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D-DIMENSIONAL MODEL OF DEVELOPED TURBULENCE WITH A PASSIVE VECTOR ADMIXTURE

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1 Introduction

Renormalization group (RG) methods have been widely used to the analysis of fully developed 3-dimensional hydrodynamic (HD) turbulence [1, 2, 3] as well as magnetohydrodynamic (MHD) turbulence [4, 5]. In these investigations the randomly forced HD an MHD equations have been used to obtain a regular expansion of scaling exponents in the small parameter $\epsilon = 2 - \lambda$. It is the deviation of the power of wave-number λ in the correlation function of the random force from the critical value $\lambda_c = 2$, at which the corresponding field theory is logarithmic.

The value of the expansion parameter ϵ has been chosen such that the only dimensional parameters of the model are the viscosity and the energy injection rate. From the RG equations it then follows that the asymptotic behaviour of the model at small wave-numbers is independent on the viscosity, and the powerlike wave-number dependence on the equal-time pair correlation function of velocity corresponds to the famous scaling law [6] due to Kolmogorov. Therefore, the RG approach provides a kind of mean-field description of turbulence with built-in Kolmogorov scaling, in which various correlation functions may be calculated in the form of asymptotic series in the parameter ϵ.

In the RG approach to the fully developed turbulence most work has been carried out for the 3-dimensional case, and only recently this approach has been applied to the analysis of 2-dimensional turbulence [7], or in general, d-dimensional ($d \geq 2$) turbulence [8]. The authors notice that at two dimensions an additional class of divergences appears, which has to be included in the renormalization procedure. Namely, the long-range correlation function of the random force is a power-like function of the wave-number proportional to $k^{4-d-2\epsilon}$. In general, it is a singular function of k^2 at the origin supposing $d = 2$. At two dimensions this correlation function is renormalized by counter-terms proportional to k^2, which are added to the force correlation function at the outset. In d-dimensional case one must use a two-parameter expansion (double expansion) of scaling exponents and scaling functions, the parameter of which are 2ϵ and $2\delta = d - 2$.

Existence of discrepancy on the value of inverse Prandtl number u corresponding to nontrivial stable fixed point (FP) of RG in the limit $d \to 3$, that has been obtained in the double expansion scheme in earlier paper [9] in comparison to the usual ϵ-expansion scheme [4, 5], as well as corresponding values of coupling constants, was the initial reason of the present analysis. Here we apply a modified minimal substraction scheme [10] based on the fact that the tensor structure of counter-terms is left generally d-dependent in the calculation of divergent part of Green's functions, and it allows us to investigate behaviour of the system under continual transition to $d = 3$ beginning from $d = 2$. Here we attempt to restore the limit Prandtl number for $d \to 3$ and also to establish the stability region supposing an arbitrary dimension d, $2 \leq d \leq 3$.

In this paper we have carried out an analysis of the randomly forced MHD equations with the proper account of the additional UV - divergences (appeared in $d = 2$)

in the case of the presence of magnetic field as a passive vector field in the developed HD turbulence. Notation the passive magnetic field means that the Lorentz force acting on medium can be neglected at large spatial scales, so, the Lorentzian term can be omitted in the Navier-Stokes equation. Using the double expansion approach the procedure of minimal substraction in the renormalization of the corresponding field-theoretic model has been applied in one loop approximation.

2 Formulation of problem

In the present paper we have studied the universal statistical features of the model of stochastic MHD described by the system of equations for the fluctuating local incompressible velocity field, $\mathbf{v}(x)$, $x \equiv (\mathbf{x}, \mathbf{t})$, $\nabla \cdot \mathbf{v} = 0$, and magnetic field, $\mathbf{b}(x)$, [4, 5, 11]:

$$\partial_t \mathbf{v} + (\mathbf{v} \cdot \nabla)\mathbf{v} - \nu \nabla^2 \mathbf{v} = \mathbf{f^v}\,, \tag{1}$$

$$\partial_t \mathbf{b} + (\mathbf{v} \cdot \nabla)\mathbf{b} - (\mathbf{b} \cdot \nabla)\mathbf{v} - \nu u \nabla^2 \mathbf{b} = \mathbf{f^b}\,, \tag{2}$$

with $\nabla \cdot \mathbf{f^v} = 0$ and $\nabla \cdot \mathbf{f^b} = 0$. Transversality of $\mathbf{f^v}, \mathbf{f^b}$ is a consequence of the equations $\nabla . \mathbf{v} = \nabla . \mathbf{b} = 0$. It was mentioned in Introduction that Lorentz force $(\mathbf{b} \cdot \nabla)\mathbf{b}$ is neglected because the magnetic field is treated as a passive vector field. The statistics of $\mathbf{v}$, $\mathbf{b}$ is completely determined by the non-linear equations (1), (2) and by certain assumptions about the statistics of the external large-scale random forces $\mathbf{f^v}$, $\mathbf{f^b}$. The dissipation $\nu \nabla^2 \mathbf{v}$ is controled by the parameter of kinematic viscosity ν, and, u denotes inverse Prandtl number.

As usually, statistical properties of the Gaussian forcing with zero mean values of $\langle \mathbf{f^v} \rangle = 0$, $\langle \mathbf{f^b} \rangle = 0$ are determined by relations:

$$\begin{aligned} \langle f^v_j(x_1) f^b_s(x_2) \rangle &= 0\,, \\ \langle f^v_j(x_1) f^v_s(x_2) \rangle &= u\, \nu^3\, D_{js}\,(x_1 - x_2; [1, g_{v1}, g_{v2}]) \\ \langle f^b_j(x_1) f^b_s(x_2) \rangle &= u^2\, \nu^3\, D_{js}\,(x_1 - x_2; [a, g_{b10}, g_{b20}])\,, \end{aligned} \tag{3}$$

where the correlation matrix

$$\begin{aligned} D_{js}\,(x; [a, g_1, g_2]) &= \delta(t_1 - t_2) \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d}\, P_{js}(\mathbf{k}) \exp\left[\, \mathbf{ik.x} \,\right] \\ &\quad \times \left[g_1\, k^{2-2\delta-2\,a\epsilon} + g_2\, k^2 \right] \end{aligned} \tag{4}$$

with transverse second-rank projector $P_{js}(\mathbf{k}) = \delta_{js} - k_j k_s / k^2$, is determined by constants g_1, g_2, and, the relation $d = 2 + 2\delta$ was used in exponent of k. The free parameter a controls the power form of magnetic forcing. The necessity to introduce a combined forcing and also to include the additional couplings (g_{v2}, g_{b2}) for obtaining of multiplicatively renormalizable two dimensional stochastic MHD, is absent in traditional formulation of stochastic hydrodymamics. The definition (4) includes two principal – low- and high-wave number – scale kinetic forcing separated by a transition region at the vicinity of the characteristic wave-number of order $O([g_{v10}/g_{v20}]^{\frac{1}{5}})$. In language of classical hydrodynamics the forcing contribution $\propto k^2$ corresponds to the appearence of large eddies convected by small and active ones and it is represented by the local term of $\mathbf{v}'\nabla^2\mathbf{v}'$. In its analogy the term $\mathbf{b}'\nabla^2\mathbf{b}'$ is added to the magnetic forcing. So, our stochastic MHD system can be described by the field-theoretical action

$$
\begin{aligned}
S &= \frac{1}{2}\int \mathrm{dx}_1 \int \mathrm{dx}_2 \\
& \Big\{u_0\,\nu_0^3\, v_j'(x_1) D_{js}\,(x_1-x_2;[1,g_{v10},g_{v20}])\, v_s'(x_2) + \\
&+ u_0^2\,{\nu_0}^3\, b_j'(x_1) D_{js}\,(x_1-x_2;[a,g_{b10},g_{b20}])\, b_s'(x_2)\Big\} + \\
&+ \int \mathrm{dx}\,\mathbf{v}'\cdot\Big(-\partial_t\mathbf{v}+\nu_0\,\nabla^2\mathbf{v}-(\mathbf{v}\cdot\nabla)\mathbf{v}\Big) \\
&+ \mathbf{b}'\cdot\Big(-\partial_t\mathbf{b}+u_0\,\nu_0\nabla^2\mathbf{b}+(\mathbf{b}\cdot\nabla)\mathbf{v}-(\mathbf{v}\cdot\nabla)\mathbf{b}\Big)\,. \qquad (5)
\end{aligned}
$$

All dimensional constants $g_{v10}, g_{b10}, g_{v20}$ and g_{b20}, which control the amount of randomly injected energy given by (3), (4), play the role of coupling constants of the perturbative expansion. Their universal values have been determined after the parameters ϵ, δ have been choosen to give the desired power form of forcing and desired dimension.

For the convenience of further calculations the factors $\nu_0^3 u_0$ and $\nu_0^3 u_0^2$ including the "bare" (molecular) viscosity ν_0 and the "bare" (molecular or microscopic) magnetic inverse Prandtl number u_0 have been extracted. The bare (non-renormalized) quantities are denoted by subscript "0".

3 One loop order renormalization

The most important measurable quantities in the study of a fully developed turbulence are considered to be the statistical objects represented by correlation and response functions of the fields. They are equivalent to functional averages – Green functions commonly expressed as a terms of Taylor series in A's about $A = 0$ by means of generating functional

$$G\left(\mathbf{A}^{\mathbf{v}},\mathbf{A}^{\mathbf{v}'},\mathbf{A}^{\mathbf{b}},\mathbf{A}^{\mathbf{b}'}\right)=\int \mathcal{D}\mathbf{v}\,\mathcal{D}\mathbf{v}'\,\mathcal{D}\mathbf{b}\,\mathcal{D}\mathbf{b}'\,e^{\left[S+A_j^v v_j+A_j^{v'}v_j'+A_j^b b_j+A_j^{b'}b_j'\right]}, \qquad (6)$$

where A_j are source fields, $\mathcal{D}\mathbf{v}\,\mathcal{D}\mathbf{v}'\,\mathcal{D}\mathbf{b}\,\mathcal{D}\mathbf{b}'$ denotes the measure of functional integration. One can derive unlimited number of identities for Green functions from this expression, so the developed turbulence problem can be formulated such as the calculation of functional integral. Such functional formulation is advantageous since the Green functions of the Fourier-decomposed stochastic MHD can be calculated by means of Feynman diagrammatic technique.

So, we apply usual RG procedure and corresponding pertubative techniques described elsewhere in details [12]. The model (5) is renormalizable by the standard power-counting rules, and for limits $\epsilon \to 0$, $\delta \to 0$ possesses the ultraviolet (UV) divergences which are present in one-particle irreducible two-point Green functions $\Gamma^{vv'}, \Gamma^{v'v}, \Gamma^{bb'}, \Gamma^{b'b}, \Gamma^{b'b'}, \Gamma^{v'v'}$

Free propagators $\hat{\Delta}$ can be calculated from the quadratic part of (5) written in the form of $-(1/2)\Phi\hat{\mathcal{K}}\Phi$, $\Phi \equiv \{\mathbf{v}, \mathbf{b}, \mathbf{v}', \mathbf{b}'\}$, using the relationship $\hat{\mathcal{K}}\hat{\Delta} = \hat{1}$, $\hat{1}$ is the unit matrix. One obtains in result,

$$\hat{\Delta}_{js} = \begin{pmatrix} \Delta^{vv}_{js} & 0 & \Delta^{vv'}_{js} & 0 \\ 0 & \Delta^{bb}_{js} & 0 & \Delta^{bb'}_{js} \\ \Delta^{v'v}_{js} & 0 & 0 & 0 \\ 0 & \Delta^{b'b}_{js} & 0 & 0 \end{pmatrix} \tag{7}$$

with the elements

$$\begin{aligned} \Delta^{vv'}_{js}(\vec{k},\omega) &= \Delta^{v'v}_{js}(-\vec{k},-\omega) = \frac{P_{js}(\vec{k})}{-i\omega + \nu_0\, k^2}\,, \\ \Delta^{bb'}_{js}(\vec{k},\omega) &= \Delta^{b'b}_{js}(-\vec{k},-\omega) = \frac{P_{js}(\vec{k})}{-i\omega + u_0\,\nu_0\, k^2}\,, \\ \Delta^{vv}_{js}(\vec{k},\omega) &= u_0\,\nu_0^3\, k^2\, \frac{g_{v10}\, k^{-2\delta-2\epsilon} + g_{v20}}{|-i\omega + \nu_0\, k^2|^2}\, P_{js}(\vec{k})\,, \\ \Delta^{bb}_{js}(\vec{k},\omega) &= u_0^2\,\nu_0^3\, k^2\, \frac{g_{b10}\, k^{-2\,a\delta-2\epsilon} + g_{b20}}{|-i\omega + u_0\,\nu_0\, k^2|^2}\, P_{js}(\vec{k})\,. \end{aligned} \tag{8}$$

Neglecting the finite parts of the one loop Feynman diagrams, we perform the integration over internal momentum space in an arbitrary dimension $d = 2 + 2\delta$ and integration over internal frequency, and then we separate the divergent (for $\epsilon \to 0, \delta \to 0$) contributions proportional to $1/\epsilon, 1/\delta$, $1/(2\epsilon + \delta)$, $1/(2a\epsilon + \delta)$, $1/(\epsilon(1+a) + \delta)$, according to the minimal subtraction scheme [13]. After the UV divergences have been removed, the continuation back to the original or "physical" values $\delta \to \frac{1}{2}$, $\epsilon \to 2$, $(d \to 3)$ is possible.

The UV divergences can be removed by adding suitable counter terms to the basic action obtained from (5) by the substitution: $g_{v10} \to \mu^{2\epsilon} g_{v1}$, $g_{v20} \to \mu^{-2\delta} g_{v2}$, $g_{b10} \to \mu^{2a\epsilon} g_{b1}$, $g_{b20} \to \mu^{-2\delta} g_{b2}$, $\nu_0 \to \nu$, $u_0 \to u$, where μ is a scale setting parameter having the same canonical dimension as the wave number.

The original form of the action S implies the counter terms

$$\begin{aligned} S_{count} &= \int \mathrm{dx}\,[\nu\,(1 - \mathrm{Z}_1)\,\mathbf{v}'\nabla^2\,\mathbf{v} + \mathrm{u}\nu\,(1 - \mathrm{Z}_2)\,\mathbf{b}'\nabla^2\,\mathbf{b} + \\ &+ \frac{1}{2}\,(Z_4 - 1) u\nu^3 g_{v2}\,\mu^{-2\delta}\,\mathbf{v}'\nabla^2\mathbf{v}' + \frac{1}{2}\,(Z_5 - 1)\,u^2\nu^3 g_{b2}\,\mu^{-2\delta}\,\mathbf{b}'\nabla^2\mathbf{b}']\,, \end{aligned} \tag{9}$$

determined to cancel the superficial UV divergences of the Green functions.

Within UV renormalization the divergences appearing in form of Laurent series in the poles are contained in the constants Z_1, Z_2, Z_4, Z_5 renormalizing the "bare" parameters $e_0 \equiv \{g_{i0}, \nu_0, u_0\}$.

Renormalized Green functions are expressed in terms of the renormalized parameters

$$\begin{aligned} g_{v1} &= g_{v10}\,\mu^{-2\epsilon}\,Z_1^2 Z_2, & g_{v2} &= g_{v20}\,\mu^{2\delta}\,Z_1^2 Z_2 Z_4^{-1}, \\ g_{b1} &= g_{b10}\,\mu^{-2\,a\epsilon}\,Z_1 Z_2^2, & g_{b2} &= g_{b20}\,\mu^{2\delta}\,Z_1 Z_2^2 Z_5^{-1}, \\ \nu &= \nu_0\,Z_1^{-1}, & u &= u_0\,Z_2^{-1} Z_1 \end{aligned} \tag{10}$$

appearing in the renormalized action S^R connected with the action (5) by the relation of multiplicative renormalization: $S^R\{\mathbf{e}\} = S\{\mathbf{e_0}\}$. The renormalized action S^R, which depends on the renormalized parameters $e(\mu)$, yields renormalized Green functions without UV divergences. The RG is mainly concerned with the prediction of the asymptotic behavior of correlation functions expressed in terms of anomalous dimensions γ_j by the use of β functions, both defined via differential relations

$$\gamma_j = \mu \frac{\partial \ln Z_j}{\partial \mu}\Big|_{e_0}, \quad \beta_g = \mu \frac{\partial g}{\partial \mu}\Big|_{e_0}, \text{ with } g \equiv \{g_{v1}, g_{v2}, g_{b1}, g_{b2}, u\}\,. \tag{11}$$

Using the RG routine the anomalous dimensions $\gamma_j(g_{v1}, g_{v2}, g_{b1}, g_{b2})$ can be extracted from one loop diagrams. The definitions (11) and expressions (10) for $d = 2 + 2\delta$ yield

$$\begin{aligned} \beta_{gv1} &= g_{v1}\,(-2\epsilon + 2\gamma_1 + \gamma_2)\,, \quad \beta_{gv2} = g_{v2}\,(2\delta + 2\gamma_1 + \gamma_2 - \gamma_4)\,, \\ \beta_{gb1} &= g_{b1}\,(-2\,a\epsilon + \gamma_1 + 2\gamma_2)\,, \quad \beta_{gb2} = g_{b2}\,(2\delta + \gamma_1 + 2\gamma_2 - \gamma_5) \\ \beta_u &= u\,(\gamma_1 - \gamma_2). \end{aligned} \tag{12}$$

The partial derivative with respect to μ in (11) generates δ and ϵ −dependent terms which cancel some mixed poles. The calculation of UV divergences gives Z-constants in the form

$$\begin{aligned} Z_1 &= 1 + \frac{S_d}{(2\pi)^d}\,u\,\lambda_5 \left(\frac{g_{v2}}{2\delta} - \frac{g_{v1}}{2\epsilon}\right), \qquad Z_2 = 1 + \frac{S_d}{(2\pi)^d}\frac{\lambda_1}{(u+1)}\left(\frac{g_{v2}}{2\delta} - \frac{g_{v1}}{2\epsilon}\right), \\ Z_4 &= 1 + \frac{S_d}{(2\pi)^d}\frac{u\lambda_4}{g_2}\left(\frac{g_{v1}^2}{2\delta + 4\epsilon} + \frac{2g_{v1}g_{v2}}{2\epsilon} - \frac{g_{v2}^2}{2\delta}\right), \\ Z_5 &= 1 + \frac{S_d}{(2\pi)^d}\frac{\lambda_2}{(u+1)g_{b2}}\left(\frac{g_{v1}\,g_{b1}}{2\delta + 2\epsilon(1+a)} + \frac{g_{v1}\,g_{b2}}{2\epsilon} + \frac{g_{v2}\,g_{b1}}{2a\epsilon} - \frac{g_{v2}\,g_{b2}}{2\delta}\right), \end{aligned} \tag{13}$$

and in consequence one obtains γ-functions :

$$\begin{aligned} \gamma_1 &= \frac{S_d}{(2\pi)^d} u\,\lambda_5\,g_v\,, \qquad \gamma_2 = \frac{S_d}{(2\pi)^d}\frac{\lambda_1\,g_v}{u+1}, \\ \gamma_4 &= \frac{S_d}{(2\pi)^d}\frac{\lambda_4}{g_2}\,u\,g_v^2\,, \qquad \gamma_5 = \frac{S_d}{(2\pi)^d}\frac{\lambda_2}{(1+u)}\frac{g_v\,g_b}{g_4}, \end{aligned} \tag{14}$$

where S_d denote d-dimensional sphere, $S_d = 2\pi^{d/2}/\Gamma(d/2)$, and

$$\lambda_1 = \frac{d-1}{2d}\,, \qquad \lambda_2 = \frac{d-2}{2d}\,, \qquad g_v = g_{v1} + g_{v2}\,,$$
$$\lambda_4 = \frac{d^2-2}{4d(d+2)}\,, \qquad \lambda_5 = \frac{d-1}{4(d+2)}\,, \qquad g_b = g_{b1} + g_{b2}\,.$$

3.1 RG equations

Correlation functions of the fields are expressed in terms of scaling functions of the variable $s = \frac{k}{\mu}$, $s \in \langle 0, 1 \rangle$. Then the asymptotic behaviour and the universality of MHD statistics stem from the existence of a stable FP. The continuous RG transformation is an operation linking a sequence of invariant parameters $\overline{g}(s)$ determined by the Gell-Mann Law equation

$$\frac{d\overline{g}(s)}{d\ln s} = \beta_g\left(\overline{g}(s)\right) \quad \text{with the abbrevation } \overline{g} \equiv \{\overline{g}_{v1}, \overline{g}_{v2}, \overline{g}_{b1}, \overline{g}_{b2}, \overline{u}\}\,, \tag{15}$$

where the scaling variable s parameterizes RG flow with the initial conditions $\overline{g}|_{s=1} \equiv g$ (the critical behaviour corresponds to infrared limit $s \to 0$). The expression of the $\beta(\overline{g}(s))$ function is known in the framework of the δ, ϵ expansion (see Eqs.(14) and also (12)). The FP $g^*(s \to 0)$ satisfies a system of equations $\beta_g(g^*) = 0$, while a stable FP, weakly dependent on initial conditions, is defined by positive definiteness of the real part of the matrix $\Omega = (\partial\beta_g/\partial g)|_{g^*}$. In other words, a FP is stable if all the trajectories $g(s)$ in its vicinity approach the FP.

The initial conditions $\overline{g}|_{s\to 1} = g$ of the equations (15), dictated by a micromodel, are insufficient since our aim is the large-scale limit of statistical theory, where $g^* \equiv \overline{g}|_{s\to 0}$. The RG fixed point is defined by the equation

$$\beta(g^*) = 0\,. \tag{16}$$

For $\overline{g}(s)$ close to g^* we obtain a system of linearized equations

$$\left(I\, s\frac{\mathrm{d}}{\mathrm{d}s} - \Omega\right)(\overline{g} - g^*) = 0, \tag{17}$$

where I is (5×5) unit matrix. Solutions of this system behave like $\overline{g} = g^* + \mathcal{O}(s^{\lambda_j})$ if $s \to 0$. The exponents λ_j are the elements of the diagonalized matrix $\Omega^{diag} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5)$ and can be obtained as roots of the characteristic polynomial $\mathrm{Det}(\Omega - \lambda I)$. The positive defineteness of Ω represented by the conditions $\mathrm{Re}_j(\lambda) \geq 0, j = 1, 2, ...5$ is the test of the infrared asymptotical stability of discussed theory.

4 Fixed points

Within the approach discussed above we have found three nontrivial fixed points of RG equations which give $g^*_{b1} = g^*_{b2} = 0$ and nonzero universal inverse Prandtl

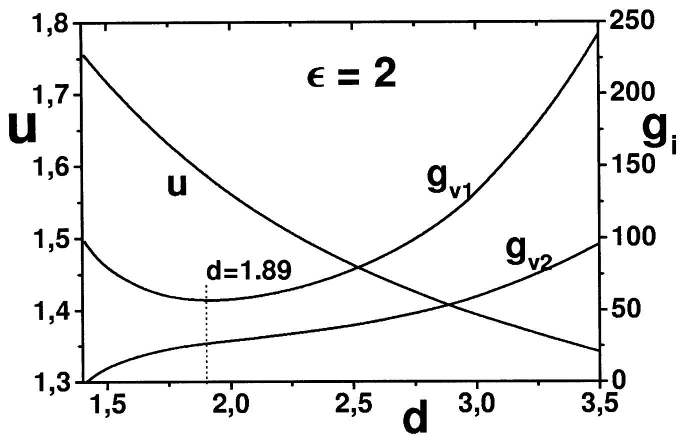

Figure 1: *Dependence of the parameters $\{g_1, g_2, u\}$ on the dimension d for physical value of $\epsilon = 2$ in the third fixed point (23).*

number

$$u^* = \frac{1}{2}\left(\sqrt{\frac{16+9d}{d}} - 1\right) . \qquad (18)$$

Namely, the values of the coupling parameters are:

(1.FP)

$$g_{v1}^* = 0 \,, \qquad g_{v2}^* = -\frac{(2\pi)^d}{S_d}\frac{2(u^*+1)(2d^2-d^3)}{d^2(d-2)} . \qquad (19)$$

(2.FP)

$$g_{v1}^* = \frac{(2\pi)^d}{S_d}\frac{4\epsilon d(d+2)(u^*+1)}{(d-1)(d+2+2du^*(u^*+1))} , \qquad g_{v2}^* = 0 ; \qquad (20)$$

(3.FP)

$$\begin{aligned} g_{v1}^* &= \frac{(2\pi)^d}{S_d}\frac{8\epsilon(u^*+1)[3d^3-(9-4\epsilon)d^2-6d(\epsilon-1)+4\epsilon]}{9(d-1)^2(d+2\epsilon-2)} , \\ g_{v2}^* &= \frac{(2\pi)^d}{S_d}\frac{8\epsilon^2(u^*+1)(d^2-2)}{9(d-1)^2(d+2\epsilon-2)} . \end{aligned} \qquad (21)$$

Setting $\epsilon = 2$ one obtains expressions for the second and third FP,

(2.FP)

$$g_{v1}^* = \frac{(2\pi)^d}{S_d}\frac{8d(d+2)(u^*+1)}{(d-1)(d+2+2du^*(u^*+1))} , \qquad g_{v2}^* = 0 ; \qquad (22)$$

(3.FP)

$$g^*_{v1} = \frac{(2\pi)^d}{S_d}\frac{16(u^*+1)(3d^3-d^2-6d+8)}{9(d-1)^2(d+2)}\,, \quad g^*_{v2} = \frac{(2\pi)^d}{S_d}\frac{32(u^*+1)(d^2-2)}{9(d-1)^2(d+2)}\,. \quad (23)$$

The fixed point (19) is non-physical because of negative value of g^*_{v2} in dimension range of $d > 2$. Detailed numerical calculations have shown that the region of stability of other fixed points, (22 - 23), is limited by the value of parameter $a < 1$ and this limiting value does not depend on the dimension d. For zero inverse Prandtl number $u = 0$ the fixed point exists,

(4.FP)

$$g^*_{v1} = \frac{(2\pi)^d}{S_d}\frac{4\epsilon d}{(d-1)}\,, \quad g^*_{v2} = 0\,, \quad g^*_{b1} = 0\,, \quad g^*_{b2} = 0\,, \quad (24)$$

but it is unstable.

5 Conclusions

In this paper we revised the calculations of stability ranges of developed magnetohydrodynamic turbulence [14] in the case when the magnetic field behaves as a passive vector admixture, i.e. the Lorentz force acting to the conductive medium can be neglected. We have used the modificated standard minimal substraction scheme [10] in the dimension $d \geq 2$ up to $d = 3$. Stability of the Kolmogorov scaling regime, which is governed by renormalization group fixed point, becomes for the parameter $a < 1$, and is unstable for $a \geq 1$ independently on dimension d. Limit value of the inverse Prandtl number at $d = 3$ restores the value of $u = 1.393$ which is known from usual ϵ-expansion, and it fluently rises to $u = 1.562$ at $d = 2$, see Fig.1. Note that numerical solution of Gell-Mann-Low equations shows that the fixed point (22) is local and the system tends to FP (23) in large attractive region. Stable "magnetic" FP with $u = 0$ [9] has not been found.

Stability of the Kolmogorov scaling regime in d-dimensional model of developed magnetohydrodynamic turbulence including back reaction of magnetic field to medium velocity field in the present minimal substraction scheme will be analyzed in the next preprint.

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D-мерная модель развитой турбулентности с пассивной векторной примесью

На основе метода ренормализационной группы рассматривается развитая магнитогидродинамическая турбулентность с двойным разложением в окрестности двумерного пространства для размерности пространства в интервале $d=\langle 2,3\rangle$ в случае, когда магнитное поле является пассивной векторной примесью, что означает пренебрежение влиянием силы Лоренца на среду. Для анализа устойчивости режима колмогоровского скейлинга, который управляется ренормгрупповыми фиксированными точками, была использована некоторая модификация стандартной минимальной схемы вычитаний. Было найдено универсальное значение обратного магнитного числа Прандтля $u=1{,}562$ для $d=2$. При переходе от $d=2$ к $d=3$ было установлено его непрерывное изменение к хорошо известному значению $u=1{,}393$ в случае ϵ-разложения.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2001

Jurcisin M., Stehlik M. E17-2001-20
D-Dimensional Model of Developed Turbulence with a Passive Vector Admixture

Developed magnetohydrodynamic turbulence near dimension $d=2$ up to dimension $d=3$ has been investigated by means of renormalization group approach and double expansion regularization in the case when the magnetic field behaves as a passive vector admixture, i.e. the Lorentz force acting to the conductive medium can be neglected. Some modification of standard minimal substraction scheme has been used to analyze the stability of the Kolmogorov scaling regime which is governed by renormalization group fixed point. The universal value of the inverse Prandtl number $u=1.562$ has been determined at $d=2$. Its continuation up to $d=3$ restores the value of $u=1.393$ which is known from usual ϵ-expansion.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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