ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

Дубна

E4-2000-276

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RADIATIVE CORRECTIONS TO THE $K^{\pm}_{e3}$ DECAY REVISED

Submitted to «Physics Letters B»

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2000

Introduction

The K_{e3} decay is important since it is the cleanest way to measure the V_{us} matrix element of the CKM matrix. If one uses the current values for V_{ud}, V_{us}, and V_{ub} taken from the PDG then $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ misses unity by 2.2 standard deviations which contradicts the unitarity of the CKM matrix and might indicate physics beyond the Standard Model. The uncertainty brought to the above expression by V_{us} is about the same as uncertainty that comes from V_{ud}, therefore reducing the error in the V_{us} matrix element would reduce substantially the error in the whole unitarity equation. In order to extract the V_{us} matrix element from the K_{e3} decay width with high precision one needs a good estimate of the radiative corrections that in general could be of the order of few percent.

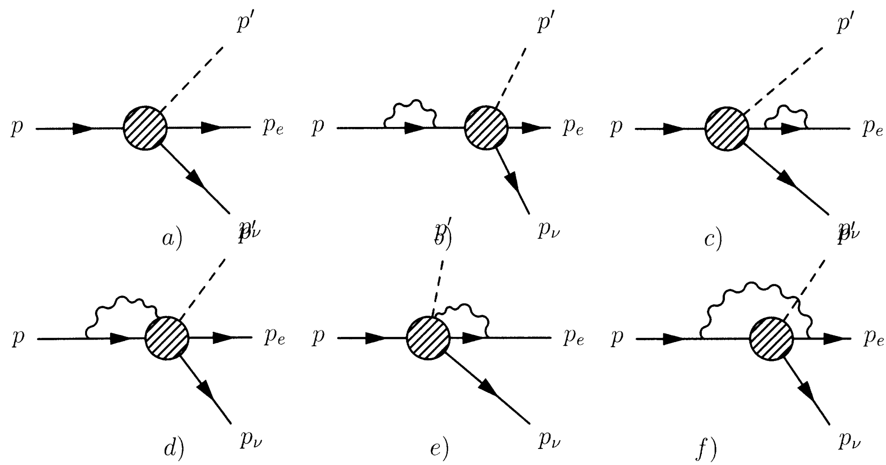

Figure 1: virtual photons

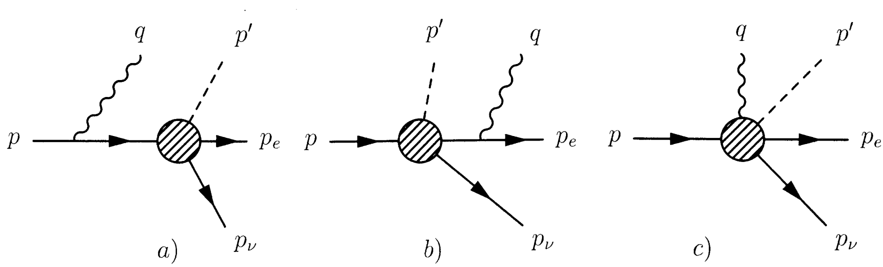

Figure 2: real photons

For corrections due to virtual photons see figure 1, for corrections due to real photons see figure 2.

The calculations of the radiative corrections to the K_{e3} decay were performed independently by E.S.Ginsberg and T.Becherrawy in the late 60's [2, 1]. Their results for corrections to the decay rate, Dalitz plot, pion and positron spectra disagree, in some places quite sharply; for example Ginsberg's correction to the decay rate is -0.45% while that of Becherrawy is -2%. We have decided to perform a new calculation because of the results of new experiments will become available soon and because of the existing descrepancies in the previous calculations. This is the motivation for the present paper.

The lowest order perturbation theory (PT) matrix element of the process

$$K^+(p) \to \pi^0(p') + e^+(p_e) + \nu(p_\nu)$$

has the form

$$M = \frac{G_F}{\sqrt{2}} V_{us}^* F_\nu(t) \bar{u}(p_\nu)\gamma_\nu(1+\gamma_5)v(p_e) \tag{1}$$

where $F_\nu(t) = \frac{1}{\sqrt{2}}(p+p')_\nu f_+(t)$. Dalitz plot density which takes into account the radiative corrections (RC) of the lowest order PT is

$$\frac{d^2\Gamma}{dydz} = \rho(y,z) = \rho_0(y,z)(1+\delta), \tag{2}$$

where

$$\rho_0(y,z) = \frac{M_K^5 G_F^2 |V_{us}|^2}{256\pi^3} |f_+|^2 A_0(y,z),$$

$$A_0(y,z) = 4[(z+y-1)(1-y) - r_\pi] + O(r_e). \tag{3}$$

Here we follow the notation of [4]:

$$r_e \equiv m_e^2/M_K^2, r_\pi \equiv m_\pi^2/M_K^2 \ ; \tag{4}$$

where m_e, m_π, and M_K are the masses of electron, pion, and kaon;

$$y \equiv 2pp_e/M_K^2, z \equiv 2pp'/M_K^2 \ . \tag{5}$$

In the kaon's rest frame, which we'll imply throughout the paper, y and z are the energy fractions of the positron and pion,

$$y = 2E_e/M_K \ , z = 2E_\pi/M_K \ . \tag{6}$$

The physical region for y and z is

$$
\begin{aligned}
2\sqrt{r_e} \leq y \leq 1 + r_e - r_\pi \ , \\
F_1(y) - F_2(y) \leq z \leq F_1(y) + F_2(y) \ , \\
F_1(y) = (2-y)(1+r_e+r_\pi - y)/\left[2(1+r_e-y)\right] \ , \\
F_2(y) = \sqrt{y^2 - 4r_e}(1+r_e-r_\pi-y)/\left[2(1+r_e-y)\right] \ . \qquad (7)
\end{aligned}
$$

For our aims we use the simplified form of physical region (omitting the terms of the order of r_e):

$$1 - y + \frac{r_\pi}{1-y} < z < 1 + r_\pi \ , \quad y_0 < y < 1 - r_\pi \ ; \qquad (8)$$

and choose the values of y and z inside this region with an additional requirement that the value of $A_0/4$ is larger than 0.07 (see table 1). In terms of the kinematic variables the momentum transfer squared between kaon and pion is:

$$t = (p - p')^2 = M_K^2(1 + r_\pi - z)$$

We accept here the following form for the strong interactions induced formfactor $f_+(t)$:

$$f_+(t) = \frac{1}{1 - \frac{t}{M^2} - i\frac{\Gamma}{M}}, \qquad (9)$$

which takes into account the K_+^*-meson intermadiate state with quantum numbers $I(J^P) = \frac{1}{2}(1^-)$, mass of $M = 892MeV$, and the width of $\Gamma = 50MeV$.

Taking into account the accuracy level of 0.1% for determination of ρ/ρ_0 we will drop the terms of order r_e. We will distinguish 3 kinds of contributions to δ: from emission of virtual, soft real, and hard real photons in the rest frame of kaon: $\delta = \delta_V + \delta_S + \delta_H$. Standard calculation (see Appendix A for details) allows one to obtain the contribution from the soft photons δ_S:

$$\delta_S = \frac{\alpha}{\pi}\left[(L_\beta - 2)\ln\frac{2\Delta\varepsilon}{\lambda} + \frac{1}{2}L_\beta - \frac{1}{4}L_\beta^2 + 1 - \frac{\pi^2}{6}\right] \ , \qquad (10)$$

where $L_\beta = 2\ln y + \ln\frac{1}{r_e}$, λ is (fictituous) photon mass and $\Delta\varepsilon$ is the maximal energy (in the rest frame of kaon) of a real soft photon. We imply $\Delta\varepsilon << M_K/2$.

When calculating hard real photons we must distinguish between inner bremsstrahlung (IB) and the structure-dependent (SD) contributions: $\delta_H = D_{IB} + D_{int} + D_{SD}$, where D_{int} is the interference term between the two. The terms D_{int} and D_{SD} we consider in the frame of the chiral perturbation theory (ChPT) to the orders of (p^2) and (p^4) and find their contribution to be at the level of 0.2%. The natural suggestion is that the contribution from SD part in virtual photon emission (which we do not calculate here) is of the same order of magnitude. The uncertainty in the SD contributions restricts the level of accuracy of our calculation. So the result of our calculation may be written in the form (which is our final result):

$$\frac{\rho}{\rho_0} = (1 + \delta_S + (\delta_V + \delta_H)_{IB}) \left(1 + O\left(2 \times 10^{-3}\right)\right). \qquad (11)$$

The total 1–loop RC may be written in form(details in Appendices A,B):

$$\begin{aligned}
&\delta_S + (\delta_V + \delta_H)_{IB} = \\
&\quad \frac{\alpha}{2\pi}(L_\beta - 1)\left[2\ln\Delta + \frac{3}{2} + \int\limits_{\Delta}^{N} \frac{dx}{y}\frac{1+(\frac{y}{y+x})^2}{1-\frac{y}{y+x}}\frac{A_0(y+x,z)}{A_0(y,z)}\right]_{\Delta\to 0} \\
&\quad + \frac{\alpha}{\pi}\left[\frac{3}{4}L - L_\beta \ln y - \ln N + \frac{1}{2}\int_0^N \frac{dx}{y}\frac{(2y+x)A_0(y+x,z)}{(y+x)A_0(y,z)}\right. \\
&\quad \left. - 4\int\limits_0^N dx \frac{2-2y-z-x}{A_0(y,z)} - \frac{3}{8} - \frac{\pi^2}{6} + \frac{1}{2}\ln y - Li_2(1-y) + \frac{1}{2}D_{IB}\right], \qquad (12)
\end{aligned}$$

where

$$L = \ln\frac{M_W^2}{M_K^2},$$

$$Li_2(z) = -\int\limits_0^z \frac{\ln(1-x)}{x}dx,$$

and

$$N = min[1, 2-y-z].$$

The explicit form of D_{IB} is given in Appendix B. The Dalitz plot corrections for $(\pi/\alpha)((\rho/\rho_0) - 1$ and D_i are presented in the tables. The first term on the right hand side of the eqn(12)is exactly the result expected from the structure function (SF) approach in leading order (LO) [10]:

$$d\Gamma^{LO}(y,z) = \int\limits_0^N \frac{dx}{y} d\Gamma_0(y+x,z)\, D\left(\frac{y}{y+x}, L_\beta\right), \qquad (13)$$

$$D(z,L) = \delta(1-z) + \frac{\alpha}{2\pi}(L-1)P^{(1)}(z) + \frac{1}{2!}(\frac{\alpha(L-1)}{2\pi})^2 P^{(2)}(z) + \ldots, \qquad (14)$$

$$P^{(1)}(z) = \left[\frac{1+z^2}{1-z}\theta\left(1-z-\Delta\right) + \delta\left(1-z\right)\left(2\ln\Delta + \frac{3}{2}\right)\right]_{\Delta\to 0} , \qquad (15)$$

$$P^{(i)}(z) = \int\limits_z^1 \frac{dx}{x} P^{(1)}(x) P^{(i-1)}(\frac{z}{x}) \ , \ i = 2, 3, ... \qquad (16)$$

However we see some deviation from the SF approach based on twist–2 evolution equations even in the LO. Namely the term $(3\alpha/(4\pi))\ln(M_W^2/M_K^2)$ which arises from very small distances of the order of $1/M_W$ and the term $(\alpha/\pi)\ln(1/y)L_\beta$, whose appearence reflects essentially the 3 particle character of the final state.

Discussion

When calculating δ_V we neglect SD part in virtual photon emission. The effect of SD contribution to the virtual photons is of the same order of magnitude as SD contribution to the real photons which is about 0.2% . We assume such a setup of experiment in which only one positron in the final state is present (we assume no limits on the number of photons). The ratio of the LO contributions in the first order to the Born contribution is about 3.5%, for the second order it is about 0.1%. We omit them in our accuracy frames.

We accept here the approach of papers [8, 9] in description of the effects of the strong interactions. Namely we consider them as deviations from point-like mesons. This deviations are belived to be described properly in the frame of the chiral perturbation theory [4-6]. We estimate their contribution for the case of real hard photon emission at the level of 0.2% in the main part of the Dalitz plot and omit them in calculations of virtual corrections.

The contribution of the $O(p^4)$ terms [5] turns out to be small. Really,one can see that they are of the order $O(L_9^r, (\bar{p}/\Lambda)^2) \leq O(10^{-2})$, $\Lambda = 4\pi F_\pi \approx 1.2GeV$, where $\bar{p}$ is the characteristic momentum of a final particle in the given reaction, $\bar{p}^2 \leq M_K^2/16 \sim F_\pi^2$. So the terms of the orders $O(p^4)$ and $O(p^6)$ can be omitted within the accuracy of $O\left(\frac{\alpha}{\pi}\times 10^{-2}\right) \leq O\left(10^{-4}\right)$.

The Dalitz plot distributions $(\pi/\alpha)\delta; (\pi/\alpha)D_i$ are presented in the tables 1-4.

We are grateful to S.I.Eidelman for interest to this problem and to O.N.Ivanov for help. The work of one of us (EAK) was supported partly by grant RFFI 99-02-17730.

Appendix A

Contribution from emission of a soft real photon can be written in a standard form in terms of the classical currents:

$$\delta_s = -\frac{4\pi\alpha}{(4\pi)^4}\int\frac{d^3q}{2\omega}\left(\frac{p}{pq} - \frac{p_e}{p_e q}\right)^2\Bigg|_{\omega=\sqrt{\vec{q}^2+\lambda^2}<\Delta\epsilon} \qquad (17)$$

We use the following formulas:

$$\frac{1}{2\pi}\int\frac{d^3q}{2\omega}\left(\frac{p}{p\cdot q}\right)^2=\ln\left(\frac{2\Delta\epsilon}{\lambda}\right)-1\ ;$$

$$\frac{1}{2\pi}\int\frac{d^3q}{2\omega}\left(\frac{p_e}{p_e\cdot q}\right)^2=\ln\left(\frac{2\Delta\epsilon}{\lambda}\right)-\frac{1}{2}L_\beta\ ;$$

$$\frac{1}{2\pi}\int\frac{d^3q}{2\omega}\frac{2p\cdot p_e}{p\cdot q p_e\cdot q}=L_\beta\ln\left(\frac{2\Delta\epsilon}{\lambda}\right)-\frac{\pi^2}{6}-\frac{1}{4}L_\beta^2\ . \qquad (18)$$

We obtain the result given above.

Consider now radiative corrections that arise from emission of virtual photons. Feynman graphs containing self–energy insertion to positron and kaon Green functions may be taken into account by introducing the wave function renormalization constants Z_e and Z_K: $M_0\to M_0(Z_KZ_e)^{1/2}$. The remaining Feynman graph contain the virtual photon emitted by positron and absorbed by kaon or by W–boson in the intermediate state.

The long distance part is calculated using a phenomenological model with point–like mesons as a relevant degrees of freedom. To calculate the contribution from the region $|k|^2<Q^2$ (Q^2 is ultra violet cutoff parameter) we use the following expressions for loop momenta scalar, vector, and tensor integrals:

$$Re\left\{\int\frac{d^4}{i\pi^2}\frac{1,\ k^\mu,\ k^\mu k^\nu}{(k^2-\lambda^2)((k-p)^2-M_K^2)((k-p_e)^2-m_e^2)}\right\}=J,\ J^\mu,\ J^{\mu\nu}\ .$$

A standard calculation yields:

$$J=\frac{-1}{yM_K^2}\left\{\frac{1}{2}\ln\frac{M_K^2}{\lambda^2}\ln\frac{y^2}{r_e}+\ln^2y+Li_2(1-y)-\frac{1}{4}\ln^2r_e\right\}\ ; \qquad (19)$$

$$J^\mu=\frac{-1}{yM_K^2}\left\{\frac{-y\ln y}{\bar y}p^\mu+p_e^\mu\left(\frac{y\ln y}{\bar y}+\ln\frac{y}{r_e}\right)\right\}\ ; \qquad (20)$$

$$J^{\mu\nu}=\frac{1}{4}g^{\mu\nu}\left(L_\Lambda+\frac{1}{2}+\frac{\bar y+y\ln y}{\bar y}\right)-$$
$$\frac{1}{2M_K^2}\left[p^\mu p^\nu\frac{\bar y+y\ln y}{\bar y^2}+(p^\mu p_e^\nu+p^\nu p_e^\mu)\left(-\frac{\bar y+y\ln y}{\bar y^2}-\frac{\ln y}{\bar y}\right)+\right.$$
$$\left.p_e^\mu p_e^\nu\left(\frac{1}{y}\ln\frac{y^2}{r_e}+2\frac{\ln y}{\bar y}+\frac{\bar y+y\ln y}{\bar y^2}\right)\right]\ ; \qquad (21)$$

with $\bar{y} = 1 - y$ with $L_\Lambda = \ln(\Lambda^2/M_K^2)$,$\Lambda$-is the ultraviolet cut-off parameter. As a result we obtain

$$\int \frac{d^4}{i\pi^2} \frac{(1/4) Sp\, p_\nu(p+p')(-p_e+k)(2p-k)p_e(p+p')}{(k^2-\lambda^2)((k-p)^2-M_K^2)((k-p_e)^2-m_e^2)} = 2M_K^4 \times$$

$$((1-y)(y+z-1)-r_\pi)\left\{-L_\Lambda - \frac{1}{2}\ln^2 r_e - 2\ln\frac{y^2}{r_e} + \ln\frac{M_K^2}{\lambda^2}\ln\frac{y^2}{r_e} + \psi(y)\right\} \quad (22)$$

where

$$\psi(y) = -1 + 2\ln^2 y + 2\ln y + 2Li_2(1-y)\ .$$

Counterterms contribution are

$$A_0 \times \frac{\alpha}{\pi}\left\{\frac{1}{2}\left[-\frac{1}{2}L_\Lambda + \frac{3}{2}\ln r_e + \ln\frac{M_K^2}{\lambda^2} - \frac{9}{2}\right] + \frac{1}{2}\left[L_\Lambda + \ln\frac{M_K^2}{\lambda^2} - \frac{3}{4}\right]\right\}\ . \quad (23)$$

The total contribution of virtual and real soft photon emission RC may be parametrized as:

$$A_0 \to A_0(1 + \delta_S + (\delta_V)_{IB})\ ;$$

with

$$\delta_S + (\delta_V)_{IB} = \frac{\alpha}{\pi}\left\{\frac{3}{4}L_\Lambda + (L_\beta - 2)\ln\Delta + \left(\frac{3}{4} - \ln y\right)L_\beta - \frac{9}{8} - \frac{\pi^2}{6} - Li_2(1-y) + \frac{1}{2}\ln y\right\}\ , \quad (24)$$

In a series of papers [7] A.Sirlin has conducted a detailed analysis of UV behaviuor of amplitudes of processes with hadrons in 1–loop level. He showed that they are UV finite (if considered on the quark level), but the effective cutoff scale on loop momenta is of the order M_W. This is the reason for our choice $\Lambda^2 = M_W^2$. The first term in the brackets of eq.(24) being included as a general factor which takes into account the short–distance contributions is cancelled in the ratio of 2-body decay widths $R_\pi = \Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ [12] and $R_K = \Gamma(K \to e\nu)/\Gamma(K \to \mu\nu)$ [9].

Appendix B

The matrix element of the radiative K_{e3} decay

$$K^+(p) \to \pi^0(p') + e^+(p_e) + \nu(p_\nu) + \gamma(q) \quad (25)$$

with terms up to $O(p^2)$ in CHPT [3, 4, 5, 6] has the form

$$
\begin{aligned}
M &= \frac{G_F}{2} e V_{us}^* [f_+(t) M_{IB} + M_{SD}], \\
M_{IB} &= \bar{u}(p_\nu)(p+p')(1-\gamma_5)\left[\frac{p_\mu}{pq} - \frac{(p_e+q)\gamma_\mu}{2p_e q}\right] v(p_e)\varepsilon^\mu(q); \\
M_{SD} &= \bar{u}(p_\nu)\gamma^\nu(1-\gamma_5)v(p_e)R_{\mu\nu}\varepsilon^\mu(q),
\end{aligned} \tag{26}
$$

where $\epsilon^\mu(q)$ is the polarization vector of hard photon (with energy $\omega > \Delta\epsilon$) and

$$
R_{\mu\nu} = g_{\mu\nu} - \frac{q_\nu (p-q)_\mu}{pq} . \tag{27}
$$

Singular at $\chi = 2p_e q \to 0$ terms which provide contribution containing large logarithm L_β arises only from $\sum |M_{IB}|^2$. To extract the corresponding terms we introduce four–vector $v = (x/y)p_e - q$. Note that $v \to 0$ when $\chi \to 0$. Using the neutrino mass–shell condition $p_\nu^2 = 0$ one has

$$
2p'p_e = \frac{y}{y+x}\left[M_K^2(x+y+z-r_\pi-1) - \chi + 2p'v\right] , \tag{28}
$$

and using the above we obtain:

$$
\begin{aligned}
\sum |f_+(t)M_{IB} + M_{SD}|^2 &= \frac{16M_K^2}{x^2}|f_+(t)|^2 \times \\
&\left[\frac{y}{y+x} A_0(y+x,z)\left(-1 - 2\frac{y+x}{y}\frac{1-\beta_e}{(1-\beta_e c)^2} + \frac{y^2+(y+x)^2}{y^2(1-\beta_e c)}\right)\right. \\
&\left. + d_{IB} + d_{SD} + d_{int}\right] ,
\end{aligned} \tag{29}
$$

with $\beta_e^2 = 1 - (4r_e)/y^2$, c being the cosine of the angle between the 3–momenta of posirton and photon and:

$$
\begin{aligned}
d_{IB} = \frac{2}{M_K^4}\left[\frac{y^2+(y+x)^2}{y^2(1-c)} - 1\right]&\left[\frac{y(2-y-x)}{y+x}M_K^2(-\chi+2p'v) - (2-y-z)\chi M_K^2\right. \\
&\left. + qp_\nu\chi\right] + \frac{x^2}{M_K^2\chi}\left[T_v - \frac{2}{M_K^2 x}T_{1v}\right] ;
\end{aligned} \tag{30}
$$

$$
d_{SD} = -\frac{x^2}{2M_K^2|f_+(t)|^2}T_{RR} ; \tag{31}
$$

$$
d_{int} = -\frac{x^2}{M_K^2} Re\frac{1}{f_+(t)^*}T_R, \tag{32}
$$

with

$$T_v = \frac{1}{4} Sp(p+p')p_\nu(p+p')v \; ;$$

$$T_{1v} = \frac{1}{4} Sp(p+p')p_\nu(p+p')vpp_e \; ;$$

$$T_{RR} = R_{\mu\lambda}R_{\mu\sigma}\frac{1}{4} Spp_\nu\gamma_\lambda p_e\gamma_\sigma ;$$

$$T_R = R_{\mu\lambda}\frac{1}{4} Spp_\nu(p+p')\left[\frac{p_\mu}{pq} - \frac{(p_e+q)\gamma_\mu}{\chi}\right] p_e\gamma_\lambda \; . \tag{33}$$

We see that in the limit $\chi \to 0$ the result is in agreement with quasi–real electron method of Baier, Fadin, and Khoze [11].

The non–leading contributions from hard photon emission we put in form:

$$D_H = D_{IB} + D_{SD} + D_{int} \; , \tag{34}$$

$$D_i = \frac{1}{A_0(y,z)} \int \frac{dO_q}{2\pi} \int\limits_0^N d_i \frac{dx}{x} \; . \tag{35}$$

To perform the integration over the phase volume of final states it is convenient to use

$$\int d\Gamma_{LIPS} = \int \frac{d^3p' d^3p_e d^3p_\nu d^3q}{2\epsilon' 2\epsilon_e 2\epsilon_\nu 2\omega} \delta^4\left(p - p' - p_e - p_\nu - q\right) = \frac{\pi^2}{16} M_K^4 dydzxdx \frac{d\rho_e d\rho_\pi}{\sqrt{D}} \; , \tag{36}$$

with

$$\begin{aligned}
D &= A\rho_\pi^2 + 2B\rho_\pi + C \; , \\
A &= -\left[(1-b)^2 - 2b\rho_e\right] < 0 \; ; \\
B &= b\left(\rho_e(1-d) - a\right) + 1 - \left(1-\rho_e\right)\left(1 - a - d\rho_e\right) \; ; \\
C &= -\left(1-\beta_\pi^2\right)\left[1 - (1-\rho_e)^2\right] - \left[\rho_e(1-d) - a\right]^2 \; ; \\
\beta_\pi^2 &= 1 - \frac{4r_\pi}{z^2} \; , \\
a &= \frac{2}{yz}\left(x + y + z - 1 - r_\pi\right) \; ; \\
b &= -\frac{x}{y} \; , \\
d &= -\frac{x}{z} \; .
\end{aligned} \tag{37}$$

Table 1: the corrections to the chosen 13 points of the kinematic region

y	z	$\mathcal{D}_{SD} \times 10^6$	$\mathcal{D}_{int} \times 10^6$	δ_{SV}	$\mathcal{D}_{ib} \times 10^4$	$\delta_{SV} + \mathcal{D}_{ib}$
0.75	0.85	-4.11	58	0.0659	37	0.069
0.75	0.95	-1.45	77	0.0371	22	0.039
0.25	0.95	-2.85	220	0.342	906	0.433
0.3	0.95	-3.17	251	0.234	581	0.293
0.4	0.95	-3.47	277	0.141	270	0.168
0.4	0.85	-6.41	207	0.238	338	0.272
0.5	0.95	-3.17	257	0.096	135	0.110
0.5	0.85	-6.48	204	0.147	178	0.165
0.6	0.95	-2.60	200	0.068	68	0.074
0.6	0.85	-5.88	164	0.099	96	0.109
0.7	0.95	-1.84	120	0.046	33	0.049
0.7	0.85	-4.78	97	0.073	52	0.079
0.7	0.8	-6.81	78	0.093	61	0.099

The scalar products of 4–momenta are:

$$2pp_e = M_K^2 y; \quad 2pp' = M_K^2 z; \quad 2pp_\nu = M_K^2 \left(2 - y - z - x\right) \quad ; 2pq = M_K^2 x;$$

$$2p_e p_\nu = M_K^2 \left[1 - x - z + r_\pi + \frac{1}{2} z x \rho_\pi\right] \; ;$$

$$2p_\nu q = x M_K^2 \left[1 - \frac{1}{2} z \rho_\pi - \frac{1}{2} y \rho_e\right] \; ;$$

$$2p_e q = \chi = M_K^2 \frac{xy}{2} \rho_e; \quad 2p'q = M_K^2 \frac{xz}{2} \rho_\pi;$$

$$2pv = 0; 2p_e v = -\chi; 2qv = -b\chi;$$

$$2p'v = -\frac{1}{2} M_K^2 x z \left[\rho_\pi (1 - b) - a - d\rho_e\right] \; ;$$

$$2p_\nu v = -2p'v + (1 + b)\chi \; ;$$

$$2p'p_e = \frac{M_K^2}{2} yz\left[\rho_\pi b + a + d\rho_e\right];$$

$$2p'p_\nu = \frac{M_K^2}{2}\left[2z - 4r_\pi - yz(\rho_\pi b + a + d\rho_e) - xz\rho_\pi\right] .$$

The ρ_π integration may be performed using the formulas

$$\int \frac{d\rho_\pi}{\sqrt{D}} = \frac{\pi}{\sqrt{-A}} ; \quad \int \frac{\rho_\pi d\rho_\pi}{\sqrt{D}} = \frac{\pi B}{(\sqrt{-A})^3} . \tag{38}$$

The remaining 2–fold integration was performed numerically. Results of numerical calculations are given in table 1.

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Received by Publishing Department
on November 17, 2000.

Баратт А., Кураев Е.А., Самойлов В.Н. E4-2000-276
Новый расчет поправок к ширине $K^{\pm}_{e3}$-распада

Рассмотрены радиационные поправки (РП) низшего порядка к дифференциальной ширине $K^{\pm}_{e3}$-распада. Важность знания РП обусловлена тем, что $K^{\pm}_{e3}$-распад является наиболее перспективным способом извлечения элемента V_{us} матрицы ККМ. Существующие в литературе 60-х годов расчеты противоречивы и зависят от параметра ультрафиолетового обрезания. Эта зависимость отсутствует при расчете в рамках СМ. Вычислены РП к распределениям по энергии пиона и позитрона. Для описания структурно-зависящего излучения использовалась киральная теория возмущений в низшем порядке. Вклады высших порядков по нашим оценкам выходят за рамки принятой точности, которая составляет порядка двух десятых процента. Приведены результаты численного анализа.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2000

Baratt A., Kuraev E.A., Samoilov V.N. E4-2000-276
Radiative Corrections to the $K^{\pm}_{e3}$ Decay Revised

We consider the lowest order radiative corrections for the decay $K^{\pm} \to \pi^0 e^{\pm} \nu$, usually referred as $K^{\pm}_{e3}$ decay. This decay is the best way to extract the value of the V_{us} element of the CKM matrix. The radiative corrections become crucial if one wants a precise value of V_{us}. The existing calculations were performed in the late 60's contradict each other and, besides, depend on ultraviolet cut-off parameter. The necessity of precise knowledge of V_{us} and the mentioned contradiction between the existing results constitutes the motivation of our paper. We calculate corrections to its Dalitz plot density of $K^{\pm}_{e3}$ decay and they in turn may be used to calculate corrections to the energy spectra of pion and positron, and to the total decay rate. We estimate the accuracy of our results on the level 0.2 %. Numerical results are presented.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2000

Макет Т.Е.Попеко

Подписано в печать 09.02.2001
Формат 60 × 90/16. Офсетная печать. Уч.-изд. листов 1,35
Тираж 360. Заказ 52492. Цена 1 р. 60 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области