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LIGHT CONE DISTRIBUTION AMPLITUDES OF PSEUDOSCALAR MESONS WITHIN THE CHIRAL QUARK MODEL

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1. INTRODUCTION

Interest in the form factor $T_P(q_1^2, q_2^2)$ for the transition processes $\gamma^*(q_1)\gamma(q_2) \to P(p)$ and $\gamma^*(q_1)\gamma^*(q_2) \to P(p)$, where the final state mesons with the momentum p are, respectively, $P = \pi^0$, η, η', and q_1 and q_2 are photon momenta, has again increased recently. Recent data on the form factors T_P for small virtuality of one of the photons, $q_2^2 \approx 0$, with the virtuality of the other photon being scanned up to 8, 22, 30 GeV^2, correspondingly, becomes available from CLEO [1] Collaboration. Theoretically, at zero virtualities, the form factor $T_P(0,0)$ is related to the axial anomaly. At asymptotically large photon virtualities, its behaviour is predicted by perturbative QCD (pQCD) [2,3] and depends crucially on the internal meson dynamics which is parameterized by a nonperturbative distribution amplitude (DA), $\varphi_P^A(x)$, with x being a fraction of the meson momentum, p, carried by a quark.

In the following, it is convenient to parameterize the photon virtualities as $q_1^2 = -(1+\omega)Q^2/2$ and $q_2^2 = -(1-\omega)Q^2/2$, where Q^2 and ω are, respectively, the total virtuality of the photons and the asymmetry in their distribution:

$$Q^2 = -(q_1^2 + q_2^2) \geq 0, \quad \text{and} \quad \omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2). \tag{1}$$

The experimental data from CLEO [1] for the process $\gamma^*\gamma \to P$ ($|\omega| = 1$) can be fitted by a monopole form factor:

$$T_P(q_1^2 = -Q^2, q_2^2 = 0)\big|_{\text{fit}} = \frac{g_{P\gamma\gamma}}{1 + Q^2/\mu_P^2}, \tag{2}$$
$$g_{\pi\gamma\gamma} \simeq 0.27 \text{ GeV}^{-1}, \quad g_{\eta\gamma\gamma} \simeq 0.26 \text{ GeV}^{-1}, \quad g_{\eta'\gamma\gamma} \simeq 0.34 \text{ GeV}^{-1},$$
$$\mu_\pi \simeq 0.78 \text{ GeV}, \quad \mu_\eta \simeq 0.77 \text{ GeV}, \quad \mu_{\eta'} \simeq 0.86 \text{ GeV},$$

where $g_{P\gamma\gamma}$ are the two-photon meson decay constants. In the lowest order of pQCD, the light-cone Operator Product Expansion (OPE) predicts the high Q^2 behaviour of the form factor as follows [2,4]:

$$T_P(q_1^2, q_2^2)\big|_{Q^2\to\infty} = J_P(\omega)\frac{f_P}{Q^2} + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^4}\right), \tag{3}$$

with the asymptotic coefficient given by

$$J_P(\omega) = \frac{4}{3}\int_0^1 \frac{dx}{1 - \omega^2(2x-1)^2}\varphi_P^A(x), \tag{4}$$

where f_P is a weak pseudoscalar meson decay constant, defined through the well know PCAC relation. For the pion, $f_\pi = 93$ MeV (for the definition of the other constants see Sect. 2). The leading-twist meson light-cone DA is normalized by $\int_0^1 dx\, \varphi_P^A(x) = 1$. Since the meson DA reflects the internal nonperturbative meson dynamics, the prediction of the value of $J_P(\omega)$ is rather a nontrivial task, and its accurate measurement would provide quite valuable information.

It is important to note that, for the considered transition process, the leading asymptotic term of pQCD expansion (3) is not suppressed by the strong coupling constant α_s. Hence, the pQCD prediction (3) may become reasonable at the highest of the presently accessible momenta $Q^2 \sim 10$ GeV^2. At asymptotically high Q^2, the DA evolves to $\varphi_P^{A,\mathrm{asympt}}(x) = 6x(1-x)$ and $J_P^{\mathrm{asympt}}(|\omega| = 1) = 2$. The fit of CLEO data for the pion corresponds to $J_\pi^{\mathrm{CLEO}}(|\omega| \approx 1) \simeq 1.6$, indicating that already at moderately high momenta this value is not too far from its asymptotic limit.

However, since the pQCD evolution of DA reaches the asymptotic regime very slowly, its exact form at moderately high Q^2 may not coincide with $\varphi_P^{A,\mathrm{asympt}}(x)$. At lower Q^2, the power corrections to the form factor become important. Thus, to study the behaviour of the transition form factor at experimentally accessible Q^2 is the subject of nonperturbative dynamics, where the same as in (3) type of the leading high Q^2 behaviour was obtained by different methods. So, the theoretical determination of the transition form factor is still challenging, and it is desirable to perform direct calculations of $T_P(q_1^2, q_2^2)$ without *a priori* assumptions about the shape of the meson DA.

The transition form factor in the symmetric kinematics, $q_1^2 = q_2^2$, $(\omega = 0)$ at high virtualities, is in accordance with the norm of a wave function (see (4)), and is thus trivial. In the other extreme limit, where one photon is almost real $(\omega = 1)$, the asymptotic coefficient is proportional to $\int_0^1 \frac{dx}{x} \varphi_P^A(x)$ and thus is very sensitive to a detailed form of the DA. Therefore, much more detailed information about the nonperturbative QCD vacuum is necessary to have control over the operator expansion.

In ref. [5], some progress was achieved by using a refined technique based on the OPE with nonlocal condensates [6], which is equivalent to the inclusion of the whole series of power corrections. By means of the QCD sum rules with nonlocal condensates, it was shown that this approach works in almost the whole kinematic region $|\omega| \lesssim 1$, and that for high values of the asymmetry parameter $|\omega| \gtrsim 0.8$ the pion transition form factor is very sensitive to the nonlocal structure of the QCD vacuum. The latter is characterized by the average quark virtuality in the vacuum [6], λ_q^2, and, within the instanton model [7], may be expressed through the average instanton size, ρ_c, as $\lambda_q^2 \approx 2\rho_c^{-2}$ [8,9]. In [10], the form factor $\gamma^*\gamma \to \pi^0$ was directly calculated from a QCD sum rule for the three-point function, leading to the estimate $J_{\mathrm{QCDsr}}(\omega = 1) \approx 1.6 \pm 0.3$. A similar result was obtained within the quark model with nonlocality induced by instantons [11], where the factorization expression for the form factor was derived and the DA was expressed in terms of a nonlocal quark-pion vertex.

Chiral models of the quark dynamics, based on the strong interaction symmetries, have many attractive features, as this approach is consistent with the low-energy theorems. In particular, the Abelian axial anomaly is within this approach, and the standard result for $T_{\pi^0}(0,0) \equiv g_{\pi^0\gamma\gamma} = (4\pi^2 f_\pi)^{-1}$ is reproduced exactly. Within this nonperturbative model of quark-meson interaction, both the small mass and the composite structure of a light pseudoscalar meson are realistically described. Furthermore, the intrinsic nonlocal structure of the model may be motivated by fundamental QCD processes like the instanton and gluon exchanges.

In this paper, within a covariant nonlocal low-energy model of the quark-meson interaction, we study the high Q^2 behaviour of the meson transition form factor $\gamma^*\gamma^* \to P$ in the general kinematics. We show that the asymptotic coefficient $J_P(\omega)$ depends on the kinematics of the transition process and on the internal meson dynamics induced by the nonlocal structure of the QCD vacuum. The dynamic dependence of J_P is governed by the parameter M_q/Λ, where M_q is the constituent quark mass, and Λ is the UV regulator. When considering the model dependence of the asymptotic coefficient J_P, experimental data can be very useful to distinguish between different assumptions, made on the nonperturbative dynamics of the QCD vacuum. Within the nonlocal quark-meson model, the expression for the asymptotic coefficient J_P is found in the whole kinematic region of ω. Moreover, from this dependence, the pseudoscalar meson DA is reconstructed in terms of the quark-meson vertex function.

The structure of the paper is following. In Sect. 2, two models of quark-meson interaction are described: the Nambu–Jona-Lasinio model and the model where the quark-meson interaction is induced by instantons. In Sect. 3, we describe the transition $\gamma\gamma \to P$ process and calculate its asymptotic for high Q^2. A relation between the DA of meson and the vertex function describing the quark-meson interaction is also obtained. The meson DA amplitudes are calculated in Sect. 4. There, the model results are compared with the experimental data from CLEO. In the last section, we give a brief discussion of the obtained results.

2. EFFECTIVE QUARK-MESON MODELS.

The effective quark-meson dynamics can be summarized in the covariant nonlocal action given by

$$S_{\rm int} = -\int d^4x\, d^4y\; F\left[x + y/2, x - y/2; \Lambda^{-2}\right]$$
$$\times \bar{q}(x + y/2)\; i\gamma_5 [g_{\pi\bar{q}q} \sum_{a=1}^{3} \lambda^a \pi^a(x) + g_{\eta_{\rm u}\bar{q}q}\lambda_{\rm u}\eta_{\rm u}(x) + g_{\eta_{\rm s}\bar{q}q}\lambda_{\rm s}\eta_{\rm s}(x)]\; q(x - y/2), \quad (5)$$

where λ_a are the Gell-Mann matrices, $\lambda_{\rm u} = (\sqrt{2}\lambda_0 + \lambda_8)/\sqrt{3}$, $\lambda_{\rm s} = (-\lambda_0 + \sqrt{2}\lambda_8)/\sqrt{3}$. The dynamic vertex $F\left[x + y/2, x - y/2; \Lambda^{-2}\right]$ with the nonlocality size Λ^{-1} depends on the coordinates of the quark and antiquark; $q(x)$, $\pi(x)$, $\eta_{\rm u}(x)$, and $\eta_{\rm s}(x)$ are, respectively, the quark, pion, and $\eta_{\rm u}$ and $\eta_{\rm s}$ pure $\bar{u}u$ and $\bar{s}s$ pseudoscalar meson states.

The fields $\eta_{\rm u}$ and $\eta_{\rm s}$ in (5) are not physical because they are subject to singlet-octet mixing:

$$\eta_{\rm s} = \eta\cos(\theta_0 - \theta) - \eta'\sin(\theta_0 - \theta), \quad (6)$$
$$\eta_{\rm u} = \eta\sin(\theta_0 - \theta) + \eta'\cos(\theta_0 - \theta), \quad (7)$$

where $\theta = -19^\circ$ is the singlet-octet mixing angle, and $\theta_0 \approx 35.5^\circ$ is the ideal mixing angle [12,13], and η, η' are physical meson fields.

In the following calculations, we restrict ourselves to the approximation (see, *e.g.* [14])

$$F\left[x + y/2, x - y/2; \Lambda^{-2}\right] \to F(y^2, \Lambda^{-2}), \tag{8}$$

where the dynamic quark-meson vertex depends only on the relative coordinate of the quark and antiquark squared, y^2, with the dependence of the vertex on angular variable (yx) neglected. The Fourier transform of the vertex function in the Minkowski space is defined as $\tilde{F}(k^2; \Lambda^2) = \int d^4y F(y^2; \Lambda^{-2}) \exp(-ik\,y)$ with normalization $\tilde{F}(0; \Lambda^2) = 1$, and we assume that it rapidly decreases in the Euclidean region ($k^2 = -k_E^2 \equiv -u$). We also approximate the momentum-dependent quark self-energy in the quark propagator $S^{-1}(k) = \widehat{k} - M$ by a constant quark mass [14] and neglect meson mass effects. Here, M is a diagonal 3×3 flavour matrix, $M = \mathrm{diag}(M_\mathrm{u}, M_\mathrm{d}, M_\mathrm{s})$, $M_\mathrm{u} = M_\mathrm{d}$. We have to note that the approximations used here are not fully consistent. Further, as we show below, the choice of the model for the quark-meson vertex (8), depending only on the relative coordinate, induces a certain artifact in the x behaviour of the DA. However, these deficiencies of the chosen approximation are not essential for the present purpose and do not lead to large numeric errors.

The quark-meson interaction is described by the coupling constants $g_{\pi\bar{q}q}$, $g_{\eta_\mathrm{u}\bar{q}q}$, and $g_{\eta_\mathrm{s}\bar{q}q}$. Let us consider their definition in the NJL and instanton induced quark-meson (IQM) models.

NJL model. In the NJL model, the quark-meson coupling constants (see [12]) are defined as follows:

$$g_{\pi\bar{q}q} = g_{\eta_\mathrm{u}\bar{q}q} \equiv g_\mathrm{u}, \quad g_{\eta_\mathrm{s}\bar{q}q} \equiv g_\mathrm{s}, \tag{9}$$

where we introduced g_u and g_s for convenience,

$$g_a^{-2} = \frac{N_\mathrm{c}}{4\pi^2} \int du\, \tilde{F}(-u; \chi_a^{-1}) \frac{u}{(1+u)^2}, \quad (a = \mathrm{u}, \mathrm{s}), \tag{10}$$

In (10), we have rescaled the integration variable by the quark mass squared. In the NJL model, the vertex function $\tilde{F}(-u; \chi_a^{-1})$ is chosen in the form of the step function:

$$\tilde{F}(-u; \chi_a^{-1}) = \theta(1 - \chi_a u), \tag{11}$$

where we introduced the parameters $\chi_a = M_a^2/\Lambda^2$. As one can see, the introduction of the vertex function of this specific form is equivalent to implementing an ultraviolet cut-off in quark-loop integrals in the standard NJL model.

The UV cut-off parameter Λ characterizes the size of the SBCS domain. For the distances larger then Λ^{-1}, SBCS takes place. The Compton length of a quark or a meson should be greater then this length. Given the values of Λ and M_a, one can estimate χ_a. Let us say some words about these parameters. The values of Λ and $M_{\rm u}$ are fixed by two conditions [12, 13]: i) the Goldberger-Treiman relation $M_{\rm u} = g_{\pi\bar{q}q} f_\pi$ 1) and ii) the ρ-meson decay constant g_ρ whose value 6.1 is well known from experiment. The mass of the strange quark is fixed by the kaon mass. In [12], the $\Lambda = 1.05$ GeV, $M_{\rm u} = 234$ MeV, and $M_{\rm s} = 467$ MeV were obtained 2). This corresponds to the quark condensate $\langle \bar{q}q \rangle \approx -(250\ {\rm MeV})^3$. Therefore, one can see that the parameters χ_q are small: $\chi_{\rm u} = 0.05$ and $\chi_{\rm s} = 0.2$.

Instanton-induced quark-meson model (IQM). In the constant quark mass approximation to IQM that we use here, the quark-meson coupling is given by the compositeness condition [14] in the form

$$g_a^{-2} = \frac{N_c}{8\pi^2} \int_0^\infty du\, \tilde{F}^2(-u; \chi_a^{-1}) \frac{(3+2u)u}{(1+u)^3}; \tag{12}$$

and the meson weak decay constants $f_\pi \equiv f_{\rm u}$, and $f_{\rm s}$ are expressed by

$$f_a = \frac{N_c g_a}{4\pi^2} M_a \int_0^\infty du\, \tilde{F}(-u; \chi_a^{-1}) \frac{u}{(1+u)^2}. \tag{13}$$

Within the instanton vacuum model, the size of nonlocality of the nonperturbative gluon field, $\rho_c \sim \Lambda^{-1}$, is much smaller than the quark Compton length M_a^{-1}; thus, χ_a are small parameters [7] as well as in the NJL model. The parameter χ_a characterizes the diluteness of the instanton vacuum. We have to note that in the exact IQM the Goldberger-Treiman relation and other low energy theorems are fulfilled [9]. Within the constant quark mass approximation to IQM, the low-energy relation following from the Adler - Bell - Jackiw (ABJ) axial anomaly $[f_\pi g_{\pi\gamma\gamma} = 1/(4\pi^2)]$ is well reproduced numerically with a relative error smaller than 10% [14]. In the formal limit of a very dilute vacuum medium $\chi_a \ll 1$, the results of this approximation are consistent with the ABJ anomaly and the Goldberger-Treiman relation.

For the IQM estimates, we take the vertex function in the Gaussian form, like in [11]:

$$\tilde{F}(k^2; \Lambda^2) = \exp(k^2/\Lambda^2), \tag{14}$$

and parameters $\Lambda^2 = 0.55 \pm 0.05\ {\rm GeV}^2$ and $M_{\rm u} = 275 \pm 25$ MeV. This choice of parameters correspond to the average quark virtuality $\lambda_{\rm u}^2 = 0.65 \pm 0.05\ {\rm GeV}^2$, and quark condensate $\langle \bar{q}q \rangle = -\,(205 \pm 15\ {\rm MeV})^3$.

1) This relation is naturally reproduced in the IQM [9] and NJL [12] models from the $\pi \to \mu\nu_\mu$ decay amplitude.

2) Here we use the parameters that had been obtained with the $\pi - A_1$-transition not taken into account.

3. MESON TRANSITION $\gamma^*\gamma^* \to P$ FORM FACTOR AT MODERATELY HIGH Q^2

Let us consider the contribution to the $\gamma^*\gamma^* \to P$ invariant amplitude as calculated from the triangle diagrams (see Fig. 1):

$$M\left(\gamma^*\left(q_1, e_1\right)\gamma^*\left(q_2, e_2\right) \to P\left(p\right)\right) = m_{P\gamma\gamma}(q_1, e_1; q_2, e_2) + m_{P\gamma\gamma}(q_2, e_2; q_1, e_1) \quad (15)$$

where $e_i (i = 1, 2)$ are the photon polarization vectors, and

$$\begin{aligned} m_{P\gamma\gamma}(q_1, e_1; q_2, e_2) &= -\frac{N_c}{3} g_{Pqq} \int \frac{d^4k}{(2\pi)^4} \tilde{F}(k^2; \Lambda^2) \\ &\times \mathrm{tr}\{i\gamma_5 S(k - p/2)\hat{e}_2 S[k - (q_1 - q_2)/2]\hat{e}_1 S(k + p/2)\} \\ &= \frac{1}{2} T_P(q_1^2, q_2^2)\epsilon_{\mu\nu\rho\sigma} e_1^\mu e_2^\nu q_1^\rho q_2^\sigma. \end{aligned} \quad (16)$$

If the tensor $\epsilon_{\mu\nu\rho\sigma} e_1^\mu e_2^\nu q_1^\rho q_2^\sigma$ is factorized from this amplitude, the form factor can be expressed as

$$T_P(q_1^2, q_2^2) = \frac{g_{Pqq}}{2\pi^2} M_a I_{P\gamma\gamma}(q_1^2, q_2^2, p^2), \quad (17)$$

where $M_a = M_\mathrm{u}$ for $P = \pi$ or $P = \eta_\mathrm{u}$ and $M_a = M_\mathrm{s}$ for $P = \eta_\mathrm{s}$. The Feynman integral $I_{P\gamma\gamma}(q_1^2, q_2^2, p^2)$ is given by

$$I_{P\gamma\gamma}(q_1^2, q_2^2, p^2) = \quad (18)$$

$$= \int \frac{d^4k}{i\pi^2} \frac{\tilde{F}(k^2; \Lambda^2)}{[M_a^2 - (k + p/2)^2 - i\varepsilon][M_a^2 - (k - p/2)^2 - i\varepsilon][M_a^2 - (k - (q_1 - q_2)/2)^2 - i\varepsilon]}.$$

Let us note that integral (18) is similar in its structure to the integral arising in the lowest order of pQCD, treating the quark-photon interaction perturbatively. In the latter case, its asymptotic behaviour is due to the subprocess $\gamma^*(q_1) + \gamma^*(q_2) \to \bar{q}(\bar{x}p) + q(xp)$ with x $(\bar{x})$ being the fraction of the meson momentum p carried by the quark produced at the q_1 (q_2) photon vertex. The relevant diagram is similar to the handbag diagram for hard exclusive processes, with the main difference that one should use, as a nonperturbative input, a quark-meson vertex instead of the meson DA. As we see below, this similarity allows one to translate the form of the quark-meson vertex into a specific shape of the meson DA.

Let us now estimate the asymptotics of the transition form factor. To this end, we rewrite the expression for integral (18) in the form that is obtained after rotating to the Euclidean space $[k^2 \to -u,\ -id^4k \to \pi^2 u du,\ \tilde{F}(k^2; \Lambda^2) \to \tilde{F}(-u; \Lambda^2)]$, by using the Feynman $\alpha-$parameterization for the denominators and integrating over the angular variables. Then, the corresponding integral $I_{P\gamma\gamma}$ is given by

$$I_{P\gamma\gamma}(q_1^2, q_2^2, p^2) = \int_0^\infty \frac{du\, u\tilde{F}(-u; \Lambda^2)}{M_q^2 + u - \frac{p^2}{4}}$$

$$\times \int_0^1 d\alpha \left[\frac{1}{\sqrt{b^4 - a_+^4}\left(b^2 + \sqrt{b^4 - a_+^4}\right)} + \frac{1}{\sqrt{b^4 - a_-^4}\left(b^2 + \sqrt{b^4 - a_-^4}\right)}\right], \quad (19)$$

where

$$b^2 = M_q^2 + u + \frac{1}{2}\alpha Q^2 - \frac{1}{4}(1-2\alpha)p^2, \quad a_\pm^4 = 2u\alpha Q^2(\alpha \pm \omega(1-\alpha)) - (1-2\alpha)up^2. \tag{20}$$

In this way, the expression (19) can be safely analyzed in the asymptotic limit of high total virtuality of the photons $Q^2 \to \infty$. Moreover, the integral over α can be taken analytically, leading, in the chiral limit $m_\pi = 0$, to the asymptotic expression given by (3), where (see [11])

$$J_P(\omega) \equiv J_{P,\mathrm{np}}(\omega) = \frac{2N_P}{3\omega}\left\{\int_0^\infty du\, \frac{\tilde{F}(-u;\chi_a^{-1})}{1+u}\ln\left[\frac{1+u(1+\omega)}{1+u(1-\omega)}\right]\right\} \tag{21}$$

$$N_P = \left[\int_0^\infty du u \frac{\tilde{F}(-u;\chi_a^{-1})}{(1+u)^2}\right]^{-1},$$

$$a = \mathrm{u} \text{ if } P = \pi, \eta_\mathrm{u}, \text{ and } \quad a = \mathrm{s} \text{ if } P = \eta_\mathrm{s}. \tag{22}$$

The η and η' mesons appear as mixed η_u and η_s states (see (7)), and for them, one has:

$$J_\eta(\omega) = c_1\, J_{\eta_\mathrm{u}}(\omega) + c_2\, J_{\eta_\mathrm{s}}(\omega), \tag{23}$$

$$J_{\eta'}(\omega) = c_3\, J_{\eta_\mathrm{u}}(\omega) + c_4\, J_{\eta_\mathrm{s}}(\omega), \tag{24}$$

where the coefficients c_i are:

$$\begin{aligned} c_1 &= \frac{5f_\mathrm{u}}{3f_\eta}\sin(\theta_0-\theta) \quad & c_2 &= -\frac{\sqrt{2}f_\mathrm{s}}{3f_\eta}\cos(\theta_0-\theta) \\ c_3 &= \frac{5f_\mathrm{u}}{3f_{\eta'}}\cos(\theta_0-\theta) \quad & c_4 &= \frac{\sqrt{2}f_\mathrm{s}}{3f_{\eta'}}\sin(\theta_0-\theta), \end{aligned} \tag{25}$$

and the constants f_η and $f_{\eta'}$ are defined as

$$f_\eta = \frac{5}{3}f_\mathrm{u}\sin(\theta_0-\theta) - \frac{\sqrt{2}}{3}f_\mathrm{s}\cos(\theta_0-\theta) \tag{26}$$

$$f_{\eta'} = \frac{5}{3}f_\mathrm{u}\cos(\theta_0-\theta) + \frac{\sqrt{2}}{3}f_\mathrm{s}\sin(\theta_0-\theta). \tag{27}$$

Here, the constants f_u and f_s are calculated using (13).

From (21) it is clear, that the prediction of the nonperturbative approach to the asymptotic coefficient is rather sensitive to χ_a, the product of the value of the constituent mass M_q and the size of nonlocality Λ^{-1} of the vertex $F(x^2;\Lambda^{-2})$ and to the relative distribution of the total virtuality among photons, ω. In particular, for the off-shell process $\gamma^*\gamma^* \to \pi^0$ in the kinematic case of symmetric distribution of photon virtualities, $q_1^2 = q_2^2 \to -\infty$ ($\omega \to 0$), the result $J(|\omega| = 0) = 4/3$ obtained from (21) is in agreement with the OPE prediction.

Let us note, that we use an approximation to the model with constant constituent quark masses for all three quark lines in the diagrams of the process (see Fig. 1). However, the asymptotic result (21) is independent of the mass parameter in the quark propagator with hard momentum flow, as it should be. The other two quark lines remain soft during the process; thus, the mass parameter M_a can be considered as given on a certain characteristic soft scale in the momentum-dependent case $M_a\left(\lambda_a^2\right)$. It means that the dynamic and kinematic dependence of $J_{P,\mathrm{np}}\left(\omega\right)$ found in (21) will be unchanged, even if one includes the momentum dependence of the quark mass and considers the dressed quark-photon vertex which goes into the bare one, γ^μ, as one of the squared quark momenta becomes infinite.

Both the expressions for J_P derived within the nonlocal quark-meson model (21) and from the light-cone OPE (4) can be put into the common form

$$J_P\left(\omega\right) = \frac{2}{3\omega}\int_0^1 d\xi R_P(\xi)\ln\left[\frac{1+\xi\omega}{1-\xi\omega}\right] \tag{28}$$

with

$$R_P^{\mathrm{pQCD}}(\xi) = -\frac{d}{d\xi}\varphi_P^A\left(\frac{1+\xi}{2}\right) \quad \text{and} \quad R_{P,\mathrm{np}}(\xi) = N_P\tilde{F}\left(\frac{-\xi}{1-\xi};\chi_a^{-1}\right)\frac{1}{1-\xi}, \tag{29}$$

$$\text{where}\quad 0 \le \xi \equiv (2x-1) \le 1$$

and similar expressions for $-1 \le \xi \le 0$. Equating both contributions, we find the meson DA in terms of the vertex function on a certain low-energy scale $\mu_0^2 \sim \Lambda^2$ [11]

$$\varphi_P^A(x) = N_P\int_{|2x-1|}^1 \frac{dy}{1-y}\tilde{F}\left(\frac{-y}{1-y};\chi_a^{-1}\right). \tag{30}$$

Thus, we show that (21) obtained within the nonlocal quark-meson model is equivalent to the standard lowest-order pQCD result (4), with the only difference that the nonperturbative information accumulated in the meson DA $\varphi_P^A(x)$ is represented by the quark-meson vertex function $\tilde{F}(-u;\chi_a^{-1})$.

To compare our results with CLEO data, one needs to investigate the asymptotic $Q^2 \to \infty$ behaviour of the following magnitude:

$$\mathcal{T}_P(\omega) = \lim_{Q^2\to\infty} Q^2 T_P(q_1^2,q_2^2), \tag{31}$$

using the leading term in the asymptotic expression (3) expressed through the asymptotic coefficient J_P (see (4)), where the meson DA are calculated from (30) in a certain model of the quark-meson interaction.

We have to note that an explicit form of the asymptotic coefficient (21) and the relation between the DA and the vertex function depend on the model of quark-meson interaction (5). In particular, the expression (30) is obtained within the approximation (8), when the quark-meson vertex depends only on the relative coordinate. This approximation results in the artificial dependence of DA on the modulo function of x and leads to the nonsmooth behaviour of the distribution at $x = 1/2$. These peculiarities disappear if the angular dependence of the vertex motivated by, *e.g.*, the instanton model is recovered [3)].

4. THE MESON DISTRIBUTION AMPLITUDES AND NUMERICAL PREDICTIONS OF THE TRANSITION FORM FACTORS AT MODERATELY HIGH Q^2.

The quark-meson interaction in the Nambu–Jona-Lasinio model [12] is described by the vertices like (5). The only difference from IQM is that the vertex function $\tilde{F}(-u, \chi_a^{-1})$ is taken in the form of the step function: $\tilde{F}(-u, \chi_a^{-1}) = \theta(1 - u\chi_a)$. One just have to calculate the integral (18) with the vertex function $\tilde{F}(-u, \chi_a^{-1})$ thus chosen. Formally, the integral (18) is convergent, and the UV cut-off is not necessary here, however, the extracting of its asymptotic behavior at large Q^2 would lead to logarithmic dependence $\sim \ln Q^2/Q^2$, which is not expected, as it is known from QCD. Therefore, the UV cut-off in the NJL model should be treated not only as a trick to make the integrals convergent, but also as a way to take into account the non-trivial nonlocal vacuum structure.

Let us note also that all calculations are performed in the chiral limit (zeroth meson masses). An on-mass-shell calculation entails problems with unphysical $\bar{q}q$ thresholds for the η'-meson [4)].

Within the NJL model, one can easily obtain from (30) analytical expressions for two special DA describing the distribution of $u(s)$- and s-quarks, respectively,

$$\varphi_{\rm u}^A(x) = \begin{cases} \dfrac{\ln\left|\dfrac{1-\xi_{\rm u}}{1-|2x-1|}\right|}{\xi_{\rm u} + \ln|1-\xi_{\rm u}|}, & |2x-1| \le \xi_{\rm u} \\ 0, & |2x-1| > \xi_u \end{cases} \tag{32}$$

where the constant $\xi_{\rm u}$ is defined as follows:

$$\xi_u = \frac{\Lambda^2}{\Lambda^2 + M_{\rm u}^2}; \tag{33}$$

$$\varphi_{\rm s}^A(x) = \begin{cases} \dfrac{\ln\left|\dfrac{1-\xi_{\rm s}}{1-|2x-1|}\right|}{\xi_{\rm s} + \ln|1-\xi_{\rm s}|}, & |2x-1| \le \xi_{\rm s}\ , \\ 0, & |2x-1| > \xi_{\rm s} \end{cases} \tag{34}$$

3) In [9], emerging of a similar cusp for the pion distribution function and its disappearance, if the angular dependence in the vertex is taken into account, were demonstrated.

4) Authors already have a model describing the meson spectrum, where unphysical quark-antiquark thresholds are eliminated [17]. However, this model in not quite consistent with the calculation of transition form factors.

$$\xi_s = \frac{\Lambda^2}{\Lambda^2 + M_s^2}. \quad (35)$$

For the pion, η, and η' mesons, one has:

$$\varphi_\pi(x) = \varphi_u(x), \quad (36)$$

$$\varphi_\eta(x) = c_1\, \varphi_u(x) + c_2 \varphi_s(x), \quad (37)$$

$$\varphi_{\eta'}(x) = c_3\, \varphi_u(x) + c_4 \varphi_s(x), \quad (38)$$

where the coefficients c_i are defined in (25). The DA calculated in the NJL and IQM models are plotted in Figs. 2–5.

Now we compare our results for the case $\omega = 1$ ($\gamma^*\gamma \to P$) with those given by CLEO collaboration [1]. In the NJL model, we have $\mathcal{T}_\pi(1) \approx 0.197$ GeV, $\mathcal{T}_\eta(1) \approx 0.19$ GeV, and $\mathcal{T}_{\eta'}(1) \approx 0.26$ GeV. The IQM predicts $\mathcal{T}_\pi(1) \approx 0.16$ GeV, $\mathcal{T}_\eta(1) \approx 0.17$ GeV, and $\mathcal{T}_{\eta'}(1) \approx 0.23$ GeV. CLEO collaboration gives $\mathcal{T}_\pi(1) = 0.17 \pm 0.3$ GeV at $Q^2 = 7.0 - 9.0$ GeV^2, $\mathcal{T}_\eta(1) \approx 0.16$ GeV at $Q^2 \sim 22$ GeV^2, $\mathcal{T}_{\eta'}(1) \approx 0.25$ GeV at $Q^2 \sim 30$ GeV^2.

5. DISCUSSION AND CONCLUSIONS.

Within the two covariant nonlocal models under consideration that describe the quark-meson dynamics, we obtained the $\gamma^*\gamma^* \to P$ transition form factor at moderately high momentum transfers squared, where the perturbative QCD evolution does not yet reach the asymptotic regime. From the model calculations, it is possible to find the normalization coefficient at the leading Q^{-2} term. The asymptotic normalization coefficient $J_P(\omega)$, given in (21), depends on the ratio of the constituent quark mass on a certain soft scale to the characteristic size of QCD vacuum fluctuations (or the UV cut-off) Λ and also on the kinematics of the process. When considering the dependence of the asymptotic coefficient on the internal dynamics, the CLEO data are consistent with a small value of the diluteness parameter, which confirms the hypothesis about the small density of the instanton liquid vacuum [7] or approximate locality of the quark-meson interaction in NJL. From the comparison of the kinematic dependence of the asymptotic coefficient of the transition pion form factor, given by pQCD and the nonperturbative models (IQM and NJL), the relation (30) between the meson distribution amplitude and the dynamical quark-meson vertex function is derived. In the specific case of symmetric kinematics ($q_1^2 = q_2^2$), our result agrees with the one obtained by OPE.

We obtained explicit expressions for the DA of pion, η, and η' meson. Comparing with CLEO data has shown that the model in the chiral limit (massless pseudoscalar mesons) is in satisfactory agreement with experiment. However, we would like to make some notes regarding the definition of f_η and $f_{\eta'}$ used by authors of [1]. In [1], f_P are obtained from the tabulated data on the decays $P \to \gamma\gamma$, using the low-energy limit of the process amplitude:

$$T_P(0,0) \simeq 1/(4\pi^2 f_P) \quad (39)$$

(see (6) in [1]). This works well for the pion, but the case of the η and η' mesons is rather different because of the singlet-octet mixing. The quark-meson model calculation gives for f_P defined by (39) in the limit $Q^2 \to 0$ the following:

$$f_{\eta}^{(0)} = \left[\frac{5}{3} f_{\rm u}^{-1} \sin(\theta_0 - \theta) - \frac{\sqrt{2}}{3} f_{\rm s}^{-1} \cos(\theta_0 - \theta)\right]^{-1} \quad (40)$$

$$f_{\eta'}^{(0)} = \left[\frac{5}{3} f_{\rm u}^{-1} \cos(\theta_0 - \theta) + \frac{\sqrt{2}}{3} f_{\rm s}^{-1} \sin(\theta_0 - \theta)\right]^{-1} . \quad (41)$$

We marked these constants by the superscript (0) to distinguish them from those defined in (26) and (27). In the limit $Q^2 \to \infty$, one should expect:

$$\lim_{Q^2 \to \infty} Q^2 T_P(Q^2, 1) = 2 f_P, \quad (42)$$

where f_P are not the same as $f_P^{(0)}$ as one can see from comparing (26) and (27) with (40) and (41). That is why the CLEO fit noticeably disagrees with the limit $2f_{\eta'}$ (see (5) in [1]) drawn in Fig. 23 [1]. Therefore, it is not correct to use the equation (7) in [1] to perform a fit. From our calculation, we see that, taking into account the singlet-octet mixing, one can avoid the discrepancy in the description of the $P\gamma\gamma$ interaction both in the low and high Q^2.

The results presented in our paper are in accordance with the conclusions made in [5, 10, 18] within the QCD sum rules. A more complete analysis of the light pseudo-scalar meson transition form factors will be done later, where effects of the finite hadron masses, nonlocality of the quark-photon vertex, $\pi - A_1$ transitions *etc.* will be considered. It is of interest also to consider the transition form factors on a wide energy scale, including not only the small and large Q^2 but also intermediate.

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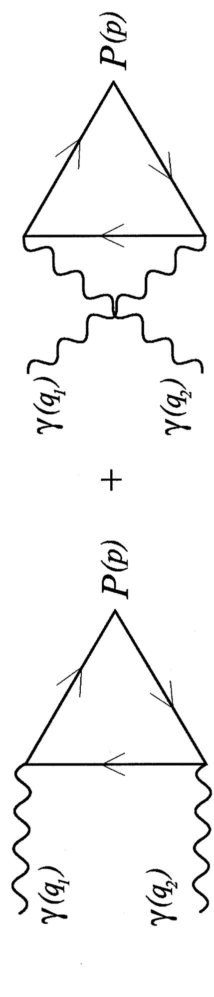
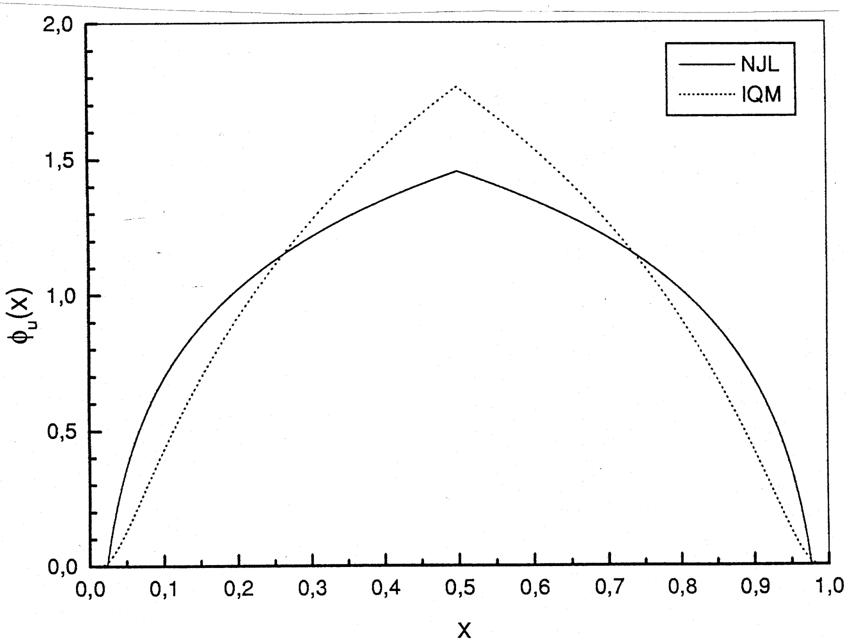
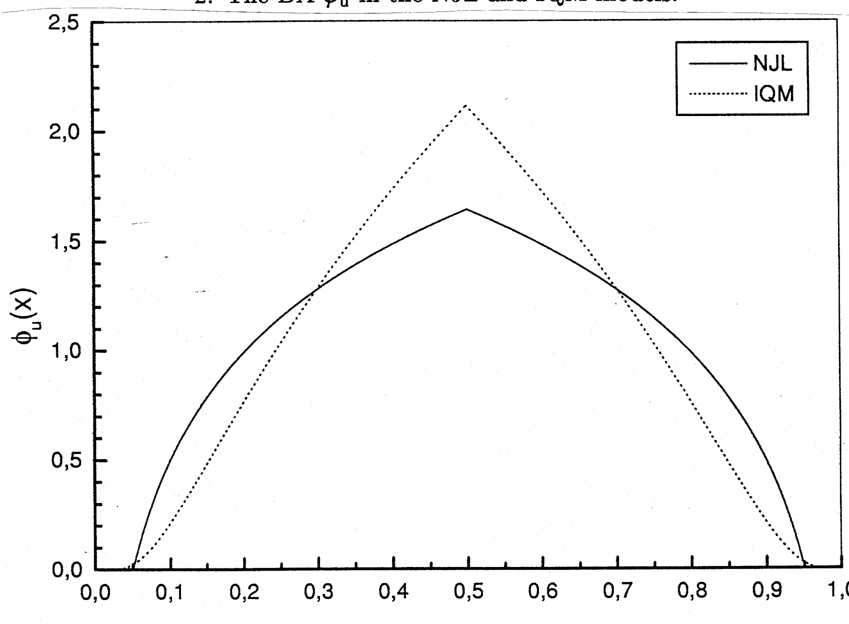


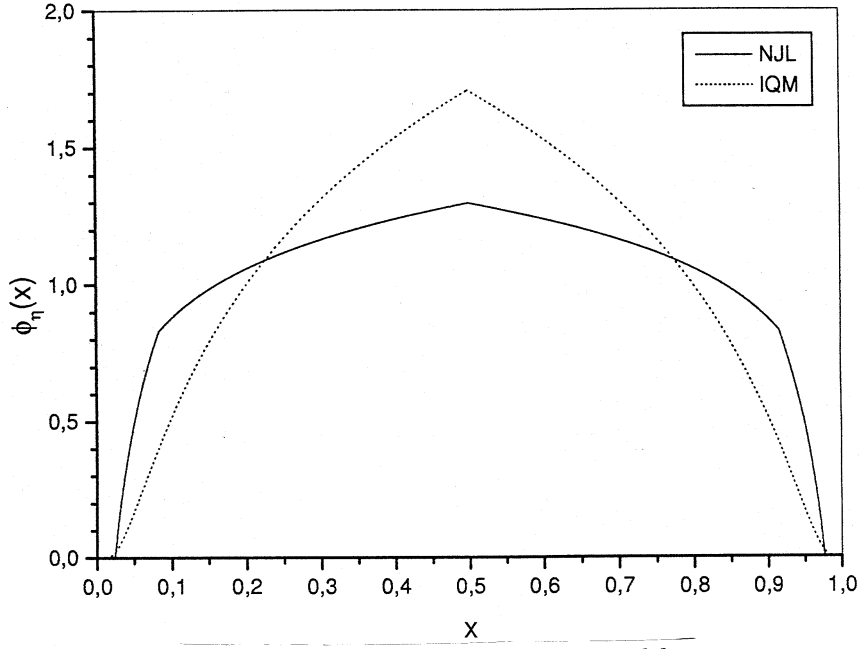
1. The diagrams contributing to the process $\gamma^*\gamma^* \to P$ amplitude.



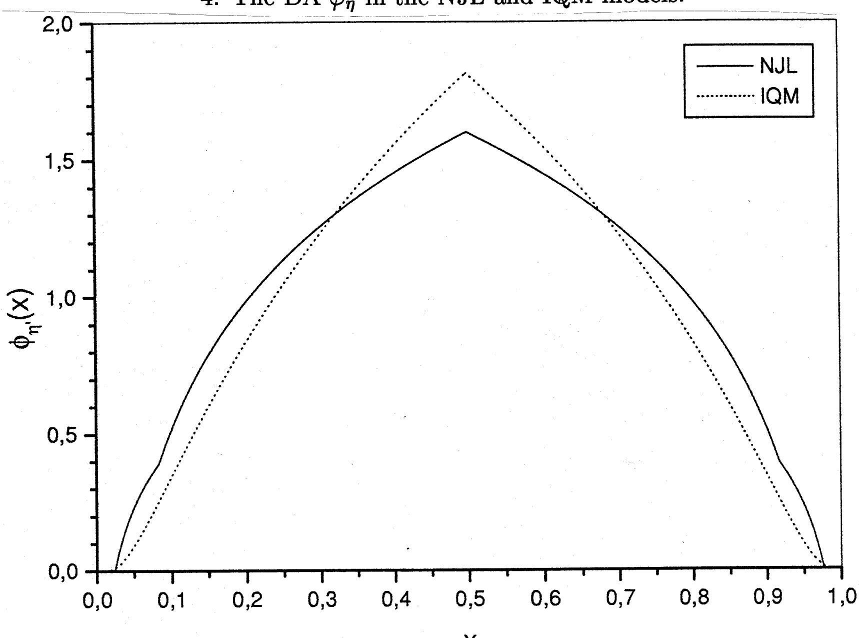
2. The DA φ_u in the NJL and IQM models.



3. The DA φ_s in the NJL and IQM models.



4. The DA φ_η in the NJL and IQM models.



5. The DA $\varphi_{\eta'}$ in the NJL and IQM models.

REFERENCES

1. J. Gronberg *et al.*, *CLEO Collaboration*, Phys. Rev. D **57**, 33 (1998).
2. G. P. Lepage and S. J. Brodsky, Phys. Lett. B **87**, 359 (1979); Phys. Rev. D **22**, 2157 (1980).
3. F. Del Aguila, M.K. Chase, Nucl.Phys. B **193**, 517 (1981); E. Braaten, Phys. Rev. D **28**, 524 (1983); E.P. Kadantseva, S.V. Mikhailov, A.V. Radyushkin, Sov. J. Nucl. Phys. **44**, 326 (1986); P. Gosdzinsky, N. Kivel, Nucl.Phys. B **521**, 274 (1998).
4. M.K. Chase, Nucl.Phys. B **167**, 125 (1980).
5. S.V. Mikhailov and A. V. Radyushkin, Sov. J. Nucl. Phys. **52**, 697 (1990).
6. S.V. Mikhailov and A.V. Radyushkin, JETP Lett. **43**, 712 (1986); Sov.J.Nucl. Phys. **49**, 494 (1989); Phys. Rev. D **45**, 1754 (1992).
7. T. Schäfer and E.V. Shuryak, Rev. of Mod. Phys. **70**, 323 (1998) (and references therein).
8. A.E. Dorokhov, S.V. Esaibegyan, and S.V. Mikhailov, Phys. Rev. D **56**, 4062 (1997).
9. A.E. Dorokhov, Lauro Tomio, Phys. Rev. D **62**, 014016 (2000).
10. A. V. Radyushkin and R. T. Ruskov, Nucl. Phys. B **481**, 625 (1996); *The asymptotics of the transition form factor* $\gamma\gamma^{*} \to \pi^{0}$ *and QCD sum rules*, hep-ph/9706518.
11. I.V. Anikin, A.E. Dorokhov, Lauro Tomio, Phys. Let. B **475**, 361 (2000).
12. M.K. Volkov, Sov. J. Part. Nucl. **17**, 186 (1986); D. Ebert, H. Reinhardt, M.K. Volkov, Prog. Part. Nucl. Phys. **35**, 1 (1994).
13. M.K. Volkov, V.L. Yudichev, Nuovo Cim. A **112**, 225 (1999).
14. H. Ito, W.W. Buck, F. Gross, Phys. Rev. C **45**, 1918 (1992); Phys. Lett. B **287**, 23 (1992); I. Anikin, M. Ivanov, N. Kulimanova, V. Lyubovitskii, Phys. Atom. Nucl. **57**, 1082 (1994);
15. D. Ebert, M.K. Volkov, Sov. J. Nucl. Phys. **36**, 736 (1982).
16. I. V. Musatov and A. V. Radyushkin, Phys. Rev. D **56**, 2713 (1997) (and references therein).
17. M.K. Volkov, V.L. Yudichev, Yad.Fiz. **63**, 536 (2000).
18. A.P. Bakulev, S.V. Mikhailov, Phys. Lett. B **436**, 351 (1998).

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Дорохов А.Е., Волков М.К., Юдичев В.Л. E4-2001-162
Амплитуды распределения псевдоскалярных мезонов на световом конусе в киральной кварковой модели

$U(3)\times U(3)$ киральная кварковая модель использована для вычисления амплитуд распределения π-, η- и η'-мезонов. Чтобы найти эти амплитуды распределения, рассмотрена асимптотика формфактора перехода псевдоскалярных мезонов (P) для процессов $\gamma^*\gamma \to P$ и $\gamma^*\gamma^* \to P$ при больших пространственноподобных фотонных импульсах. Показано, что в асимптотическом режиме происходит факторизация больших и малых расстояний. Коэффициентная функция дается стандартным выражением пертурбативной КХД, а динамика больших расстояний сосредоточена в амплитудах распределения. В рамках используемой модели вид волновой функции зависит от отношения параметра УФ регуляризации Λ к конституентной массе кварка. Показано, что результаты киральной кварковой модели близки к вычислениям в модели инстантонного вакуума. Численные предсказания киральной кварковой модели для асимптотик псевдоскалярных формфакторов для процессов $\gamma^*\gamma \to P$ близки к данным коллаборации CLEO.

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Dorokhov A.E., Volkov M.K., Yudichev V.L. E4-2001-162
Light Cone Distribution Amplitudes of Pseudoscalar Mesons within the Chiral Quark Model

A $U(3)\times U(3)$ chiral quark model is used to calculate the light cone distribution amplitudes of the π, η, and η' mesons. To find these distribution amplitudes, the asymptotics of the transition pseudoscalar meson (P) form factor for the processes $\gamma^*\gamma \to P$ and $\gamma^*\gamma^* \to P$ at large space-like photon momenta were considered. It is shown that in the asymptotic regime a factorization of large and small distances occurs. The coefficient function is given by the standard pQCD expression, and the large distance dynamics is accumulated in distribution amplitudes. Within the model used, the form of a wave function depends on the ratio of the UV regulator parameter Λ to the constituent quark mass. It is shown that the results of the chiral quark model are close to the instanton model calculations. The numerical predictions of the chiral quark model for the asymptotics of the transition pseudoscalar form factors for the processes $\gamma^*\gamma \to P$ are close to the CLEO data.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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