E4-2002-52

P. N. Bogolubov, P. S. Isaev

ON THE HISTORY OF CREATION OF THE MICROSCOPIC THEORIES OF SUPERFLUIDITY AND SUPERCONDUCTIVITY

1.Introduction

There are two wonderful physical phenomena discovered one after the other almost 30 years later, but they turned out to be so close in nature that they can be called by a common word—superfluidity: this is superconductivity discovered by the Holland physicist H. Kammerlingh-Onnes in 1911 and superfluidity discovered by the Russian physicist Kapitsa in 1938. These phenomena were completely understood after brilliant publications of our contemporary of genius Nikolai Nikolaevich Bogoliubov.

Half a century ago, the journal "Izv. Akad. Nauk SSSR" (Ser. Fiz., vol. 11, no. 1 (January–February), 1947) published the paper by N.N. Bogoliubov "On the theory of superfluidity" that gave the profound explanation of the phenomenon of superfluidity at the microscopic level.

Ten years later in October, 1957, in the JINR preprint P-94 (Laboratory of Theoretical Physics) entitled "On a new method in the theory of superconductivity", Bogoliubov gave the microscopic substantiation for the phenomenon of superconductivity. In the first lines of that preprint, Bogoliubov wrote: "In this paper, we show that by developing the method we proposed earlier for the theory of superfluidity, we can construct a consistent theory of superconductivity...".

At the 3rd session of the Scientific Council of JINR (November 20–25, 1957), on the 3rd day, November 22, Bogoliubov gave a talk "On a new method in the theory of superconductivity". The brilliant work by Bogoliubov produced strong impression on members of the Council.

A year later, in the journal "Vestn. Akad. Nauk SSSR" (vol. 28, no. 4, 1958), Nikolai Nikolaevich published the paper "Problems of the theory of superfluidity of Bose- and Fermi systems". In the title, both the phenomena are combined into one phenomenon—superfluidity. Superconductivity is a phenomenon caused by the behavior of a collective of spin 1/2 particles, i.e. by the behavior of Fermi-systems; whereas superfluidity is a phenomenon caused by the behavior of a collective of particles with integer spin (=0, 1,...), i.e by the behavior of Bose-systems. This explains the title of the Bogoliubov's paper "Problems of the theory of superfluidity of Bose- and Fermi-systems.

It has long been known that with decreasing temperature, the effective resistance of metals diminishes. But nobody knew what resistance will be when temperature decreases down to absolute zero, i.e to -273.15^o in the Celsium scale (C) or to 0^o in the Kelvin scale (K). (The relation between these two scales can be written as $T_K = T + 273.15$ K.) In 1908, Kammerlingh-Onnes succeeded in reaching a very low temperature of $\sim$1 K at the Leiden University. (At present, very low temperatures are reached that differ from the absolute zero only in million fractions of degree.) And naturally, one of the first experiments was the experiment on determination of the resistance of conductors at low temperatures. As it was expected, the electric resistance decreased with lowering temperature. But three years later (in 1911), Kammerlingh-Onnes discovered that the resistance of mercury, instead of being gradually diminished, sharply fell to zero at a temperature of 4.15 K and below. That year became the year of discovery of superconductivity. It was established experimentally that the resistance of metals in a superconducting state diminishes to fantastically small values $\leq 10^{-23}$ Ohm/cm. The electric current in conductors, without suffering the resistance, exists for a long time. For instance, a superconducting ring with induced current was conserved during more than two years at a temperature below the critical one (Collins). For comparison, we mention that the resistance of the most pure samples of copper at low temperatures equals $\sim 10^{-9}$ Ohm/cm.

Somewhat later, Kammerlingh-Onnes ascertained that the electric resistance of mercury in a superconducting state can be regenerated in a sufficiently strong magnetic field that is called the critical field. Experimental studies of the phenomenon carried out since 1911 revealed many facts of the dependence of superconductivity on the properties of conductors, the influence of magnetic and electric fields and on their frequency. In this short note, we do not present all the discovered facts. Being a pure laboratory phenomenon, superconductivity is now actively introduced into industrial technologies ; in particular, superconducting magnets for accelerators are constructed. But this is beyond the scope of our paper.

A phenomenon similar in its physical nature was discovered by P.L. Kapitsa in the behavior of liquid helium at low temperatures. At $T \leq 2.17$ K, liquid helium ^4He becomes superfluid (it was called He II unlike nonsuperfluid He I). As it was elucidated in the process of studies, He II consists of two mutually penetrating components, superfluid and normal ones. The superfluid component is completely nonviscous and freely flows through narrow slits and capillaries. As it is known, the viscosity is the result of interaction of molecules of a liquid between themselves and with walls of a vessel. The absence of viscosity can signify that the liquid molecules are the ”ideal gas” of noninteracting molecules that are in the state of ”condensation” (we explain this term below). The absence of viscosity of the superfluid component of He II is in a sense the absence of ”resistance” of fluidity of helium, which makes the behavior of the ”fluid” component at low temperatures to be like the electronic superconductivity of mercury discussed above. It is today when theoretical results by Bogoliubov become known, we see the identity in the behavior of the Bose- and Fermi systems at low temperatures. But in 1938, when the phenomenon of superfluidity of ^4He was discovered by Kapitsa, this was far from being evident.

When the absolute zero is approached, the motion of atoms in any body is slowed down and is reduced to small vibrations around some position of equilibrium. Thus, any body should harden. Helium turned out to be a single liquid that has no time to be ”frozen” and to turn into a solid—even at very low temperatures it remains the liquid with strongly pronounced quantum properties. The de Broglie wavelength of helium atoms $((\lambda = \frac{\hbar}{m_{H_e} v}))$ at the temperature $\sim 1 - 2$ K is comparable with the intermolecular distance $(\lambda \sim 10^{-8}$ cm). This means that He II should be a quantum object and is not a usual classical liquid, and the quantum deep essence of superfluidity cal be understood only in the framework of microscopic theory.

2. Theory of superfluidity

Bogoliubov wrote in his the above paper published in the journal ”Vestn. Akad. Nauk SSSR” in 1958 that ”Though the phenomenon of superfluidity of helium was discovered much later, it was understood considerably earlier, evidently, due to specific properties of the Bose statistics. So, the microscopic theory of superfluidity of Bose systems was developed by the author on the basis of the model of Bose-gas with weak interaction". It is known from the kinetic theory of gases that for an ideal homogeneous gas, the temperature and velocity of gas molecules are connected by the formula $\frac{mv^2}{2} = \frac{1}{3}kT$ where m is the mass of a particle of the gas; v — the velocity of the particle; T — temperature in the Kelvin scale; k — the Boltzmann constant $\sim 1,38 \cdot 10^{-23}$ Joule/Kelvin. If the temperature T tends to zero, then, as we said above, the motion of gas molecules slow down, and at the absolute zero, all the gas molecules stop and are "condensed" forming a "condensate" (called in analogy with the condensation of water vapor into water). However, quantum field theory asserts that even at the zeroth temperature, there are "zeroth vibrations" of particles, but this is not required for us now. The helium condensate at 0 K would be an ideal Bose–Einstein gas. The "ideal" gas is the gas in which there is no interaction between particles. The "ideal" liquid is the liquid in which there is no interaction between molecules of the liquid, no viscosity, and no thermal conductivity. Bogoliubov has shown that if a condensate is the ideal Bose-gas with noninteracting particles, it cannot exhibit the property of superfluidity.

Let us assume that the gas as a whole moves with a certain velocity U, and consequently all its particles move with the same velocity. Nothing prevents these particles from being slowed down because of collisions with walls of a vessel or with particles of an admixture and dropping out of the condensate. If a particle loses its velocity upon collision, then from the condensate with total energy $1/2Nmu^2$ where N is the number of particles; m — their mass, the particle falls out of the condensate, and the condensate energy becomes $1/2(N-1)mu^2 + 1/2mV^2$ where V is the velocity of the particle upon collision. In the process of collision, the particle velocity decreases ($V < u$), and the condensate starts to slow down. The process of slowing down the motion is energetically profitable, the ideal Bose-gas stops, and the property of fluidity gets lost. This occurs owing to the property of the gas being ideal, since the particles of the ideal gas do not "feel" each other, do not interact with each other, the condensate do not reacts on falling the particles out of the collective and its slowing down.

In his report on the General meeting of the Academy of Sciences of the USSR on June 20, 1958, Bogoliubov said that "To explain the phenomenon of superfluidity, one should take account of the interaction between particles and therefore consider nonideal Bose-systems".

The phenomenon of superfluidity of liquid helium attracted attention of many theorists, and a lot of theoretical works appeared with its explanation. The phenomenological or macroscopic theory of superfluidity of He II was proposed by Laslo Tissa in 1938 and was essentially improved by Lev Landau in 1941. It was called the two-component theory. In his report on the common meeting of the Academy of sciences of the USSR on June 20, 1958,Bogoliubov said that "In the Tissa theory, the liquid helium below the temperature of phase transition was considered as a combination of superfluid and normal components each of which can move with its own velocity. According to this, the equations of hydrodynamics were composed as equations of hydrodynamics of two liquids. In the Landau theory, attention was paid to the fact that motion of the superfluid component differs from that of the normal component not only because it is nonviscous but also because it is necessarily potential. As a result, somewhat different hydrodynamic equations were derived. The Landau and Tissa hydrodynamic equations led to different dependences of the velocity of the so-called second sound on temperature. Excellent experimental studies by V.Peshkov confirmed the validity of the Landau equations".

The potential character of motion of He II mentioned by Bogoliubov means the vortex-free motion of the liquid. In the same report, Bogoliubov emphasized the difference between macroscopic (phenomenological) and microscopic approaches to the construction of the theory of superfluidity: "...The macroscopic theory is aimed at deriving equations of the type of classical equations of mathematical physics that would describe the whole set of experimental facts related to macroscopic objects. These equations are derived on the basis of certain assumptions, so-called axioms that relate different macroscopic quantities.

The microscopic theory sets a deeper problem to understand the intrinsic mechanism of the phenomenon on the basis of quantum-mechanical laws. To construct the microscopic theory, one should consider an appropriate dynamic system characterized by a definite Hamiltonian and determine basic properties of the phenomenon under consideration by analyzing the corresponding equations of quantum mechanics. In particular, it is necessary to establish those relations that result in the equations of the macroscopic theory...".

The phenomenological theory of superfluidity of He II proposed By Landau was constructed on the basis of several assumptions. Let us make several remarks. Let us consider a crystal at very low temperatures. "Frozen" atoms of the crystal lattice will oscillate near the position of equilibrium, so that the self-energy of the crystal will be a sum of energies of quantum oscillators. In the crystal theory, quantum oscillators are associated with the so-called phonons. The energy of a phonon equals $\hbar\omega$ where ω is the frequency of oscillations, $\hbar$ is the Planck constant divided by $2\pi(h/2\pi)$. Propagation of vibrations in a crystal is treated as the propagation of acoustic waves, phonons, and the very phonons are treated as quasiparticles with energy ε, momentum p, and mass m that are not ever connected by a classical expression of the kinetic energy $mv^2/2$ and momentum mv where v denoted the "sound" velocity of propagation of a phonon. Every particle in a crystal possesses three spatial degrees of freedom; the number of particles per 1 cm^3 amounts to about $10^{22} - 10^{23}$. Thus, the number of simplest vibrations in a crystal equals $3 \cdot 10^{22} - 3 \cdot 10^{23}$. These vibrations exist simultaneously and independently.

He II is a liquid rather than a crystal. However, liquids can be also described in terms of phonons. Elementary (simplest) excitations in He II are also called phonons (or quasiparticles).

L.Landau suggested that the energy ε of sound quanta (phonons) is a linear function of the momentum p, i.e. $\varepsilon(p) = c \cdot p$ where c is the sound velocity. However, to explain superfluidity, its temperature dependence, heat capacity, and the dependence of a superfluid liquid on the flux velocity, phonons are insufficient. Landau assumed that there exist the so-called rotons whose energy ε is expressed through the momentum of rotons p in the form $\varepsilon = \Delta + \frac{(p-p_0)^2}{2}$ where Δ is a constant equal to a minimal energy $\varepsilon_{\min}$ (at $p = p_0$), p_0 is a fitting parameter of the theory. The analytic dependence of $\varepsilon(p)$ on the momentum p for rotons is of the form given above only around $\varepsilon_{\min}$. Physically, the phenomenon of superfluidity in the Landau theory was as follows: At the temperature of absolute zero, helium is in the ground state—the "condensate" state. With increasing temperature, there arise vibrations of molecules—excitations (or phonons) with energy $\varepsilon = c \cdot p$, and as long as their number is small, phonons can be considered as an ideal gas. From the very beginning, the Landau theory left unclear the following problem: Why should the energy dependence of a level of elementary excitation be of the form $\varepsilon = c \cdot p$? With further increasing temperature, the number of phonons increases, and at a certain critical temperature T_λ, the liquid is no longer superfluid. According to the Landau theory, the liquid stops to be superfluid also in the case when its velocity exceeds some critical value at which there occurs the formation of the above-mentioned rotons, which slows down the liquid flux. Thus, for superfluidity it is necessary that the traveling speed of He II would in any case be smaller than a certain critical value $v_{cr} = \frac{\min \varepsilon(p)}{p} \neq 0$. A crucial point in the explanation of superfluidity in the energy dependence of elementary excitations (phonons or quasiparticles) on the momentum (see Fig. 1). Owing to the minimum in the dependence of the phonon energy on the momentum, the retardation of movement of quasiparticles in a definite regions of velocities $0 < V < V_{cr}$ becomes energetically unprofitable. The same concerns also the formation of excitations in that region of velocities. The liquid in this region is flowing without energy loss, is not slowing down, and becomes superfluid. Unfortunately, the critical velocity in the Landau theory is much larger than the experimentally observed values.

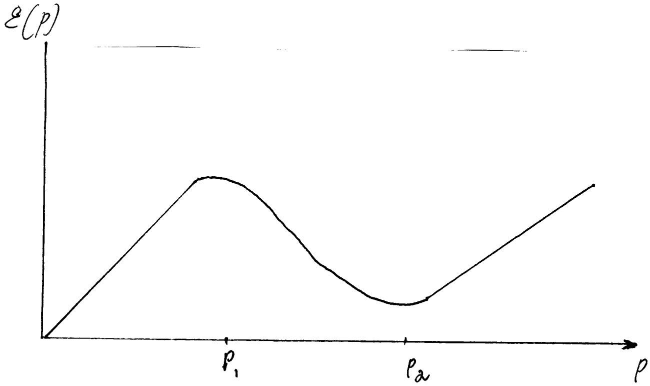

Fig. 1. The energy dependence of quasiparticles on the momentum

To construct the microscopic theory of superconductivity, "... it would be necessary to consider the liquid helium as an assembly of interacting atoms, and on the basis of solution of the quantum-mechanical problems of many interacting bodies, to determine the corresponding structure of energy levels..." (Bogoliubov "Lectures on quantum statistics", Selected papers, "Naukova dumka", 1970, vol. II, p. 371). Nikolai Nikolaevich constructed a model of nonideal Bose–Einstein gas with weak interaction between particles and, upon a consistent and rigorous solution of the system of equations of this model, derived the following expression for the spectrum of excited states of He II:

$$\varepsilon(p) = \sqrt{2T(p)\frac{N}{V}\nu(p) + T^2(p)} = \sqrt{\frac{|p|^2\nu(p)}{m\upsilon} + \frac{|p|^4}{4m^2}},$$

where p is the momentum of a particle (quasiparticle); $T(p)$ — the kinetic energy of the particle; V — the volume of the whole system; $v = V/N$ — the elementary volume per particle; N — the number of particles in the whole system, m — the phonon mass; $\nu(p)$ — a component of the Fourier expansion of the function of potential interaction of a pair of helium molecules $\nu(0) > 0$. The value $\nu(p) > 0$ provides a real value for the roots of $\varepsilon(p)$. If we agree to ascribe only positive values to the square root and consider two possibilities:

1. extremely small momenta p and

2. very large momenta p, we obtain approximate values of the spectrum of excited states:

1. for small p : $\varepsilon(p) = C|p|(1 + \ldots)$,

2. for large p : $\varepsilon(p) = \frac{p^2}{2m} + \frac{1}{v}\nu(p) + \ldots$

where C is the sound velocity in helium.

Since $\nu(p)$ with increasing p tends to zero, at large momenta p, the value of $\varepsilon(p)$ approaches the kinetic energy of one molecule $\varepsilon(p) = p^2/2m$.

The total number of quasiparticles (phonons) is not conserved, they can create and annihilate. In the state of statistical equilibrium, the average occupation number $\bar{n}_p$ (the average number of particles with momentum p) are determined by the formula

$$\bar{n}_p = \left[\exp\left(\frac{\varepsilon(p) - p \cdot u}{\Theta} - 1\right)\right]^{-1},$$

where u is an arbitrary vector. Since $\bar{n}_p$ should be a positive number, for all $p \neq 0$, the inequality

$$\varepsilon(p) - p \cdot u > 0,$$

should be satisfied, from which it follows that the value of $\varepsilon(p)$ is to be larger than the product of absolute values of u and p: $\varepsilon(p) > |p| \cdot |u|$, or in other words,

$$|u| < \min \frac{\varepsilon(p)}{p},$$

Thus, the condition for superconductivity of helium according to Landau

$$v_{cr} = \frac{\min \varepsilon(p)}{p} \neq 0$$

according to Bogoliubov follows from the microscopic theory. The expression for $\bar{n}_p$ gives the distribution of momenta in the gas of quasiparticles, at which the gas as a whole moves with the velocity u. The theoretical consideration was carried out in the coordinate system, in which the condensate, i.e. an aggregate of helium molecules, is at rest in the ground state. If, on the contrary, we take the coordinate system, in which the gas of quasiparticles as a whole is at rest, the condensate would move with the velocity u. "Since this relative movement is stationary in the state of statistical equilibrium without external forces, we see that it is not accompanied by friction, and consequently, is the property of superfluidity" (Bogoliubov "To the theory of superfluidity", Izv. Akad. Nauk, ser. fiz. vol. 11, no 1, pp. 77–90). Bogoliubov concludes the first section of this paper with the words: "... a quasiparticle at small momenta is nothing else than a phonon. With increasing momenta when the kinetic energy $T(p)$ becomes high as compared with the binding energy of a molecule, the energy of the quasiparticle continuously transforms to the individual energy of the molecule $T(p)$. Thus, the division of quasiparticles into two sorts — phonons and rotons — is out of the question".

The method of solving the system of equations for a weakly interacting Bose-system applied by Bogoliubov for considering the phenomenon of superfluidity of He II contained an extremely important mathematical transformation — extraction of a certain special quadratic part (in creation and annihilation of Bose-quasiparticles) from the interaction Hamiltonian by a canonical transformation later called the Bogoliubov transformation.

The transformation eliminated, from the ground state of the system, those states that corresponded to the virtual creation from vacuum of a pair of the part with equal and opposite-directed momenta, whereupon use could be made of the method of perturbation theory for solving a new system of equations. Ten years later, Bogoliubov used this method for constructing the theory of superconductivity, which will be discussed below.

The graph of the Bogoliubov spectrum $\varepsilon(p)$ under certain assumptions on the dependence of $\nu(p)$ on p is shown in Fig. 1. In the region $0 \leq p \leq p_1$, the energy dependence of the phonon spectrum on the momentum p is linear: $\varepsilon(p) = C \cdot p$. The curve in Fig. 1 has a relative minimum at point p_2 (it corresponds to point $\varepsilon_{\min} = \Delta$ in the Landau theory). The minimum of $\varepsilon(p)$ signifies that at this value of the momentum, slowing down of particles in the flux of superfluid liquid is energetically unprofitable, since at any value of p from the interval $p_1 < p < p_2$, the energy of quasiparticles $\varepsilon(p)$ is higher than $\varepsilon(p_2)$. So, the system exhibits the property of superfluidity. For an ideal gas, $\varepsilon(p) = \frac{p^2}{2m}$, $\min \frac{\varepsilon(p)}{|p|} = \frac{p^2}{2m} \to 0$, as $p \to 0$ and there would be no superfluidity. "From a physical point of view, this property (superfluidity) in a system of interacting Bose-particles of the considered type results from that a condensate in systems of that sort forms a bound collective that reacts by increasing energy when particles fall out of it" (Bogoliubov. From a report on the General meeting of the Academy of Sciences of the USSR, June 20, 1958).

In October, 1946, Bogoliubov reported his work on the theory of superfluidity of helium on the General meeting of the phys.-math. section of the Acad. of Sci. of the USSR.

Academician Shirkov wrote in his memories about Bogoliubov: "... To this moment, Dau (L.Landau) was about 5 years a classic of superfluidity, the author of the known semiphenomenological theory based on the representation of quanta he introduced for special collective excitations of the vortex nature—rotons... According to recollections of participants of the General meeting, Dau sharply criticized the speaker. However, he rapidly understood and estimated what he heard, and 2–3 weeks later, he sent a paper [5] for publication, in which he proposed adhoc the curve with inflection for the excitation spectrum (see Fig. 1, this paperP.N.B. and P.S.I.), and in particular, it was said there: "... For a spectrum like that one, of course, cannot consider phonons and rotons as strictly different types of elementary excitations". The Landau phenomenological curve follows from the N.N. formula under a certain assumption on the character of interaction between atoms of He II. However, there is no reference to N.N. in the publication by Landau". [1]

The shape of dependence of the energy spectrum of quasiparticles on their momentum was verified in experiments on scattering of a monochromatic beam of neutrons in He II. This process of scattering can be represented as collision of neutrons with quasiparticles. If we know the initial energy of the monochromatic beam and change the energy of neutrons after scattering under different angles, we can calculate the energy and momentum of quasiparticles. In Fig. 2, experimental data are presented on the measurement of the spectrum of elementary excitations. [2] The experimental curve for $\varepsilon(p)$ is in excellent agreement with the predictions of the Bogoliubov formula and the Landau phenomenological curve (see Fig. 1) mentioned by D.V.Shirkov.

---

[1] See Collected Papers entitled "Nikolai Nikolaevich Bogoliubov. Mathematician, mechanician, physicist", Dubna, 1994, pp.194–195. Academician D.V.Shirkov was a near colleague of Bogoliubov since 1950 and later became a co-author of papers and monographs.

[2] See the paper by D.Kukin "Superfluidity" in the book "Physics of Microcosm. Small Encyclopaedia" (p.361) edited by D.V.Shirkov, Izdat. "Sovetskaya entsiklopediya", Moscow, 1980.

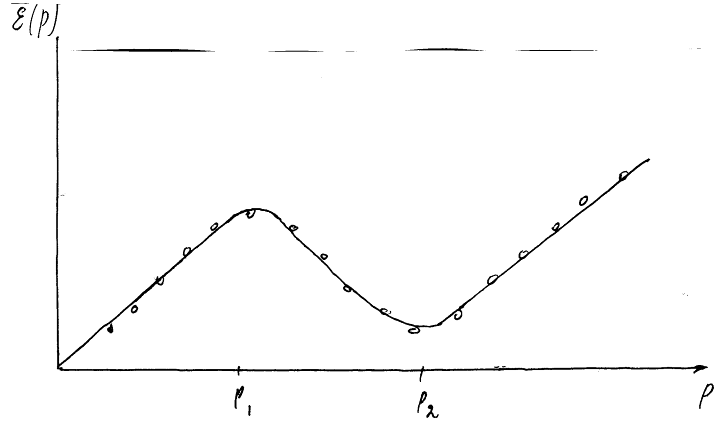

Fig. 2. Energy dependence of elementary excitations (quasiparticles) $E(f)$ on the momentum p. Circles stand for experimental points.

3. Theory of superconductivity

Academician Shirkov continued in his memories about Bogoliubov mentioned above: "It was a severe test for the Dau's self-respect in 1957 when N.N. Bogoliubov suddenly intruded into the superconductivity theory; the phenomenon of superconductivity discovered in 1911, from the late twenties, became a painful challenge to leading theoreticians. It was clear that superconductivity is a macroscopic manifestation of the laws of quantum mechanics. It was intensively studied by experimenters, however, theoretical understanding could not be reached yet. Dau worked in this field from the mid thirties and in collaboration with V.Ginzberg, he constructed the phenomenological theory of superconductivity.

A starting point for Bogoliubov was a paper by Cooper in which it was assumed that the phenomenon is originated by the correlation of electron pairs caused by interaction with an ion core. There, Cooper gave an expressions for the energy gap with a specific dependence $\exp\left(-\frac{1}{g}\right)$ on the coupling constant g of electron–phonon interaction. Bogoliubov immediately saw the analogy with the phenomenon of pair correlation of the boson type, he discovered when constructing the superfluidity theory.Taking the electron–phonon interaction Hamiltonian (the Fröhlich Hamiltonian) as a basis and reformulating his (u, v)-transformation for fermions, he analyzed the condition of compensation of possible singularities in the vicinity of Fermi surface and derived an expression of the type of the Cooper formula.

When N.N. completed his studies and started to speak at meetings, it became known that a large preprint written by Bardeen, Cooper, and Schrieffer appeared in Western countries, but it did not reach Moscow. Dau rapidly estimate the work by Bogoliubov. It was even agreed to make a joint seminar on the superconductivity theory. At the first seminar, upon the N.N. report, Dau said:" Nikolaj Nikolaevich, I don't know what is the content of the work by Bardeen with colleagues, but I think that it has no such beautiful and convincing result..." The Dau words turned out to be prophetic..." [3]

---

[3] See footnote 1 from memories by Academician Shirkov, pp.195–196.

The history of construction of the superconductivity theory was richer than that of the superfluidity theory. The superconductivity theory went through the Gorter–Casimir two-liquid model (1934), F. and G. London equations (1935), the Pipperd nonlocal theory (1950–1953), the Ginzburg–Landau phenomenological theory (1950), and completed with the microscopic theory includings studies by Fröhlich (1950), Bardeen (1950), Cooper (1956), and the main work by Bardeen, Cooper, and Schrieffer (1957).

What are experimental facts of the phenomenon of superconductivity? Let us list some well-known facts:

1. There is a certain critical temperature T_c (for different substances and pure elements, it is different), below of which there occurs the phenomenon of superconductivity. The critical temperature for most substances changes within the limits 0,5 $\mathrm{K} \leq T_c \leq 10$ K.

2. There is a critical magnetic field H_c that destroys superconductivity if a superconductor is placed in an external magnetic field. For instance, for niobium, $H_c \sim 1950$ Gs, for aluminum, $H_c \sim 100$ Gs.

3. The magnetic flux is forced out of the interior of a superconductor (the Meissner effect).

4. The heat capacity in a normal metal C_n is a sum of the heat capacity of conductivity electrons C_{ne} and that of the lattice C_{ng}: $C_n = C_{ne} + C_{ng}$. For a superconductor, the heat capacity C_s is also a sum of the heat capacity of conductivity electrons C_{se} and that of the lattice C_{sg}: $C_s = C_{se} + C_{sg}$. It is assumed that the lattice heat capacity slightly changes when a metal goes from a normal state to a superconducting one, i.e. $C_{ng} \sim C_{sg}$. Then the difference $C_s - C_n \sim C_{es} - C_{en}$. It can be found, since C_{ne} can be measured in an independent way. It turned out that

$$\frac{C_{es}}{C_{en}(T = T_c)} = A \exp\left(-\frac{B}{t}\right),$$

where $t = \frac{T}{T_c}$, and A and B are some constants. This behavior of the ratio of heat capacities is indicative of the so-called "gap" in the energy spectrum of electrons. This gap separates the ground state from the lowest excited state. The number of electrons excited above the ground state changes exponentially with changing temperature. A gap of that sort determines thermodynamic properties of all superconductors, and it is necessary for any theory of superconductivity.

5. In a superconducting state, there is no Thomson thermal effect that consists in extraction or absorption of the heat in a conductor with current, along which the temperature changes (except for the extraction of the Joule heat). The absence of the Thomson effect signifies that in a superconducting state, electrons that are in the lowest state are separated in energy from excited states by a gap of the dimension $k_b T_c$ where k_b is the Boltzmann constant.

It is established experimentally that the magnitude of the gap for different superconductors little changes within the limits $3 < k_b T_c < 4$.

6. The gap size changes with changing temperature, and when $t = \frac{T}{T_c} \to 1$, it tends to zero.

A specific feature of a great number of experimental data on superconductivity is a certain similarity of superconducting properties of metals having very different crystalline and atomic characteristics. Let us present an example of a characteristic of that sort: this is the difference of energies between a superconducting and a normal phase. A superconductor can be transformed from a superconducting state to a normal one with the help of a magnetic field whose critical value H_c at absolute zero amounts to about hundreds of gausses. If the quantity $\frac{H_c^2}{8\pi}$ (the energy of magnetic field per unit volume) is divided by the number of atoms (everything is measured per unit volume), we obtain $\sim 10^{-8}$ eV per atom, which seems to be a very small value as compared, for instance, with the correlation energy of the Coulomb interaction of conductivity electrons (~ 1 eV). A sharp phase transition from a normal state of a conductor to a superconducting state corresponds to a high correlation of electrons switching on in the state of superconductivity.

In 1950, Maxwell and Reynolds established independently that the critical temperature of transition of mercury isotopes into a superconducting state is connected with the mass of isotopes M_I by the relation $T_c \sim \frac{\text{const}}{M_I^{1/2}}$. This effect was called the isotopic effect. It signified that an oscillating crystal lattice is important for the phenomenon of superconductivity. Since the mass of an isotope can influence only the spectrum of phonons, the isotopic effect suggested an idea that superconductivity essentially depends on the interaction of conductivity electrons with a lattice, or in other words, on the interaction of electrons with phonons.

We already mentioned the phonon when describing the phenomenon of superfluidity. Let us briefly recall the history of this conception. The dynamic theory of crystal lattice as an ensemble of bound quantum oscillators of different frequencies was constructed by the Dutch physicist P.Debye (1912), the German physicists M.Born and T.Karman (1913, the USA), and by the Austrian physicist E.Schroedinger (1914) in a form close to the modern one. In 1929, I.Tamm called the quantum oscillatory motions of atoms of the crystal lattice the phonons (see "Physical encyclopaedia". Published by "Soviet encyclopaedia", Moscow, 1983, p.735). The American physicist Bloch explained the origin of electrical resistance as follows: moving electrons colliding with atoms of the lattice give a part of their energy to it and heat the metal. In view of the lattice oscillations being quantum in nature, Bloch interpreted the interaction of conductivity electrons with lattice as the exchange by phonons.

The isotopic effect observed by Maxwell and Reynolds in 1950 was in the same year, but slightly earlier, theoretically predicted by Fröhlich. He has shown that two electrons can interact with each other through the exchange by a phonon, and in this case, there can even arise a weak attraction between them, despite the Coulomb repulsion, which can produce the energy gap. The exchange of electrons by phonons is equivalent to the interaction of electrons with the lattice, which allowed Fröhlich to predict the isotopic effect prior to its experimental discovery. "... Fröhlich's bold idea put forward in 1950 suggested that just this interaction that is in general rather weak and produces the resistance to systematic motion of electrons can result in superconductivity under certain conditions. He mathematically formulated this idea with the help of second quantization...".

"... New important ideas were proposed by Schafroth, Butler and Blatt. Since 1954, they have systematically developed the idea of importance of pair correlations of electrons and of formation of peculiar "quasimolecules" consisting of two electrons and thus obeying the Bose statistics....". In the conception of Schafroth, Butler and Blatt the basic origin of the formation of quasimolecules is the Fröhlich attraction of two electrons in the vicinity of the Fermi surface. "...However, Schafroth, Butler and Blatt did not dispose of an appropriate mathematical method and could not construct a convincing scheme..." (From the report by Bogoliubov at the General meeting of the Acad. of Sci. of the USSR, June 20, 1958).

Let us interrupt the Bogoliubov citation and explain what does the "Fermi surface" mean? The gas of conductivity electrons at the temperature of absolute zero condenses and combines into Cooper pairs with spins and momenta equal and opposite in direction $(p, -p)$. In accordance with the Pauli principle, two identical electrons cannot simultaneously be in the same state. It is stated in quantum mechanics that all particles of the same sort are identical; so are also all electrons. As a result, every pair of bound electrons with momenta p and $-p$ occupies its own energy level starting from the ground level. Thus, the condensed gas of electrons fills, from the bottom, a certain energy region, or in other words, there arises a certain energy boundary (a surface in the three-dimensional momentum space) $\varepsilon(p) = \varepsilon_p$ that separates the filled region from the empty one. This surface is called the Fermi surface. The greatest number of electron pair are located near the Fermi surface, since this region possesses the largest phase volume. These electrons (or Fermi surface) determine the majority of properties of metals. As a result of excitation, one electron of a pair can be knocked out beyond the limits of the surface. Then inside the surface, there will occur a free place, a "hole", that can, in principle, be occupied by another electron. It is important to notice that when a single electron is removed from the system, there is also removed a great number of pairs of electrons that would tend to occupy empty states (with $\vec{K}, -\vec{K}$). So, the energy difference between the occupied state of a pair system and the state with one excited electron turns out to be many times as large as the binding energy of a single pair; nevertheless, the superconductivity theory that treats only a single excitation correctly determines the magnitude of the energy gap. We should not detail this mechanism; it suffices to understand what is meant by the Fermi surface.

Let us turn back to the report by Bogoliubov: "...Further development of the theory was achieved in the works by Cooper, and then, by Bardeen, Cooper, and Schrieffer. The work by Cooper appeared in 1956; a short note by Bardeen, Cooper, and Schrieffer was published in April, 1957; and a detailed paper, in December, 1957...".

Bardeen, Cooper, and Schrieffer have made a number of ungrounded simplifications of the mathematical scheme in the process of solution and have drawn the conclusions which do not look reliable. Bogoliubov continued: "... They established that the energies of excited states are separated from the ground states energy by an energy gap, and the property of superconductivity is obtained in their scheme. Main characteristics of superconductivity in their theory are simply expressed through two parameters... and are in satisfactory qualitative agreement with experiment...".

"... Before the detailed paper by Bardeen, Cooper, and Schrieffer became known in Moscow, at the end of September, 1957, I succeeded to show that the method we developed for constructing the microscopic theory of superconductivity of Bose-systems can be generalized to a consistent construction of superconductivity on the basis of the Fröchlich original model. The corresponding equations can be solved without any ungrounded assumptions...".

To construct the microscopic theory of superconductivity, Nikolai Nikolaevich took the Hamiltonian of a dynamic system as a sum of effective energies of electrons, phonons and the Hamiltonian of electron–phonon interaction. Difficulties of the conventional perturbation theory turned out to be of the same type as for Bose-systems in the superfluidity theory. The method of canonical transformations called in the world scientific literature the "Bogoliubov canonical transformations" used by Bogoliubov in constructing the superfluidity theory was generalized to the Fermi amplitudes describing the behavior of systems with half-integer spin. Matrix elements describing the process of virtual production of particles (phonons) from vacuum can contain denominators that vanish under some conditions. This would lead to infinite values of certain integrals. Canonical transformations are to be chosen so as to provide mutual cancellation of the matrix elements that correspond to virtual creation of a pair of particles with opposite momenta and spins from vacuum. Just this compensation of diverging classes of diagrams was realized by Bogoliubov in constructing the theory of superfluidity of Bose-systems. Now, he generalized this transformation to the Fermi-systems. Calculating the energy of the ground state of a system in the second approximation, Nikolai Nikolaevich derived mathematically strict results that explain the origin of the energy gap, its dependence on the coupling constant of phonons with conductivity electrons, and the energy spectrum of excited states. "... Let us stress that the performed studies confirmed the validity, in the first approximation, of the formulae derived by Bardeen, Cooper, and Schrieffer. Using the method of approximate second quantization, we carried out a systematic study of the Fröhlich model and found that besides excitations of the fermion type, it also contains collective boson excitations without energy gap..." (Bogoliubov; from the report on...).

The Fröhlich model takes no account of the Coulomb repulsion between electrons of a Cooper pair. The Bardeen theory took into account the Coulomb repulsion simply as a certain averaged constant I_{coul} that does not take into consideration the character of the Coulomb interaction $\frac{1}{r}$ where r is the distance between electrons. These drawbacks were removed in the work by Shirkov and Tolmachev who supplemented the interaction Lagrangian with the terms describing the realistic Coulomb repulsion between electrons. "... As a result of their investigations, it turned out that their condition of superconductivity differs from the Bardeen one in that the constant I_{coul} should first be divided by a quantity of an order of the logarithm of the ratio of the Fermi energy of the phonon mean energy..." (From the Bogoliubov report on the General meeting ...). Before long, the results of studies by Bogoliubov with colleagues were collected in the monograph by Bogoliubov, Shirkov, and Tolmachev "A new method in the superconductivity theory" (ed. by Akad.Nauk SSSR, Moscow, 1958). "... In general, the physical picture of the structure of a superconducting or superfluid state of a Fermi system looks as follows: Interactions present in such systems always initiate correlations, specifically, between pairs of fermions with momenta K and $(-K+q)$. In this case, nothing distinguishes the value $q=0$ from others. In the case of transition to a superfluid state or to a superconducting case, there appears a discontinuity at $q=0$ (of course, we are specking about a system at rest as a whole). For a given particle with momentum K, there is a nonzero probability of finding another particle with momentum $-K$. Evidently, in the normal state, such probability is equal to zero. This situation can be interpreted as formation of a system of bound pairs at rest—quasimolecules, i.e. as formation of the Bose–Einstein condensate of such pairs. In this connection, one-fermion excitations should be considered as gas generated by dissociation, and the energy gap is interpreted as the energy necessary for dissociation.

Besides of the excitations accompanied by the discontinuity of pairs, there are collective excitations of the whole bound collective (on the whole of the same type like in the usual Bose gas).

Note that when we are talking about systems of molecules or, in general, about a system consisting of bound complexes of particles, we always mean a situation when the binding energy of complexes is much higher than the energy of interaction between the complexes.

According to the fundamental principles of quantum mechanics, we can never consider that particles of one molecule differ from analogous particles of another molecule of the same sort and cannot change places. However, since the interaction between molecules is small as compared to the binding energy of individual particles in a molecule, these effects of exchange can be neglected. In the case under consideration of a superconducting state or a superfluid state of a system of fermions, the situation is completely different: the binding energy of pairs can be of the order of or even lower than the energy of interaction between them. Here, the exchange effects are essential.

So, we are concerned with new notions about the dynamics of a system. The quasimolecules we are speaking about are in the process of constant very fast exchange by individual particles entering into them. In essence, we can no longer speak about individual molecules but should mean the collective of particles as a whole. The process of interaction of particles consists in that at every moment of time, they in different combinations are bound into complexes. In contrast to realistic molecules, the lifetime of every complex of that sort, a quasimolecule, and the mean free time, the time between separate acts of interaction of this complex with other complexes, are of the same order...".

And, finally, the last two paragraphs from the mentioned Bogoliubov report point to good prospects of the use of the method of the theories of superfluidity and superconductivity in the nuclear theory: "It was suggested in the nuclear theory that nuclear matter should be thought of to be constructed out of complexes of neutrons and protons—deuterons, α- particles, etc.—rather than of them themselves. However, such ideas raised serious objections, because the binding energy of such complexes is of the order of the energy of interaction between them. It is just this situation that is typical of the electron system in a superconducting metal. Apparently, in both the cases, we are dealing with the same dynamic mechanism. Therefore, it may be hoped that the method worked out for constructing the theories of superfluidity and superconductivity turns out to be fruitful in the nuclear theory, as well. At present, we are engaged in solving this problem".

Indeed, the journal "Doklady Akademii nauk SSSR" (Reports of the Acad of Sci. of the USSR), in the same year 1958, published the Bogoliubov paper "To the problem of the condition of superfluidity in the theory of nuclear matter" (vol. 119, N. 1, pp. 52–55) that laid foundations for the nuclear superfluid model developed further by V.G.Soloviev (JINR, BLTP).

Thus, we observe astounding creative work of the great scientist: Having declared, in his report on the problem of superfluidity of Bose- and Fermi- systems delivered on June 20, 1958, about a possibility to use his method for constructing the nuclear superfluid model, only 3–4 months later, he published a new fundamental work giving complete solution to the new important problem.

"To solve a problem", as Bogoliubov said in his report, means for him to give an adequate formulation of the problem, to write an exact system of equations, to derive a complete exact solution to that system, to obtain strict results, and to provide the interpretation of a phenomenon that leaves no doubts in the complete solution of the problem. This was the case with the creation of the microscopic theory of superfluidity (1947), with the proof of dispersion relations in the theory of strong interactions (1956), with the construction of the axiomatic field theory (1955–1957), with the creation of the microscopic theory of superconductivity (1957), with the creation of the superfluid nuclear theory (1958), with the proof of the quasiaverage theorem (1961), with the solution of the problem of quark "color" (1965).

The scientific work of Bogoliubov, his methodology, his approaches to solving important scientific problems produced a huge influence on the development of the world theoretical physics not only by a great number of the above-mentioned solved problems of world class, but also by complete mathematical strictness of their solution. Works by Bogoliubov have become, figuratively speaking, a definite height of the standard of theoretical thinking and solving problems that should be the ultimate aim of every theoretician in his development and creative work.

The authors are highly grateful to A.T.Filippov for useful comments.

Received on March 20, 2002.

Боголюбов П. Н., Исаев П. С. E4-2002-52
К истории создания микроскопических теорий
сверхтекучести и сверхпроводимости

Изложена история создания микроскопической теории сверхтекучести (1947 г.) и микроскопической теории сверхпроводимости (1957 г.).

Работа посвящается 90-летию со дня рождения нашего гениального современника академика Николая Николаевича Боголюбова.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Сообщение Объединенного института ядерных исследований. Дубна, 2002

Bogolubov P. N., Isaev P. S. E4-2002-52
On the History of Creation of the Microscopic Theories
of Superfluidity and Superconductivity

The history of creation of the microscopic theory of superfluidity (1947) and the microscopic theory of superconductivity (1957) is expounded.

The paper is dedicated to the 90th anniversary of the birth of our genius contemporary Academician Nikolai Nikolaevich Bogoliubov.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Communication of the Joint Institute for Nuclear Research. Dubna, 2002

Макет *Т. Е. Попеко*

ЛР № 020579 от 23.06.97.
Подписано в печать 22.04.2002.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 1,18. Уч.-изд. л. 2,09. Тираж 380 экз. Заказ № 53251

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.