E2-2001-209

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# ON UNCONSTRAINED $SU(2)$-GLUODYNAMICS WITH $\theta$-ANGLE

Submitted to «European Physics Journal C»

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## I. INTRODUCTION

The gauge- and Poincare invariant action of Yang-Mills theory depends on two parameters, the coupling constant $g$ and so-called $\theta$-angle, as coefficients in front of the CP even part $S^{(+)}$

$$S^{(+)} = \frac{1}{2g^2} \int d^4x \, \mathrm{tr} F_{\mu\nu} F^{\mu\nu} \,, \tag{1.1}$$

and the CP odd part $S^{(-)}$

$$S^{(-)} = \frac{\theta}{32\pi^2} \int d^4x \, \mathrm{tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \,, \tag{1.2}$$

respectively. At the classical level neither the value of the coupling constant nor that of the theta angle effect the observables, because the complete information for the description of the classical behaviour of the gauge fields is coded entirely in the extremum of the action. When all components of the gauge potential entering the action are varied as independent variables the topological charge density term $Q(x) = \mathrm{tr} F_{\mu\nu} \tilde{F}^{\mu\nu}$ can be discarded as a total divergence

$$\mathrm{tr} F_{\mu\nu} \tilde{F}^{\mu\nu} = \partial_\mu K^\mu \,, \tag{1.3}$$

with the Chern-Simons current $K^\mu$ [1]

$$K^\mu = \varepsilon^{\mu\alpha\beta\gamma} \mathrm{tr} \left( A_\alpha \partial_\beta A_\gamma + \frac{2}{3} A_\alpha A_\beta A_\gamma \right) \tag{1.4}$$

and thus the extremal curves are independent of both the coupling constant and the theta angle.

Passing to the quantum theory it is generally believed [2–4] that the physical observables become theta dependent. Although in perturbative calculations all diagrams with vertex $Q(x)$ vanish, nonperturbative phenomena such as tunneling between the above topologically distinct classical vacua, labeled by the integer value of the winding number functional

$$W[A] = \int d^3x K^0 \,, \tag{1.5}$$

leads to the appearance of theta-vacua. Configurations with different winding number are related to each other by large gauge transformations reflecting the fact that the topological current $K_\mu$ is not gauge invariant.

We therefore pose at this place the question whether it is possible to express the Chern-Simons term in the classical action as a total divergence of a gauge invariant current using the unconstrained formulation of gauge theories [5]- [20]. In the hope to obtain such a representation of the Chern-Simons term we would like to generalize in the present notes the Hamiltonian reduction of classical $SU(2)$ Yang-Mills field theory given in [18] to arbitrary theta angle by including the CP odd part (1.2) of the action. We shall reformulate the original degenerate Yang-Mills theory as an unconstrained nonlocal theory of selfinteracting second rank symmetric tensor fields.

Carrying out such a reduction in the presence of a total divergence term in the action one can meet so called *"divergence problem"* specific for the field theory with constraints which has no analog for finite-dimensional mechanical systems. This problem has first been formulated explicitly in the context of the canonical reduction of General Relativity[1]. Forty years ago R. Arnowitt, S. Deser and C.W. Misner [22] gave a clear and vivid formulation of the phenomenon: *" a term which in the original Lagrangian (or Hamiltonian) is a pure divergence, may cease to be a divergence upon elimination of the redundant variables and hence may contribute to the equations of motion obtained from the reduced Lagrangian (Hamiltonian)"*. A simple *ad hoc* example from [22] explains the idea of this statement. Consider a theory where among the variables there is a redundant variable satisfying the constraint

$$\nabla^2\Phi = \chi^2 \,. \tag{1.6}$$

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[1]Presumably, the idea of the importance of the careful consideration of terms which are total spatial divergences goes back to P.Dirac in 1959 when he constructed the reduced Hamiltonian in general relativity as a certain surface integral at spatial infinity [21].

A term $\nabla^2\Phi$ added to the degenerate Lagrangian being a divergence has no influence on the classical equation of motion, while after projection onto the constraint shell it appears as $\chi^2$ and would contribute to equations of motion.

We shall demonstrate that the Hamiltonian reduction of $SU(2)$ Yang-Mills gauge theory is free of the above mentioned divergence problem due to the Bianchi identities. Equivalence of constrained and unconstrained formulations of gauge theories on the classical level requires the demonstration of the agreement between reduced and original non-Abelian Lagrangian equations of motion. We shall explicitly construct the canonical transformation, well defined on the reduced phase space, that eliminates the theta dependence of the classical equations of motion for the unconstrained variables.

## II. THETA INDEPENDENCE ON THE CONSTRAINED LEVEL

Let us first review the case of the original constrained theory and demonstrate that under the special boundary conditions for the fields at spatial infinity (see Eq. (2.9) bellow) there exists a canonical transformation which completely eliminates the theta-dependence from the classical degenerate theory.

### A. Hamiltonian fomulation of the constrained theory

Both parts of action $S = S^{(+)} + S^{(-)}$ are invariant under the local gauge transformations

$$A_\mu \;\to\; A'_\mu = U^{-1}(x)\,(A_\mu - \partial_\mu)\,U(x)\;, \qquad (2.1)$$

with an arbitrary space-time depended element $U(x)$ of the gauge group. This means that the Lagrangian theory is degenerate and the standard Hamiltonian description needs to be generalized. We shall follow the Dirac Generalized Hamiltonian approach [23,24].

Inclusion of the CP odd part of the action $S^{(-)}$ leads to the modification of the canonical momenta

$$\Pi_a = \frac{\partial L}{\partial \dot{A}_{a0}} = 0\,, \tag{2.2}$$

$$\Pi_{ai} = \frac{\partial L}{\partial \dot{A}_{ai}} = \frac{1}{g^2}\left(\dot{A}_{ai} - \left(D_i(A)\right)_{ac} A_{c0}\right) - \frac{\theta}{8\pi^2} B_{ai}\,, \tag{2.3}$$

where the covariant derivative $D_i$ reads

$$\left(D_i(A)\right)_{mn} = \delta_{mn}\, \partial_i + \left(J^c\right)_{mn}\, A_{ci}\,, \tag{2.4}$$

with the $3 \times 3$ matrix generators of $SO(3)$ group, $\left(J_s\right)_{mn} := \epsilon_{msn}$, and non-Abelian magnetic fields

$$B_{ai} = \varepsilon_{ijk}\left(\partial_j A_{ak} + \frac{1}{2}\epsilon_{abc} A_{bj} A_{ck}\right) \tag{2.5}$$

has been introduced. Independently of this modification the phase space spanned by the variables $(A_{a0}, \Pi_a)$ and $(A_{ai}, \Pi_{ai})$ is restricted by the three primary constraints $\Pi_a(x) = 0$ .

The canonical Hamiltonian is

$$H_C = \int d^3x \left[\frac{g^2}{2}\left(\Pi_{ai} + \frac{\theta}{8\pi^2} B_{ai}\right)^2 + \frac{1}{2g^2} B_{ai}^2 + \Pi_{ai}\left(D_i A_0\right)_a\right]\,, \tag{2.6}$$

where we have used that the topological charge density $Q(x) = \mathrm{tr} F_{\mu\nu}\tilde{F}^{\mu\nu}$ can be rewritten in terms of the non-Abelian electric and magnetic fields as

$$Q = -\frac{1}{2\pi} E_{ai} B_{ai}\,. \tag{2.7}$$

The standard way in the Hamiltonian approach to proceed further, is to perform a partial integration in the last term in expression (2.6) for the canonical Hamiltonian

$$\int_{V_R} d^3x \Pi_{ai}\left(D_i A_0\right)_a = -\int_{V_R} d^3x A_{a0}\left(D_i \Pi_i\right)_a + \oint_{\Sigma_R} d^2\sigma_i A_{a0} \Pi_{ai}\,, \tag{2.8}$$

where according to the Gauss theorem the surface integral is over the two-dimensional closed surface covering the three-dimensional volume $V_R$ ( for simplicity we assume that it is a ball with radius $R$ ). Supposing that

$$\lim_{R\to\infty} \oint_{\Sigma_R} d^2\sigma_i A_{a0} \Pi_{ai} = 0\,, \tag{2.9}$$

we obtain the non-Abelian Gauss law constraint

$$(D_i)_{ac}\,\Pi_{ic} = 0\,, \tag{2.10}$$

as the condition to maintain the primary constraints $\Pi_a = 0$ during the evolution. According to the Dirac prescription the generator of time translation is the total Hamiltonian

$$H_T = \int d^3x \left[\frac{g^2}{2}\left(\Pi_{ai} + \frac{\theta}{8\pi^2}B_{ai}\right)^2 + \frac{1}{2g^2}B^2_{ai} - A_{a0}D_i\Pi_{ai} + \lambda_a\Pi_a\right]\,, \tag{2.11}$$

depending on three arbitrary functions $\lambda_a(x)$ and the Poisson brackets have a canonical structure

$$\{A_{ai}(\vec{x},t), \Pi_{bj}(\vec{y},t)\} = \delta^{ab}\delta_{ij}\delta^3(\vec{x}-\vec{y})\,, \tag{2.12}$$

$$\{A_{a0}(\vec{x},t), \Pi_{b}(\vec{y},t)\} = \delta^{ab}\delta^3(\vec{x}-\vec{y})\,. \tag{2.13}$$

### B. Canonical equivalence of constraint theory with different theta angle

Based on the representation (2.11) for the total Hamiltonian one can immediately verify the equivalence of classical theories with different value of parameter $\theta$. To convince let us perform the transformation to a new coordinates $A_{ai}$ and $E_{bj}$

$$A_{ai}(x) \to A_{ai}(x) = A_{ai}(x)\,, \tag{2.14}$$

$$\Pi_{bj}(x) \to E_{bj} = \Pi_{bj}(x) + \frac{\theta}{8\pi^2}B_{bj}(x)\,. \tag{2.15}$$

One can easily check that this transformation is canonical, the new coordinates $A_{ai}$ and $E_{ai}$ satisfy the same canonical Poisson brackets relations (2.12) as the original one. And noticed that by virtue of the Bianchi identity

$$\epsilon^{\mu\nu\lambda\rho}D_\nu F_{\lambda\rho} = 0\,, \tag{2.16}$$

one can conclude that the $\theta$-dependence completely disappears from the Hamiltonian (2.11).

Note that the canonical transformation (2.14) can be represented in the form

$$E_{ai} = \Pi_{ai} - \frac{\theta}{8\pi^2}\frac{\delta}{\delta A_{ai}}W[A]\,, \tag{2.17}$$

where $W[A]$ denotes the winding number functional (1.5).

## III. THETA INDEPENDENCE ON THE UNCONSTRAINED LEVEL

We shall now derive the unconstrained version of Yang-Mills theory with theta angle and then give the analog of the transformation (2.14) after projection to the reduced phase space, thus checking the consistency of the unconstrained canonical formulation of Yang-Mills theory.

### A. Hamiltonian formulation of the unconstrained theory

For the reduction of $SU(2)$ Yang Mills theory we shall follow the method developed in [18] for the CP even part of action. To reduce the CP odd part one can proceed similarly.

Let us therefore perform the following point transformation to the new set of Lagrangian coordinates $q_j \quad (j = 1, 2, 3)$ and the six elements $S_{ik} = S_{ki} \quad (i, k = 1, 2, 3)$ of the positive definite symmetric $3 \times 3$ matrix $S$

$$A_{ai}(q, S) = O_{ak}(q) S_{ki} - \frac{1}{2}\epsilon_{abc}\left(O(q)\,\partial_i O^T(q)\right)_{bc}, \tag{3.1}$$

where $O(q)$ is an orthogonal $3 \times 3$ matrix parameterized by the three fields $q_i$.

The first term in (3.1) corresponds to the so-called polar decomposition for arbitrary quadratic matrices. The inclusion of the additional second term is motivated by the inhomogeneity of the gauge transformation (2.1). [2] The transformation (3.1) induces a point canonical transformation linear in the new conjugated momenta $P_{ik}$ and $p_i$. Using the corresponding generating functional depending on the old momenta and the new coordinates,

$$F_3[\Pi; q, S] = \int d^3z\; \Pi_{ai}(z) A_{ai}(q(z), S(z)), \tag{3.2}$$

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[2] One can treat equation (3.1) as gauge transformation to new field configuration $S(x)$ which satisfy the so-called symmetric gauge condition $\epsilon_{abc}S_{bc} = 0$. The uniqueness and regularity of the transformation (3.1) depends on the boundary conditions imposed.

one can obtain the transformation to new canonical momenta $p_i$ and $P_{ik}$

$$p_j(x) = \frac{\delta F_3}{\delta q_j(x)} = -\Omega_{jr}\left(D_i(Q)S^T\Pi\right)_{ri}\,, \tag{3.3}$$

$$P_{ik}(x) = \frac{\delta F_3}{\delta S_{ik}(x)} = \frac{1}{2}\left(\Pi^T O + O^T\Pi\right)_{ik}\,. \tag{3.4}$$

Here

$$\Omega_{ji}(q) := -\frac{1}{2}\mathrm{Tr}\left(O^T(q)\,\frac{\partial O(q)}{\partial q_j}\,J_i\right). \tag{3.5}$$

The symplectic structure of new variables is encoded in the fundamental Poisson brackets[3]

$$\{S_{ij}(x), P_{kl}(y)\} = \frac{1}{2}\left(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right)\delta^{(3)}(x-y)\,. \tag{3.6}$$

A straightforward calculation based on the linear relations (3.3) and (3.4) between the old and the new momenta leads to the following expression for old momenta $\Pi_{ai}$ in terms of the new canonical variables

$$\Pi_{ai} = O_{ak}(q)\left[P_{ki} + \epsilon_{kis}P_s\right], \tag{3.7}$$

where the vector $P_s$ is a solution to the system of first order partial differential equations

$$^*D_{ks}(S)P_s = -s_k(x) + \Omega^{-1}_{kl}p_l\,. \tag{3.8}$$

In (3.8) the $^*D$ denotes the matrix operator

$$^*D_{ik}(S) = -i\left(J^m D_m(S)\right)_{ik}\,, \tag{3.9}$$

and one can verify that vector

$$s_k(x) = (D_i(S))_{lk}P_{il} \tag{3.10}$$

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[3]These new brackets take into account the symmetry constraints $S_{ij} = S_{ji}$ and $P_{kl} = P_{lk}$ and rigorously speaking are the Dirac brackets.

coincides up to a divergence term with the spin density part of the Noetherian angular momentum calculated in terms of the new variables and projected onto the constraint shell. Using the representations (3.1) and (3.7) one can easily convince oneself that the new variables $S$ and $P$ make no contribution to the Gauss law constraints (2.10)

$$O_{as}(q)\Omega^{-1}_{sj}(q)p_j = 0\,. \tag{3.11}$$

Here and in (3.7) we assume that the matrix $\Omega$ is invertible and thus the equivalent set of Abelian constraints is

$$p_a = 0\,. \tag{3.12}$$

The Abelian form of Gauss law constraints is the main advantage of new variables. In terms of this coordinates the projection to the constraints shell is achieve by vanishing value of momenta $p_a$ in all expressions.

The reduced Hamiltonian is defined as projection of total Hamiltonian to the constraint shell $p_a = 0$. In terms of the unconstrained canonical variables $S$ and $P$ it reads

$$H^* = \int d^3x \Big[ \frac{g^2}{2}\left(P_{ai} + \frac{\theta}{8\pi^2}B_{(ai)}\right)^2 + \frac{g^2}{2}\left(P_a + \frac{\theta}{8\pi^2}B_a\right)^2 + \frac{1}{2g^2}B^2_{ai}\Big]\,. \tag{3.13}$$

Here $B_{(ai)}$ and $B_a$ denote the symmetric tensor $B_{(ai)} = 1/2(B_{ai} + B_{ia})$ and vector $B_a = 1/2\epsilon_{abc}B_{bc}$ constructed from chromomagnetic field

$$B_{sk} = \epsilon_{klm}\left(\partial_l S_{sm} + \frac{1}{2}\,\epsilon_{sbc}\, S_{bl}S_{cm}\right)\,. \tag{3.14}$$

The vector $P_a$ representing the nonlocal term in the Hamiltonian (3.13) is given as the solution to the system of differential equations

$$^*D_{ks}(S)P_s = -s_k(x)\,, \tag{3.15}$$

which is the projection of Eqs.(3.8) to the constraint surface $p_a = 0$.

### B. Canonical equivalence of the unconstrained theory with different theta angles

For the original degenerate action in terms of the $A_\mu$ fields the equivalence of classical theories with arbitrary value of theta angle has been reviewed in Section II. Let us now examine the same problem for the derived unconstrained theory considering the analog of the canonical transformation (2.14) after projection onto the constraint surface:

$$S_{ai}(x) \to S_{ai}(x) = S_{ai}(x)\,, \tag{3.16}$$

$$P_{bj}(x) \to E_{bj}(x) = P_{bj}(x) + \frac{\theta}{8\pi^2} B_{(bj)}(x)\,. \tag{3.17}$$

First of all one can easily check that this transformation to new variables $S_{ai}$ and $E_{bj}$ is canonical with respect to the Dirac brackets (3.6). The Hamiltonian (3.13) in terms of the new variables $S_{ai}$ and $E_{bj}$ is therefore $\theta$ -independent. It looks as

$$H^* = \int d^3x \Big[ \frac{g^2}{2} E_{ai}^2 + \frac{g^2}{2} E_a^2 + \frac{1}{2g^2} B_{ai}^2 \Big]\,. \tag{3.18}$$

where $E_a$ is a solution to equation (3.15) with the replacement $P_{ai} \to E_{ai}$. This follows from the observation, that if $P_a$ is a solution to equation (3.15) then expression

$$E_a = P_a + \frac{\theta}{8\pi^2} B_a \tag{3.19}$$

is a solution to the same equation with the replacement $P_{ai} \to E_{ai}$ . This is indeed valid because $B_{ai}$ field satisfies the identity

$$^{*}D_{ks}(S) B_s = (D_i(S))_{kl} B_{(li)}\,. \tag{3.20}$$

Equation (3.20) is the Bianchi identity $(D_i)_{ab} B_{bi} = 0$ rewritten in terms of the symmetric $B_{(ai)}$ and antisymmetric $B_a$ parts of the chromomagnetic field strength.

The reduced form of the generating functional (1.5) corresponding to the transformation (3.16) is the same functional $W$ evaluated for the symmetric tensor $S_{ik}$. One can convince oneself that the symmetric part of the magnetic field $B_{(ij)}(S)$ can be written as the functional derivative of this functional $W[S]$

$$\frac{\delta}{\delta S_{ij}(x)} W[S] = B_{(ij)}(x)\,, \tag{3.21}$$

and thus the canonical transformation that eliminates the theta dependence from the Hamiltonian can be represented in the same from as (2.17) with the nine gauge fields $A$ replaced by the six unconstrained fields $S_{ik}(x)$.

## IV. CONCLUDING REMARKS

We have explored the question of theta-independence of classical unconstrained $SU(2)$ Gluodynamics in order to build the basis for passing to the quantum level. We have shown that the exact projection of $SU(2)$ Gluodynamics to the reduced phase space leads to an unconstrained system whose classical equations of motion are consistent with the original degenerate theory in the sense that they are theta-independent. The crucial point is that the fulfillment of this condition is due to properly taking into account the Bianchi identity for the magnetic field. As a consequence of the independence of the classical equations of motion of the gauge invariant local fields, the parity odd term in the Yang-Mills action is a total divergence of some gauge-invariant current, in contrast to the original unconstrained theory, where it was the total divergence of the gauge variant Chern-Simons current $K_\mu$. The explicit construction of the gauge invariant current in the unconstrained theory remains a topic for further investigation. Furthermore, to deal practically with such a complicated non-local Hamiltonian as (3.13) one would have to use some approximation, because the the exact solution to equation (3.15) is unknown. Implementing the one or another approximating solution, it is desirable to be consistent with the theta-independence of classical theory.

## V. ACKNOWLEDGEMENTS

It is a pleasure to thank A.N. Kvinikhidze, M.D.Mateev, V.N. Pervushin, P. Schuck and A.N.Tavkhvelidze for illuminating discussions. D.M. and A.K. would like to thank Prof. G. Röpke for kind hospitality at the AG "Theoretische Vielteilchenphysik" Rostock where part of this work has been done and the Deutsche Forschungsgemeinschaft for providing a stipendium. H.-P. P. acknowledges financial support by the Bundesministerium für Forschung und Technologie.

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Received by Publishing Department
on October 4, 2001.

Хведелидзе А.М. и др. E2-2001-209
О редуцированной $SU(2)$-глюодинамике с $\theta$-углом

В рамках обобщенного гамильтонова формализма с использованием метода пуассоновой редукции получена гамильтонова система без связей, эквивалентная $SU(2)$ калибровочной теории поля Янга–Миллса с $\theta$-углом. Для любого $\theta$-угла получено, что после устранения калибровочных степеней свободы редуцированная нелокальная система выражается в терминах самодействующего поля симметрического тензора второго ранга. Показано, что каноническая эквивалентность глюодинамик с различными $\theta$-углами остается в силе и после процедуры редукции.

Работа выполнена в Лаборатории теоретической физики им. Н.Н.Боголюбова ОИЯИ и на кафедре физики Университета г. Росток (Германия).

Препринт Объединенного института ядерных исследований. Дубна, 2001

Khvedelidze A.M. et al. E2-2001-209
On Unconstrained $SU(2)$-Gluodynamics with $\theta$-Angle

The Hamiltonian reduction of classical $SU(2)$ Yang–Mills field theory to the equivalent unconstrained theory of gauge invariant local dynamical variables is generalized to the case of nonvanishing $\theta$-angle. It is shown that for any $\theta$-angle the elimination of the pure gauge degrees of freedom leads to a corresponding unconstrained nonlocal theory of self-interacting second rank symmetric tensor fields, and that the obtained classical unconstrained gluodynamics with different $\theta$-angles are canonically equivalent as on the original constrained level.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR and at the Fachbereich Physik der Universität Rostock (Germany).

Preprint of the Joint Institute for Nuclear Research. Dubna, 2001

Макет Т.Е.Попеко

Подписано в печать 29.11.2001
Формат 60 × 90/16. Офсетная печать. Уч.-изд. л. 1,04
Тираж 425. Заказ 52980. Цена 1 р. 30 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области