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ON THE MIXED SYMMETRY IRREDUCIBLE
REPRESENTATIONS OF THE POINCARÉ GROUP
IN THE **BRST** APPROACH

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1 Introduction

It is well known that the particles with the value of spin more than two arise naturally when quantizing such classical objects as the relativistic oscillator, string or discrete string. The challenging problem for these kinds of theories is to construct the lagrangian description both for free and for interacting particles with the higher spins.

Recent developments in this activity have revealed, that the particles with the higher spins can propagate through the background having the constant curvature, in particular through the AdS space (see [1] and the references therein), as well as interact with the constant electromagnetic field or symmetrical Einstein spaces [2]. The utilization of the technique of the Supersymmetric Quantum Mechanics leads also to the description of the particle with spin 2 on the background of the constant curvature [3], and on the background being real “Kahler – like” manifold [4]. In all these approaches corresponding lagrangians possess some gauge invariance in order to remove the states with the negative norm – the ghosts – from the physical spectrum.

The same problem, i.e., the inclusion into the theory of necessary gauge invariance is present also in the description of free particles which belong to irreducible representations of the Poincare group. The possible way out of this problem is the method of the BRST constructions which naturally leads to the desired gauge invariant hermitian lagrangians [5]. However, the construction of the corresponding nilpotent BRST charges for the system of constraints describing either reducible massive [6] or irreducible massless [7] representations of the Poincare group with the Young tableaux having one row is not straightforward due to the presence of the second class constraints. This problem can be solved after the introduction into the theory of the additional bosonic oscillator and the construction of the auxiliary representations for the second class constraints in terms of this oscillator. Then after the partial gauge fixing [7] the BRST invariant lagrangian leads to the one constructed by Fronsdal [8] for the irreducible massless higher spins.

The construction of lagrangians describing an arbitrary representations of the Poincare group is complicated due to the necessity of the construction of the auxiliary representations of the Lie algebras having the rank more than one in the framework of the BRST approach. The general method for such constructions where some generators from the Cartan subalgebra are excluded from the total system of constraints forming an arbitrary Lie algebra was given in [9]. Below we use this method for the derivation of the lagrangians for the massless particles belonging to the irreducible representations of the Poincare algebra with the corresponding Young tableaux having two rows. The complete generalization of this procedure to the arbitrary irreducible representation of the Poincare group will be given elsewhere. Let us note that the approach used is different from the one given in [10] and leads to the different form of the final lagrangian which describes only irreducible representations, corresponding to Young tableaux with two rows.

The paper is organized as follows.

In Sec.2 we quote the main results of [9] needed for our study.

In Sec.3 we explicitly derive the lagrangians describing the irreducible representations of the Poincare group with corresponding Young tableaux having two rows.

In Sec.4 we present some explicit examples of our construction.

Sec.5 contains our conclusions and discussion of open problems. Some complicated expressions and formulae are carried out to Appendix.

2 The BRST construction

The fields we are going to consider correspond to the following Young tableaux

$$\begin{array}{|c|c|c|c|c|c|c|c|c|c|} \hline \mu_1 & \mu_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \mu_{n_1} \\ \hline \nu_1 & \nu_2 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \nu_{n_2} \\ \hline \end{array} \quad (2.1)$$

and are described by the field $\Phi_{\mu_1\mu_2\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x)$ which is the $n_1 + n_2$ ($n_1 \geq n_2$) rank tensor field symmetrical with respect to the permutations of each type indices. The correspondence with a given Young tableaux implies, that after the symmetrization of all indices of the first row with one index of the second row the basic field vanishes, i.e.,

$$\Phi_{\{\mu_1\mu_2\cdots\mu_{n_1},\nu_1\}\nu_2\cdots\nu_{n_2}}(x) = 0. \quad (2.2)$$

Further, all traces of the basic field must vanish:

$$\begin{aligned} \Phi_{\rho\rho\mu_3\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x) &= 0, \\ \Phi_{\rho\mu_2\mu_3\cdots\mu_{n_1},\rho\nu_2\cdots\nu_{n_2}}(x) &= 0, \\ \Phi_{\mu_1\mu_2\cdots\mu_{n_1},\rho\rho\nu_3\cdots\nu_{n_2}}(x) &= 0. \end{aligned} \quad (2.3)$$

In addition, this field is subject to the following system of equations, namely the mass shell and transversality conditions for each type of indices. In the massless case we have

$$p_\rho^2 \Phi_{\mu_1\mu_2\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x) = 0, \quad (2.4)$$

$$p_\rho \Phi_{\rho\mu_2\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x) = 0, \quad (2.5)$$

$$p_\rho \Phi_{\mu_1\mu_2\cdots\mu_{n_1},\rho\nu_2\cdots\nu_{n_2}}(x) = 0. \quad (2.6)$$

To describe all irreducible representations of the Poincare group simultaneously it is convenient to introduce an auxiliary Fock space generated by the creation and annihilation operators a_μ^{i+}, a_μ^j with Lorentz index $\mu = 0, 1, 2, \dots, D-1$ and additional internal index $i = 1, 2$. These operators satisfy the following commutation relations

$$[a_\mu^i, a_\nu^{j+}] = -g_{\mu\nu}\delta^{ij}, \quad g_{\mu\nu} = \text{diag}(1, -1, -1, \dots, -1), \quad (2.7)$$

where δ^{ij} is usual Cronecker symbol.

The general state of the Fock space depends on the space-time coordinates x_μ

$$|\Phi\rangle = \sum \Phi_{\mu_1\mu_2\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x) a_{\mu_1}^{1+} a_{\mu_2}^{1+} \cdots a_{\mu_{n_1}}^{1+} a_{\nu_1}^{2+} a_{\nu_2}^{2+} \cdots a_{\nu_{n_2}}^{2+} |0\rangle \quad (2.8)$$

and the components $\Phi_{\mu_1\mu_2\cdots\mu_{n_1},\nu_1\nu_2\cdots\nu_{n_2}}(x)$ are automatically symmetrical under the permutations of indices of the same type.

The conditions (2.2) – (2.6) can be easily expressed in this auxiliary Fock space as

$$T|\Phi\rangle = 0, \quad (2.9)$$

$$L^{ij}|\Phi\rangle = 0, \quad (2.10)$$

$$L^0|\Phi\rangle = L^i|\Phi\rangle = 0, \quad (2.11)$$

where the operators

$$T = a_\mu^{+2} a_\mu^2, \quad L^{11} = \frac{1}{2} a_\mu^1 a_\mu^1, \quad L^{12} = a_\mu^1 a_\mu^2, \quad L^{22} = \frac{1}{2} a_\mu^2 a_\mu^2, \quad L^i = p_\mu a_\mu^i, \quad (2.12)$$

along with their conjugates

$$T^+ = a_\mu^{+2} a_\mu^1, \quad L^{11+} = \frac{1}{2} a_\mu^{1+} a_\mu^{1+}, \quad L^{12+} = a_\mu^{1+} a_\mu^{2+}, \quad L^{22+} = \frac{1}{2} a_\mu^{2+} a_\mu^{2+}, \quad L^{i+} = p_\mu a_\mu^{i+} \quad (2.13)$$

and operators

$$L^0 = -p_\mu^2, \quad H^i = -a_\mu^{i+} a_\mu^i + \frac{D}{2} \quad (2.14)$$

satisfy the following nonzero commutation relations

$$\begin{aligned} [L^{11}, L^{11+}] &= H^1, & [L^{22}, L^{22+}] &= H^2, & [L^{12}, L^{12+}] &= H^1 + H^2, \\ [T^+, L^{11+}] &= -L^{12+}, & [T^+, L^{12+}] &= -2L^{22+}, \\ [T, L^{11}] &= L^{12}, & [T, L^{12}] &= 2L^{22}, & [T, T^+] &= H^1 - H^2, \\ [L^{22}, L^{12+}] &= -T, & [L^{22}, T^+] &= -L^{12}, & [T, L^{22+}] &= -L^{12+}, \\ [L^{11}, L^{12+}] &= -T^+, & [L^{12}, L^{11+}] &= -T, & [L^{12}, L^{22+}] &= -T^+, \\ [T, L^{12+}] &= -2L^{11+}, & [T^+, L^{12}] &= 2L^{11} \end{aligned} \quad (2.15)$$

and

$$\begin{aligned} [L^i, T^+] &= -\delta^{i2} L^1, & [L^i, T] &= -\delta^{i1} L^2, \\ [L^{i+}, T^+] &= \delta^{i1} L^{2+}, & [L^{i+}, T] &= \delta^{i2} L^{1+}, \\ [L^i, L^{11+}] &= -\delta^{i1} L^{1+}, & [L^{i+}, L^{11}] &= \delta^{i1} L^1, \\ [L^i, L^{22+}] &= -\delta^{i2} L^{2+}, & [L^{i+}, L^{22}] &= \delta^{i2} L^2, \\ [L^i, L^{12+}] &= -\delta^{i1} L^{2+} - \delta^{i2} L^{1+}, & [L^{i+}, L^{12}] &= \delta^{i1} L^2 + \delta^{i2} L^1, \\ [L^i, L^{j+}] &= \delta^{ij} L^0. \end{aligned} \quad (2.16)$$

Note the asymmetry between the operators a_μ^{1+} and a_μ^{2+} in (2.9). The later equation means in turn that just (2.2) is fulfilled where one index of the second row is symmetrized with indices of the first one. In order to restore the mentioned symmetry, the following equation

$$T^+|\Phi\rangle = 0, \quad (2.17)$$

should be added to the system of constraints. The consistency of the equations (2.9) and (2.17)

$$[T, T^+]\Phi\rangle = (H^1 - H^2)|\Phi\rangle = (n_1 - n_2)|\Phi\rangle = 0 \quad (2.18)$$

implies the equality of numbers $n_1 = n_2 = n$ of the operators a_μ^{1+} and a_μ^{2+} in the Fock space vector (3.17). However, in this case one can show using only equation (2.9) the following property of the basic field

$$\Phi_{\mu_1\mu_2\cdots\mu_n,\nu_1\nu_2\cdots\nu_n}(x) = (-1)^n \Phi_{\nu_1\nu_2\cdots\nu_n,\mu_1\mu_2\cdots\mu_n}(x) \quad (2.19)$$

and, as a consequence, the equation (2.17) is also fulfilled. Moreover, there is no nonzero solution of the equation (2.9) for the case of $n_1 < n_2$. All this means that the Fock space vector under the constraint (2.9) contains all possible Young tableaux of the type (2.1) and only once each of them.

The set of operators $L^{ij}, T, L^{ij+}, T^+, H^i$ form the algebra (2.15) of the group $SO(3, 2)$, H^i being the Cartan generators. Since the conditions $H^i|\Phi\rangle = 0$ can not be satisfied for any nonzero vector $|\Phi\rangle$, the generators H^i must be excluded from the total set of constraints, and therefore one obtains the system of the first and the second class constraints. The corresponding nilpotent BRST charge can be constructed as follows [9].

The first step is to construct the auxiliary representations of the algebra $SO(3, 2)$ using the Verma module after introduction of the additional creation and annihilation operators $[b_I, b_J^\dagger] = \delta_{IJ}$, $I, J = 1, \dots, 4$ (The general method for such constructions was given in [11]). Note that the number of the oscillators is equal to the number of positive roots of algebra $SO(3, 2)$ and the vector $|\Phi\rangle$ depends also on the creation operators b_I^\dagger . The auxiliary representation has the form:

$$\begin{aligned} L_{aux.}^{11+} &= b_1^\dagger, & L_{aux.}^{12+} &= b_2^\dagger, & L_{aux.}^{22+} &= b_3^\dagger, \\ T_{aux.}^+ &= b_4^\dagger - b_2^\dagger b_1 - 2b_3^\dagger b_2, \\ T_{aux.} &= (h_1 - h_2 - b_4^\dagger b_4) b_4 - 2b_1^\dagger b_2 - b_2^\dagger b_3, \\ H_{aux.}^1 &= 2b_1^\dagger b_1 + b_2^\dagger b_2 - b_4^\dagger b_4 + h_1, \\ H_{aux.}^2 &= b_2^\dagger b_2 + 2b_3^\dagger b_3 + b_4^\dagger b_4 + h_2, \\ L_{aux.}^{11} &= (b_1^\dagger b_1 + b_2^\dagger b_2 - b_4^\dagger b_4 + h_1) b_1 - b_4^\dagger b_2 + b_3^\dagger b_2 b_2, \\ L_{aux.}^{12} &= (2b_1^\dagger b_1 + b_2^\dagger b_2 + 2b_3^\dagger b_3 + h_1 + h_2) b_2 + b_4^\dagger b_4 b_4 b_1 \\ &\quad + b_2^\dagger b_3 b_1 - b_4^\dagger b_3 + (h_2 - h_1) b_4 b_1, \\ L_{aux.}^{22} &= (b_2^\dagger b_2 + b_3^\dagger b_3 + b_4^\dagger b_4 + h_2) b_3 \\ &\quad + (h_2 - h_1) b_4 b_2 + b_1^\dagger b_2 b_2 + b_4^\dagger b_4 b_4 b_2, \end{aligned} \quad (2.20)$$

where h_1 and h_2 are parameters and the auxiliary representations of operators H^i depend on them linearly.

Further, let us denote $E^\alpha \equiv (L^{ij}, T)$ ($\alpha > 0$), and define

$$\mathcal{H}^i = H^i + \tilde{H}^i_{aux.} + h^i, \quad \mathcal{E}^\alpha = E^\alpha + E^\alpha_{aux.}(h), \quad (2.21)$$

where we have explicitly extracted the dependence on parameters h_i in the auxiliary representations of Cartan generators. After writing the $SO(3, 2)$ algebra in the compact form

$$\begin{aligned} [\mathcal{H}^i, \mathcal{E}^\alpha] &= \alpha(i)\mathcal{E}^\alpha, \\ [\mathcal{E}^\alpha, \mathcal{E}^{-\alpha}] &= \alpha^i \mathcal{H}^i, \\ [\mathcal{E}^\alpha, \mathcal{E}^\beta] &= N^{\alpha\beta} \mathcal{E}^{\alpha+\beta}, \end{aligned} \quad (2.22)$$

we introduce the anticommuting variables $\eta_\alpha \equiv (\eta_{ij}, \eta_T)$ $\eta_{-\alpha} = \eta_\alpha^+$, having ghost number one and corresponding momenta $\mathcal{P}_{-\alpha} = \mathcal{P}_\alpha^+$ and \mathcal{P}_α , having ghost number minus one with the commutation relations:

$$\{\eta_\alpha, \mathcal{P}_{-\beta}\} = \{\eta_{-\alpha}, \mathcal{P}_\beta\} = \delta_{\alpha\beta}. \quad (2.23)$$

The “ghost vacuum” is defined as

$$\eta_\alpha|0\rangle = \mathcal{P}_\alpha|0\rangle = 0 \quad (2.24)$$

for positive roots α . The nilpotent BRST charge for the above subsystem of constraints with no H^i dependence has the form

$$Q_1 = \sum_{\alpha>0} (\eta_\alpha \mathcal{E}^{-\alpha} + \eta_{-\alpha} \mathcal{E}^\alpha) - \frac{1}{2} \sum_{\alpha\beta} N^{\alpha\beta} \eta_{-\alpha} \eta_{-\beta} \mathcal{P}_{\alpha+\beta} \quad (2.25)$$

where the parameters h^i have to be substituted by the expressions [9]

$$-\pi^i = -H^i - \tilde{H}^i_{aux.} - \sum_{\beta>0} \beta(i) (\eta_\beta \mathcal{P}_{-\beta} - \eta_{-\beta} \mathcal{P}_\beta) \quad (2.26)$$

The inclusion of the constraints $\mathcal{L}^A \equiv (L^0, L^i, L^{i+})$ into the total BRST charge Q is trivial, namely

$$Q = Q_1 + Q_2 \quad (2.27)$$

where

$$Q_2 = \eta_0 L^0 + \eta_i L^{i+} + \eta_i^+ L^i - \eta_i^+ \eta_i \mathcal{P}_0 + \sum_{\alpha>0, A, B} (\eta_A \eta_\alpha^+ \mathcal{P}_B C_{-\alpha, A}^B + \eta_A \eta_\alpha \mathcal{P}_B C_{\alpha, A}^B) \quad (2.28)$$

in self explanatory notations. This completes the procedure of constructing nilpotent BRST charge for our system.

3 The lagrangian and the partial gauge fixing

The BRST invariant lagrangian which describes the massless irreducible representations of the Poincare group of the form (2.1) can be written as

$$-L = \int d\eta_0 \langle \chi | K Q | \chi \rangle, \quad (3.1)$$

being invariant under the gauge transformations

$$\delta | \chi \rangle = Q | \Lambda \rangle \quad (3.2)$$

with a parameter of gauge transformations $|\Lambda\rangle$. The Kernel operator K in the scalar product (3.1) is necessary to make the lagrangian hermitian [9], since the BRST charge Q is not hermitian as one can conclude from the explicit form of the auxiliary representations (2.20). The operator K is constructed as follows. Let us introduce the vector in the space of Verma module

$$|n_1, n_2, n_3, n_4\rangle_V = (L^{11+})^{n_1} (L^{12+})^{n_2} (L^{22+})^{n_3} (T^+)^{n_4} |0\rangle_V \quad (3.3)$$

where $\alpha_1, \alpha_2, \dots, \alpha_r$ is some ordering of positive roots, $n_i \in N$ and $E^\alpha |0\rangle_V = 0$. The corresponding vector in the Fock space generated by the creation and annihilation operators b_I, b_I^+ is

$$|n_1, n_2, n_3, n_4\rangle = (b_1^+)^{n_1} (b_2^+)^{n_2} (b_3^+)^{n_3} (b_4^+)^{n_4} |0\rangle. \quad (3.4)$$

The Kernel operator K defining the scalar product of two vectors $|\Phi_1\rangle$ and $|\Phi_2\rangle$ as $\langle \Phi_2 | K | \Phi_1 \rangle$ has the form

$$K = Z^+ Z, \quad (3.5)$$

where the operator Z transforms the given state from the Fock space to the corresponding state in the Verma module and have the following form

$$Z = \sum_{n_i} \frac{1}{\prod n_i!} (L^{11+})^{n_1} (L^{12+})^{n_2} (L^{22+})^{n_3} (T^+)^{n_4} |0\rangle_V \langle 0 | (b_1)^{n_1} (b_2)^{n_2} (b_3)^{n_3} (b_4)^{n_4}. \quad (3.6)$$

Such form of the operators Z and K guarantees the identity of the scalar products of the corresponding vectors in the Fock space and in the Verma module. Then the following modified hermiticity relation is satisfied [9]

$$Q^+ K = K Q \quad (3.7)$$

In order to be physical the lagrangian (3.1) must have ghost number equal to zero and therefore the ghost number of the vector $|\chi\rangle$ which in turn is the series expansion with respect to creation operators $\eta_0, \eta_i^+, \mathcal{P}_i^+, \eta_\alpha^+, \mathcal{P}_\alpha^+$, along with operators a_μ^{i+} and b_I^+ must be zero as well. The same applies to the parameter of the gauge transformations $|\Lambda\rangle$ which has the ghost number equal to -1 .

Below we prove, following a slightly different way than in [7], that the vector $|\chi\rangle$ contains the only physical field $|S_0\rangle$ with no b_I^+ dependence. The other fields can be either excluded using the equations of motion or gauged away.

First let us write explicitly the \mathcal{P}_α^+ dependence of the vector $|\chi\rangle$ and the parameter $|\Lambda\rangle$

$$|\chi\rangle = |\chi_0\rangle + \mathcal{P}_\alpha^+|\chi_\alpha\rangle + \mathcal{P}_\alpha^+\mathcal{P}_\beta^+|\chi_{\alpha\beta}\rangle + \dots \quad (3.8)$$

$$|\Lambda\rangle = |\Lambda_0\rangle + \mathcal{P}_\alpha^+|\Lambda_\alpha\rangle + \mathcal{P}_\alpha^+\mathcal{P}_\beta^+|\Lambda_{\alpha\beta}\rangle + \dots \quad (3.9)$$

From the gauge transformation law (3.2) and the explicit form of the auxiliary representations of $SO(3, 2)$ algebra (2.20) one can see that the field $|\chi_0\rangle$ transforms through parameters $|\Lambda_\alpha\rangle$ as

$$\begin{aligned} \delta|\chi_0\rangle = & (L^{11+} + b_1^+)|\Lambda_{11}\rangle + (L^{12+} + b_2^+)|\Lambda_{12}\rangle \\ & + (L^{22+} + b_3^+)|\Lambda_{22}\rangle + (T^+ + b_4^+ - b_2^+b_1 - 2b_3^+b_2)|\Lambda_T\rangle. \end{aligned} \quad (3.10)$$

As it was shown in [7] one can use this gauge freedom to gauge away all b_I^+ dependence in $|\chi_0\rangle$ and therefore

$$b_I|\chi_0\rangle = 0 \quad (3.11)$$

Let us note, that one is left with the residual gauge freedom in transformations of $|\chi_0\rangle$ with the parameter Λ_0 . However this parameter is irrelevant to the proof of b_I^+ independence of the vector $|\chi_0\rangle$.

As the second step we use the ideology of auxiliary BRST conditions [12]. Namely since the equations of motion resulting from the lagrangian (3.1) have the form¹

$$Q|\chi\rangle = 0 \quad (3.12)$$

one can also impose on $|\chi\rangle$ the auxiliary conditions

$$R_I|\chi\rangle = [M_I, Q]_\pm|\chi\rangle = QM_I|\chi\rangle = 0 \quad (3.13)$$

for some operators M_I . Such conditions do not affect the physical content of the theory since they remove the states with the zero norm. The choice of the operators M_I is actually the choice of the BRST gauge.

Taking $M_I = b_I$, one obtains

$$(\eta_\alpha + A_\alpha(\eta_\alpha^+, b))|\chi\rangle = 0 \quad (3.14)$$

where the operators A_α are written in the Appendix. Their explicit form leads to the following solution of equations (3.14)

$$|\chi\rangle = (1 + \mathcal{P}_{11}^+A_{11})(1 + \mathcal{P}_{12}^+A_{12})(1 + \mathcal{P}_{22}^+A_{22})(1 + \mathcal{P}_T^+A_T)|\chi_0\rangle, \quad (3.15)$$

¹Since the Kernel operator K is nondegenerate one can multiply the equation $KQ|\chi\rangle = 0$ on K^{-1} to obtain (3.12)

and from (3.11) and (A1)–(A4) one obtains

$$|\chi\rangle = |\chi_0\rangle. \quad (3.16)$$

Until now we have proven that the all b_i^+ and \mathcal{P}_α^+ dependence in the vector $|\chi\rangle$ is the BRST gauge artifact. However one has to keep in mind the equations of motion resulting from the lagrangian (3.1) after the variation with respect to this “pure gauge” fields (see the Appendix).

Inserting the vector $|\chi\rangle$ of the form (3.16) into (3.1) and performing explicit calculation one obtains that the part $|\chi\rangle$ which contains η_α^+ dependence completely decouples from the lagrangian, therefore the state vector has effectively the form

$$\begin{aligned} |\chi\rangle = & |S_0\rangle + \eta_1^+ \mathcal{P}_1^+ |S_1\rangle + \eta_2^+ \mathcal{P}_2^+ |S_2\rangle + \eta_1^+ \mathcal{P}_2^+ |S_3\rangle \\ & + \eta_2^+ \mathcal{P}_1^+ |S_4\rangle + \eta_1^+ \eta_2^+ \mathcal{P}_1^+ \mathcal{P}_2^+ |S_5\rangle + \eta_0 \mathcal{P}_1^+ |R_1\rangle \\ & + \eta_0 \mathcal{P}_2^+ |R_2\rangle + \eta_0 \eta_1^+ \mathcal{P}_1^+ \mathcal{P}_2^+ |R_3\rangle + \eta_0 \eta_2^+ \mathcal{P}_1^+ \mathcal{P}_2^+ |R_4\rangle \end{aligned} \quad (3.17)$$

with vectors $|S_i\rangle$ and $|R_i\rangle$ having ghost number zero and depending only on bosonic creation operators a_μ^{i+} .

Finally using the residual gauge freedom with the parameter $|\Lambda_0\rangle$ and equations of motion one can gauge away the field $|S_3\rangle - |S_4\rangle$, express the other fields in terms of the single field $|S_0\rangle$ and put them into the lagrangian. The final expression for the lagrangian describing all massless irreducible representations of the Poincare group with the corresponding Young tableaux having two rows has the form

$$\begin{aligned} -L = & \langle S_0 | L^0 - L^{+1} L^1 - L^{+2} L^2 - L^{+1} L^{+1} L^{11} \\ & - L^{+1} L^{+2} L^{12} - L^{+2} L^{+2} L^{22} - 2L^{+11} L^0 L^{11} - L^{+11} L^1 L^{11} \\ & - L^{+12} L^0 L^{12} - L^{+12} L^1 L^2 - 2L^{+22} L^0 L^{22} - L^{+22} L^2 L^2 \\ & - L^{+1} L^{+11} L^1 L^{11} - L^{+1} L^{+12} L^2 L^{11} + L^{+1} L^{+22} L^1 L^{22} \\ & - L^{+1} L^{+22} L^2 L^{12} - L^{+2} L^{+11} L^1 L^{12} + L^{+2} L^{+11} L^2 L^{11} \\ & - L^{+2} L^{+12} L^1 L^{22} - L^{+2} L^{+22} L^2 L^{22} + L^{+1} L^{+1} L^{+22} L^{11} L^{22} \\ & - L^{+1} L^{+2} L^{+12} L^{11} L^{22} + L^{+2} L^{+2} L^{+11} L^{11} L^{22} + 3L^{+11} L^{+22} L^0 L^{11} L^{22} \\ & + L^{+11} L^{+22} L^1 L^1 L^{22} - L^{+11} L^{+22} L^1 L^2 L^{12} + L^{+11} L^{+22} L^2 L^2 L^{11} \\ & + L^{+1} L^{+11} L^{+22} L^1 L^{11} L^{22} + L^{+2} L^{+11} L^{+22} L^2 L^{11} L^{22} | S_0 \rangle \end{aligned} \quad (3.18)$$

where the field $|S_0\rangle$ is constrained as

$$T|S_0\rangle = 0 \quad (3.19)$$

$$\begin{aligned} L^{11} L^{11} |S_0\rangle & = L^{11} L^{12} |S_0\rangle = L^{22} L^{22} |S_0\rangle = L^{12} L^{22} |S_0\rangle \\ = (L^{12} L^{12} + 2L^{11} L^{22}) |S_0\rangle & = 0. \end{aligned} \quad (3.20)$$

From the algebra (2.15) one can conclude that the constraints given above are consistent with each other. Actually there are two independent constraints on the

basic field, namely the constraint (3.19) and first of the constraints (3.20), the other ones can be considered as the consistency conditions for this system.

The lagrangian (3.18) is invariant under the transformations

$$\delta|S_0\rangle = L^{i+}|\lambda_i\rangle \quad i = 1, 2. \quad (3.21)$$

parameters $|\lambda_i\rangle$ coming from $\mathcal{P}_1^+|\lambda_1\rangle + \mathcal{P}_2^+|\lambda_2\rangle$ term in $|\Lambda_0\rangle$ are constrained as

$$L^{ki}|\lambda_i\rangle = T|\lambda_2\rangle = 0 \quad T|\lambda_1\rangle = |\lambda_2\rangle \quad (3.22)$$

The conditions (3.22) are necessary to maintain the gauge fixed form (3.17) (or equivalently the gauge invariance of constraints (3.19) –(3.20)) of the wavefunction $|\chi\rangle$ with respect the residual gauge transformations (3.2). Finally one arrives at the transformations with the single gauge parameter $|\lambda\rangle = |\lambda_1\rangle$

$$\delta|S_0\rangle = (L^{1+} + L^{2+}T)|\lambda\rangle \quad (3.23)$$

constrained as follows

$$L^{ij}|\lambda\rangle = T^2|\lambda\rangle = 0 \quad (3.24)$$

4 Examples

In this section we construct the explicit form of the lagrangians for some simple Young tableaux which correspond to lower orders in the expansion of the field $|S_0\rangle$.

- $\begin{array}{|c|} \hline \square \\ \hline \end{array} : |S_0\rangle = \Phi_{\mu,\nu}(x)a_\mu^{1+}a_\nu^{2+}|0\rangle$

The equation (3.19) means antisymmetry of the field: $\Phi_{\mu,\nu}(x) = -\Phi_{\nu,\mu}(x)$. The lagrangian (3.18) for this antisymmetric field (the notoph in four dimensions [13])

$$L = -\Phi_{\mu,\nu}\partial_\rho^2\Phi_{\mu,\nu} + 2\Phi_{\mu,\nu}\partial_\mu\partial_\rho\Phi_{\rho,\nu} \quad (4.1)$$

in terms of the field strength $F_{\mu\nu\rho} = \partial_\mu\Phi_{\nu,\rho} + \partial_\nu\Phi_{\rho,\mu} + \partial_\rho\Phi_{\mu,\nu}$ has the standard form

$$L = \frac{1}{3}F_{\mu\nu\rho}^2 \quad (4.2)$$

and is invariant under the well known gauge transformations

$$\delta\Phi_{\mu,\nu}(x) = \partial_\mu\lambda_\nu(x) - \partial_\nu\lambda_\mu(x). \quad (4.3)$$

- $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} : |S_0\rangle = \Phi_{\mu\nu,\rho}(x)a_\mu^{1+}a_\nu^{1+}a_\rho^{2+}|0\rangle$

The symmetry with respect to the first two indices $\Phi_{\mu\nu,\rho} = \Phi_{\nu\mu,\rho}$ which is

guaranteed by the construction and the condition (3.19) lead to the following property of the field $\Phi_{\mu\nu,\rho}$:

$$\Phi_{\mu\nu,\rho} + \Phi_{\rho\nu,\mu} + \Phi_{\mu\rho,\nu} = 0. \quad (4.4)$$

Taking all this into account the lagrangian (3.18) for the third rank tensor field $\Phi_{\mu\nu,\rho}$ corresponding to the considered Young tableaux can be written in the form

$$\begin{aligned} L = & 2\Phi_{\mu\nu,\rho}\partial_\sigma^2\Phi_{\mu\nu,\rho} - 3\Phi_{\mu\mu,\rho}\partial_\sigma^2\Phi_{\nu\nu,\rho} - 4\Phi_{\mu\nu,\rho}\partial_\mu\partial_\sigma\Phi_{\sigma\nu,\rho} - 2\Phi_{\mu\nu,\rho}\partial_\rho\partial_\sigma\Phi_{\mu\nu,\sigma} \\ & + 6\Phi_{\mu\nu,\rho}\partial_\mu\partial_\nu\Phi_{\sigma\sigma,\rho} + 3\Phi_{\mu\mu,\rho}\partial_\rho\partial_\sigma\Phi_{\nu\nu,\sigma}, \end{aligned} \quad (4.5)$$

and is invariant under the following gauge transformations

$$\delta\Phi_{\mu\nu,\rho}(x) = \partial_\mu\lambda_{\nu,\rho}(x) + \partial_\nu\lambda_{\mu,\rho}(x) - \partial_\rho\lambda_{\mu,\nu}(x) - \partial_\rho\lambda_{\nu,\mu}(x), \quad (4.6)$$

the gauge transformation parameter being traceless $\lambda_{\mu,\mu} = 0$.

- $\begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} : |S_0\rangle = \Phi_{\mu\nu,\rho\sigma}(x)a_\mu^{1+}a_\nu^{1+}a_\rho^{2+}a_\sigma^{2+}|0\rangle$

The field $\Phi_{\mu\nu,\rho\sigma}$ is symmetrical with respect to the permutations of the indices (μ, ν) and (ρ, σ) by the construction and obeys the relations

$$\Phi_{\mu\nu,\rho\sigma} + \Phi_{\rho\nu,\mu\sigma} + \Phi_{\mu\rho,\nu\sigma} = 0 \quad (4.7)$$

and

$$\Phi_{\mu\mu,\rho\rho} = -2\Phi_{\mu\rho,\mu\rho} \quad (4.8)$$

which is the only nontrivial among the constraints (3.19) as a consequence of (4.7). Moreover, according to (2.19) it is also symmetrical under the permutation of pairs of indices

$$\Phi_{\mu\nu,\rho\sigma} = \Phi_{\rho\sigma,\mu\nu}. \quad (4.9)$$

The lagrangian for this field has the form

$$\begin{aligned} L = & -4\Phi_{\mu\nu,\rho\tau}\partial_\sigma^2\Phi_{\mu\nu,\rho\tau} + 12\Phi_{\mu\nu,\rho\rho}\partial_\sigma^2\Phi_{\mu\nu,\tau\tau} - 3\Phi_{\mu\mu,\nu\nu}\partial_\rho^2\Phi_{\sigma\sigma,\tau\tau} \\ & + 16\Phi_{\mu\nu,\rho\sigma}\partial_\mu\partial_\tau\Phi_{\tau\nu,\rho\sigma} - 24\Phi_{\mu\nu,\rho\sigma}\partial_\mu\partial_\nu\Phi_{\tau\tau,\rho\sigma} \\ & - 24\Phi_{\mu\nu,\rho\rho}\partial_\mu\partial_\sigma\Phi_{\sigma\nu,\tau\tau} + 12\Phi_{\mu\nu,\rho\rho}\partial_\mu\partial_\nu\Phi_{\sigma\sigma,\tau\tau} \end{aligned} \quad (4.10)$$

and is invariant under the transformations

$$\delta\Phi_{\mu\nu,ab}(x) = \partial_\mu\lambda_{\nu,ab}(x) - 2\partial_a\lambda_{\mu,\nu b}(x) \quad (4.11)$$

where the symmetrization over couples of Greek and Latin indices is assumed. The parameter of the gauge transformations $\lambda_{\mu,\nu\rho}$ is symmetrical with respect to the indices ν and ρ and obeys the constraints (3.24)

$$\lambda_{\mu,\mu\nu} = \lambda_{\mu,\nu\nu} = 0, \quad (4.12)$$

$$\lambda_{\mu,\nu\rho} + \lambda_{\nu,\rho\mu} + \lambda_{\rho,\mu\nu} = 0 \quad (4.13)$$

and, therefore it is described by the Young tableaux of type $\square\square$.

Although the symmetry properties of the tensor $\Phi_{\mu\nu,\rho\sigma}$ do not coincide with those for Weyl tensor $C_{\mu\nu,\rho\sigma}$.

$$C_{\mu\nu,\rho\sigma} = -C_{\nu\mu,\rho\sigma} = -C_{\mu\nu,\sigma\rho}, \quad C_{\mu\nu,\rho\sigma} = C_{\rho\sigma,\mu\nu}, \quad (4.14)$$

they can be related to each other with the help of the following transformations

$$\Phi_{\mu\nu,\rho\sigma} = \frac{1}{4}(C_{\mu\rho,\nu\sigma} + C_{\mu\sigma,\nu\rho}) \quad (4.15)$$

$$C_{\mu\rho,\nu\sigma} = \frac{4}{3}(\Phi_{\mu\nu,\rho\sigma} - \Phi_{\rho\nu,\mu\sigma}). \quad (4.16)$$

Strictly speaking $C_{\mu\nu,\rho\sigma}$ is not a Weyl tensor because its traces do not vanish. However, one can obtain from (4.10) the following lagrangian

$$\begin{aligned} L = & -\frac{1}{2}C_{\mu\rho,\nu\tau}\partial_\sigma^2 C_{\mu\rho,\nu\tau} - \frac{1}{2}C_{\mu\tau,\nu\rho}\partial_\sigma^2 C_{\mu\rho,\nu\tau} + 3C_{\mu\rho,\nu\rho}\partial_\sigma^2 C_{\mu\tau,\nu\tau} \\ & -\frac{3}{4}C_{\mu\nu,\mu\nu}\partial_\sigma^2 C_{\rho\tau,\rho\tau} + 2C_{\mu\rho,\nu\sigma}\partial_\mu\partial_\tau C_{\tau\rho,\nu\sigma} + 2C_{\mu\rho,\nu\sigma}\partial_\mu\partial_\tau C_{\tau\sigma,\nu\rho} \\ & -6C_{\mu\rho,\nu\sigma}\partial_\mu\partial_\nu C_{\tau\rho,\tau\sigma} - 6C_{\mu\rho,\nu\rho}\partial_\mu\partial_\sigma C_{\sigma\tau,\nu\tau} + 3C_{\mu\rho,\nu\rho}\partial_\mu\partial_\nu C_{\sigma\tau,\sigma\tau} \end{aligned} \quad (4.17)$$

from which the vanishing of all traces of this tensor on mass shell follows. Therefore one can conclude that the lagrangian (4.17) consistently describes the free field theory of the Weyl tensor.

5 Conclusions

In the present paper we have explicitly constructed the lagrangians describing massless irreducible representations of the Poincare group of the form (2.1). As it was mentioned in the introduction and is clear from the calculations above the approach used for this construction can be directly applied for the lagrangian description of an arbitrary representation of the Poincare group as well.

It seems to be interesting to generalize this technique also for the description of the interaction of higher spin fields with some gravitational background. This will obviously lead to the modification of the system of constraints present in the BRST charge. The problem of constructing of the nilpotent BRST charge for this kinds of physical systems can in turn reveal an allowed types of gravitational backgrounds where the higher spin fields can propagate consistently.

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Appendix

In this Appendix we present some formulae implicitly used in the main body of the paper.

The explicit form of the operators $A_\alpha(\eta_\alpha^+, b)$ in (3.14) shows that they indeed depend only on the operators η_α^+ and b_I – the property crucial for the establishing of the relation (3.16) :

$$A_{11} = -2\eta_T^+ b_2 - 2\eta_T^+ b_1 b_4 - \eta_{11}^+ b_1 b_1 + \eta_{22}^+ b_2 b_2 + 2\eta_{12}^+ b_1 b_1 b_4 + 2\eta_{22}^+ b_1 b_2 b_4, \quad (A1)$$

$$A_{12} = -\eta_T^+ b_3 - \eta_{11}^+ b_1 b_2 - \eta_{12}^+ b_2 b_2 + \eta_T^+ b_1 b_4 b_4 - \eta_{12}^+ b_1 b_1 b_4 b_4 - \eta_{22}^+ b_1 b_2 b_4 b_4, \quad (A2)$$

$$A_{22} = 2\eta_T^+ b_3 b_4 - \eta_{11}^+ b_2 b_2 - 2\eta_{12}^+ b_2 b_3 - \eta_{22}^+ b_3 b_3 + 2\eta_T^+ b_2 b_4 b_4 - 2\eta_{12}^+ b_1 b_3 b_4 - 2\eta_{22}^+ b_2 b_3 b_4 - 2\eta_{12}^+ b_1 b_2 b_4 b_4 - 2\eta_{22}^+ b_2 b_2 b_4 b_4, \quad (A3)$$

$$A_T = -\eta_{11}^+ b_2 - \eta_{12}^+ b_3 + \eta_T^+ b_4 b_4 - \eta_{12}^+ b_1 b_4 b_4 - \eta_{22}^+ b_2 b_4 b_4. \quad (A4)$$

The lagrangian (3.1) after using the explicit expression of the state vector (3.17) and of the BRST charge (2.27) takes the form

$$\begin{aligned} -L = & \langle R_1 || R_1 \rangle + \langle R_2 || R_2 \rangle - \langle R_3 || R_3 \rangle - \langle R_4 || R_4 \rangle - \langle R_1 | L^1 | S_0 \rangle \\ & + \langle R_1 | L^+ | S_1 \rangle + \langle R_1 | L^+ | S_3 \rangle - \langle R_2 | L^2 | S_0 \rangle + \langle R_2 | L^+ | S_4 \rangle \\ & + \langle R_2 | L^+ | S_2 \rangle - \langle R_3 | L^1 | S_4 \rangle + \langle R_3 | L^2 | S_1 \rangle + \langle R_3 | L^+ | S_5 \rangle \\ & - \langle R_4 | L^1 | S_2 \rangle + \langle R_4 | L^2 | S_3 \rangle - \langle R_4 | L^+ | S_5 \rangle + \langle S_0 | L^0 | S_0 \rangle \\ & - \langle S_0 | L^+ | R_1 \rangle - \langle S_0 | L^+ | R_2 \rangle - \langle S_1 | L^0 | S_1 \rangle + \langle S_1 | L^1 | R_1 \rangle \\ & + \langle S_1 | L^+ | R_3 \rangle - \langle S_2 | L^0 | S_2 \rangle + \langle S_2 | L^2 | R_2 \rangle - \langle S_2 | L^+ | R_4 \rangle \\ & - \langle S_3 | L^0 | S_4 \rangle + \langle S_3 | L^2 | R_1 \rangle + \langle S_3 | L^+ | R_4 \rangle - \langle S_4 | L^0 | S_3 \rangle \\ & + \langle S_4 | L^1 | R_2 \rangle - \langle S_4 | L^+ | R_3 \rangle + \langle S_5 | L^0 | S_5 \rangle \\ & - \langle S_5 | L^1 | R_4 \rangle + \langle S_5 | L^2 | R_3 \rangle. \end{aligned} \quad (A5)$$

The equations of motion resulting from the lagrangian (A5) are

$$S0 : \quad L^+ | R_1 \rangle + L^+ | R_2 \rangle - L^0 | S_0 \rangle = 0,$$

$$S1 : \quad L^1 | R_1 \rangle + L^+ | R_3 \rangle - L^0 | S_1 \rangle = 0,$$

$$S2 : \quad L^2 | R_2 \rangle - L^+ | R_4 \rangle - L^0 | S_2 \rangle = 0,$$

$$\begin{aligned}
S3 : & \quad L^2|R_1\rangle + L^{+2}|R_4\rangle - L^0|S_4\rangle = 0, \\
S4 : & \quad L^1|R_2\rangle - L^{+1}|R_3\rangle - L^0|S_3\rangle = 0, \\
S5 : & \quad L^2|R_3\rangle - L^1|R_4\rangle + L^0|S_5\rangle = 0, \\
R1 : & \quad |R_1\rangle - L^1|S_0\rangle + L^{+1}|S_1\rangle + L^{+2}|S_3\rangle = 0, \\
R2 : & \quad |R_2\rangle - L^2|S_0\rangle + L^{+2}|S_2\rangle + L^{+1}|S_4\rangle = 0, \\
R3 : & \quad |R_3\rangle - L^2|S_1\rangle + L^1|S_4\rangle - L^{+2}|S_5\rangle = 0, \\
R4 : & \quad |R_4\rangle + L^1|S_2\rangle - L^2|S_3\rangle + L^{+1}|S_5\rangle = 0, \tag{A6}
\end{aligned}$$

while the equations of motion obtained from the variation of the initial lagrangian (3.1) with respect to other fields in the expansion (3.8) which couple to the $|\chi_0\rangle$ look as follows

$$\begin{aligned}
T|S_0\rangle = 0, & \quad T|R_1\rangle - |R_2\rangle = 0, & \quad T|R_2\rangle = 0, \\
T|S_1\rangle - |S_3\rangle - |S_4\rangle = 0, & \quad |S_2\rangle - T|S_3\rangle = 0, & \quad L^{11}|S_1\rangle = 0, \\
L^{11}|S_3\rangle = 0, & \quad L^{12}|S_3\rangle + |S_5\rangle = 0, & \quad L^{12}|S_1\rangle = 0, \\
L^{22}|S_1\rangle - |S_5\rangle = 0, & \quad |S_2\rangle - T|S_4\rangle = 0, & \quad L^{22}|S_3\rangle = 0, \\
T|S_2\rangle = 0, & \quad L^{11}|S_2\rangle - |S_5\rangle = 0, & \quad L^{11}|S_4\rangle = 0, \\
L^{12}|S_4\rangle + |S_5\rangle = 0, & \quad L^{12}|S_2\rangle = 0, & \quad L^{22}|S_4\rangle = 0, \\
L^{22}|S_2\rangle = 0, & \quad L^{11}|R_2\rangle + |R_3\rangle = 0, & \quad L^{11}|R_1\rangle = 0, \tag{A7} \\
L^{12}|R_1\rangle - |R_3\rangle = 0, & \quad L^{12}|R_2\rangle + |R_4\rangle = 0, & \quad L^{22}|R_2\rangle = 0, \\
L^{22}|R_1\rangle - |R_4\rangle = 0, & \quad T|R_3\rangle - |R_4\rangle = 0, & \quad T|S_5\rangle = 0, \\
L^{11}|S_5\rangle = 0, & \quad L^{12}|S_5\rangle = 0, & \quad L^{22}|S_5\rangle = 0, \\
L^{11}|R_3\rangle = 0, & \quad L^{12}|R_3\rangle = 0, & \quad L^{22}|R_3\rangle = 0, \\
T|R_4\rangle = 0, & \quad L^{11}|R_4\rangle = 0, & \quad L^{12}|R_4\rangle = 0, \\
L^{12}|S_0\rangle + |S_3\rangle + |S_4\rangle = 0, & \quad L^{22}|R_4\rangle = 0.
\end{aligned}$$

Due to this large but nevertheless, consistent system of equations all auxiliary fields entering in the expansion of the state vector (3.17) are expressed via the basic field $|S_0\rangle$

$$|S_1\rangle = -L^{11}|S_0\rangle, \quad |S_2\rangle = -L^{22}|S_0\rangle, \quad |S_3\rangle = -\frac{1}{2}L^{12}|S_0\rangle,$$

$$\begin{aligned}
|S_4\rangle &= -\frac{1}{2}L^{12}|S_0\rangle, & |S_5\rangle &= -L^{11}L^{22}|S_0\rangle, \\
|R_4\rangle &= (L^1L^{22} - \frac{1}{2}L^2L^{12} + L^{+1}L^{11}L^{22})|S_0\rangle, \\
|R_3\rangle &= (\frac{1}{2}L^1L^{12} - L^2L^{11} - L^{+2}L^{11}L^{22})|S_0\rangle, \\
|R_2\rangle &= (L^2 + \frac{1}{2}L^{+1}L^{12} + L^{+2}L^{22})|S_0\rangle, \\
|R_1\rangle &= (L^1 + L^{+1}L^{11} + \frac{1}{2}L^{+2}L^{12})|S_0\rangle.
\end{aligned} \tag{A8}$$

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Бурдик Ч., Пашнев А., Цулая М.
О неприводимых представлениях группы Пуанкаре
со смешанной симметрией в БРСТ-подходе

E2-2001-4

Дано лагранжево описание неприводимых безмассовых представлений группы Пуанкаре, которым соответствуют схемы Юнга с двумя строчками, и приведено несколько содержательных примеров, включая нотопф и тензор Вейля. Для данного описания использован метод БРСТ-конструкций, применимый для систем со связями второго рода. Данная схема предполагает построение вспомогательных представлений алгебры с помощью модулей Верма.

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Burdik Č., Pashnev A., Tsulaia M.
On the Mixed Symmetry Irreducible Representations
of the Poincare Group in the BRST Approach

E2-2001-4

The lagrangian description of irreducible massless representations of the Poincare group with the corresponding Young tableaux having two rows along with some explicit examples including the notoph and Weyl tensor is given. For this purpose the method of the BRST constructions is used adopted to the systems of second class constraints by the construction of auxiliary representations of the algebras of constraints in terms of Verma modules.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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