

E2-2001-221

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**ELECTRICALLY CHARGED  
EINSTEIN-BORN-INFELD BLACK HOLES  
WITH MASSIVE DILATON**

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# 1 Introduction

In recent years there has been an increasing interest in the Born-Infeld (BI) type of generalizations of Abelian and non-Abelian gauge theories. Such generalizations appear naturally in the context of the (super)string theory. The BI action including a dilaton and an axion field, appear in the couplings of an open superstring and an Abelian gauge field [1]. This action, describing a BI-dilaton-axion system coupled to Einstein gravity, can be considered as a non-linear extension in the Abelian field of Einstein-Maxwell-dilaton-axion (EMDA) gravity. It has also been shown that the world volume action of a D-brane is describe by a kind of BI action arising as a sum over disc string diagrams [2].

Without any doubt the black holes are one of the most interesting objects in both pure Einstein-Born-Infeld (EBI) and Einstein-Born-Infeld-dilaton(-axion) (EBID(A)) gravity. Black holes in pure EBI theory were studied earlier in [3]-[5]. The motivation for considering EBID(A) black holes is to understand the possible causal structures in the presence of a dilaton (and axion) coupled to a nonlinear electromagnetic field. The nonlinearity of the electromagnetic field may lead to surprising phenomena. It was shown that a kind of nonlinear electromagnetism produces nonsingular black holes satisfying the weak energy condition [6] - [8]. It should be noted that this may contradict the strong version of the cosmic censorship conjecture.

Recently EBID black holes in four dimensions have been considered in [9], [10], and [11]. The three dimensional EBID black holes are studied in [12]. It has been shown that the four-dimensional EBID black holes have remarkable properties which are not observed for the corresponding EMD and EBI solutions. By examining the internal structure of the black holes, it has been found that there is no inner horizon and that the global structure is the same as the Schwarzschild one in both electrically and magnetically charged case.

In the black hole investigations performed so far the dilaton has been considered as massless. The massless dilaton, however, contradicts the experiments. That is why, it is the EBI black holes coupled to a massive dilaton, which are the most interesting from a physical point of view.

The purpose of the present letter is to present numerical EBI black

hole solutions with a massive dilaton and to show that the massive dilaton allows many more black hole causal structures than the massless dilaton. We find that depending on the black hole mass and charge and the dilaton mass the EBI black holes can have either one, two or three horizons. The extremal black hole solutions were also found out.

It should be noted that black holes coupled to a massive dilaton but within the framework of ordinary EMD gravity were studied earlier in [13] and [14]. The authors of [13] using a qualitative analysis argue that there may exist EMD black holes with three horizons and black holes with a triply degenerated horizon in presence of a massive dilaton. The authors of [14] present numerical EMD black hole solution with one and two horizons and also argue that space-times with three horizons cannot be ruled out. Note, however, that no black solutions with three horizons or with a triply degenerated horizon were presented in [13] and [14]. We hope that the presented (numerical) black holes solutions (within the framework of EBID gravity) in this letter are the first explicit examples of black holes with three horizons and triply degenerated horizon in presence of a massive dilaton.

## 2 Basic equations

We consider the following EBID action

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} [R - 2\nabla^\mu \varphi \nabla_\mu \varphi - U(\varphi) + \mathcal{L}_{BI}], \quad (1)$$

where  $R$  is Ricci scalar curvature with respect to the space-time metric  $g_{\mu\nu}$ ,  $\varphi$  is the dilaton field and  $U(\varphi)$  is the dilaton potential. The BI part of the action is given by (see [9] and references therein):

$$\mathcal{L}_{BI} = 4be^{2\alpha\varphi} \left\{ 1 - \left[ 1 + \frac{e^{-4\alpha\varphi}}{2b} F^2 - \frac{e^{-8\alpha\varphi}}{16b^2} (F \star F)^2 \right]^{\frac{1}{2}} \right\}. \quad (2)$$

Here  $\star F$  is the dual to the Maxwell tensor and  $\alpha$  is the dilaton coupling parameter. In the context of the string theory, the BI parameter  $b$  is related to the string tension  $\alpha'$  by  $b = 1/2\pi\alpha'$ . Note that in the  $b \rightarrow \infty$  limit our action reduces to the EMD system with massive dilaton.

It should be pointed that EBID action does not possess an electric-magnetic duality. That is why one may expect that the electrically and magnetically charged black holes will be different.

We are interested in static and spherically symmetric configurations and therefore assume the general static, spherically symmetric parameterization of the space-time metric

$$ds^2 = -f e^{-2\delta} dt^2 + f^{-1} d\rho^2 + \rho^2 d\Omega, \quad (3)$$

where  $f$  and  $\delta$  depend on  $\rho$  only. As boundary conditions at infinity, we require the metric be asymptotically flat and the dilaton vanish, i.e.,  $\varphi(\infty) = 0$ . Note that the condition  $\varphi(\infty) = 0$  is consistent with the asymptotic flatness when and only when the minimum of the dilaton potential is chosen to be  $\varphi = 0$ .

In the present letter we consider only electrically charged configurations. In this case the gauge potential has the form

$$A = -\Phi(\rho) dt \quad (4)$$

where  $\Phi$  is the electric potential. From the BI equations we obtain

$$\frac{d\Phi}{d\rho} = -\frac{Q_e}{\sqrt{\rho^4 + \frac{Q_e^2}{b}}} e^{2\alpha\varphi} e^{-\delta}. \quad (5)$$

The electric charge  $Q_e$  is defined by  $Q_e = -\lim_{\rho \rightarrow \infty} \rho^2 \frac{d\Phi(\rho)}{d\rho}$ .

Using metric (3) and the expression for the electric field (5) in the field equations determined by action (1) we get the equations for the structure of a spherically symmetric black hole. Before we explicitly write them, we are going to introduce a dimensionless radial coordinate

$$r = \sqrt{b}\rho,$$

and a dimensionless dilaton potential given by

$$U(\varphi) = 2m_D^2 V(\varphi),$$

where  $m_D$  is the dilaton mass. From now on, the prime will denote a differentiation with respect to the dimensionless coordinate  $r$ . We also define other dimensionless quantities by

$$q_e = \sqrt{b} Q_e, \quad \gamma = \frac{m_D}{\sqrt{b}}. \quad (6)$$

It is also convenient to introduce the local mass  $m(r)$  defined by  $f = 1 - 2m(r)/r$ .

With all these definitions the equations for a spherically symmetric black hole are reduced to the following

$$\begin{aligned}
m'(r) &= \frac{1}{2}fr^2(\varphi')^2 + \frac{1}{2}\gamma^2V(\varphi)r^2 + e^{2\alpha\varphi} \left( \sqrt{r^4 + q_e^2} - r^2 \right), \\
\delta'(r) &= -r(\varphi')^2, \\
\varphi''(r) &= -\frac{2}{r}\varphi' + f^{-1} \left( \frac{f-1}{r} + \gamma^2V(\varphi)r + 2e^{2\alpha\varphi} \frac{\sqrt{r^4 + q_e^2} - r^2}{r} \right) \varphi' \\
&\quad + f^{-1} \left( \frac{\gamma^2}{2} \frac{dV(\varphi)}{d\varphi} + 2\alpha e^{2\alpha\varphi} \frac{\sqrt{r^4 + q_e^2} - r^2}{r} \right).
\end{aligned} \tag{7}$$

In order to satisfy the asymptotic flatness we impose the following boundary conditions at the spatial infinity:

$$m(\infty) = M, \quad \delta(\infty) = 0, \quad \varphi(\infty) = 0. \tag{8}$$

Here  $M$  is the dimensionless black hole mass which is related to the dimensionfull mass  $\mathcal{M}$  by  $\mathcal{M} = M/\sqrt{b}$ .

We also assume the existence of a regular or degenerated event horizon at  $r = R_h$  and we have

$$\begin{aligned}
f_h &= 0, \quad \varphi_h < \infty, \quad \delta_h < \infty, \\
f'_h \varphi'_h &= \frac{1}{2}\gamma^2 \frac{dV}{d\varphi}(\varphi_h) + 2\alpha e^{2\alpha\varphi_h} \frac{\sqrt{R_h^4 + q_e^2} - R_h^2}{R_h^2}.
\end{aligned} \tag{9}$$

where the variables with subscript  $h$  shows that they are evaluated at the horizon.

Under this conditions we obtain the black hole solutions numerically using the continuous analogue of Newton method [15], [16]. We have carefully tested our numerical code reproducing independently the results from the article [9]-[10].

### 3 Numerical results and discussions

In what follows we consider black holes with a dilaton coupling parameter  $\alpha = -1$  and dilaton potential  $V(\varphi) = \varphi^2$ .

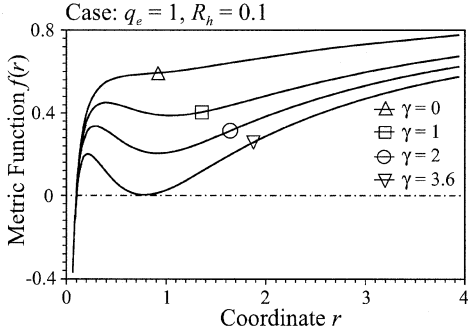


Figure 1: Radial dependence of the metric function  $f(r)$  for different values of the parameter  $\gamma$ . The solution corresponding to  $\gamma = 3.6$  is extremal.

In Fig.1 we show the results of the numerical integration for the radial dependence of the metric function  $f$  for different values of the parameter  $\gamma$  and for a black hole charge  $q_e = 1$ . The curves  $f(r)$  for  $\gamma = 0$ ,  $\gamma = 1$ , and  $\gamma = 2$  represent EBI black holes with only one horizon and mass  $M = 0.6$ ,  $M = 0.76$ ,  $M = 0.86$ , respectively and almost the same radius. The curve  $f(r)$  for  $\gamma = 3.6$  represents an extremal black hole solution with mass  $M = 0.96$ . As an interesting peculiarity we note that this extremal black hole has an inner horizon.

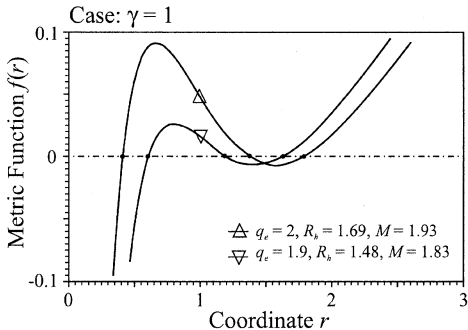


Figure 2: Radial dependence of the metric functions  $f(r)$  for black holes with three horizons.

Solutions representing EBI black holes with three horizons are shown in Fig.2. The parameters  $q_e = 2$  and  $R_h = 1.69$  correspond to a black hole with mass  $M = 1.93$ , while the others correspond to black hole mass  $M = 1.82$ .

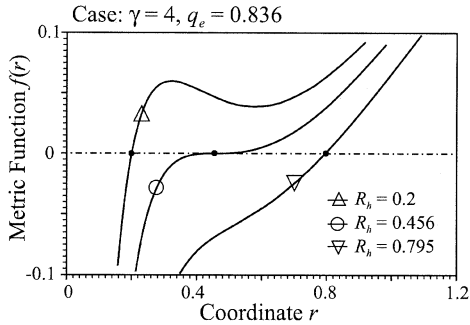


Figure 3: The curve corresponding to  $R_h = 0.456$  represents an extremal black hole with a triply degenerated horizon.

In Fig.3 we show the radial dependence of the metric function  $f(r)$  for an extremal black hole with a triply degenerated horizon. The mass of this black hole is  $M = 0.79$ .

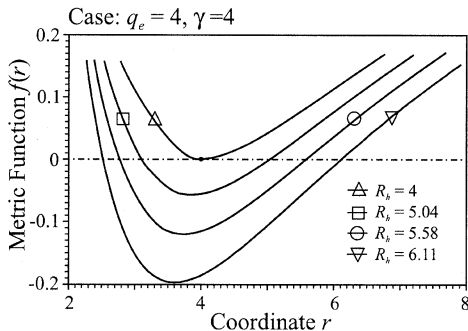


Figure 4: The function  $f(r)$  represents Reissner-Nordström type black holes. The solution with  $R_h = 4$  is extremal.

We have also found EBI black holes with two horizons whose causal structure is of the Reissner-Nordström type. The metric function  $f(r)$  for these black holes is presented in Fig.4. The curves with  $R_h = 5.04$ ,  $R_h = 5.58$ , and  $R_h = 6.11$  correspond to black holes with masses  $M = 4.1$ ,  $M = 4.22$ , and  $M = 4.36$ , respectively. The solution with radius  $R_h = 4$  represents an extremal black hole with  $M = 4$ .

We see that the spectrum of black hole solutions in EBI gravity with a massive dilaton is much richer than the one in the case of a

massless dilaton. In contrast to the massless dilaton case where no extremal electrically charged black hole solutions exist, the massive dilaton do admits extremal black holes. The existence of black holes with three horizons and with a triply degenerated horizon is genuine nonlinear effect due to both nonlinear electrodynamics and dilaton potential. We have carefully examined the black hole solutions within framework of EMD gravity with a massive dilaton and we have found no numerical evidence for the existence of EMD black holes with three horizons and with a triply degenerated horizon. This shows that the nonlinear electrodynamics is substantial for the existence of black holes with three horizons and with a triply degenerated horizon and it is not an effect due to only the dilaton potential.

So far our considerations of the black holes causal structures have been performed in Einstein frame. The Einstein frame and the string frame are related via the conformal factor  $e^{-2\varphi}$ . Since the dilaton field and its derivatives are regular everywhere except at the center where they diverge, the black hole causal structures will remain the same in the string frame.

## Conclusion

In this letter we have presented numerical EBI black hole solutions with a massive dilaton. These solutions show that the massive dilaton allows many more black hole causal structures than the massless one. Depending on the black hole mass, charge and the dilaton mass, EBI black holes can have either one, two, or three horizons. Extremal black hole solutions have been presented, too. There are extremal black holes with an inner horizon as well as extremal black holes with a triply degenerated horizon.

## Acknowledgments

We would like to thank R. Rashkov for the useful comments. We would also like to thank D. Wiltshire, G. Clement and D. Gal'tsov for pointing out some useful references.



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Received by Publishing Department  
on September 16, 2001.

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E2-2001-221

Электрически заряженные черные дыры в рамках теории  
Эйнштейна–Борна–Инфельда

Проводится численное моделирование статических сферически-симметричных заряженных черных дыр в рамках обобщенной теории Эйнштейна–Борна–Инфельда с массивным дилатоном. Численные результаты показывают, что наличие дилатонного потенциала приводит к существенно более сложной структуре черной дыры в сравнении с безмассовым случаем. В частности, показано, что в зависимости от соотношения между массой черной дыры, ее зарядом и массой дилатона возможны состояния с одним, двумя и тремя горизонтами, а также состояние с трижды вырожденным горизонтом.

Работа выполнена в Лаборатории информационных технологий ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2001

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E2-2001-221

Electrically Charged Einstein–Born–Infeld Black Holes  
with Massive Dilaton

We numerically construct static and spherically symmetric electrically charged black hole solutions in Einstein–Born–Infeld gravity with massive dilaton. The numerical solutions show that the dilaton potential allows many more black hole causal structures than the massless dilaton. We find that depending on the black hole mass and charge and the dilaton mass the black holes can have either one, two, or three horizons. The extremal solutions are also found out. As an interesting peculiarity we note that there are extremal black holes with inner horizons and with triply degenerated horizon.

The investigation has been performed at the Laboratory of Information Technologies, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2001

Макет Т.Е.Попеко

Подписано в печать 02.11.2001

Формат 60 × 90/16. Офсетная печать. Уч.-изд. л. 0,77

Тираж 425. Заказ 52935. Цена 95 к.

Издательский отдел Объединенного института ядерных исследований  
Дубна Московской области