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PARTIAL WIDTHS OF NONMESONIC WEAK DECAYS OF Λ-HYPERNUCLEI

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1 Introduction

Recent papers [1] consider the possibility of studying the spin-isospin dependence of the matrix elements of non-leptonic weak interactions $\Lambda N \to NN$ registering two correlated α-particles created in the reactions

$$\begin{array}{lcccl} {}^{10}_{\Lambda}\mathrm{Be} & \to & n+p & + & ({}^{8}\mathrm{Li} \xrightarrow{\beta^-} {}^{8}\mathrm{Be} \to \alpha\alpha\,) \\ & \to & n+n & + & \boxed{{}^{8}\mathrm{Be}^* \to \alpha\alpha} \\ {}^{10}_{\Lambda}\mathrm{B} & \to & n+p & + & \boxed{{}^{8}\mathrm{Be}^* \to \alpha\alpha} \\ & \to & n+n & + & ({}^{8}\mathrm{B} \xrightarrow{\beta^+} {}^{8}\mathrm{Be} \to \alpha\alpha\,) \end{array} \tag{1}$$

by the decay of excited states of product nucleus ^{8}Be. It was stated, that the knowledge of experimental $\alpha\alpha$-decay widths (4 for protons and 4 for neutrons) is sufficient for complete determination of all four (eight) matrix elements of effective weak interaction

$$w^{SJ}_{l=1,\tau} = \left| \sum_{L'S'} \langle NN : L'S'T' : J|V_{\mathrm{w}}|\tau p s_{\Lambda} : L = 1SJ\rangle \right|^2 , \tau = n,p. \tag{2}$$

In this short note we will concentrate on two important questions:
i) Is it possible to extract 4 matrix elements $w^{SJ}_{l\,\tau}$ from the experimental data?
ii) How do the obtained matrix elements depend on the nuclear "residual" interaction employed in calculations of the wave functions for $A = 9$ and $A = 8$ nuclei?

To answer these questions, we will use standard approaches of the many-particle nuclear shell model: the spectra and wave functions of nuclear excited states are obtained by the diagonalization of the Hamiltonian matrix. The parameters of the used residual two-body interaction were determined by the fit of a large amount of experimental spectroscopic data on a wide range of nuclei.

2 Partial width of nonmesonic decays

The main observable in weak nonmesonic decays of a hypernucleus is the total decay width

$$\Gamma_{\mathrm{nm}} = \sum_{\tau=n,p} \Gamma^{\tau}_{\mathrm{nm}} = \sum_{\tau=n,p} \sum_{E_i,J_i,T_i} \left| \left\langle \left[\Psi^{(A-2)}(\{i\}) \cdot \Psi^{(NN)}(JT) \right]^{\mathcal{J}} |V_{\mathrm{w}}| \left[\Psi^{(A-1)}(\{c\}) \cdot \psi^{\Lambda}(\tfrac{1}{2}) \right]^{\mathcal{J}} \right\rangle \right|^2 , \tag{3}$$

here $\{c\} \equiv \{E_c, J_c, T_c, \tau_c\}$ and $\{i\} \equiv \{E_i, J_i, T_i, \tau_i\}$ are sets of quantum numbers describing the states of initial and final nuclei. The hypernucleus decays from its ground state in which the nucleons form a core nucleus in its ground state, $\Psi^{(A-1)}(E_c, J_c, T_c, \tau_c)$, and Λ-hyperon has a minimal energy. In the shell model representation:

$$|I\rangle = |l^k, E_c = 0, J_c, T_c, \tau_c; s_\Lambda : \mathcal{J}\rangle.$$

We consider two decay channels: $n\Lambda \to nn$ and $p\Lambda \to pn$, in which the Λ-hyperon picks-up one $1p$-nucleon of the core-nucleus. Employing the technique of fractional parentage coefficients (FPC), the ground state wave function of the initial hypernucleus is decomposed [2] into a complete set of wave functions of excited states in the residual $(A-2)$ nucleus $\Psi^{(A-2)}(E_i, J_i, T_i, \tau_i)$ coupled to the complete set of wave function of states of an $N\Lambda$ pair, $|\tau l, s_\Lambda : L = lSJ\rangle$. So, the wave function of the initial state of the hypernucleus is

$$\begin{aligned} |I\rangle = &\sum_{\tau,j,l} \sum_{E_i,J_i,T_i} \sum_{J,S} \sqrt{k}\,(T_i\tau_i \tfrac{1}{2}\tau|T_c\tau_c)\, U(J_i j \mathcal{J} \tfrac{1}{2} : J_c J) \\ &\times U(l\tfrac{1}{2}J\tfrac{1}{2} : jS)\, g^{E_c J_c T_c}_{E_r J_r T_r} \left[|k^{k-1} E_r J_r T_r \tau_r\rangle \otimes |\tau l\, s_\Lambda : L = l, SJ\rangle\right]^{\mathcal{J}} \end{aligned} \tag{4}$$

and

$$\Gamma_{\rm nm} = \sum_\tau \sum_i \Gamma_i^\tau$$

where

$$\Gamma_i^\tau = \sum_{SJ} G^2_{\mathcal{J}}(\{c\}, \{i\}, \tau lSJ)\, w^{SJ}_{l\,\tau} \tag{5}$$

Here

$$w^{SJ}_{l\,\tau} = \left| \sum_{L',S'} \langle l_1 l_2 : L'S'JT|V_{\rm w}|\tau l,\ s_\Lambda : L = lSJ\rangle \right|^2 \tag{6}$$

are unknown matrix elements of "weak interaction" to be extracted from partial transition widths. The factor $G_{\mathcal{J}}$ is equal to

$$G_{\mathcal{J}}(\{c\}, \{i\}, \tau lSJ) = \sum_j U(J_i j \mathcal{J}\tfrac{1}{2} : J_c J)\, U(l\tfrac{1}{2}J\tfrac{1}{2} : jS)\, S_i(\tau lj)\,, \tag{7}$$

where $S_i(\tau lj)$ are spectroscopic amplitudes for the separation of one nucleon from the ground state of the nucleus

$$S_i(\tau lj) = \sqrt{k}\ (T_i\tau_i\ \tfrac{1}{2}\tau \mid T_c\tau_c)\, g^{\,E_c J_c T_c}_{\,E_i J_i T_i}\ (lj), \tag{8}$$

and $g_i^c(lj)$ is a one-nucleon FPC in the intermediate coupling

$$g_i^c(lj) = \sum_{f_cL_cS_c} \sum_{f_iL_iS_i} a_{f_cL_cS_c}^{E_cJ_cT_c} a_{f_iL_iS_i}^{E_iJ_iT_i} \times \langle l^k[f_c]L_cS_cT_c\{| l^{k-1}[f_i]L_iS_iT_i\rangle \begin{pmatrix} L_i & S_i & J_i \\ l & \frac{1}{2} & j \\ L_c & S_c & J_c \end{pmatrix}. \qquad (9)$$

The coefficients $a_{f_cL_cS_c}^{E_cJ_cT_c}$ and $a_{f_iL_iS_i}^{E_iJ_iT_i}$ are results of the shell-model Hamiltonian diagonalization, e.g. [3].

The partial widths of nonmesonic decay of $1p$-shell hypernuclei for transition into natural parity states are linear combinations of four matrix elements ($^1P_1$, $^3P_0$, $^3P_1$ and $^3P_2$) only. From the equation (4) one can easily see that partial widths corresponding to different J_i values are determined by quite definite (and different) combinations of matrix elements $w_{l\,\mathcal{T}}^{SJ}$. The coefficients of these combinations for our case ($J_c = \frac{3}{2}$, $\mathcal{J} = 1$) are given below.

	$^3P_0$	$^1P_1$	$^3P_1$	$^3P_2$
$J_i = 0$		$\sqrt{\frac{2}{3}}g_{\frac{3}{2}}$	$\sqrt{\frac{1}{3}}g_{\frac{3}{2}}$	
$J_i = 1$	$\sqrt{\frac{2}{3}}g_{\frac{1}{2}}$	$-\sqrt{\frac{1}{9}}g_{\frac{1}{2}} + \sqrt{\frac{5}{9}}g_{\frac{3}{2}}$	$\sqrt{\frac{2}{9}}g_{\frac{1}{2}} + \sqrt{\frac{5}{18}}g_{\frac{3}{2}}$	$\sqrt{\frac{1}{6}}g_{\frac{3}{2}}$
$J_i = 2$		$-\sqrt{\frac{1}{3}}g_{\frac{1}{2}} + \sqrt{\frac{1}{3}}g_{\frac{3}{2}}$	$\sqrt{\frac{2}{3}}g_{\frac{1}{2}} + \sqrt{\frac{1}{6}}g_{\frac{3}{2}}$	$\sqrt{\frac{1}{2}}g_{\frac{3}{2}}$
$J_i = 3$				$g_{\frac{3}{2}}$

As a result, in an ideal case when the transitions to final states with $J_i = 0, 1, 2$ and 3 are observed, one can unambiguously determine all four matrix elements $w_{l\,\mathcal{T}}^{SJ}$. The nuclear residual interaction accounted by many-particle shell model influences the relative $g_{\frac{1}{2}}$ and $g_{\frac{3}{2}}$ quantities.

It is supposed that from the measurements of correlated α-particles emitted in decay of ^{8}Be* (either direct or delayed, after $\beta^\pm$-decay of ^{8}B and ^{8}Li) one can extract the partial widths related to the following states of ^{8}Be: $(0^+0)_{\text{g.s.}}$, $(2^+0)_1$, $(2^+1)_1$, $(2^+0)_2$, and $(1^+1)_1$ (see Fig. 1). The latter transition goes through β-decay of ^{8}Li. There is no observed transition to the 3^+ state, but the 2^+ states with excitation energies near to 3 MeV and 16 MeV relate with **different** configurations: $^1D_2$ and $^3P_2$ respectively. Therefore, **we have a necessary number** of linear independent equations for determination of **all four (eight) matrix elements** $w_{1\,\mathcal{T}}^{SJ}$.

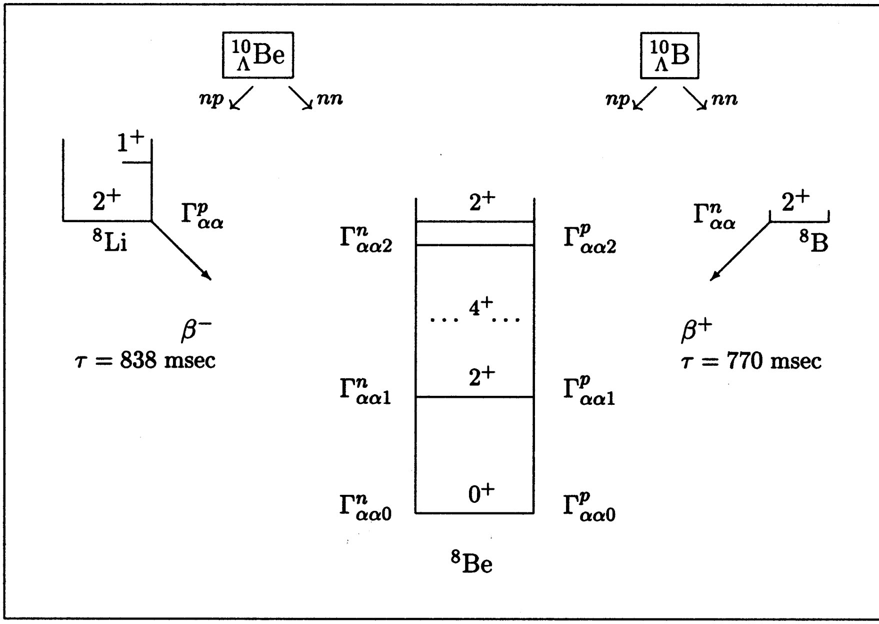

Fig. 1: The notation of the partial widths $\Gamma^{\tau}_{\alpha\alpha i}$.

Table 1 illustrates the influence of a nuclear model on the calculated one-body spectroscopic amplitudes. The used models employ the full $1p$-shell space and differ mainly in the nuclear residual interactions. We have selected several models used usually in the calculations of characteristics of $1p$-shell nuclei. The calculations were performed with the many-particle shell model code OXBASH [4].

The main attention in this paper is paid to the theoretical analysis, nevertheless, a few words should be said about the experimental situation. It is quite possible that $\Gamma_{\alpha\alpha 1}$ and $\Gamma_{\alpha\alpha 2}$ widths could be measured at the Dubna Nuclotron [5]. The fact that

$$\Gamma_{\alpha\alpha 2} = \Gamma_{\alpha\alpha}\left((2^+1)_1\right) + \Gamma_{\alpha\alpha}\left((2^+0)_2\right),$$

does not complicate the analysis. We would like to stress that the measurement of $\Gamma^p_{\alpha\alpha}$ ($^{10}_{\Lambda}$Be) is very important, because only this partial width contains the information about w^{10}_{1p}.

It should be noted that $\Gamma^{\tau}_{\alpha\alpha 0}$ does not practically depend on details of the ^{8}Be structure.

Table 1: Neutron spectroscopic factors in ^{9}Be

Model	Quantity	States of ^{8}Be				
		$(0^+0)_{g.s.}$	$(2^+0)_1$	$(2^+0)_2$	$(2^+1)_1$	$(1^+1)_1$
Experi-	E_x (MeV) [6]	0.0	3.04	16.92	16.63	17.64
ment	$S^2_{1/2}+S^2_{3/2}$ [7]	0.67(14)	1.49(23)	1.14(19)		0.27(5)
	$S^2_{1/2}+S^2_{3/2}$ [8]	0.60(17)	1.20(21)	0.65(15)		0.10(7)
CK_{pot}	E_x	0.0	3.82	12.87	16.19	17.10
[9]	$S_{1/2}$		−0.2045	0.3186	0.1936	0.2068
	$S_{3/2}$	0.7534	0.8578	0.5973	0.6463	−0.1664
	$S^2_{1/2}+S^2_{3/2}$	0.5676	0.7775	0.4582	0.5129	0.0705
CK I	E_x	0.00	3.41	14.43	15.80	16.88
[9]	$S_{1/2}$		−0.2424	0.2881	0.2257	−0.3144
	$S_{3/2}$	0.7616	0.8166	0.5886	0.6698	0.3210
	$S^2_{1/2}+S^2_{3/2}$	0.5800	0.7256	0.4295	0.4996	0.2018
CK II	E_x	0.00	3.55	13.36	16.19	16.93
[9]	$S_{1/2}$		−0.2428	0.3107	0.2025	0.2369
	$S_{3/2}$	0.7620	0.8180	0.6112	0.6776	−0.1758
	$S^2_{1/2}+S^2_{3/2}$	0.5806	0.7281	0.4700	0.5002	0.0870
PBA I	E_x	0.00	3.39	14.33	15.73	16.77
[10]	$S_{1/2}$		−0.2715	0.2937	0.2160	−0.3180
	$S_{3/2}$	0.7689	0.7942	0.5899	0.6649	0.3263
	$S^2_{1/2}+S^2_{3/2}$	0.5912	0.7045	0.4342	0.4888	0.2076
PBA III	E_x	0.00	2.86	16.39	15.95	16.95
[10]	$S_{1/2}$		−0.2827	0.2512	0.2308	−0.3329
	$S_{3/2}$	0.7703	0.7766	0.5611	0.6525	0.3336
	$S^2_{1/2}+S^2_{3/2}$	0.5934	0.6830	0.3779	0.4791	0.2221
WB(5−16)	E_x	0.00	3.57	15.44	15.85	17.39
[11]	$S_{1/2}$		−0.1676	0.2699	0.2260	0.2836
	$S_{3/2}$	0.7409	0.8649	0.5904	0.6812	−0.2999
	$S^2_{1/2}+S^2_{3/2}$	0.5489	0.7761	0.4215	0.5151	0.1704

Table 2: Weights of $w_{1\,n}^{SJ}$ in the α-decay widths.

State in ^{8}Be	Model	$^{2S+1}L_J$ $^3P_0$	$^1P_1$	$^3P_1$	$^3P_2$	Norm factor
$(0^+0)_1$	CK_{pot}		0.6667	0.3333		0.5676
	CK I		0.6667	0.3333		0.5800
	CK II		0.6667	0.3333		0.5806
	PBA I		0.6667	0.3333		0.5912
	PBA III		0.6667	0.3333		0.5934
	WB(5−16)		0.6667	0.3333		0.5489
$(2^+0)_1$	CK_{pot}		0.4837	0.0432	0.4731	0.7775
	CK I		0.51523	0.0253	0.4595	0.7256
	CK II		0.5152	0.0253	0.4595	0.7281
	PBA I		0.5374	0.0149	0.4477	0.7045
	PBA III		0.5476	0.0109	0.4415	0.6830
	WB(5−16)		0.4579	0.0602	0.4819	0.7761
$(2^+0)_2$	CK_{pot}		0.0565	0.5542	0.3893	0.4582
	CK I		0.0701	0.5266	0.4033	0.4295
	CK II		0.0640	0.5387	0.3973	0.4700
	PBA I		0.0673	0.5321	0.4006	0.4342
	PBA III		0.0847	0.4988	0.4165	0.3779
	WB(5−16)		0.0812	0.5052	0.4136	0.4215
$(2^+1)_1$	CK_{pot}		0.1598	0.3767	0.4635	0.5129
	CK I		0.1316	0.4194	0.4490	0.4996
	CK II		0.1505	0.3905	0.4590	0.5002
	PBA I		0.1374	0.4103	0.4523	0.4888
	PBA III		0.1238	0.4319	0.4444	0.4791
	WB(5−16)		0.1341	0.4155	0.4504	0.5151
$(1^+1)_1$	CK_{pot}	0.4047	0.5285	0.0014	0.0655	0.0705
	CK I	0.3264	0.5864	0.0022	0.0851	0.2018
	CK II	0.4299	0.5068	0.0041	0.0592	0.0870
	PBA I	0.3247	0.5874	0.0023	0.0855	0.2076
	PBA III	0.3326	0.5822	0.0016	0.0835	0.2221
	WB(5−16)	0.3147	0.5938	0.0035	0.0880	0.1704

3 Conclusion

The properties of nonmesonic decays of $1p$-shell nuclei can be described in terms of few weak interaction phenomenological matrix elements $w_{1}^{SJ}{}_{T}$ defined by Eq. (6) [1]. The present consideration shows that these matrix elements can be extracted from the measured values of $\Gamma^{\tau}_{\alpha\alpha i}$, partial widths of nonmesonic decays ${}^{10}_{\Lambda}$Be and ${}^{10}_{\Lambda}$B. Also it is shown that the uncertainties related to the description of nuclear structure are not essential for this task.

The relation between $w_{1}^{SJ}{}_{T}$ and "elementary" weak $\Lambda N \rightarrow NN$ interaction (exchange by one pion, exchange by one kaon, two-meson exchange, etc) will be discussed in subsequent papers together with possible reasons for the known problems in explanations of the experimental ratio Γ^n/Γ^p.

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Парциальные ширины безмезонных слабых распадов гиперядер

Показано, что феноменологические матричные элементы, которые полностью описывают слабое взаимодействие $\Lambda N \to NN$ в ядрах $1p$-оболочки, могут быть извлечены из ширин безмезонных распадов гиперядер $^{10}_{\Lambda}$Be и $^{10}_{\Lambda}$B. При этом влияние неопределенностей, связанных с описанием структуры ядер, невелико.

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Partial Widths of Nonmesonic Weak Decays of Λ-Hypernuclei

It is shown that the phenomenological matrix elements completely describing $\Lambda N \to NN$ weak interaction in the $1p$-shell nuclei can be obtained from the partial widths of nonmesonic decays of $^{10}_{\Lambda}$Be and $^{10}_{\Lambda}$B hypernuclei. It is shown that the uncertainties related to the description of nuclear structure are not essential for this task.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR and at the Nuclear Physics Institute, Academy of Sciences of Czech Republic (Řež, Czech Republic).

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