$\mathrm{sn}\bar{\omega}t = \frac{2\pi}{k\mathbf{K}} \sum_{n=1}^{\infty} \frac{q^{n-1/2}}{1-q^{2n-1}} \sin(2n-1)\frac{\bar{\omega}\pi}{2\mathbf{K}} t.$$

Here $q = \exp(-\pi\mathbf{K}'/\mathbf{K})$, $\mathbf{K}'(k) = \mathbf{K}(\sqrt{1-k^2})$. This formula involves only the frequencies proportional to odd numbers of the basic frequency $\Omega = \frac{\bar{\omega}\pi}{2\mathbf{K}}$. It is obvious that sn^2 includes the frequencies $n\Omega$ with n even only. So the Fourier expansion of the function $J(t)$ will involve only one basic frequency 2Ω and its satellites 4Ω, 6Ω, etc. Numerically, the frequency $2\Omega = \frac{\bar{\omega}\pi}{\mathbf{K}}$ can be rather different from the result of the harmonic problem (24). So the effect of including the anharmonic term $\sim J^2(t)$ into the system (12) is the transformation of the basic frequency $\tilde{\Omega}$ into 2Ω and the appearance of satellites $n2\Omega$. The equidistance of such a spectrum is evident, characteristic of a bounded classical motion.

It is necessary to note the dependence of Ω on IC (also characteristic of a classical motion). The roots of the polynomial (28) depend on c, E. These constants together with the phase χ are determined by $J(0)$, $\dot{J}(0)$ and $\Pi(0)$. Examples of such a dependence are demonstrated in Table 1.

A very interesting situation arises at a sufficiently large value of $\dot{J}(0)$ when the constant c' becomes negative. If $\bar{\kappa} > 1/2$ and $0 > c' > (1-2\bar{\kappa})^3/(27\bar{\kappa}^2)$, the polynom (20) has two positive roots (Fig.2) with $r^2 < 1$: the bigger one corresponds to the minimum of the potential well and the smaller one corresponds to the top of the barrier. In this case the time dependent single particle potential (9) is always repulsive. This fact becomes obvious after rewriting (9) as

$$V(x,t) = \frac{m}{2}\omega^2[1 + 2\bar{\kappa}(r^2-1)]x^2 - m\omega^2\bar{\kappa}(r^2-1)x_0^2/A$$

and noting that $[1 + 2\bar{\kappa}(r^2-1)] < 0$ for the considered area $0 < r^2 < \frac{2\bar{\kappa}-1}{2\bar{\kappa}}$. Nevertheless, the system can possess a collective dynamic potential whose bottom is lower than that of the equilibrium state. The corresponding condition $V_{bot} < V_{eq}$

can be written as

$$2r_{bot}^2 + \bar{\kappa}(3r_{bot}^2 - 1)(r_{bot}^2 - 1) < 2r_{eq}^2 + \bar{\kappa}(3r_{eq}^2 - 1)(r_{eq}^2 - 1),$$

where r_{bot} is the position of the minimum of the dynamic potential. After simple transformations one gets

$$(r_{bot}^2 - r_{eq}^2)\{2 + \bar{\kappa}[3(r_{bot}^2 + r_{eq}^2) - 4]\} < 0.$$

Taking into account that r_{bot}^2 is determined by $c' < 0$ and r_{eq}^2 is determined by $c' > 0$, it is easy to see from Fig.2 (the case $\bar{\kappa} > 1/2$) that r_{bot}^2 is always smaller than r_{eq}^2. Hence, one has

$$2 + \bar{\kappa}[3(r_{bot}^2 + r_{eq}^2) - 4] > 0.$$

Substituting into this formula the minimal values for r_{bot}^2 and r_{eq}^2

$$r_{bot}^2(min) = \frac{2\bar{\kappa} - 1}{3\bar{\kappa}}, \quad r_{eq}^2(min) = \frac{2\bar{\kappa} - 1}{2\bar{\kappa}},$$

one finally obtains the condition $\bar{\kappa} > 1/2$ that is right our case.

In an analogous way one can derive the condition for the top of the barrier V_{top} to be higher than V_{eq}

$$(r_{top}^2 - r_{eq}^2)\{2 + \bar{\kappa}[3(r_{top}^2 + r_{eq}^2) - 4]\} > 0.$$

It is obvious that $r_{top}^2 < r_{eq}^2$; so one has

$$2 + \bar{\kappa}[3(r_{top}^2 + r_{eq}^2) - 4] < 0.$$

Taking into account that $r_{top}^2 \sim 0$, one obtains the following condition:

$$r_{eq}^2 < \frac{4}{3}\frac{2\bar{\kappa} - 1}{2\bar{\kappa}} = \frac{4}{3}r_{eq}^2(min).$$

Such a potential is shown in Fig.1 by the dashed curve. The eigenfrequencies calculated for this potential well are shown in Table 1. The limits of variation of c' are determined by the input excitation energy E: at some value of c' the energy

$E_{eq} + E$ turns lower than the bottom of the potential well or higher than the top of the potential barrier.

Table 1

Dependence of eigenfrequencies on the initial conditions.

(I) $\bar{\kappa} = 2$, $E_{HF} = 24$ Mev, $r^2_{eq} = 1$. (II) $\bar{\kappa} = 2$, $E_{HF} = 24$ Mev, $r^2_{eq} = 0.752$.

(III) $\bar{\kappa} = -0.5$, $E_{HF} = 9.5$ Mev, $r^2_{eq} = 1$. (IV) $\bar{\kappa} = -2$, $E_{HF} = 15$ Mev, $r^2_{eq} = 0.5$.

I	c'	0	0.01	0.1	0.5	1
	$\hbar\Omega$	9.25	14.13	17.91	21.84	23.98
II	c'	-.0001	-0.04	-0.08	-0.1	-0.11
	$\hbar\Omega$	16.89	15.96	14.55	13.31	12.18
III	c'	0.94	0.945	0.95	1	1.005
	$\hbar\Omega$	5.53	7.04	7.58	9.50	9.60
IV	c'	.47	.5	.7	.75	.755
	$\hbar\Omega$	10.66	14.68	18.90	19.42	19.46

3.4 Quantization

Solving nonlinear equations of motion, one expects to find out anharmonicity effects. We have already observed the main effect of anharmonicity - the satellites of the basic frequency that form the equidistant spectrum. However, such a result is contradictory to the practice of quantum mechanical calculations, where one usually has some deviation from the precise equidistance. Hence, to obtain the anharmonicity of the spectrum, it is necessary to quantize this model.

Its quantization is elementary because we have already the expression for the energy of vibrations (17). Choosing $q = J$ and $p = \dfrac{m\dot{J}}{4J}$ as the canonically conjugate variables, one can represent the Hamilton function in the form

$$H = \frac{p^2}{2m^*} + V(q) \tag{30}$$

with

$$V(q) = \frac{m}{2}\omega^2 q + \frac{c}{q} + \frac{\kappa}{2}(q - J_0)^2, \qquad m^* = \frac{m}{4q}. \tag{31}$$

It is easy to see that equation (15) coincides with the Hamilton equations $\dot{q} = \frac{\partial H}{\partial p}$, $\dot{p} = -\frac{\partial H}{\partial q}$, that justifies our choice of canonical variables.

The quantum Hamiltonian can be produced following the Pauli [18, 19] prescription

$$H = -\frac{\hbar^2}{m}(\frac{\partial}{\partial q} + 2q\frac{\partial^2}{\partial q^2}) + V(q).$$

This operation, however, does not complete the construction of the quantum Hamiltonian because it is necessary to solve the initial condition problem. Our quantum Hamiltonian will contain the constant c which is determined by IC. Thus, the variety of initial conditions of the classical problem will generate a variety of quantum Hamiltonians. However, the Hamiltonian which ideally describes the dynamics of the nucleus should be unique.

We suppose that the solution of this problem can be found by taking into account the principal difference between the classical and quantum descriptions of excitations. Being an integral of motion (energy), the classical Hamiltonian changes each time the initial conditions change. Hence, strictly speaking, all excited states and the equilibrium (ground) state are described by different Hamiltonians. An absolutely different situation prevails in a quantum case. Here all states (ground and excited) are obtained as eigenstates of only one Hamiltonian. The ground state is the only state that is described by the same Hamiltonian in both the cases. So it is natural to use for quantization the classical Hamiltonian, derived for IC, which correspond to the ground state. That means, that for our model we have to take the equilibrium value of the constant c. This statement agrees with the conclusion of A. Klein [5] that "the value of c is related to the equilibrium value of q". Furthermore, it bears a strong resemblance to the stationarity conditions of Kan [20].

Two methods will be used to analyse the spectrum. The first one is the Bohr-Sommerfeld quantization rule

$$\int_{q_1}^{q_2} P(q)\,dq = \pi\hbar(n + \frac{1}{2}), \tag{32}$$

where q_1 and q_2 are the classical turning points, $P(q) = \sqrt{2m^*(E - V)}$.

Another method was suggested by Cambiaggio [21]. Its idea is in the self-consistency prescription: The input energies must coincide with the Fourier spectrum of the action.

The results strongly depend on the values of c_{eq} and $\bar{\kappa}$. In accordance with the results of the previous analysis, three domains of $\bar{\kappa}$ values must be considered separately: $\bar{\kappa} > 0$, $-1 < \bar{\kappa} < 0$ and $\bar{\kappa} < -1$.

Let us consider first the case: $\bar{\kappa} > 0$. The potential well here has infinite walls and a minimum at the point $J = J_0$ (for $c'_{eq} = 1$) which corresponds to an equilibrium state of the harmonic oscillator, i.e. the inclusion of the residual interaction does not change the equilibrium state of the system that is characterized by the inertia tensor $J_{eq} = J_0$ and by the energy $E_{eq} = m\omega^2 J_0$. The spectrum, being infinite, has very small anharmonicity. The calculations with $\bar{\kappa} = 2$ show that the levels E_n are positioned equidistantly with good accuracy up to a rather large n. For example, the difference $E_1 - E_0 = 23.984$ practically coincides with $E_{RPA} = 23.971$ Mev. Small anharmonicity can be noticed at $n \approx 100$. So the difference $E_{101} - E_{100} = 26.017$ Mev demonstrates the anharmonicity $Anh = (E_{101} - E_{100} - E_{RPA})/E_{RPA} \approx 8\%$.

The second case ($-1 < \bar{\kappa} < 0$) is more interesting. Here the potential (31) has a minimum at the same point $J = J_0$ (for $c'_{eq} = 1$) which also corresponds to an equilibrium state of the harmonic oscillator. However, this state is metastable because now the potential has the finite height barrier whose top lies at the point $J = -J_0(1 + \sqrt{1 - 8\bar{\kappa}})/(4\bar{\kappa}) > J_0$. So the inclusion of the residual interaction with $-1 < \bar{\kappa} < 0$ changes the equilibrium state of the system qualitatively without changing its quantitative characteristics J_{eq} and E_{eq}. The barrier height decreases from ∞ to 0 when $\bar{\kappa}$ changes from 0 to -1. Hence, the anharmonicity can be rather large when $\bar{\kappa}$ is close to -1. For example, at $\bar{\kappa} = -0.5, c'_{eq} = 1$ the barrier height is ≈ 50 Mev. The potential well has four bound states, and the deviation of the spectrum from the equidistant one is appreciable right from the beginning (Table 2). Taking $c'_{eq} = 1.05$, one obtains the barrier with the height ≈ 22 Mev.

The potential has only two bound states and the anharmonicity slightly increases (Table 2).

Table 2

The spectra calculated by the Bohr-Sommerfeld (B) and Cambiaggio (C) methods.

(IIa) $c' = 1$, $\bar{\kappa} = -0.5$, $E_{RPA} = 9.786$ Mev, $r^2_{eq} = 1$. (IIb) $c' = 1.05$, $\bar{\kappa} = -0.5$, $E_{RPA} = 8.975$ Mev, $r^2_{eq} = 1.053$. (III) $c' = 1$, $\bar{\kappa} = -2$, $E_{RPA} = 14.891$ Mev, $r^2_{eq} = 0.64$.

		$E_1 - E_0$	$E_2 - E_1$	$E_3 - E_2$	$E_4 - E_3$
IIa	B	9.489	9.151	8.749	8.243
	C	9.496	9.166	8.777	8.299
IIb	B	8.557	8.032	-	-
	C	8.572	8.073	-	-
III	B	14.339	13.686	12.858	11.657
	C	14.355	13.723	12.939	11.870

The third case ($\bar{\kappa} < -1$) is of special interest because here the potential has a maximum at the point $J = J_0$ (for $c'_{eq} = 1$). Its minimum lies at $J = J_{eq} = -J_0(1 + \sqrt{1 - 8\bar{\kappa}})/(4\bar{\kappa}) < J_0$. The well depth (or barrier height) increases from 0 to ∞ when $\bar{\kappa}$ changes from -1 to $-\infty$. Hence, a remarkable anharmonicity can be observed in the vicinity of $\bar{\kappa} = -1$. For example, at $\bar{\kappa} = -2$ the well depth is ≈ 67 Mev. Here there are four bound states and the deviation of the spectrum from the equidistant one is strong, exactly as in the previous case, already for the low lying states (Table 2).

It is seen from the table that the results found by the Bohr-Sommerfeld and Cambiaggio methods are quite close, the difference between them increases together with the anharmonicity. Such a behaviour is naturally explained by the fact that both the methods are approximate ones.

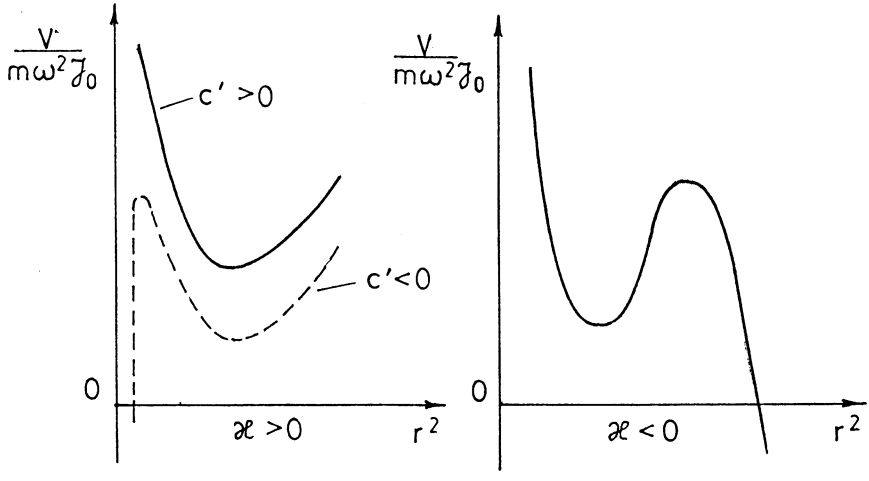

Fig.1. The r-dependence of the potential $V(r) = \frac{m}{2} J_0 \omega^2 [r^2 + c'/r^2 + \bar{\kappa}(r^2-1)^2]$ for various values of $\bar{\kappa}$ and c'.

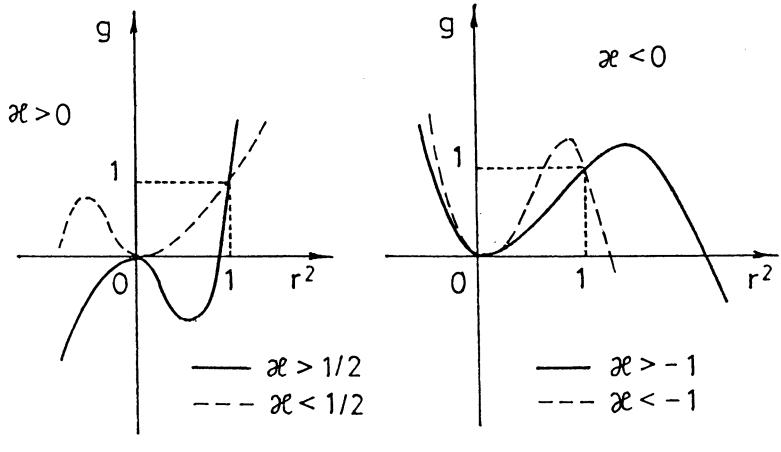

Fig.2. The function $g(r) = r^4[1 + 2\bar{\kappa}(r^2-1)]$ for various values of $\bar{\kappa}$.

4 Conclusion

Let us list the main results of the paper.

The classical and quantum aspects of the analytically solvable one-dimensional monopole model of Suzuki are revisited. The set of nonlinear dynamic equations for monopole collective characteristics of a nucleus is derived from the TDHF equation by using the method of the Wigner function moments. The WFM method reproduces the exact results for the collective properties of the model. It allows one to perform a more extensive analysis of the classical aspects of the problem, to look at the model from new sides (c-dependence of the solutions of the equations of motion). The collective Hamiltonian, which generates these equations, is constructed. It is shown that the anharmonicity of the collective spectrum, being the specific property of quantum systems, cannot be observed in classical ones. The Hamiltonian is quantized by two methods. The choice of the initial conditions, necessary for quantization of the model, is established. The calculations show that the anharmonicity of quantum spectra depends strongly on the strength constant of the residual interaction, being negligible for $\kappa > 0$ and rather big for $\kappa < 0$.

References

[1] Suzuki T.// Nucl. Phys. 1973. A217. 182.

[2] Popov V. S., Perelomov A. M.// JETP. 1969. V. 56. P. 1375.

[3] Blaizot J. P., Schulz H.// Nucl. Phys. 1982. V. A377. P. 237.

[4] Reinhardt H., Schulz H.// Nucl. Phys. 1982. V. A391. P. 36.

[5] Klein A.// Nucl. Phys. 1984. V. A431. P. 90.

[6] Kirson M. W.// Nucl. Phys. 1979. V. A317. P. 388.

[7] Frascaria N.// Nucl. Phys. 1988. V. A482. P. 245c.

[8] Chomaz Ph., Frascaria N.// Phys. Rep. 1995. V. 252. P. 278.

[9] Emling H.// Prog. Part. Nucl. Phys. 1994. V. 33. P. 729.

[10] Wigner E.// Phys. Rev. 1932. V. 40. P. 749.

[11] Ring P., Schuck P.// The Nuclear Many Body Problem (Springer, Berlin, 1980)

[12] Groenewold H. J.// Physica 1946. V. 12. P. 405.

Moyal J. E.// Proc. Cambr. Phil. Soc. 1949. V. 45. P. 99.

[13] Balbutsev E. B. et al.// Phys. Lett. 1981. V. B105. P. 84.

[14] Balbutsev E. B.// Sov. J. Part. Nucl. 1991. V. 22. P. 159.

[15] Chandrasekhar S.// Ellipsoidal Figures of Equilibrium (Yale University Press, New Haven, Conn. 1969)

[16] Smirnov V. I.// Course of higher mathematics, V. 3. Part 2 (Fizmatgiz, Moscow, 1958)

[17] Gradshtein I. S., Ryzhik I. M.// Tables of integrals, sums, series and products (Fizmatgiz, Moscow, 1962)

[18] Pauli V.// Obscie printsipi volnovoi mekhaniki (Gostekhizdat, Moscow, 1947); Handb. d. Phys., Bd. XXIV/1, Berlin, 1933.

[19] Bohr A.// Kong. Danske Vidensk. Selsk., Mat.-fys Medd. 1952. V. 26. N14.

[20] Kit-Keung Kan // Phys. Rev. 1981. V. A24. P. 2831.

[21] Cambiaggio M. C.// in The building blocks of nuclear structure, Amalfi, Italy, 1992. ed. Covello A.(World Scientific, Singapore, 1993) P. 293.

Received by Publishing Department
on November 3, 2000.

Бальбуцев Е.Б.
Е4-2000-267

Движения большой амплитуды в модели Сузуки

Изучаются классические и квантовые аспекты аналитически решаемой одномерной чисто монопольной модели Сузуки, чтобы прояснить проблему квантования классического коллективного движения. Система динамических уравнений для монопольного момента ядра выводится из TDHF уравнения с помощью метода моментов функции Вигнера. Она позволяет описывать монопольные колебания большой амплитуды. Построен и проквантован соответствующий коллективный гамильтониан. Дан детальный анализ ангармоничности коллективного спектра.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2000

Balbutsev E.B.
E4-2000-267

Large Amplitude Motion in the Suzuki Model

The classical and quantum aspects of the analytically solvable one-dimensional pure monopole Suzuki model are studied to clarify the problem of quantization of classical collective motion. A set of dynamic equations for a monopole moment of a nucleus are derived from the TDHF equation using the Wigner function moments. They provide the description of large amplitude monopole vibrations. The corresponding collective Hamiltonian is constructed and quantized. The anharmonicity of the collective spectra is analyzed in detail.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2000

Макет Т.Е.Попеко

Подписано в печать 24.11.2000
Формат 60 × 90/16. Офсетная печать. Уч.-изд. листов 1,8
Тираж 360. Заказ 52361. Цена 2 р. 15 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области