E4-2001-57

Yu.N.Uzikov[1], C.Wilkin[2]

MODEL-INDEPENDENT ANALYSIS OF THE NEUTRON-PROTON FINAL-STATE INTERACTION REGION IN THE $pp \to pn\pi^+$ REACTION

Submitted to «Physics Letters B»

[1]Kazakh State University, Almaty, 480121 Kazakhstan;
E-mail: uzikov@nusun.jinr.dubna.su
[2]University College London, London, WC1E 6BT, UK

Inclusive data on the $pp \to \pi^+ X$ reaction in the 400 MeV to 1 GeV range generally show two structures. There is firstly a broad peak corresponding to the quasi-free production of the Δ-isobar. In addition, there is an enhancement near the edge of phase space arising from the strong neutron-proton final-state interaction (fsi) in either the spin-triplet or singlet S-wave which comes when the np excitation energy, E_{np}, is only a few MeV. The details of the enhancement region are hard to investigate in a single-arm experiment, where only the π^+ is measured, because of contamination from the much larger two-body $pp \to d\,\pi^+$ reaction [1]. The simplest way to overcome this background is by measuring in coincidence the final proton and pion in the exclusive $pp \to pn\,\pi^+$ reaction. The most complete examples of such an experiment were carried out at LAMPF, where both the five-fold differential cross section [2] and the proton analysing power [3] were studied at a proton beam energy of $T_p = 800$ MeV.

LAMPF data taken at one pair of proton/pion angles are shown in Fig. 1 as a function of the detected proton momentum [2]. The large peak on the right is associated with protons and pions formed in the $pp \to \Delta^{++}n/\Delta^+p$ reactions and its magnitude and width can be explained in different versions of one-meson-exchange models [2, 4]. At the maximum of the smaller peak, E_{np} is about 1 MeV and the cross section is strongly influenced by the np fsi. The form of this peak is normally parameterised by Watson final-state interaction factors [5, 6] which take into account the nearby poles in the scattering amplitudes due to the deuteron bound state in the spin-triplet case and the anti-bound state for the spin-singlet. Since the latter is closer to $E_{np} = 0$, the singlet fsi peak is expected to be narrower than that of the triplet. Though this difference in shape can, in principle, be used to extract the relative amount of the np spin-triplet and singlet in the final-state peak, the limited number of points in the peak and the modest resolution in E_{np} makes this impractical for the LAMPF data.

In analysing their experiment, the authors of Ref. [2] made the *ad hoc* assumption that the one-meson-exchange model for quasi-free Δ production was valid for E_{np} above 10 MeV and that, below this excitation energy, the prediction could be smoothly joined onto Watson fsi factors. The relative spin-singlet strength depends, of course, upon the kinematics and their analysis suggested that the fraction was about 15-25% of the total for the different angle pairs measured. This is close to a statistical mixture of 25% and is in complete contrast to measurements at lower energies, where the singlet fraction is typically 10% [7]. Alternative model-dependent analyses of the LAMPF data do lead to smaller singlet production, but they depend upon other assumptions made [8]. In view of these differing conclusions, it is worthwhile to seek a different way of deducing the spin-singlet contribution from these data.

When the square of the low energy np triplet S-state scattering wave function at energy $E_{np} = k^2/m_N$, where m_N is twice the n-p reduced mass, is analytically continued to negative energy, it manifests the deuteron pole at $k^2 = -\alpha_t^2$, where $\alpha_t = 0.232\ \text{fm}^{-1}$. It has recently been shown [9] that the relative normalisation of the scattering, $\Psi_k(r)$, and bound-state, $\Psi_{\alpha_t}(r)$, wave function depends purely upon the deuteron binding energy and is independent of the np potential [9]. Using real boundary conditions, it follows that at short distances

$$[\Psi_k(r)]^2 \approx \frac{2\pi}{\alpha_t(k^2+\alpha_t^2)}\,[\Psi_{\alpha_t}(r)]^2\ . \tag{1}$$

Apart from questions associated with the D-state contributions which are small at low k^2, this relation becomes exact as $k^2 \to -\alpha_t^2$. In the scattering region where $E_{np} \geq 0$, it allow one to estimate the spin-triplet amplitude for $pp \to pn\,\pi^+$ in terms of that for $pp \to d\,\pi^+$. Thus, for any value of the initial (σ_i) and final spin-triplet projection λ, the matrix elements are related by

$$M^{\lambda}_{\sigma_1\sigma_2}(pp \to \{np\}_t\,\pi^+) \approx f(k^2)\, M^{\lambda}_{\sigma_1\sigma_2}(pp \to d\,\pi^+)\ , \tag{2}$$

where

$$f^2(k^2) = \frac{2\pi\, m_N}{\alpha_t(k^2+\alpha_t^2)}\ \cdot \tag{3}$$

We are here using a normalisation where the unpolarised $pp \to d\,\pi^+$ cross section is given by

$$\frac{d\sigma}{d\Omega^*} = \frac{1}{64\pi^2 s_{pp}}\,\frac{q^*_{\pi d}}{q^*_p}\,\frac{1}{4}\sum_{\sigma_1\sigma_2\lambda} \mid M^{\lambda}_{\sigma_1\sigma_2}(pp \to d\,\pi^+)\mid^2\ , \tag{4}$$

s_{pp} is the square of the centre-of-mass (cm) energy, and q_p^* and $q_{\pi d}^*$ are the cm momenta in the initial and final states respectively. At low E_{np} the approximation of eq. (2) reproduces very well the results of single-arm experiments where only the pion is detected [7].

We now extend this approach to treat two-arm experiments. Starting from eq. (2), the triplet contribution to the laboratory five-fold differential cross section for the detection of a pion at an angle θ_π and a proton at an angle θ_p becomes

$$\frac{\mathrm{d}^5\sigma_t(pp \to pn\pi^+)}{\mathrm{d}p_p\,\mathrm{d}\Omega_p\,\mathrm{d}\Omega_\pi} = \frac{1}{16\pi^3}\,s_{pp}\frac{q_p^*}{q_{\pi d}^*}\,\Phi\,f^2(k^2)\,\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega_\pi^*}(pp \to d\pi^+)\,, \tag{5}$$

where the phase-space factor is

$$\Phi = \frac{p_p^2 p_\pi^3}{p_0\,m_p\,E_p\,|p_\pi^2\,E_n - \mathbf{p}_\pi \cdot \mathbf{p}_n E_\pi|}\,. \tag{6}$$

Here p_0 is the beam momentum, E_i and p_i $(i = p, n, \pi)$ are the laboratory energy and momentum of the i-th particle in the final state.

The pion production angles in the laboratory (θ_π) and cm (θ_π^*) systems are related by

$$E_0\,E_\pi - p_0\,p_\pi\cos\theta_\pi = \varepsilon_0\,\varepsilon_\pi - q_p^*\,q_\pi^*\cos\theta_\pi^*\,, \tag{7}$$

where $\varepsilon_\pi(\varepsilon_0)$ and q_π^* are the energy of the pion (incident proton) and 3-momentum of the pion in the overall cm system and E_0 is the total laboratory energy of the incident proton. Due to the difference between the effective mass of the final pn state and that of the deuteron, $q_\pi^* \neq q_{\pi d}^*$. However, this effect is quite small at low E_{np}.

The results of eq. (5), which should be valid at low relative energies E_{np}, do not depend upon the details of the pion production dynamics and automatically include the final state interaction in the triplet pn system. The input $pp \to d\pi^+$ cross sections are taken from the SAID SP96 parameterisation [10] and this procedure should involve errors that are smaller than those of an individual experiment. The predictions of our approach for the triplet $pp \to pn\,\pi^+$ cross section are shown at one angle pair in Fig. 1 together with an evaluation of quasi-free Δ-production [4]. The values of the np excitation energies are indicated and from this it is seen that when E_{np} is below about 5 MeV the magnitude and shape of the fsi peak are both well reproduced. For comparison the fsi parameterisation of Goldberger and Watson [6], multiplied by phase space and normalised to the peak value, is also illustrated.

Results at different angles in the fsi region are shown in Fig. 2. Larger deviations from the predictions are to be found when the minimum value of the excitation energy, E_{np}^{min}, is increased. This is the case at $(\theta_p, \theta_\pi) = (25°, 40°)$ and $(30°, 28°)$ where E_{np}^{min} are respectively 13 and 28 MeV. Even in this last case, the changes induced by the modified kinematics are quite small, as can be judged from the dot-dash curve shown in the figure. A potentially more serious effect arises from averaging the estimates over the experimental angular acceptance. As shown in Fig. 2, this tends to reduce a little the predictions when $E_{np}^{min} \leq 1$ MeV but can increase them otherwise. After smearing, there seems to be some underprediction in Fig. 2b, though it should be stressed that these data were taken from the polarisation experiment [3] where the consistency with the earlier unpolarised cross section run was found only on the 15% level.

Since the predictions of the Fäldt-Wilkin extrapolation theorem [9] reproduce most of the magnitude and angular dependence of the cross section leading to the fsi peaks, this is strong evidence that the vast bulk of the LAMPF data corresponds to np triplet final states. Extra confirmation of this interpretation is found from the proton analysing power which was measured at two pairs of angles in the fsi region [3]. In the fsi peak we expect A_y to be essentially constant at the value corresponding to that of $pp \to d\pi^+$ at the appropriate pion angle. These values are shown with the experimental data in Fig. 3. Though at $(14.5°, 21°)$ the data seem to fall a little above the prediction, deviations of this size are not unknown in polarisation measurements.

Although we have reproduced well the LAMPF data at low E_{np} with just triplet terms, it may be helpful to try to estimate an upper bound on the possible spin-singlet contribution from the areas under the peaks. The overall systematic error due to beam normalisation and detector efficiency in the measurement was estimated to be about 7% [2], to which must be compounded some error coming from the $pp \to d\pi^+$ input [10]. The error arising from using the extrapolation theorem in the scattering domain is likely to be rather smaller than this, with variations in the wave functions at short distances being on the 1–2% level for $E_{np} < 10$ MeV [11]. Further work is needed to include the D-state effects more consistently, though it has been shown that the extrapolation theorem is valid in this coupled-channel case [12]. Under the present conditions, we would estimate the accuracy of eq. (5) to be better than 5% for $E_{np} \leq 3$ MeV [13]. We therefore conclude that the singlet contribution to the unpolarised LAMPF data at low E_{np} in Fig. 1 is below about 10%. This upper bound is about a factor of two less than the average quoted in the experimental paper [2], though it must be stressed that this involved considerable model dependence, including the choice of a 10 MeV matching point.

There is a 15% normalisation uncertainty between the first measurements of the unpolarised cross section at LAMPF [2] and their later experiment, where the primary purpose was the determination of the analysing power [3]. If, nevertheless, we take seriously the disagreement with the latter data shown in fig. 2b, it is possible that this is due to a spin-singlet contribution. Assuming that the singlet cross section to be of the form

$$\frac{\mathrm{d}^5\sigma_s(pp \to pn\pi^+)}{\mathrm{d}p_p\,\mathrm{d}\Omega_p\,\mathrm{d}\Omega_\pi} = \zeta\,\frac{(k^2+\alpha_t^2)}{(k^2+\alpha_s^2)} \times \frac{\mathrm{d}^5\sigma_t(pp \to pn\pi^+)}{\mathrm{d}p_p\,\mathrm{d}\Omega_p\,\mathrm{d}\Omega_\pi}\,, \tag{8}$$

where $\alpha_s = -0.04\ \mathrm{fm}^{-1}$ then, after averaging over acceptance, a value of $\zeta = 0.03$ restores the agreement with the data. This would correspond to a singlet fraction integrated over the range $0 \leq E_{np} \leq 3$ MeV of about 10%. However, such a fraction would make the agreement worse in fig. 2c, where the data were taken at a rather similar cm angle but in the other hemisphere.

To improve the sensitivity to the singlet/triplet ratio using the extrapolation theorem, the ratio of the $pp \to pn\pi^+$ and $pp \to d\pi^+$ cross sections has to be established better by measuring both reactions in the same experiment. Data with better resolution on E_{np} would also allow one to investigate the singlet/triplet ratio from the shape of the fsi peak. A new measurement of the $pp \to pn\pi^+$ reaction at the

ANKE spectrometer of the proton synchrotron COSY-Jülich, where both protons and pions were detected near the forward direction at $T_p = 492$ MeV, involved both these improvements and can therefore put more stringent limits on the spin ratio [14].

In conclusion, we have generalised the use of the extrapolation theorem linking np scattering and bound-state wave functions to describe five-fold differential cross sections. In so doing, we have shown that the LAMPF $pp \to pn\,\pi^+$ cross section data at 800 MeV are consistent with there being no final-state spin-singlet contribution. This smallness should not come as a complete surprise. If one assumes that pion production passes through an intermediate Δ isobar [2, 4, 8], then the S-wave $\Delta\,N$ intermediate state cannot lead to singlet np final states. These must be produced through higher partial waves or from non-resonant pion production.

This work was initiated within the framework of the ANKE collaboration at COSY and the authors would like to thank all their colleagues, especially V. Abaev, V.I. Komarov, V. Koptev, and V. Kurbatov, for many fruitful discussions. Financial support from the Forschungszentrum Jülich and BMBF (WTZ grant KAZ 99/001) and the generous hospitality of Institut für Kernphysik 2, where the work was carried out, are gratefully acknowledged.

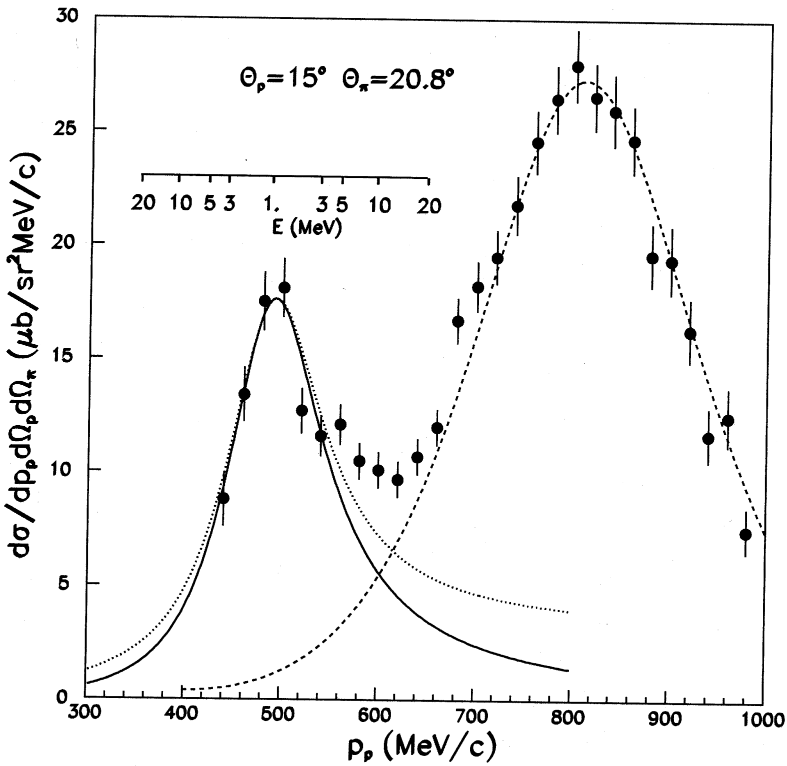

Figure 1: Differential cross section for the $pp \to pn\pi^+$ reaction at 800 MeV at fixed proton and pion laboratory angles of $(\theta_p, \theta_\pi) = (15°, 20.8°)$ as a function of the measured proton momentum [2]. The $pp \to \Delta N$ peak on the right can be described in a one-meson-exchange model (dashed curve) [4]. The peak on the left is a reflection of the strong np fsi, which can be calculated (solid curve) from the $pp \to d\pi^+$ cross section, as discussed in the text. A scale is given showing the np excitation energy at the geometry corresponding to the centres of the counters. For comparison, the triplet fsi factor in the Goldberger and Watson form [6], multiplied by the phase space factor of eq. (6) and normalised to the peak, is shown as the dotted curve.

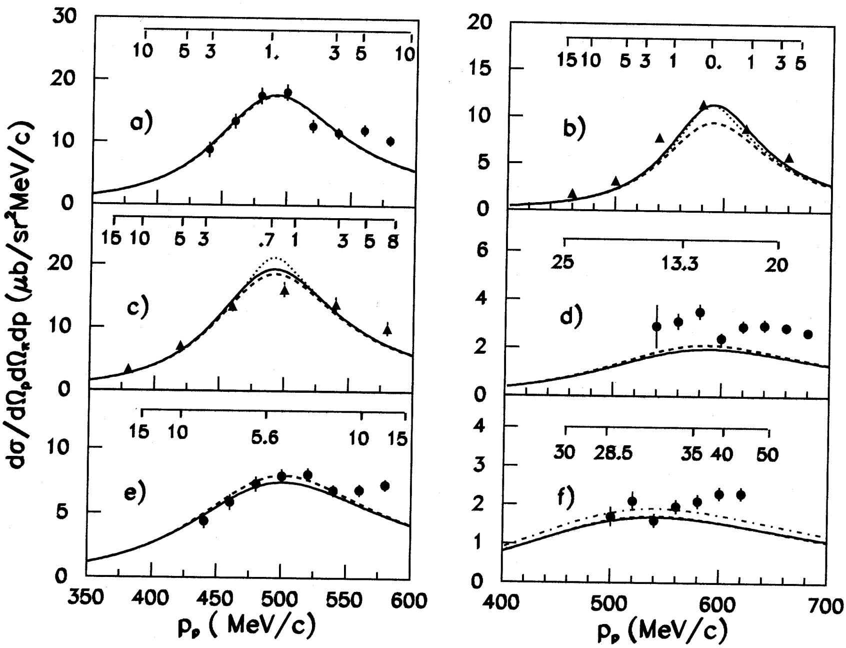

Figure 2: Differential cross section for the $pp \to pn\pi^+$ reaction at 800 MeV in the neighbourhood of the np fsi region for different angular positions, a) $(\theta_p, \theta_\pi) = (15°, 20.8°)$, b) $(14.5°, 42°)$, c) $(14.5°, 21°)$, d) $(25°, 40°)$, e) $(20°, 22°)$, f) $(30°, 28°)$. The np excitation energies at the counter centres are indicated; the number with the decimal corresponds to the minimum value of E_{np}. The experimental data are taken from Refs. [2] (circles) and [3] (triangles) and it should be noted that these may differ in normalisation by 15%. The data are compared to the spin-triplet final-state predictions of eq. (5) without acceptance corrections (solid curve) and with (broken curve). An attempt to resolve the apparent discrepancy in b), through the introduction of a singlet contribution by eq. (8) with $\zeta = 0.03$ makes the situation worse in c) (dotted curves). The dot-dash curve in f) was obtained using the deuteron mass rather than the invariant mass of the np system when evaluating the pion production angle θ^*_π in eq. (7).

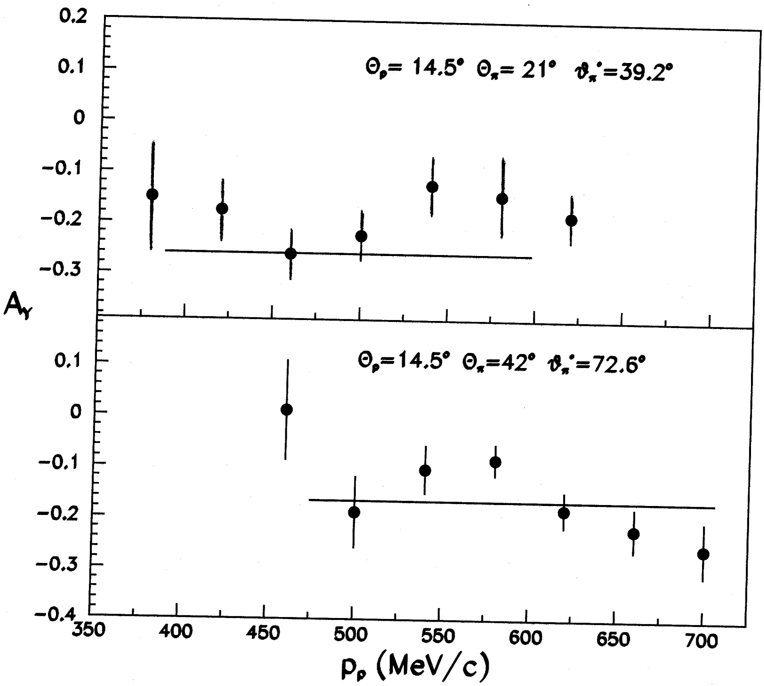

Figure 3: Proton analysing power for the $\vec{p}p \to pn\,\pi^+$ reaction at 800 MeV in the neighbourhood of the np fsi region. The experimental data of Refs. [3] are compared to the SAID predictions of A_y (horizontal lines) for the $\vec{p}p \to d\,\pi^+$ reaction [10]. It should be noted that SAID uses the opposite convention for the sign of A_y.

References

[1] See for example W.R. Falk *et al.*, Phys. Rev. C **32** (1985) 1972; R.G. Pleydon *et al.*, Phys. Rev. C **59** (1999) 3208.

[2] J. Hudomalj-Gabitzch *et al.*, Phys. Rev. C **18** (1978) 2666.

[3] A.D. Hancock *et al.*, Phys. Rev. C **27** (1983) 2742.

[4] O. Imambekov and Yu.N. Uzikov, Sov. J. Nucl. Phys. **47** (1988) 695.

[5] K.M. Watson, Phys. Rev. **88** (1952) 1163.

[6] M.L. Goldberger and K.M. Watson, *Collision Theory* (John Wiley, N.Y.) 1964.

[7] A. Boudard, G. Fäldt and C. Wilkin, Phys. Lett. **B389** (1996) 440.

[8] J. Dubach, W.M. Kloet, and R.R. Silbar, Phys. Rev. C **33** (1986) 373.

[9] G. Fäldt and C. Wilkin, Physica Scripta **56** (1997) 566.

[10] R.A. Arndt *et al.*, Phys. Rev. C **48** (1993) 1926; `http://said.phys.vt.edu/said_branch.html/`

[11] G. Fäldt and C. Wilkin, Phys. Rev. C **56** (1997) 2067.

[12] A.V. Smirnov, Communication 34 of the Lebedev Physical Institute, Moscow, Russia, 1997.

[13] A.V. Smirnov and Yu.N. Uzikov, Yad. Fiz. **61** (1998) 421; Phys. At. Nuc. **61** (1998) 361.

[14] V. Kurbatov (private communication, 2001) and *publication in preparation.*

Received by Publishing Department
on April 4, 2001.

Узиков Ю.Н., Уилкин К. E4-2001-57

Модельно-независимый анализ реакции $pp \to pn\pi^+$
в области нейтрон-протонного взаимодействия
в конечном состоянии

Экспериментальные данные о реакции $pp \to pn\pi^+$, полученные в эксклюзивном двухплечевом эксперименте при 800 МэВ, демонстрируют узкий пик, возникающий из-за сильного протон-нейтронного взаимодействия в конечном состоянии. Ранее в рамках определенной модели утверждалось, что этот пик содержит ~ 25 %-ный вклад спин-синглетного конечного состояния. Путем сравнения с данными о реакции $pp \to d\pi^+$ на основе модельно-независимого подхода показано, что при всех измеренных углах величина и форма пика могут быть полностью объяснены вкладом спин-триплетного конечного состояния, при вкладе спин-синглетного состояния не более нескольких процентов. В рамках этого подхода получено также хорошее качественное согласие с измеренной анализирующей способностью протона.

Работа выполнена в Лаборатории ядерных проблем им. В.П.Джелепова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2001

Uzikov Yu.N., Wilkin C. E4-2001-57

Model-Independent Analysis of the Neutron-Proton Final-State Interaction Region in the $pp \to pn\pi^+$ Reaction

Experimental data on the $pp \to pn\pi^+$ reaction measured in an exclusive two-arm experiment at 800 MeV show a narrow peak arising from the strong proton-neutron final-state interaction. It was claimed, within the framework of a certain model, that this peak contained up to a 25 % spin-singlet final-state contribution. By comparing the data with those of $pp \to d\pi^+$ in a largely model-independent way, it is here demonstrated that at all the angles measured the whole of the peak could be explained as being due to spin-triplet final states, with the spin-singlet being at most a few percent. Good qualitative agreement with the measured proton analysing power is also found within this approach.

The investigation has been performed at the Dzhelepov Laboratory of Nuclear Problems, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2001

Макет Т.Е.Попеко

Подписано в печать 23.04.2001
Формат 60 × 90/16. Офсетная печать. Уч.-изд. листов 1,35
Тираж 360. Заказ 52620. Цена 1 р. 60 к.

Издательский отдел Объединенного института ядерных исследований
Дубна Московской области