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MEASUREMENT OF THE MAGNETIC MOMENT  
OF THE NEGATIVE MUON BOUND  
IN DIFFERENT ATOMS

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# Introduction

In the last years new experimental possibilities have increased interest in measurements of the magnetic moment of the electron in the 1s-state of different atoms. It was shown by Breit in 1928 [1] that the electron in the 1s-state should possess a magnetic moment different from that of a free electron due to its relativistic motion. This effect was considered in more detail in [2]. But measurement of the magnetic moment of a deeply bound electron is extremely difficult (except for the hydrogen atom) since it requires intense fluxes of multiply ionized atoms (ions with one electron). Indeed, up to the present time the g-factor of the electron in the 1s-state was only measured for the hydrogen atom [3, 4]. Only in the year 2000 there was the first publication [5] devoted to the measurement of the magnetic moment of the 1s-electron for an atom with the nuclear charge different from one. Lately the theoretical calculations of the g-factor of the electron in the 1s-state of atoms with a nuclear charge up to  $Z=92$  have been carried out [6].

In 1958 Hughes and Telegdi [7] paid attention to the fact that the relativistic variation of the magnetic moment must also take place for the negative muon in an atom and could be examined in an experiment with negative muons. The relativistic correction to the magnetic moment of the negative muon in the 1s-state can be measured for practically any atom with the zero nuclear magnetic moment, so the Z-dependence of the relativistic correction can be studied up to  $Z=82$ .

As the calculations [6, 8] show, along with the relativistic correction to the magnetic moment of the 1s-electron there are additional radiative corrections, which are due to the strong Coulomb field of the nucleus. Accordingly, the g-factor of the 1s-electron in hydrogen and hydrogen-like ions can be expressed as:

$$g_e^{1s} = 2 (1 + a_e^{\text{free}} + a_e^{\text{BS}} + a_e^{\text{rel}}) \quad (1)$$

where  $a_e^{\text{free}}$  is the radiative correction to the g-factor of the free electron;  $a_e^{\text{BS}} \equiv a_e^{\text{BS}}(\text{QED})$  is an additional radiative (quantum-electrodynamical) correction for the bound electron;  $a_e^{\text{rel}}$  is the relativistic correction for the 1s-electron.

The radiative correction to the magnetic moment of the free electron is measured with a precision close to the one of theoretical calculations [9] and is equal to  $a_e^{\text{free}} = 0.001\,159\,652\,193(10)$  [10].

As follows from the calculations of [8], the additional radiative correction for the 1s-electron is defined as:

$$a_e^{\text{BS}}(\text{QED}) = \frac{(\alpha Z)^2 \alpha}{4 \pi} + \dots \quad (2)$$

In [1, 2, 6, 8] the relation between the relativistic correction and  $Z$  was predicted to be:

$$a_e^{\text{rel}} = \frac{2}{3} (\sqrt{1 - (\alpha Z)^2} - 1) \quad (3)$$

Equation (3) shows, that the relativistic correction becomes approximately equal to the radiative correction for the free electron ( $a_e^{\text{free}}$ ) at  $Z \sim 6$  and exceeds the latter by approximately one order of magnitude at  $Z = 25$ .

At present the magnetic moment of the 1s-electron is the most precisely measured for hydrogen [4]. The ratio of the magnetic moments (g-factors) of the bound  $g_e^{1s}$  and free  $g_e^{\text{free}}$  electrons it is found to be [4]:

$$\frac{g_e^{1s}}{g_e^{\text{free}}} - 1 = -17.709(13) \cdot 10^{-6}.$$

This result is in good agreement with the calculated value of  $(g_e^{1s}/g_e^{\text{free}} - 1)$  which, according to [8], is  $-17.7051 \cdot 10^{-6}$ . But the experimental error of the measurement [4] is close to the expected value of  $a_e^{\text{BS}}$  in hydrogen, what makes it impossible to determine  $a_e^{\text{BS}}$  from this experiment.

Lately [5] the magnetic moment of the electron in the 1s-state of the fivefold ionized carbon atom has been measured. To achieve this goal, a special setup based on the continuous Stern-Gerlach effect was designed. Ions  $\text{C}^{5+}$  were held in a magnetic trap. The magnetic field in the central part of the trap was 3.8 T. The electric quadrupole field and the additional magnetic field quadratically varying along the axial axis was applied parallel to the magnetic field. The transitions between the states with the electron spin projection  $\pm 1/2$  were induced by a microwave field. The frequency of the axial movement of the carbon ions in the field used was determined by the projection of the electron spin onto the magnetic field direction. Simultaneous measurement of the axial and cyclotron frequencies of the motion of ions made it possible to determine the magnetic moment of the electron in the  $\text{C}^{5+}$  ion:  $g_e(\text{C}^{5+}) = 2.001\,042(2)$ . Accordingly,  $g_e(\text{C}^{5+}) - g_e^{\text{free}} = -0.001\,277(2)$  ( $g_e^{\text{free}} = 2.002\,319\,304\,386(20)$  [10]). The result obtained in [5] agrees with the value of the relativistic correction to the magnetic moment of the electron in the 1s-state of the carbon atom ( $-0.001\,278$ ) predicted by formula (3), but the precision of the measurement is not sufficient to prove the predictions of the theory for  $a_e^{\text{BS}}$ .

# 1 Magnetic moment of the negative muon in atoms

The corrections to the magnetic moment of the negative muon bound to a zero spin nucleus and surrounded by a zero spin electronic shell were considered in [11, 12]. According to [11, 12], the g-factor of the negative muon in the 1s-state of an atom with the diamagnetic electron shell can be expressed as:

$$g_{\mu}^{1s} = 2 \left( 1 + \sum_{i=1}^7 a_{\mu}^{(i)} \right), \quad (4)$$

where  $g_{\mu}^{1s}$  is the muon g-factor in the 1s-state of the atom;  $a_{\mu}^{(1)} \dots a_{\mu}^{(7)}$  are the corrections to the g-factor:  $a_{\mu}^{(1)}$  is the radiative correction for the free muon;  $a_{\mu}^{(2)}$  is the radiative correction due to the effect of the Coulomb field of the nucleus;  $a_{\mu}^{(3)}$  is the relativistic correction;  $a_{\mu}^{(4)}$  is the nuclear polarization correction;  $a_{\mu}^{(5)}$  is the correction for polarization of the electron shell of the atom;  $a_{\mu}^{(6)}$  is the correction for diamagnetic screening of the external magnetic field by the electron shell of the atom;  $a_{\mu}^{(7)}$  is the center-of-mass correction. For a free muon  $g_{\mu}^{\text{free}} = 2(1 + a_{\mu}^{(1)})$ . The corrections  $a_{\mu}^{(1)}$ ,  $a_{\mu}^{(2)}$  and  $a_{\mu}^{(3)}$  are analogous to the corrections  $a_e^{\text{free}}$ ,  $a_e^{\text{BS}}$  and  $a_e^{\text{rel}}$  for the 1s-electron ( $a_{\mu}^{(1)} \equiv a_{\mu}^{\text{free}}$ ;  $a_{\mu}^{(2)} \equiv a_{\mu}^{\text{BS}}$ ;  $a_{\mu}^{(3)} \equiv a_{\mu}^{\text{rel}}$ ).

The radiative correction to the magnetic moment of the free muon is known with a high accuracy:  $a_{\mu}^{(1)} = 0.001\,165\,923\,0(84)$  [10]. The radiative correction to the magnetic moment of a bound muon differs from  $a_{\mu}^{(1)}$  by  $a_{\mu}^{(2)}$ . The value of  $a_{\mu}^{(2)}$  does not exceed 2% of the relativistic correction  $a_{\mu}^{(3)}$  even in the case of large Z [12]. The center-of-mass correction is also much smaller than the relativistic correction and is about  $a_{\mu}^{(7)}/a_{\mu}^{(3)} \sim m_{\mu}/M$  [12], where  $m_{\mu}$  and  $M$  are the muon and nucleus masses, respectively.

The largest correction to the magnetic moment of a bound negative muon is due to its relativistic motion in the Coulomb field of the nucleus [2]:

$$a_{\mu}^{(3)} = -\frac{4}{3} \int F^2 dr \quad (5)$$

where  $F$  is the small component of the radial wave function of the muon.

The results of the numerical calculations show [11, 12] that the relativistic correction to the magnetic moment of a bound muon is about 0.1%, 1.1%, and 3.2% in the case of oxygen, zinc, and lead, respectively. Thus, the relativistic correction is comparable with the radiative correction for oxygen and exceeds it by one order of magnitude for zinc.

Up to now there have only been three experiments where the magnetic moment of the negative muon in the 1s-state of light (C, O, Mg, Si, S) [13, 14] and heavy (Zn, Cd, Pb) [15] atoms was measured. The accuracy of the measurement [13] of the corrections to the negative muon g-factor in Mg, Si, and S atoms was  $\sim 3\%$  and was close to the accuracy of theoretical calculations. In [13] satisfactory agreement between the experimental and calculated data was obtained for C, O, Mg, Si, and S. But the values of  $(g_{\mu}^{\text{free}} - g_{\mu}^{1s})/g_{\mu}^{\text{free}}$  measured in [14] for negative muons in Mg, Si, and S appeared to be smaller than in [13] by  $17 \cdot 10^{-4}$  in absolute value. According to [13],  $(g_{\mu}^{\text{free}} - g_{\mu}^{1s})/g_{\mu}^{\text{free}}$  is  $(29.6 \pm 0.7) \cdot 10^{-4}$ ,  $(36.3 \pm 1.1) \cdot 10^{-4}$ , and  $(48.2 \pm 1.6) \cdot 10^{-4}$  for Mg, Si, and S, respectively.

So, according to [14], the correction to the magnetic moment of the muon in the 1s-state the atom is approximately two times smaller for Mg and Si and 30 % smaller for S than the theoretical calculation.

In the case of heavy atoms the accuracy of measurements [15] is about 50%: the ratio  $(g_{\mu}^{\text{free}} - g_{\mu}^{1s})/g_{\mu}^{\text{free}}$  is equal to  $(120 \pm 62) \cdot 10^{-4}$ ,  $(201 \pm 140) \cdot 10^{-4}$ , and  $(468 \pm 220) \cdot 10^{-4}$  for Zn, Cd, and Pb, respectively. Despite the fact that the experimental data for heavy atoms do not contradict the calculated ones, they cannot be considered as a proof for the effect of a decrease in the magnetic moment of a Dirac particle at its relativistic motion in the Coulomb field of the nucleus.

The goal of this investigation was to prove the existence of the substantial discrepancy between the theoretical and the experimental data on the muon g-factor in the 1s-state of Mg, Si, and S atoms and to obtain more reliable data for the atoms with the nuclear charge  $Z \geq 30$ . Our preliminary results for C, O(H<sub>2</sub>O), Mg, and Si atoms are published in [16]. Analogous measurements are now being carried out by J.H.Brewer at TRIUMF (Canada).

When the negative muon is implanted in a medium, it slows down and is captured by a medium atom. In condensed matter the muon reaches the 1s-state within the time less than  $10^{-10}$  s. Because of its large mass, the Bohr radius of the muon is 200 times smaller than that of the K-electron. The negative muon is an unstable particle and decays mainly through the mode  $\mu^{-} \rightarrow e^{-} + \nu_{\mu} + \bar{\nu}_{e}$ . As a consequence of the parity nonconservation in this process, the spatial distribution of the decay electrons is asymmetric. This phenomenon serves as the basis for measuring the magnetic moment of the muon. In the transverse magnetic field its magnetic moment (and spin) precesses with the frequency  $\omega = 2\mu_{\mu}H/\hbar = g\mu_{\text{B}}^{\mu}H/\hbar$ , where  $\mu_{\text{B}}^{\mu}$  is the Bohr magneton for the muon. For polarized muons the exponential curve of the time distribution (with respect to muon stops in the sample) of decay electrons

is modulated by a cosine with the frequency  $\omega$ . The cosine amplitude is proportional to the muon polarization in the 1s-state. Therefore, by measuring the muon spin precession frequency one can determine the magnetic moment of the bound muon. The correction to the magnetic moment (g-factor) of a bound negative muon can be determined as:

$$(g_{\mu}^{\text{free}} - g_{\mu}^{1s})/g_{\mu}^{\text{free}} = (\omega^{\text{free}} - \omega)/\omega^{\text{free}} \quad (6)$$

where  $\omega^{\text{free}}$  and  $\omega$  are the free muon spin precession frequency and the precession frequency of the spin of  $\mu^{-}$  in the 1s-state of the atom.

## 2 Measurements

The present measurements were carried out with the ‘‘Stuttgart LFQ-spectrometer’’ [17] at the  $\mu$ E4 beamline of the Paul Scherrer Institute accelerator (Switzerland). The momentum of the muon beam was  $\sim 68$  MeV/c. The external magnetic field of (0.1–0.2) T transverse to the direction of the muon spin was produced on the sample by the Helmholtz coils. The stability of the current in the Helmholtz coils during the experiment was  $\Delta I/I \sim 2 \cdot 10^{-5}$ . The middle diameter of the coils was 510 mm, the distance between the coil centers was 240 mm. These dimensions are close to the optimal ones for obtaining the magnetic field with homogeneity not worse than  $10^{-5}$  in the volume of  $3 \times 3 \times 3$  cm<sup>3</sup> in the center of the coils. The components of the terrestrial magnetic field and stray fields from the magnetic elements nearest to the spectrometer were compensated by three pairs of additional coils with an accuracy of  $10^{-2}$  G. The residual magnetic field was measured by three mutually perpendicular permalloy sensors. The positioning of the Helmholtz coils relative to the beam axis (collimator) was done with a laser.

The samples investigated were shaped as cylinders 30 mm in diameter and 12, 18, 11, 10, 14 and 7 mm thick in the case of carbon (reactor grade graphite), oxygen (water), magnesium, silicon, sulfur, and zinc respectively. Water was packed in a cylindrical container made of phenoplast with 2 mm thick walls. The weight of the water container was 1.7 g. The silicon sample was an intrinsic silicon crystal ( $\rho \sim 10^4$   $\Omega \cdot \text{cm}$ ). The samples were positioned so that the sample axis coincided with the axis of the beam. The diameter of the beam spot on the sample was about 16 mm. The position of the sample relative to the beam axis was fixed with an accuracy better than 1 mm.

To find the muon distribution in the sample volume, the dependence of the muon stopping rate on the thickness of a copper degrader (muon range

curve) was measured for a  $1 \text{ g/cm}^2$  thick graphite sample. The muon range curve had a maximum at  $\sim 4 \text{ g/cm}^2$  and the full width at half-maximum (at a level of 5% of the maximum) was  $0.8 \text{ g/cm}^2$  ( $1.4 \text{ g/cm}^2$ ). Thus, the volume of the muon stop region in the samples was not larger than  $6 \text{ cm}^3$ .

There are a few percent of electron admixture in the muon beam, which contributes to the background in the measured spectra of the time distribution of the electrons from the decay of muons stopped in the sample. The time distribution of the beam electrons and the background due to them have a periodic structure with the accelerator frequency. The field frequency of the PSI accelerator is stabilized at the level of  $10^{-8}$  and is  $50.6330 \text{ MHz}$  (see., e.g., [18]). Thus, in the measured spectra there is always a periodic background with a well-known frequency. This fact makes it possible to check the parameters of the setup, including the parameters of the time-digital converter (TDC), in the course of the experiment (we used the EG&G' ORTEC Model 9308 TDC).

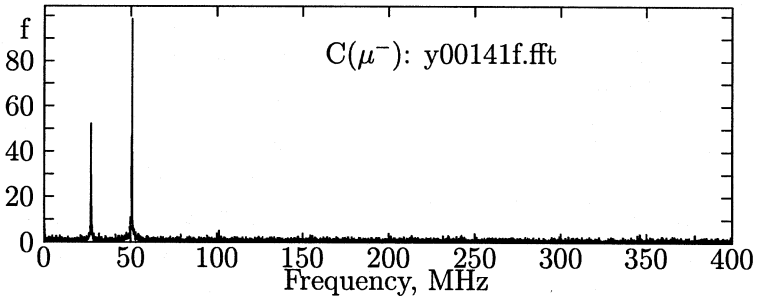


Figure 1: The result of the Fourier analysis of the experimental data ( $\mu\text{SR}$ -histogram) for graphite during the standard exposure of about three hours to the beam of negative muons in a magnetic field of  $0.2 \text{ T}$  transverse to the direction of the muon spin ( $f$  is the Fourier amplitude)

Figure 1 shows the results of the Fourier analysis of the experimental data ( $\mu\text{SR}$ -histogram) for graphite during the standard exposure of about three hours to the beam of negative muons. In the Fourier spectrum one can see the frequency of the muon-spin precession in the external magnetic field and a high frequency  $F_{\text{ac}}$  corresponding to the periodic background. The least-squares fit of the experimental  $\mu\text{SR}$ -histogram for this sample shows that  $F_{\text{ac}}$  is determined with an accuracy of  $10^{-5}$  ( $0.5 \text{ kHz}$ ) and coincides

with the accelerator frequency within the limits of experimental error. The values of  $F_{ac}$  determined from the spectra measured at different time during the 500-hour run coincide within the limits of statistical error.

The above consideration shows that the parameters of our  $\mu$ SR-spectrometer allow one to measure the frequency of the muon-spin precession with an accuracy up to  $10^{-5}$ .

The free muon spin precession frequency was determined from the precession frequency of  $\mu^+$  in copper as  $\omega^{free} = \omega(\mu^+, Cu)/(1 + K)$ , where  $K$  is the Knight shift for the positive muon in copper:  $K = (60.0 \pm 2.5) \cdot 10^{-6}$  [19]. Accordingly, the precession frequency of the positive muon in copper and carbon was measured first. After that the  $\mu$ E4 muon beamline was tuned to obtain the beam of negative muons with the same momentum as the beam of positive muons and the muon spin precession frequency was measured for C, O(H<sub>2</sub>O), Mg, Si, S, and Zn. The measurements for the O(H<sub>2</sub>O), Mg, Si, S, and Zn samples alternated with the graphite measurements in the corresponding magnetic field.

### 3 Discussion of the results

The following values were obtained for the asymmetry coefficient in the space distribution of positrons from the  $\mu^+$  decay in Cu and C, and of electrons from the  $\mu^-$  decay in C, O(H<sub>2</sub>O), Mg, Si, S, and Zn:  $0.181 \pm 0.001$  and  $0.218 \pm 0.001$ , and  $0.0486 \pm 0.0003$ ,  $0.0177 \pm 0.0004$ ,  $0.0324 \pm 0.0004$ ,  $0.0304 \pm 0.0004$ ,  $0.0213 \pm 0.0002$  and  $0.0107 \pm 0.0005$ .

Table 1 enumerates the samples in order of measurement. For each measurement the table gives the external magnetic field ( $H$ ), the muon spin precession frequency ( $\omega$ ) and the duration of the measurement. By comparing the data for the precession frequency of the positive muon spin in graphite and copper it was found that the paramagnetic frequency shift for  $\mu^+$  in carbon is  $[(1 + K) \cdot \omega(\mu^+, C) - \omega(\mu^+, Cu)]/\omega(\mu^+, Cu) = +(5.0 \pm 0.3) \cdot 10^{-4}$ .

Table 2 shows for the samples the measured negative muon spin precession frequencies and the precession frequencies of the free muon spin in the same field ( $\omega^{free}$ ).

In the case of carbon  $\omega^{free}$  was determined from the measurement of the precession frequency of the positive muon spin in copper in the same magnetic field as in the case of determination of  $\omega(\mu^-, C)$  for the negative muon in graphite:  $\omega^{free} = \omega(\mu^+, Cu)/(1 + K)$ . From the same data the ratio  $R = \omega(\mu^+, Cu)/\omega(\mu^-, C)$  independent of the magnetic field was determined.



In the case of O(H<sub>2</sub>O), Mg, Si, S and Zn  $\omega^{\text{free}}$  was determined on the basis of  $\omega(\mu^-, C)$  measured in the corresponding magnetic field:  $\omega^{\text{free}} = R \cdot \omega(\mu^-, C)/(1 + K)$ .

Table 1: The experimental conditions and the order of measurement of the muon spin precession frequency for the investigated samples

N	Sample		H, G	$\omega$ , rad/ $\mu$ s	Duration of meas.
1	Cu	$\mu^+$	1000	85.1200 $\pm$ 0.0021	2 <sup>h</sup>
2	C	$\mu^+$	1000	85.1575 $\pm$ 0.0012	4 <sup>h</sup>
3	C	$\mu^-$	1000	85.048 $\pm$ 0.006	4 <sup>h</sup>
4	S	$\mu^-$	1500	127.048 $\pm$ 0.032	18 <sup>h</sup>
5	S	$\mu^-$	1500	126.996 $\pm$ 0.036	13 <sup>h</sup>
6	C	$\mu^-$	1500	127.463 $\pm$ 0.005	3 <sup>h</sup>
7	C	$\mu^-$	2000	170.114 $\pm$ 0.007	4 <sup>h</sup>
8	Zn	$\mu^-$	2000	168.87 $\pm$ 0.61	13 <sup>h</sup>
9	Zn	$\mu^-$	2000	168.44 $\pm$ 0.55	8 <sup>h</sup>
10	Zn	$\mu^-$	2000	169.47 $\pm$ 0.77	8 <sup>h</sup>
11	C	$\mu^-$	2000	170.104 $\pm$ 0.008	3 <sup>h</sup>
12	C	$\mu^-$	1500	127.4681 $\pm$ 0.0055	3 <sup>h</sup>
13	Mg	$\mu^-$	1500	127.2544 $\pm$ 0.0076	19 <sup>h</sup>
14	Mg	$\mu^-$	1500	127.2731 $\pm$ 0.0076	17 <sup>h</sup>
15	C	$\mu^-$	1500	127.4480 $\pm$ 0.0060	3 <sup>h</sup>
16	Si	$\mu^-$	1500	127.075 $\pm$ 0.012	21 <sup>h</sup>
17	Si	$\mu^-$	1500	127.099 $\pm$ 0.012	21 <sup>h</sup>
18	C	$\mu^-$	1500	127.4406 $\pm$ 0.0071	3 <sup>h</sup>
19	O(H <sub>2</sub> O)	$\mu^-$	1500	127.455 $\pm$ 0.009	17 <sup>h</sup>

The values of the negative muon g-factor in the 1s-state of carbon, oxygen, magnesium, silicon, sulfur, and zinc atoms obtained in the present paper are compared in Table 3 with the analogous experimental data of [13, 15] and with the theoretical calculations [12]. The last column of the table shows the calculated value [12] of the relativistic correction to the magnetic moment of the bound negative muon.

The corrections to the g-factor (magnetic moment) of the negative muon

Table 2: The experimental data free muon and  $\mu^-$  spin precession frequencies in the external magnetic field  $H$  for the investigated samples

Sample	$H$ , G	$\omega$ , rad/ $\mu$ s	$\omega^{\text{free}}$ , rad/ $\mu$ s	$10^4 \cdot \frac{\omega^{\text{free}} - \omega}{\omega^{\text{free}}}$
C	1000	85.048 $\pm$ 0.006	85.115 $\pm$ 0.002	7.9 $\pm$ 0.7
O(H <sub>2</sub> O)	1000	127.455 $\pm$ 0.009	127.545 $\pm$ 0.011	7.0 $\pm$ 1.1
Mg	1000	127.264 $\pm$ 0.006	127.558 $\pm$ 0.010	23.1 $\pm$ 0.9
Si	1500	127.087 $\pm$ 0.009	127.545 $\pm$ 0.011	35.9 $\pm$ 1.1
S	1500	127.022 $\pm$ 0.025	127.563 $\pm$ 0.011	42.4 $\pm$ 2.1
Zn	2000	168.93 $\pm$ 0.38	170.243 $\pm$ 0.014	77 $\pm$ 22

in the 1s-state of carbon, oxygen, magnesium, silicon, and sulfur atoms resulting from this investigation are close to the data [13] and differ from the results [14], where the value of  $(g_{\mu}^{\text{free}} - g_{\mu}^{1s})/g_{\mu}^{\text{free}}$  for negative muons in Mg, Si, and S was smaller by  $(17 \pm 4) \cdot 10^{-4}$ . The accuracy of the present measurements of  $g_{\mu}^{1s}$  for the light atoms (C, O, Mg, Si, S) is close to the accuracy of the measurements [13], being  $\sim 3$  times higher for Mg and Si and  $\sim 1.5$  times higher for S than in [14]. For Zn the accuracy of the present measurements is three times better as compared with the literature data [15]. The present experimental data for C, Si, S, and Zn agree with the theoretical calculations within three standard deviations ( $3\sigma$ ). But in the case of O(H<sub>2</sub>O) and Mg the difference between the experimental data and the theoretical calculations [12] is about  $7\sigma$ . The cause for this disagreement could be the fact that in the calculations [12] the possible Knight shift and the chemical shift were not taken into account. Remember that after capture of the negative muon in C, H<sub>2</sub>O, Mg, Si, S, and Zn a solitary atom analogous to B, N, Na, Al, P, and Cu atom respectively is formed.

Since water is diamagnetic substance, the cause for the observed difference between the experimental and theoretical muon spin precession frequencies could be the chemical shift on nitrogen. As follows from the present data, the chemical shift on the nitrogen atom in water is  $+(6.8 \pm 1.1) \cdot 10^{-4}$ . This value does not contradict to the known NMR data on the chemical shift on nitrogen in different compounds, which is varies in a wide range from  $-400 \cdot 10^{-6}$  to  $+400 \cdot 10^{-6}$  (see., e.g, [20]).

In the case of Mg we were unable to find the Knight shift data for the

Table 3: The corrections to the g-factor of the bound negative muon for the carbon, oxygen (water), magnesium, silicon, sulfur, and zinc samples

Sample	$10^4 \cdot \frac{g_{\mu}^{\text{free}} - g_{\mu}^{\text{ls}}}{g_{\mu}^{\text{free}}}$	$10^4 \cdot \frac{g_{\mu}^{\text{free}} - g_{\mu}^{\text{ls}}}{g_{\mu}^{\text{free}}}$	theor. [12]	
	pres. exp.	exp. [13, 15]	$10^4 \cdot \frac{g_{\mu}^{\text{free}} - g_{\mu}^{\text{ls}}}{g_{\mu}^{\text{free}}}$	$a_{\mu}^{(3)} \cdot 10^4$
C(graphite)	$7.9 \pm 0.7$	$7.6 \pm 0.3$ $7.1 \pm 0.6$ $8.0 \pm 0.5$	$8.2 \pm 0.1$	6.29
O, in H <sub>2</sub> O	$7.0 \pm 1.1$	$9.4 \pm 1.0$	$14.3 \pm 0.2$	$11.04 \pm 0.01$
Mg, metal.	$23.1 \pm 0.9$	$26.4 \pm 0.7$	$29.8 \pm 0.6$	$23.79 \pm 0.06$
Mg, in MgH <sub>2</sub>		$29.6 \pm 0.7$		
Si, crystal.	$35.9 \pm 1.1$	$36.3 \pm 1.1$	$39.1 \pm 1.0$	$31.70 \pm 0.10$
S, amorphous	$42.4 \pm 2.1$	$48.2 \pm 1.6$	$49.1 \pm 1.5$	$40.35 \pm 0.15$
Zn, metal.	$77 \pm 22$	$130 \pm 63$	117.3	$112.6 \pm 1.0$

Mg+Na alloy which could have been directly taken into account in the muon spin precession frequency shift. But in the NMR experiment with the Mg<sub>17</sub>Al<sub>12</sub> alloy a large Knight shift on Mg and Al was found. The value of the Knight shift is [21]  $1.3 \cdot 10^{-3}$  on Mg and  $1.7 \cdot 10^{-3}$  on Al. Assuming that the difference between the experimental and the calculated  $g_{\mu}^{\text{ls}}(\text{Mg})$  is due to the Knight shift, the value of the Knight shift on Na in the Mg+Na alloy is  $(6.2 \pm 1.0) \cdot 10^{-4}$ . As is seen, this value is comparable in order of magnitude with the measured Knight shifts on Mg and Al in the Mg<sub>17</sub>Al<sub>12</sub> alloy.

From the NMR measurements in silicon with a boron impurity of  $2.1 \cdot 10^{19} \text{ cm}^{-3}$  it is known that the Knight shift on boron is  $(0.65 \pm 0.05) \cdot 10^{-4}$  [22]. This value is in good agreement with the estimate based on the magnetic susceptibility data for silicon with a boron impurity of  $5.2 \cdot 10^{19} \text{ cm}^{-3}$  [23]. In the present work and in the measurements [13] “pure” silicon samples with impurity concentrations below  $10^{13} \text{ cm}^{-3}$  were studied. The concentration of free charge carriers in such samples is several orders of magnitude lower than in the samples studied in the NMR experiment [22] and in the magnetic susceptibility experiment [23]. Accordingly, in our case the Knight shift is expected to be negligible (see also the estimates [13]).

The upper limit for the Knight shift, which should be taken into account in determination of the muon g-factor in Zn, follows from the NMR mea-

measurements for the  $\text{Cu}_x\text{Zn}_{(1-x)}$  alloy [24]. As follows from [24], the Knight shift on Cu decreases approximately by a factor of 3 as  $x$  decreases from 1.0 to 0.25 and is  $(7 \pm 1) \cdot 10^{-4}$  at  $x = 0.3$ . This value of the Knight shift is three times smaller than the error of the present measurement and, thus, the Knight shift on Zn is not taken into account.

Table 4: Calculated values of the corrections  $a_e^{\text{BS}}$ ,  $a_\mu^{\text{BS}}$  to the magnetic moment of the electron [8] and the muon [12] and the present-day accuracy ( $\sigma$ ) of the measurements of the corrections to the g-factor value of the electron and the muon in the 1s-state of some atoms

	$e^-$		$\mu^-$	
	$a_e^{\text{BS}} \cdot 10^6$	$\sigma \cdot 10^6$	$a_\mu^{\text{BS}} \cdot 10^6$	$\sigma \cdot 10^6$
H	0.0102	0.013	-	-
C	0.4	1.0	8	30
Si	2.9	-	40	100
Zn	$\sim 20$	-	153	2000

It would be probably appropriate to analyze the available experimental data on the magnetic moment of the bound electron and muon from the point of view of the possibility of measuring the quantum-electrodynamic correction  $a^{\text{BS}}(\text{QED})$  to the magnetic moments of the electron and the muon in the Coulomb field of the nucleus. Table 4 shows the up-to-date experimental accuracy and the expected value of this correction for some atoms. As is seen from the table, the calculated value of  $a^{\text{BS}}(\text{QED})$  is about one order of magnitude larger for the muon than for the electron, though the current accuracy ( $\sigma$ ) of determination of the corrections to the g-factor for the muon is lower than for the electron. Nevertheless, in the case of atoms with nuclear charge  $Z > 10$  measurements of  $a^{\text{BS}}(\text{QED})$  with negative muons seem to be as resultive as measurements with electrons. Besides, for a given  $Z$  the muon in the 1s-state of the atom is in about two orders of magnitude stronger Coulomb field than the electron, and thus the deviation of the experimental values of  $a^{\text{BS}}(\text{QED})$  from the theoretically predicted ones, if any, can be much larger for the muon.

## Conclusion

The results of the present investigation agree with the experimental data [13] and indicate that the magnetic moment of the negative muon in the Coulomb field of the nucleus differs from the one of the free muon. In the case of carbon, silicon, sulfur, and zinc the present experimental data agree within the experimental errors with the theoretical calculations and confirm the Z-dependence of the relativistic correction to the magnetic moment of the negative muon in the 1s-state of different atoms.

The analysis of the known experimental data shows that an increase in the accuracy the negative muon g-factor measurements in silicon and zinc can make it possible to determine  $a_{\mu}^{\text{BS}}(\text{QED})$  and to check predictions of quantum electrodynamics in the case of the presence of the strong Coulomb fields.

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Измерение магнитного момента отрицательного мюона  
в связанном состоянии в различных атомах

Теоретические расчеты показывают, что магнитный момент электрона и отрицательного мюона, связанных в атоме, должен из-за их релятивистского движения отличаться от магнитного момента свободных частиц. Кроме того, возникают дополнительные радиационные поправки к магнитному моменту, обусловленные нахождением электрона (мюона) в сильном кулоновском поле атомного ядра.

Представлены результаты измерения магнитного момента отрицательного мюона в  $1s$ -состоянии в атомах углерода, кислорода, магния, кремния, серы и цинка. Достигнутые точности позволяют проверить зависимость величины релятивистской поправки от  $Z$  атома.

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Measurement of the Magnetic Moment  
of the Negative Muon Bound in Different Atoms

Theoretical calculations show that the magnetic moment of the electron and the negative muon in a bound state in an atom should be different from the magnetic moment of the free particle due to their relativistic motion. There are also additional radiative corrections to the magnetic moment of a bound electron (muon) due to the presence of the strong Coulomb field of the atomic nucleus.

The results of the measurements of the magnetic moment of the negative muon in carbon, oxygen, magnesium, silicon, sulfur, and zinc are presented. The accuracy of the measurements makes it possible to prove the dependence of the relativistic correction to the magnetic moment of a bound muon on  $Z$  of the atom.

The investigation has been performed at the Dzhelapov Laboratory of Nuclear Problems, JINR.

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