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PROJECTIVE FORMALISM AND SOME METHODS FROM ALGEBRAIC GEOMETRY IN THE THEORY OF GRAVITATION

1 Introduction and Statement of the Investigated Problem

Variational approach is an essential and powerful method [1, 2] in the theory of gravitation, not only for obtaining the Einstein equations, but also the equations of motion in the case of some space-time decomposition, for example the (3+1) splitting [3] of space-time. Presently, another space-time splitting - (4+1) is frequently used within the framework of the Kaluza-Klein theòries, called Randall-Sundrum models [4, 5], where the main idea is that the four-dimensional Universe may have appeared as a result of a compactification of a five-dimensional one with a line element, given by

$$ds^2 = e^{-2kr_c r_5}\eta^{\mu\nu}dx^\mu dx^\nu + r_c^2 dx_5^2 \ , \tag{1}$$

where r_c is a compactification radius, $\eta^{\mu\nu}$ is the ordinary Minkowski metric, $x_5 \subset [0, \pi]$ is a periodic coordinate and $\mu\nu$ are four dimensional indices. In both cases, the gravitational Lagrangian is of the form

$$L \equiv -\sqrt{-g}R \equiv L(g_{ij}, g_{ij;k}, g_{ij;kl}) \ , \tag{2}$$

where ; may denote either a partial or a covariant derivative as a result of the standard scalar curvature representation.Let us decompose the metric tensor g_{ij} according to the known formulae

$$g_{ij} \equiv p_{ij} - \frac{1}{e}u_i u_j \quad , \tag{3}$$

where p_{ij} is the projective tensor (satisfying the projective relation $p_i^k p_k^j = p_i^j \neq \delta_i^j$), u_i are the (covariant) components of the vector field u with a lenght $e = u_i u^i$. If formulae (3) is applied in both cases,a substantial difference will be noted. In the ADM (3+1) approach, the vector field components are identified with some components of g_{ij} so that $p_i^j = \delta_i^j$ and the projective tensor components turn out to coincide with the three-dimensional components $\overset{(3)}{g_{ij}}$, defined on the three-dimensional submanifold, i.e. $p_{ij} = \overset{(3)}{g_{ij}}$. In a (4+1) space-time splitting, or in some other kind of decomposition, for example, warped compactifications [6] to four dimensional Minkowski space on seven-dimensional manifolds, this will be of course no longer the case.The already known nice geometrical meaning of the embedded space will not be valid, and one will have to deal with some kind of a multidimensional projective formalism and a decomposition

$$g_{ij} = p_{ij} + h_{ij} \quad , \tag{4}$$

where space-time is decomposed into a p-dimensional subspace and orthogonal to it $(n-p)-$dimensional space [7]. If we restrict ourselves only to the gravitational part in the action (it is present in all mentioned cases in its standard form), then the combined system of equations of motion for p_{ij} and for u_i (or respectively, for h_{ij}) have to be solved. Another example in the same spirit is from relativistic hydrodynamics, where the vector field u will be the tangent vector, defined at each point of the trajectory of motion of matter.

However, there is also another "alternative" , and it shall be investigated in the present paper. Namely, in view of the expression (3), let us simply assume that g_{ij} depends on two vector fields $u = u(x_1, x_2, ...x_n)$ and $v = v(x_1, x_2, ...x_n)$:

$$g_{ij}(x_1, x_2, ...x_n) = g_{ij}(\overrightarrow{u}, \overrightarrow{v}) \quad . \tag{5}$$

The left-hand side suggests that g_{ij} may be regarded (for each pair of i and j) as some hypersurfaces on the $n-$dimensional manifold, but they may also be unterpreted as defined on a two-dimensional manifold, represented by the vector fields u and v. In terms of the differentials du and dv, definite differential-geometric characteristics may be assigned, such as the first and the second quadratic form [8]. This is the reason why in the present paper the choice of the variables willl be related with differential quantities.

2 First and Second Differentials and Variations

In general, the differential of a vector is not necessarily a vector. Here it shall be assumed that du and dv are defined in the corresponding **tangent spaces** T_u and T_v of the vector fields u and v. The **first differential** $dg_{ij}(u,v)$ will be given by the expression

$$dg_{ij}(u,v) = \frac{\partial g_{ij}}{\partial u^k} du^k + \frac{\partial g_{ij}}{\partial v^k} dv^k = \frac{\partial g_{ij}}{\partial u} \mathbf{du} + \frac{\partial g_{ij}}{\partial v} \mathbf{dv} \tag{6}$$

(for brevity further the indices k will be omitted). In the case when $\frac{\partial g_{ij}}{\partial u^k}(x_1^0, ...x_n^0)$ and $\frac{\partial g_{ij}}{\partial v^k}(x_1^0, ...x_n^0)$ form a basis in the tangent space to the hypersurface $g_{ij}(x_1, x_2, ...x_n)$, **du** and **dv** may also be interpreted as the linear coordinates of the tangent vector. It can easily be proved that in a curved space the differentials of a vector, having vector transformation properties are only the **covariant differentials**. The kind of the differential is of no importance for the presented formalism in this paper. Also, instead of **u** and **v** one may write down some other (tensor) variable, related for example to a$k-$dimensional hypersurface, embedded in the $n-$dimensional spacetime.

Of more importance is the expression for the **second differential**

$$d^2 g_{ij}(u,v) = \frac{\partial^2 g_{ij}}{\partial u^2} (\mathbf{du})^2 + 2 \frac{\partial g_{ij}}{\partial u} . \frac{\partial g_{ij}}{\partial v} \mathbf{dudv} + \frac{\partial^2 g_{ij}}{\partial v^2} (\mathbf{dv})^2 \quad . \tag{7}$$

The expressions for the **first and second variations** of $g_{ij}(\mathbf{u},\mathbf{v})$ have the same structure. Note that $\frac{\partial^2 g_{ij}}{\partial u^2}(\mathbf{du})^2$ is the concise notation for

$$\frac{\partial^2 g_{ij}}{\partial u^2}(\mathbf{du})^2 \equiv \frac{\partial^2 g_{ij}}{\partial u^i \partial u^k} du^i du^k + \frac{\partial^2 g_{ij}}{\partial u^{i2}} (du^i)^2, \tag{8}$$

and also it has been assumed that $\delta^2 u \equiv d^2 u \equiv \delta^2 v \equiv d^2 v \equiv 0$. If $du = a_i dx^i$, then we will have

$$d^2 u = a_i d^2 x^i + da_i \wedge dx^i = a_i d^2 x^i + (\frac{\partial a_i}{\partial x^k} - \frac{\partial a_k}{\partial x^i}) dx^i dx^k = 0 . \tag{9}$$

Provided the Poincare's theorem is fulfilled and $d^2 x^i \equiv 0$ (i.e. $dx^i = const.$ – **dx** is a **coordinate line**), one would have $d^2 u = 0$ only if

$$\frac{\partial a_i}{\partial x^k} - \frac{\partial a_k}{\partial x^i} \equiv 0 \tag{10}$$

for every pair of i and k. This means that du is either an **exact differential** ($a_i = const.$), or that the vector field **du** has **zero-helicity components** a_i ($rot\mathbf{a} \equiv \mathbf{0}$, eq.(10)) in regard to the chozen basic vector field components dx^i. Note that if $rot\mathbf{a} \equiv \mathbf{0}$, but dx^i **are not** basic vectors, one would have $\mathbf{d^2u} \neq 0$. From a mathematical point of view, models with $d^2 x^i \neq 0$, when the scalar product $< e_i, dx^j >= f_i^j \neq \delta_i^j$ have been considered in [9].

3 Formulation of the Variational Problem in the Case of Different Operators of Variation and Differentiation

The gravitational part of the action, which will be investigated, is of the type

$$S \equiv \int L(g_{ij}, dg_{ij}, d^2 g_{ij}) d^n x \ . \tag{11}$$

The **first variation of the action** (provided the volume element is not being varied) is

$$\delta S \equiv \int \delta L d^n x \equiv \int \left[\frac{\partial L}{\partial g_{ij}} \delta g_{ij} + \frac{\partial L}{\partial (dg_{ij})} \delta dg_{ij} + \frac{\partial L}{\partial (d^2 g_{ij})} \delta d^2 g_{ij} \right] d^n x \ . \tag{12}$$

The operators δ and d are defined in one and the same way, but here they are distinguished. In the spirit of Cartan's works [10], they may correspond to variations and differentiations along different paths (not necessaril u and v). For example, one of the operators may be defined on a submanifold.

To find the equations of motion for u and the divergent terms, one would need to interchange the places of the operators d and δ. For that purpose, the corresponding expressions for $[\delta, d]\, g_{ij}$ and $(\delta d^2 - d\delta^2)\, g_{ij}$ have to be found. The **first** one is

$$P_{ij} \equiv [\delta, d]\, g_{ij}(\mathbf{u}, \mathbf{v}) \equiv \delta d g_{ij}(\mathbf{u}, \mathbf{v}) - d\delta g(\mathbf{u}, \mathbf{v}) \equiv$$

$$\equiv \frac{\partial g_{ij}}{\partial u} [\delta, d]\, u + (u \leftrightarrow v) \ , \tag{13}$$

where $(u \leftrightarrow v)$ means the same expression with interchanged u and v. As for the **second** expression, it shall be presented in the following compact and symmetric form

$$\left(\delta d^2 - d\delta^2\right) g_{ij}(\mathbf{u}, \mathbf{v}) \equiv Q_{ij} = \delta \widetilde{Q}_{ij}(\mathbf{du}, \mathbf{dv}) - \mathbf{d}\widetilde{\mathbf{Q}}_{ij}(\boldsymbol{\delta}\mathbf{u}, \boldsymbol{\delta}\mathbf{v}) \ , \tag{14}$$

where $\widetilde{Q}_{ij}(\mathbf{du},\mathbf{dv})$ is the quadratic form in respect to **du** and **dv** from eq.(7)

$$\widetilde{Q}_{ij}(\mathbf{du},\mathbf{dv}) \equiv d^2 g_{ij}(u,v) =$$

$$= \frac{\partial^2 g_{ij}}{\partial u^2}(\mathbf{du})^2 + 2\frac{\partial g_{ij}}{\partial u}.\frac{\partial g_{ij}}{\partial v}\mathbf{dudv} + \frac{\partial^2 g_{ij}}{\partial v^2}(\mathbf{dv})^2 \ , \qquad (15)$$

$\widetilde{\mathbf{Q}}_{ij}(\boldsymbol{\delta}\mathbf{u},\boldsymbol{\delta}\mathbf{v})$ is the same form, but with $\boldsymbol{\delta}\mathbf{u}$ and $\boldsymbol{\delta}\mathbf{v}$.

By means of the last three expressions, expression (12) for the variation of the action can be represented in the following form

$$\delta S \equiv \int \left[\frac{\delta L^{(0)}}{\delta g_{ij}}\delta g_{ij} + A_{ij} + \delta B_{ij} + dC_{ij}\right] d^n x \equiv 0 \quad , \qquad (16)$$

where

$$\frac{\delta L^{(0)}}{\delta g_{ij}} \equiv \frac{\partial L}{\partial g_{ij}} - d\left[\frac{\partial L}{\partial (d g_{ij})}\right] + \delta d\left[\frac{\partial L}{\partial (d^2 g_{ij})}\right] \ . \qquad (17)$$

This expression will represent the equation of motion for u, provided also that the variations at the endpoints vanish and $\delta B_{ij} = 0$. A_{ij} is a term, which appears in (16) due to the assumption $\delta \neq d$ and will disappear when $\delta = d$

$$A_{ij}(\mathbf{u},\mathbf{v}) \equiv -\boldsymbol{\delta}\left[\frac{\partial L}{\partial (d^2 g_{ij})}\right]\widetilde{Q}_{ij}(\mathbf{du},\mathbf{dv}) +$$

$$+\mathbf{d}\left[\frac{\partial L}{\partial (d^2 g_{ij})}\right]\widetilde{Q}_{ij}(\boldsymbol{\delta}\mathbf{u},\boldsymbol{\delta}\mathbf{v}) +$$

$$+\left(\frac{\partial g_{ij}}{\partial v}\frac{\partial^2 L}{\partial u \partial (d g_{ij})} - \frac{\partial g_{ij}}{\partial u}\frac{\partial^2 L}{\partial v \partial (d g_{ij})}\right)(du\delta v - \delta u dv) \ . \qquad (18)$$

B_{ij} is the expression

$$B_{ij} \equiv \frac{\partial L}{\partial(dg_{ij})} dg_{ij} - d(\frac{\partial L}{\partial(d^2 g_{ij})}) \delta g_{ij} + \frac{\partial L}{\partial(d^2 g_{ij})} \widetilde{Q}_{ij}(du, dv) \quad , \quad (19)$$

and C_{ij} is equal to

$$C_{ij} \equiv \frac{\partial L}{\partial(d^2 g_{ij})} \delta^2 g_{ij} - \frac{\partial L}{\partial(d^2 g_{ij})} \widetilde{Q}_{ij}(\mathbf{du}, \mathbf{dv}) = 2 \frac{\partial L}{\partial(d^2 g_{ij})} K_{ij} \delta u \delta v \ . \quad (20)$$

In (20) K_{ij} is given by

$$K_{ij} = \frac{\partial^2 g_{ij}}{\partial u \partial v} - \frac{\partial g_{ij}}{\partial u} \frac{\partial g_{ij}}{\partial v} \ . \quad (21)$$

Note that if the variations at the initial and final endpoints of the chosen curves disappear and also $\delta = d$, then the equation $K_{ij} \equiv 0$ (taking into account second variation of the metric tensor $\delta^2 g_{ij}$) appears as a necessary and sufficient condition for the fulfillment of the equations of motion (17).

4 First Variation of the Lagrangian as a Third-Rank Polynomial

From the representation (16) of the variated Lagrangian, let us express all variations and differentiations in terms of the vector fields u and v. The following notations are introduced

$$X_1^i \equiv \delta u^i \qquad X_2^i \equiv \delta v^i \qquad Z_1^i \equiv \delta du^i \qquad Z_2^i \equiv \delta dv^i$$

$$Y_1^i \equiv du^i \qquad Y_2^i \equiv dv^i \qquad T_1^i \equiv d\delta u^i \qquad T_2^i \equiv d\delta v^i \ . \tag{22}$$

After some transformations, an expression for the **first variation** of the Lagrangian will be obtained:

$$\delta L \equiv \left\{P_1^u X_1 + \widetilde{P}_1^v X_2 + Q_1^u Z_1 + \widetilde{Q}_1^v Z_2\right\} +$$

$$+\{P_2^u X_1 Y_1 + P_3^u Y_1 X_2 + \widetilde{P}_2^v X_2 Y_2 +$$

$$+\widetilde{P}_3^v Y_2 X_1 + Q_2 T_1 X_2 + Q_2 T_2 X_1\}$$

$$+\left\{P_4^u X_1 X_2 Y_1 + \widetilde{P}_4^v X_1 X_2 Y_2 + P_5^u X_1 Y_1 Y_2 + \widetilde{P}_5^v X_2 Y_1 Y_2\right\} +$$

$$+\left\{P_6^u X_1 Y_1^2 + \widetilde{P}_7^v X_1 Y_2^2 + P_7^u X_2 Y_1^2 + \widetilde{P}_6^v X_2 Y_2^2\right\} \equiv 0 \ . \tag{23}$$

If we assume (just from a general consideration and not from a concrete example) that the coefficient functions $P_1^u, \widetilde{P}_1^v, ..., P_7^u, \widetilde{P}_7^v, Q_1, Q_2$ are independent of the variables X_1, X_2........T_1, T_2 and depend only on u and v, then (23) will represent a **non-homogeneous polynomial of third degree** (the highest degree in the polynomial) and it is **linear** in respect to the variables X_1, X_2, Z_1, Z_2, **quadratic** in respect to $X_1, X_2, Y_1, Y_2, T_1, T_2$ and **cubic** *only* in respect to X_1, X_2, Y_1, Y_2. Note that the polynomial structure is rather specific, since Z_1, Z_2 enter only the **linear terms** and T_1, T_2 only the **quadratic terms** (and in combination only with X_1, X_2 and not Y_1, Y_2). In (23) P_1.......P_7, Q_1, Q_2 are functions of u and v, which are of the following form

$$P_1^u \equiv \frac{\partial L}{\partial g_{ij}}\frac{\partial g_{ij}}{\partial u} \; ; \quad P_2^u \equiv \frac{\partial L}{\partial (dg_{ij})}\frac{\partial^2 g_{ij}}{\partial u^2} \; , \tag{24}$$

$$P_3^u \equiv \frac{\partial}{\partial v}\left[\frac{\partial L}{\partial (dg_{ij})}\frac{\partial g_{ij}}{\partial u}\right] - \frac{\partial}{\partial u}\left[\frac{\partial L}{\partial (dg_{ij})}\frac{\partial g_{ij}}{\partial v}\right] + \frac{\partial L}{\partial (dg_{ij})}\frac{\partial^2 g_{ij}}{\partial u \partial v} \; , \tag{25}$$

$$P_4^u \equiv 2\frac{\partial L}{\partial (d^2 g_{ij})}\frac{\partial K_{ij}}{\partial u} \qquad P_5^u \equiv -2\frac{\partial^2 L}{\partial u \partial (d^2 g_{ij})} \quad ,26 \quad (1)$$

$$P_6^u \equiv -\frac{\partial^2 L}{\partial u \partial \{d^2 g_{ij})}\frac{\partial^2 g_{ij}}{\partial u^2} \qquad P_7^u \equiv -\frac{\partial^2 L}{\partial v \partial (d^2 g_{ij})} \; . \; .$$

$\widetilde{P}_1^v$.....$\widetilde{P}_7^v$ are the same expressions, but with u and v interchanged. It should be noted, however, that depending on the concrete Lagrangian, powers in $du, dv, \delta u, \delta v$ may come also from the coefficient functions, therefore changing the highest degree of the polynomial. Further the problem shall be formulated in terms of a third-rank polynomial, keeping in mind that the degree of the polynomial does not change the formulation, but only the technical methods for solving the algebraic equation.

5 Third–Rank Polynomials - Formulation of the Problem from an Algebro-Geometric Point of View

Let us formulate the investigated problem from the point of view of algebraic geometry, using the well-known approaches and terminology in [11-13]. For a clear and illustrated with many examples exposition of the subject, one may see also [14].

Let the defined in (22) set of variables $(X_1^i, X_2^i, Y_1^i, Y_2^i, Z_1^i, Z_2^i, T_1^i, T_2^i)$ $(i = 1, 2..., n)$ belong to the *algebraic variety* $\overline{X} \subset A^n(k)$, where $A^n(k)$ is the *n−dimensional affine space*, defined over the field $k = k[X_1, X_2, Y_1, Y_2, Z_1, Z_2, T_1, T_2]$ of the functions in 8− variables. The coefficient functions $P_1^u, \widetilde{P}_1^v, P_2^u, \widetilde{P}_2^v, ...P_7^u, \widetilde{P}_7^v, Q_1, Q_2$ are defined not on the same field k, but on the manifold M. In fact, $A^n(k)$ is the cartesian product of $n-$tuples of k. Since all the the components of the metric tensor and of the vector fields are also defined on $M \supset (x_1^0, x_2^0, ...x_n^0)$, for each point $(x_1^0, x_2^0, ...x_n^0)$ a mapping $\varphi : M \rightarrow \overline{X}$ between the elements of the *manifold* and the elements of the *algebraic variety* is also defined

$$\varphi(x_1^0, x_2^0, ..x_n^0) =$$

$$= (X_1^i(x_1^0, ...x_n^0), \ X_2^i(x_1^0, ...x_n^0), ..., T_1^i(x_1^0, ..., x_n^0), T_2^i(x_1^0, .., x_n^0) \ . \tag{27}$$

Now let $R[X_1^i, X_2^i, Y_1^i, Y_2^i, Z_1^i, Z_2^i, T_1^i, T_2^i]$ denotes the ring of all polynomials $f_1, f_2....., f_n...$, defined on the points $\overline{X}$ of the *algebraic variety* $\subset A^n(k)$ and belonging to the ideal $V(J, k)$, such as

$$V(J,k) = \left\{ \overline{X} \subset A^n(k) \ : \ (f_1(\overline{X}), \ f.(\overline{X}), ..., f_n(.\overline{X})...) = 0 \right\} . \tag{28}$$

Therefore, from (27) and (28) it is easily seen that the following sequence of mappings is defined

$$M \rightarrow \overline{X} \longrightarrow V(J,k) \ . \tag{29}$$

The considered in this paper problem can be defined in the following way:

Proposition. The variational problem $\delta L = 0$ is equivalent to **finding all the elements** $X_1^i, X_2^i, Y_1^i, Y_2^i, Z_1^i, Z_2^i, T_1^i, T_2^i$ of the algebraic variety $\overline{X}$, which satisfy **an algebraic equation** $f(\overline{X}) \equiv 0$, defined on the elements of the variety and with a finite number of coefficient functions $P_1^u, \widetilde{P}_1^v, P_2^u, \widetilde{P}_2^v, ...P_7^u, \widetilde{P}_7^v, Q_1, Q_2$ - functions of the metric tensor and the two chozen vector fields. The algebraic equation belongs to the ideal

$I = \left(P_1^u, \widetilde{P}_1^v, P_2^u, \widetilde{P}_2^v, ...P_7^u, \widetilde{P}_7^v, Q_1, Q_2 \right) \subset$
$\subset R[X_1^i, X_2^i, Y_1^i, Y_2^i, Z_1^i, Z_2^i, T_1^i, T_2^i]$,

where R is the ring of all the polynomials, defined on $\overline{X}$.

If found and considered as functions, defined on the manifold M, the elements of the algebraic variety $\overline{X}$ are no longer independent, but will represent a system of **partial differential equations** in respect to du (or δu, for example). Further, if du is known, then again the new system of partial differential equations (this time in respect to u) will give an expression for u (if v is assumed to be known), or it will give a relation between the two vector fields (if u and v are not known). Furthermore, the obtained relation between the vector fields from the variational principle $\delta L = 0$ might be used in the determination of the equation of motion for u. In an algebraic language, the simultaneous investigation of the variation equation $\delta L = 0$ and the equation of motion means that the *intersection "varieties"* of the *two algebraic surfaces* (defined by the corresponding algebraic equations) should be found.

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Проективный формализм и некоторые методы алгебраической геометрии в теории гравитации

Применяются методы алгебраической геометрии к вариационному формализму теории гравитации. Предполагается, что метрический тензор зависит от двух векторных полей u и v и что гравитационный лагранжиан зависит от метрического тензора и его дифференциалов, первого и второго.

Показано, что первая вариация гравитационного лагранжиана может быть представлена в виде полинома третьей степени от δu, δv, du, dv. Следовательно, решение вариационной задачи оказывается эквивалентным нахождению элементов алгебраического множества (δu, δv, du, dv), удовлетворяющих алгебраическому уравнению.

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Projective Formalism and Some Methods from Algebraic Geometry in the Theory of Gravitation

The purpose of this paper is to propose the implementation of some methods from algebraic geometry in the theory of gravitation, and more especially in the variational formalism. It has been assumed that the metric tensor depends on two vector fields, defined on a manifold, and also that the gravitational Lagrangian depends on the metric tensor and its first and second differentials. Assuming also different operators of variation and differentiation, it has been shown that the first variation of the gravitational Lagrangian can be represented as a third-rank polynomial in respect to the variables, defined in terms of the variated or differentiated vector fields. Therefore, the solution of the variational problem is found to be equivalent to finding all the variables — elements of an algebraic variety, which satisfy the algebraic equation.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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