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FERMION-FERMION GLUING BY MEANS OF UNOBSERVABLE SCALAR BOSONS

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1 Introduction

Field model under consideration describes interacting fermionic and bosonic fields, analogously to the Yukawa model, QED, QCD. First two theories allow to detect its bosons (mesons and photons) along with their fermions. One does not observe bosons that interact strongly with quarks. The unobservability of isolated gluons is reasoned in QCD by color confinement. This is an additional hypothesis rather than a theorem derived from QCD Lagrangian, e.g. see [1] ch. 18.7, and [2, 3]. In the suggested model bosons are unobservable and this is its exact corollary.

In the model several fermionic fields interact with several scalar fields by means of a derivative coupling, see sect. 2. For the sake of convenience of the model comparison with QCD the model fermionic fields will be called quark fields. The corresponding particles will be called quarks.

It is supposed that the model scalar fields interact only with these quark fields and do not interact with all other fields of the particle physics, e.g. electromagnetic and leptonic ones.

Let us list some other peculiarities of the model. In place of renormalizability and asymptotic freedom of QCD the model has the property of being partially solvable. This means that one can find exactly a subset of eigenvectors of the model total Hamiltonian H along with quark-quark potential, see sect. 3. The model interaction is nonlocal and we assume that the theory of interacting quarks need not be relativistic (of course the bound quarks states, i.e. hadrons, must have relativistic description). Note also that model is not gauge invariant just like the known Yukawa model of the meson-nucleon interaction.

The model is specified by postulating its Hamiltonian H, see sect. 2. The existence of the related Lagrangian is not supposed. As Weinberg noted “The point of the Lagrangian formalism is that it makes it easy to satisfy Lorentz invariance and other symmetries”, see [4], the preface to ch. 7, p. 292. As we do not assume Lorentz and gauge invariance the Lagrangian is not needed.

In sect. 3 the model is partially solved, using an unitary transformation of its Hamiltonian H.

The important point of this paper is using the particle interpretation of fields in terms of "clothed" particles and their creation-destruction operators, see the review [5] and section 4. It is the "clothed" scalar bosons which will be shown to be invisible, see sect. 5. Sect. 6 contains some concluding remarks.

2 The model Hamiltonian

The model total Hamiltonian H is the sum of its free part H_0 and the interaction H_I: $H = H_0 + H_I$. Here $H_0 = H_{0f} + H_{0b}$, where H_{0f} is the fermion part of H_0 and H_{0b} is its boson part. The operator H_{0f} is the sum of ordinary free Hamiltonians of several spinor (quark) fields, numerated by two indices: quark flavor $f = u, d, s\ldots$ (up, down, strange, ...) and quark color $i = r, y, v$ (red, yellow, violet)

$$H_{0f} = \sum_f \sum_i \int d^3x \psi^\dagger_{fi}(x)(\vec{\alpha}\vec{p} + \beta M_{fi})\psi_{fi}(x)\,. \tag{1}$$

Here ψ_{fi} is the column of Dirac components $(\psi_{if})_\mu$, $\mu = 1,2,3,4$ and $\psi^\dagger_{fi}$ is the row of Hermitian conjugated components $(\psi_{if})^+_\mu$.

The boson part H_{0b} of H_0 describes several free scalar Hermitian fields $\varphi_a(x)$, $a = 1, 2, \ldots$

$$H_{0b} = \sum_a \frac{1}{2} \int d^3x[\pi^2_a(x) + \vec{\nabla}\varphi_a(x) \cdot \vec{\nabla}\varphi_a(x) + \mu^2_a \varphi^2_a(x)]\,. \tag{2}$$

The following nonlocal interaction Hamiltonian is postulated

$$\begin{aligned} H_I \;=\; & \int d^3x \int d^3y \sum_a G_a(x-y) \sum_f \sum_{i,j} \Bigg\{ \sum_{m=1,2,3} \psi^\dagger_{fi}(x)\alpha^m g^a_{ij}\psi_{fj}(x)\frac{\partial}{\partial y_m}\varphi_a(y) \\ & +\psi^\dagger_{fi}(x) g^a_{ij}\psi_{fj}(x)\pi_a(y) \Bigg\}\,. \end{aligned} \tag{3}$$

Here $G_a(x-y)$ are formfactors, the interaction is local when $G_a(x-y) \sim \delta(x-y)$; α^m are the same Dirac matrices as in Eq. (1). In all Eqs. (1)-(3) and in the following x and y denote three-vectors: $x = (x_1, x_2, x_3)$.

The interaction H_I involves the same quarks fields as QCD. Similar to QCD the interaction conserves the flavor (there are no vertices like $\psi_u^\dagger \psi_d \varphi$, $\psi_u^\dagger \psi_s \varphi$, etc) and does not depend on flavor : the coupling constant of vertices $\psi_u^\dagger \psi_u \varphi$, $\psi_d^\dagger \psi_d \varphi$, ... are equal ($g_{if}^a$ does not depend on f), e.g. cf. [1] ch. 18.7, Eq. (18.7.5); [6] ch. 1, Eq. (1.9). In distinction to QCD the quark fields interact with derivatives $\partial_\mu \varphi_a$ of scalar fields and not with vector gluon fields B_a^μ.

The usual commutation and anticommutation relations for the Schroedinger fields ψ_{fi}, φ_a, π_a are postulated:

$$[\varphi_a(x), \pi_{a'}(x')] = i\delta_{aa'}\delta(x-x'), \tag{4}$$

$$\{\psi_{fi\mu}(x), \psi^+_{f'i'\mu'}(x')\}_+ = \delta_{ff'}\delta_{ii'}\delta_{\mu\mu'}\delta(x-x'). \tag{5}$$

Only nonvanishing commutators and anticommutators are written.

Remind that I do not suppose that a Lagrangian exists which would correspond to the model Hamiltonian. Therefore the Eqs. (4), (5) are not to be deduced from a Lagrangian using the known cannonical procedure. The equations are postulated as defining point of the model. The Eqs. (1)-(5) together with general quantum postulates allow one to perform any quantum calculations.

In the next section the following abridged notations will be used

$$J_a^0(x) = \sum_f \sum_{i,j} \sum_\mu \psi^+_{fi\mu}(x) g^a_{ij} \psi_{fj\mu}(x), \tag{6}$$

$$J_a^m(x) = \sum_f \sum_{i,j} \sum_{\mu,\nu} \psi^+_{fi\mu}(x) \alpha^m_{\mu\nu} g^a_{ij} \psi_{fj\nu}(x), \quad m = 1, 2, 3. \tag{7}$$

In terms of (6) and (7) the Eq. (3) may be written as

$$H_I = \int d^3x \int d^3y \sum_a G_a(x-y) \left\{ \sum_m J_a^m(x) \frac{\partial}{\partial y_m} \varphi_a(y) + J_a^0(x) \pi_a(y) \right\}. \tag{8}$$

3 Hamiltonian unitary transformation

Let $\exp iS$ be an unitary transformation, S being Hermitian. The unitaryly transformed total Hamiltonian $H \to H' = \exp(iS)H\exp(-iS)$ may be calculated using the equation

$$\begin{aligned} H' &= H + [iS, H] + \frac{1}{2}[iS, [iS, H]] + \ldots && (9) \\ &= H_0 + H_I + [iS, H_0] + [iS, H_I] + \ldots && (10) \end{aligned}$$

Let us assume the following ansatz for S using the notation (6)

$$S = \int d^3x \int d^3y \sum_a G_a(x-y) J_a^0(x) \varphi_a(y) \,. \tag{11}$$

Using the Eqs. (4) and (5) one may verify that the sum of the second and third terms in (10) vanishes

$$[iS, H_0] = -H_I \,. \tag{12}$$

Note. Instead of postulating Eq. (3) for H_I one may begin with postulating Eq. (11) for S and then calculate H_I using Eq. (12).

The Eq. (12) allows one to rewrite the r.h.s. of (10):

$$H' = H_0 + \frac{1}{2}[iS, H_I] + \sum_{n=2}^{\infty} \frac{1}{(n-1)!(n+1)} [iS, \ldots [iS, H_I] \ldots] \,. \tag{13}$$

Now calculate the second term in r.h.s. of Eq. (13). Using the identity

$$[AB, CD] = A[B, C]D + [A, C]BD + CA[B, D] + C[A, D]B$$

and Eq. (4) one obtains

$$
\begin{aligned}
[S, H_I] &= \int d^3x \int d^3y \sum_a G_a(x-y) \int d^3x' \int d^3y' \sum_{a'} G_{a'}(x'-y') \\
&\times \left[J_a^0(x)\varphi_a(y), \sum_m J_{a'}^m(x') \frac{\partial}{\partial y'_m} \varphi_{a'}(y') + J_{a'}^0(x')\pi_{a'}(y') \right] \\
&= \int d^3x \int d^3y \sum_a G_a(x-y) \int d^3x' \int d^3y' \sum_{a'} G_{a'}(x'-y') \\
&\times \left\{ \sum_m \left[J_a^0(x), J_{a'}^m(x') \right] \varphi_a(y) \frac{\partial}{\partial y'_m} \varphi_{a'}(y') \right. \\
&+ \left. \left[J_a^0(x), J_{a'}^0(x') \right] \varphi_a(y)\pi_{a'}(y') + J_{a'}^0(x') J_a^0(x) i\delta_{aa'}\delta(y-y') \right\} . \qquad (14)
\end{aligned}
$$

Further the commutators of "current densities" J^0 and J^m entering Eq. (14) may be calculated using the identity

$$
[AB, CD] = A\{B, C\}_+ D - \{A, C\}_+ BD - C\{D, A\}_+ B + CA\{D, B\}_+
$$

and anticommutators of the fermion fields, see Eq. (5). The result is

$$
\left[J_a^0(x), J_{a'}^0(x') \right] = \delta(x-x') \sum_f \sum_{i,j} \sum_\mu \psi_{fi\mu}^+(x)(g^a g^{a'} - g^{a'} g^a)_{ij} \psi_{fj\mu}(x) \,, \qquad (15)
$$

$$
\left[J_a^0(x), J_{a'}^m(x') \right] = \delta(x-x') \sum_f \sum_{i,j} \sum_{\mu,\nu} \psi_{fi\mu}^+(x)(g^a g^{a'} - g^{a'} g^a)_{ij} \alpha_{\mu\nu}^m \psi_{fj\nu}(x) \,. \qquad (16)
$$

Both these commutators vanish if matrices g^a, $a = 1, 2, \ldots$ mutually commute

$$
[g^a, g^{a'}] = 0, \quad \forall a, a' \,. \qquad (17)
$$

It is not difficult to verify that if (15) and (16) do not vanish then r.h.s. of Eq. (13) will be infinite series of terms, containing products of fermion and boson fields. Then the model will not be partially solvable and "clothed" bosons will not be invisible. So we require that Eq. (17) must hold. In this case $[S, H_I]$ depends only on J^0 (see the last term in curly brackets in Eq. (14), i.e.

depends only on fermion fields. Then $[S,[S,H_I]]=0$ because of $[J_a^0, J_{a'}^0]=0$ and $\sum_n$ in r.h.s. of Eq. (13) vanishes. So we obtain

$$H' = H_0 + \frac{1}{2}[iS, H_I] = H_{0b} + H_f\,, \qquad H_f = H_{0f} + V_{ff}\,, \tag{18}$$

$$V_{ff} \equiv -\frac{1}{2}\int d^3x \int d^3x' \sum_a F_a(x-x') J_a^0(x') J_a^0(x)\,, \tag{19}$$

$$F_a(x-x') \equiv \int d^3y G_a(x-y) G_a(x'-y)\,. \tag{20}$$

For H_{0f} and H_{0b} see Eqs. (1) and (2).

So H' does not contain terms describing interaction between boson and fermion fields as well as boson-boson interactions. Because of $[H_{0b}, H']=0$ all well-known eigenvectors ψ_b of H_{0b} are also H' eigenvectors. But the latter are not eigenvectors of the starting Hamiltonian H because $H \neq H'$. But if ψ_b is an eigenvector of H' then $\exp(-iS)\psi_b$ is H eigenvector:

$$He^{-iS}\psi_b = e^{-iS}H'e^{iS}e^{-iS}\psi_b = e^{-iS}H'\psi_b \sim e^{-iS}\psi_b\,.$$

A successful representation of the obtained H' eigenvector and H itself will be achieved in the next section using the so-called "clothed" boson and fermion operators.

Note. In the case when only one fermion field is present the l.h.s. of Eq. (16) turns into $[j_0(x), j_m(y)]$, where j_0 and j_m are charge and current densities of the fermionic field. S. B. Gerasimov called my attention to the paradox (anomaly) by J. Schwinger connected with the equations $[j_0(x), j_m(y)]=0$, $m=1,2,3$. Due to them the commutator $[j_0(0),[H, j_0(0)]]$ is zero. However, Schwinger had pointed out a way of the calculation of the commutator mean value in the H vacuum eigenstate Ω ($H\Omega=0$) which gives an essentially positive value $2(\Omega, j_0 H j_0 \Omega)$ for this mean, see [7]. By the way he used the hermiticy property (HP) of the kind

$$(\varphi_n, A\varphi_m) = (A\varphi_n, \varphi_m)$$

in the case when A is the Hamiltonian H. It should be stressed that HP is not valid for some states φ_m and φ_n. For example, let A be the momentum operator

$\hat{p} = -id/dx$, φ_n be its eigenvector $\varphi_0(x)$ with zero eigenvalue ($\varphi_0(x) = \text{const}$) and $\varphi_m = f(x)\varphi_0$. Then

$$(\varphi_0, \hat{p}f(x)\varphi_0) = \int_{-\infty}^{+\infty} dx(-i)df/dx = -i[f(\infty) - f(-\infty)].$$

This is finite and nonzero if e.g. $f(x) = \tanh x$. Meanwhile HP gives zero value for $(\varphi_0, \hat{p}f(x)\varphi_0)$.

To resolve Schwinger's paradox one must assume that HP is not valid in the case $A = H$, $\varphi_n = \Omega$, $\varphi_m = j_0^2(0)\Omega$:

$$(\Omega, Hj_0^2(0)\Omega) \neq (H\Omega, j_0^2(0)\Omega) = 0.$$

Indeed, it can be proved that $(\Omega, Hj_0^2\Omega) \neq 0$ if Eqs. (5) are postulated. Proof: It follows from $[j_0, [H, j_0]] = 0$ that $Hj_0^2 = 2j_0Hj_0 - j_0^2H$. Therefore,

$$(\Omega, Hj_0^2\Omega) = 2(\Omega, j_0Hj_0\Omega) > 0.$$

4 Introduction of "clothed" particles' operators

Let us introduce new boson and fermion operators

$$\tilde{\varphi} = e^{-iS}\varphi e^{iS}, \qquad \tilde{\psi} = e^{-iS}\psi e^{iS}. \tag{21}$$

Rewrite $H' = \exp(iS)H\exp(-iS)$ in the form $H = \exp(-iS)H'\exp(iS)$ and express H in terms of the operators (21):

$$H = e^{-iS}H'e^{iS} = H'(\tilde{\varphi}, \tilde{\psi}) = H_{0b}(\tilde{\varphi}) + H_f(\tilde{\psi}). \tag{22}$$

Here we used identities of the kind

$$e^{-iS}\varphi^2 e^{iS} = e^{-iS}\varphi e^{iS}e^{-iS}\varphi e^{iS} = \tilde{\varphi}^2.$$

The Eq. (22) means that the starting Hamiltonian H assumes the form of the sum of the free boson part $H_{0b}(\tilde{\varphi})$ and purely fermionic part $H_f(\tilde{\psi})$ when being expressed in terms of $\tilde{\varphi}$ and $\tilde{\psi}$. Let us stress that $H_{0b}(\varphi)$ and $H_{0b}(\tilde{\varphi})$ are the same functions of their arguments, but they are different operators because $\tilde{\varphi} \neq \varphi$.

Till now Hamiltonians had been represented as functions of fields. We can introduce creation-destruction operators instead of fields using the well-known expansions of φ, π (in terms of the operators a, $a^\dagger$) and of ψ, $\psi^\dagger$ (in terms of b, d, $b^\dagger$, $d^\dagger$), e.g. see [8]. The fields $\tilde{\varphi}$, $\tilde{\psi}$ have the same expansions in terms of $\tilde{a}$, $\tilde{a}^\dagger$ and $\tilde{b}$, $\tilde{d}$, $\tilde{b}^\dagger$, $\tilde{d}^\dagger$ respectively:

$$\tilde{a}(k) = \exp(-iS)a(k)\exp(iS)\,, \tag{23}$$

etc. All eigenvectors of $H_{0b}(\tilde{\varphi})$ (being simultaneously H eigenvectors) can be written as

$$\tilde{\Omega}, \quad \tilde{a}^\dagger(k)\tilde{\Omega}, \quad \forall k, \tag{24}$$

$$\tilde{a}^\dagger(k_1)\tilde{a}^\dagger(k_2)\tilde{\Omega}, \quad \forall k_1, k_2, \tag{25}$$

etc. Here $\tilde{\Omega}$ is no-boson state, namely $\tilde{a}(k)\tilde{\Omega} = 0$, $H_f\tilde{\Omega} = 0$.

The operators $\tilde{a}$, $\tilde{a}^\dagger$ satisfy the definition of "clothed" boson operators (see [5], sect. 2) because (24) are H eigenvectors. It was argued in [5] that just the eigenvectors (24) (and not the "bare" states $a^\dagger(k)\Omega$) are to describe the observed no-boson and one-boson states. The Eqs. (23) and (11) allow one to find $\tilde{a}^\dagger$ as functions of "bare" creation-destruction operators a, $a^\dagger$, b, d, $b^\dagger$, $d^\dagger$.

However the transformed fermion operators $\tilde{b} = \exp(-iS)b\exp(iS)$, etc. are not "clothed". Indeed, the states $\tilde{b}^\dagger(p)\tilde{\Omega}$ are not H eigenvectors: $V_{ff}(\tilde{\psi})$ (see Eq. (19)) contains the terms of the kind $\tilde{b}^\dagger\tilde{d}^\dagger\tilde{b}^\dagger b$ which transform $\tilde{b}^\dagger\Omega$ in the state "one fermion plus fermion-antifermion pair". Therefore, $H\tilde{b}^\dagger\tilde{\Omega} = H_f\tilde{b}^\dagger\tilde{\Omega}$ is not proportional to $\tilde{b}^\dagger\tilde{\Omega}$. The "clothed" fermion operators can be found by means of sequential unitary transformations of the operator H_f described in [5], sect. 2. In particular, they take away from the transformed H_f the above terms $\tilde{b}^\dagger\tilde{d}^\dagger\tilde{b}^\dagger b$. The only important point for our purpose is that solely fermion part $H_f(\tilde{\psi})$ of the total Hamiltonian $H(\tilde{\varphi}, \tilde{\psi})$ must be transformed and the obtained "clothed" fermion operator b_c, d_c, $b_c^\dagger$, $d_c^\dagger$ will be functions of the tilded ones $\tilde{b}$, $\tilde{d}$, $\tilde{b}^\dagger$, $\tilde{d}^\dagger$. Therefore, the absence of tilded boson-tilded fermion interaction in $H(\tilde{\varphi}, \tilde{\psi})$ means as well that the "clothed" (observable) bosons do not interact with "clothed" (observable) fermions.

5 "Clothed" bosons as particles of "dark matter"

I have supposed (see Introduction) that boson fields φ_a interact with the quark fields only and not with other known fields. As was demonstrated in the previous section the "clothed" bosons do not interact also with "clothed" quarks and among themselves. So, "clothed" bosons cannot be observable (detected) because measuring devices use some kind of interaction.

Let us make here a reservation. If our bosons have nonzero masses μ_a, see Eq. (2), then one must suppose that they interact gravitationally with other non-zero-mass particles. This would allow one to observe macroamounts of the bosons but not individual ones. So, our "clothed" bosons would satisfy the definition of the the particles of cosmological "dark matter" [9].

6 Concluding remarks

Model bosonic fields φ_a manifest themselves only in generating fermion-fermion interaction V_{ff}, see Eqs. (19) and (6). The interaction depends upon arbitrary elements g_{ij}^a of Hermitian matrices g^a as well as upon arbitrary functions $G_a(x-y)$. Note that the matrices g^a entering Eq. (3) can be juxtaposed to the Gell-Mann matrices λ_{ij}^a which are generators of color SU(3) group of QCD. However, unlike λ^a only mutually commuting g^a, $a = 1, 2, \ldots$ must be used in order that the model be partially solvable and "clothed" bosons unobservable. For example, one may take the unit matrix as g^1 and an arbitrary Hermitian 3×3

matrix for g^2 e.g. such that

$$\sum_{i,j} \psi_i^\dagger g_{ij}^2 \psi_j = \psi_1^\dagger \psi_2 + \psi_1^\dagger \psi_3 + \psi_2^\dagger \psi_3 + H.c.$$

(the flavor index is omitted).

Further investigations are to show if the suggested model may be used instead of QCD as a theory of strong interaction of the known physical quarks (remind that in this paper the term "quark" has been used to designate a fermion of the model). It seems that some experimental facts (e.g. quark confinement potential) can be described by the adjustment of g_{ij}^a and $G_a(x-y)$. However, the main task should be to strive for "falsification" of the model [10], looking for such model consequences which would not be compatible definitely with experiment.

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Склеивание фермионов посредством ненаблюдаемых скалярных бозонов

Предложена теория нескольких фермионных полей, взаимодействующих с несколькими скалярными полями. Модель нелокальная и частично решаемая при произвольном выборе ее формфакторов. Показано, что последние определяют фермион-фермионные потенциалы. Наблюдаемые («одетые») бозоны модели имеют свойства космологической «темной материи» и не могут детектироваться в земных лабораториях. Это является точным результатом модели, в то время как в КХД ненаблюдаемость изолированных глюонов объясняется посредством дополнительной гипотезы о конфайнменте цвета.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

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Fermion-Fermion Gluing by Means of Unobservable Scalar Bosons

A theory of several fermion fields interacting with several scalar fields is suggested. The model is nonlocal and is partially solvable under arbitrary choice of its formfactors. The latter are shown to determine the resulting fermion-fermion interaction. The observable («clothed») bosons of the model have the properties of the cosmological «dark matter» and cannot be detected in earth laboratories. This is the exact result of the model. Meanwhile in QCD the invisibility of isolated gluons is explained by using additional hypothesis of color confinement.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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