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GLOBAL DEPENDENCE OF OPTICAL POTENTIAL PARAMETERS FOR ALPHA-PARTICLES WITH ENERGIES UP TO 80 MeV

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I. INTRODUCTION

Macroscopic and semi-microscopic analyses of angular distributions of differential cross sections (ADDS) for α-particle scatterings are the important source of information about matter distribution in nuclides and properties of the potential of nucleus-nucleus interaction.

Optical potential (OP) parameters of α-particle interaction with nuclei at low and intermediate energies received from the analysis of ADDS of elastic scattering within the framework of optical model (OM) are exposed to considerable ambiguities and require reliable estimations [1, 2]. It is very important for receiving of detailed information about nuclear structure (parameters of deformation lengths, ratio of neutron and proton components, etc.) in the framework of up-to-date potential approaches (OM, methods of coupled channels and distorted waves). Global optical potential on the basis of data on elastic scattering of ^{90}Zr has been received in the work [3] for α-particles at energy above 80 MeV. At that the recommendations about parameter geometry OP and its dependencies upon energy received by Put and Paans has been used [4]. Up to present time there were no successful global potentials because of increase of strong absorption with energy up to 80 MeV. Effects of strong absorption lead to two types of OP ambiguities. It is discrete ambiguities often shown through depth value of real part of potential corresponding to changes necessary for conditions of phase equivalence [5]. Continuous ambiguities forming potential family characterized by equal behavior near nuclear surface exists for each of discrete parameter number. Pointed ambiguities of OP need a search of physical situations that could restrict them.

Differential and total reaction cross sections with energy above 80 MeV for nuclei from A=12 – 124 have been calculated and their systematic analysis has been carried out firstly within the framework of the double folding model [6] on the basis of full M3Y-effective interaction and nucleon densities calculated for all collided nuclei by the method of density functional.

Inclusion in analysis together with ADDS of elastic scattering and values of total reaction cross sections allows limiting ambiguities of OP parameters since data on differential and total reaction cross sections are the main nuclear values received from

OM. Theoretical model pretended to full description of data must render both experimental ADDS and total reaction cross sections with the same number of entering parameters. Nowadays there are not many works where analysis has two these values simultaneously in one model. Choice of optimal parameters of macroscopic OP and folding model [6] further allows receiving reliable information about structure of excited state of investigated nuclear.

This work firstly proposes the global (energy and mass) dependence of parameters of macroscopic OM and semi-microscopic folding model (SFM) for α-particles in weakly studied energy range.

2. EXPERIMENTAL DATA FOR GLOBAL ANALYSIS

We selected and compiled the initial experimental material including results of our work [7-13] and literature sources for building of desired global dependence of potential parameters of α-particle interaction.

The main characteristics of experimental conditions when ADDS have been received are shown in table 1. Information published in corresponding works [4, 7-10, 11] are shown for literature data.

Experimental ADDS from works [10,12,13] including into analysis have been received at isochronous cyclotron U-150M using the registration system and identification of reaction products on the complex base CAMAC-PC/AT [14].

In works [10,12,13] the systematic error in scattering ADDS is connected mainly with uncertainties: target thickness (4-6%), solid angle of spectrometer (1%), calibration of current integrator and did not exceed 10%. Statistic error of analyzed data comprises 1-5% and only in some cases (in minimums of ADDS at large angles) reaches 6-15%. All used targets produced by the method of thermal evaporation are self-bearing. Target thickness has been defined according to α-particle energy loss of radioactive source ^{241}Am-^{243}Am-^{244}Cm and ^{239}Pu. Errors of absolute values of cross sections in scattering ADDS in published data [4, 7-10, 11] comprise 5-10%.

Experimental values of total reaction cross sections σ_R which errors comprised 4-7% in investigated energy are taken from work [10] and other sources.

Table 1. Main characteristics of the experiment on elastic scattering of α-particles

Nucleus	E_α, MeV	Target		Angular diapason (deg., c.m.)	Reference
		thickness (mg/cm^2)	Composition (%)		
For energy dependence					
^{90}Zr	21.0	0. 730	97.0	48-177	[7]
^{90}Zr	23.4	0. 730	97.0	48-177	[7]
^{90}Zr	25.0	0. 730	97.0	48-177	[7]
^{90}Zr	31.0	1.0	98.0	10-98	[8]
^{90}Zr	35.4 35.4	0.520 0.855	97.65 97.67	10-100 6-47	[9]
^{90}Zr	40.0±0.2 40.0±0.5	0.84-2.47 2.13±0.08	97.6 95.0	5-175 10-70	[4] [10]
^{90}Zr	50.1±0.5	2.13±0.08	95.0	15-80	[10]
^{90}Zr	59.1±0.3	0.84 –2.47	97.6	5-175	[4]
^{90}Zr	65.0	5.0-8.5	95.0	10-76	[11]
^{90}Zr	79.5±0.4	0.84 –2.47	97.6	5-175	[4]
^{90}Zr	99.5±0.5	0.84 –2.47	97.6	5-175	[4]
For mass A-dependence					
^{12}C	50.5±0.5	1.1-2.0	98.9	13-173	[12]
^{24}Mg	50.5±0.5	1.0-3.2	99.1	12-172	[12]
^{28}Si	50.5±0.5	0.59 –0.76	92.17	11-171	[12]
^{48}Ti	50.5±0.5	4.5±0.08	99.2	24-64	[12]
^{50}Ti	50.5±0.5	3.15±0.08	83.2	32-84	[13]
^{58}Ni	50.5±0.5	0.56	99.5	20-65	[12]
^{68}Zn	50.5±0.5	3.48±0.08	91.2	16-76	[12]
^{70}Zn	50.5±0.5	3.10±0.08	95.0	16-66	[12]
^{90}Zr	50.1±0.5	2.13±0.08	95.0	16-75	[10]
^{94}Zr	50.1±0.5	2.60±0.08	91.2	12-75	[10]
^{120}Sn	50.5±0.5	2.20±0.08	99.2	10-63	[12]
^{124}Sn	50.5±0.5	2.00±0.08	95.1	10-65	[12]

3. DATA ANALYSIS AND GLOBAL DEPENDENCE OF POTENTIAL PARAMETERS

3.1. MACROSCOPIC OPTICAL MODEL ANALYSIS

Complex optical potential $U(\vec{r})$ with Woods-Saxon form-factor for real and imaginary parts in the case of α-particles scattering has a standard form:

$$U(\vec{r}) = V(1+\exp\frac{r-R_V}{a_V})^{-1} + iW(1+\exp\frac{r-R_W}{a_W})^{-1} + V_C(r)\,. \quad (1)$$

In this formalities OP is characterized by six parameters: depths of real V and imaginary W of particles, their corresponding to radial parameters R_v and R_w and diffuseness parameters a_v and a_w at the nucleus periphery. $V_c(r)$ corresponds to Coulomb potential of uniformly charged sphere with radius $R_c=1.25\ A^{1/3}$ fm.

Experimental ADDS of elastic scattering of α-particles in present work were analyzed within the framework of deformed OM potential using code SPI-GENOA [15]. Potential parameters met to optimal correspondence of experimental and theoretical cross section values were found by minimization of the following value

$$\chi^2 = \frac{1}{N}\sum_{i=1}^{N}[\frac{\sigma^T(\theta_i)-\sigma^E(\theta_i)}{\Delta\sigma^E(\theta_i)}]^2\,, \quad (2)$$

where N- number of experimental points in angular distribution, σ^T and σ^E- calculated and measured value of differential cross section of scattering at angle of θ_i, and $\Delta\sigma^E$- value uncertainty $\sigma^E(\theta_i)$.

Optimal OP parameters were selected in such way, in order to reach the best agreement within the framework of joint analysis ADDS scattering and data on total reaction cross sections, description of "transparency effect" [7,16] for nuclei with A=90 and tendency of value change of volume integral J_R^P from real part [2].

In concluding part of the present work the optimal OP parameters received by us keeping above mentioned criterions were described using the following expressions for all six OP parameters:

$$V(A,Z,E_\alpha)=a_0+a_1ZA^{-1/3}+a_2E_\alpha\,,\quad W(A,Z,E_\alpha)=b_0+b_1A^{1/3}+b_2E_\alpha\,, \quad (3)$$

$$r_V= c_0+c_1A^{1/3}+c_2E_\alpha\,,\quad r_W= d_0+d_1A^{1/3}+d_2E_\alpha\,, \quad (4)$$

$$a_V= e_0+e_1A^{1/3}+e_2E_\alpha\,,\quad a_W=f_0+f_1A^{1/3}+f_2E_\alpha\,. \quad (5)$$

We can note that expressions (3) for depth dependence of real and imaginary part of OP are similar to the part which was received by Nolte and Machner [3] for α- particles with energy above 80 MeV and F.G. Perey [17] for nucleons. Term which is proportional to $Z/A^{1/3}$ was adopted by F.G. Perey [17] for effect calculation appeared due to Coulomb repulsion: average coulomb potential on nuclear surface is proportional to Z/R and, correspondingly, to the value $Z/A^{1/3}$. A linear dependence from nucleus radius and energy E_α of incident particles was adopted for the depth of imaginary part of potential in the case of α- particles.

3.1.1. Energy dependence

The analysis of 12 experimental ADDS of elastic scattering of α-particles with energies from 21 to 99.5 MeV at nucleus ^{90}Zr was carried out in order to build energy constituent of global dependence of parameters OP with energy below 80 MeV. Taking into consideration the fact, that global dependence of OP with energy above 80 MeV is built on the basis of data for ^{90}Zr in work [3], the choice of the investigation object is very successful what increases essentially the value of this work in which the community of approach is kept (and possibility of result unification) at global dependence building with energy below and above 80 MeV.

We found optimal values of OP for each ADDS at different energies using recommendations from [3,4] for search of energy dependence of OP parameters of α-nuclear interaction. We began the analysis of data family from 12 ADDS from energy of α-particles 99.5 – 79.5 MeV (above 80 MeV) in order to provide overlap in energy diapason with works [3,4]. The values recommended in [3] are adopted as starting parameters of OP. One can note that the less energy of incident α-particles the worse parameters OP from [3] describe experimental ADDS (depending upon E_α) with decrease of energy of incident α-particles (below 80 MeV).

It was found in work [4] that geometry of optical potential for ^{90}Zr is almost constant at α-particles energy above 80 MeV (effect of Put – Paans). However, it becomes reliant on E_α with more low energies. We have studied energy dependence of geometric parameters OP with energies of α-particles below 80 MeV. Because of discrete ambiguity of OP the one of main difficulties at building of such dependencies is mutual parameter influence on each other (V and R_v, W и R_w: increase of one parameter results in decrease of another). For liquidation of such ambiguities we used recommendations of Hodgson and others [18].

We applied the method of two-phase adjustment of OP parameters: firstly we varied depth values V, W and diffuseness parameters a_v, a_w, and then values of OP radii r_v , r_w. A number of partial waves we took into account reached 35-75 depending on mass of nucleus-target and energy of α-particles. Optimal values of OP parameters received in present work are shown in table 2. Then the independent selection of "physically" significant potentials was carried out: one of such possibilities is use as value criterion of volume integral from real part OP which sharply changes its values at transition from one family to another because of discrete ambiguity. It is set [2] that value of volume integral J_R^P , related to a number of interacted nucleon pares i.e. $J_R^P = J_R / 4A$ is value of the same order for different types of charged particles and there are potential families with the same value J_R^P ~(330-370) MeV•fm^3 for protons, deuterons, ^{3}He-ions and α-particles scattered at the same nuclei. Holding this point of view, we were choosing OP parameters according to change tendency J_R^P [4]: its value must change weakly with energy E_α.

Table 2. Optimal values of parameters OP for α-particles at nucleus ^{90}Zr .

E_α, MeV	V, MeV	r_v , fm	a_v , fm	W, MeV	r_w , fm	a_w , MeV	χ^2/N	$J_R/4A$, MeV•fm^3
21	272.52	1.245	0.582	15.426	1.570	0.316	3.74	609
23.4	161.08	1,245	0.649	11.458	1.570	0.452	3.82	368
25	159.48	1.245	0.648	10.884	1.569	0.583	2.80	365
31	196.86	1,248	0.707	22.272	1.571	0.501	3.41	464
35.4	144.97	1.246	0.757	19.361	1.574	0.577	5.26	347
40	145.16	1.245	0.729	12.983	1.624	0.819	24.48	343
50.1	144.99	1,245	0.760	14.504	1,567	0.583	28.06	347
59.1	144.13	1.245	0.805	18.550	1.570	0.510	6.49	351
65	143.73	1.247	0.827	22.231	1.572	0.632	5.92	355
79.5	140.58	1.245	0.857	19.437	1.570	0.692	8.25	350
99.5	134.31	1.244	0.832	20.714	1.570	0.636	8.82	330

Fig. 1 presents the analysis results of ADDS of α-particle elastic scattering for various energies at ^{90}Zr at optimal values of obtained OP parameters. One can see good correspondence with experimental data in wide angular range.

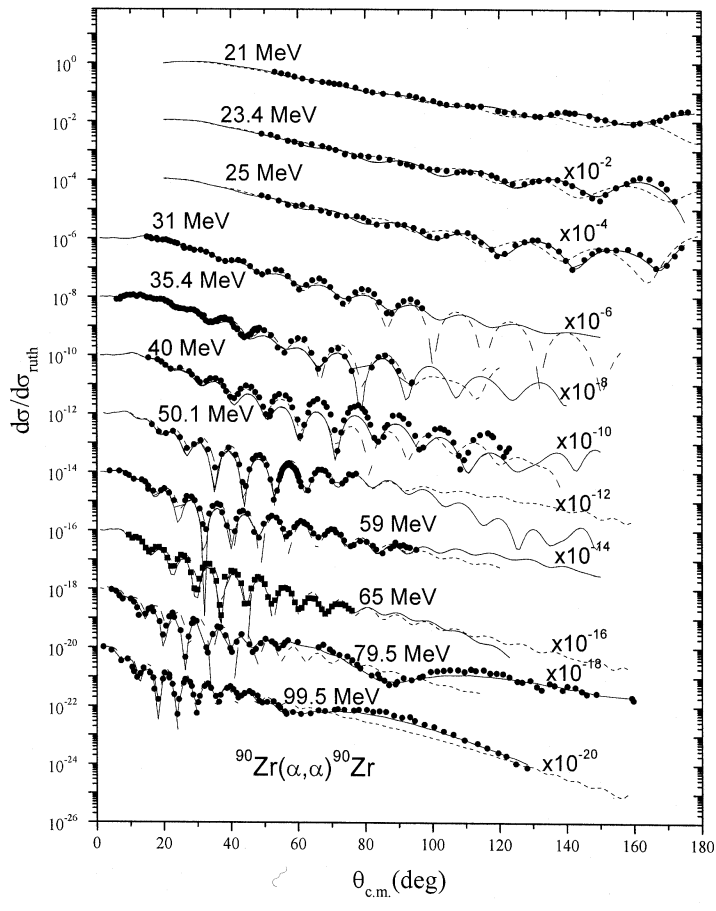

Fig. 1. Angular distributions of differential cross sections of elastic scattering for α–particles at ^{90}Zr. Points – experimental data, solid lines – macroscopic OM, dash lines – semi-microscopic folding-model.

We also tested other variants of adjustments (namely: firstly variation of V and W depth at other fixed parameters; secondly, changing radial parameters with fixed depths and diffusenesses and then changing diffusenesses with fixation of the rest OP parameters and, finally, leaving all 6 OP parameters simultaneously). But as a result of these actions there were large scattering values and it was not possible to obtain any dependencies, so the method of adjustments mentioned above is more acceptable.

We used values of total reaction cross sections σ_R with energies of α-particles up to 80 MeV as an additional criterion for selection of optimal OP parameters that are calculated using the code SPI-GENOA [15]. The figure 2 shows that received calculated values σ_R at nucleus ^{90}Zr at optimal and global parameters OP reproduce the measured values of total reaction cross section and tendency of their change with increase of α-particles energy what confirms the reality of theoretical cross sections calculated using the approach described above.

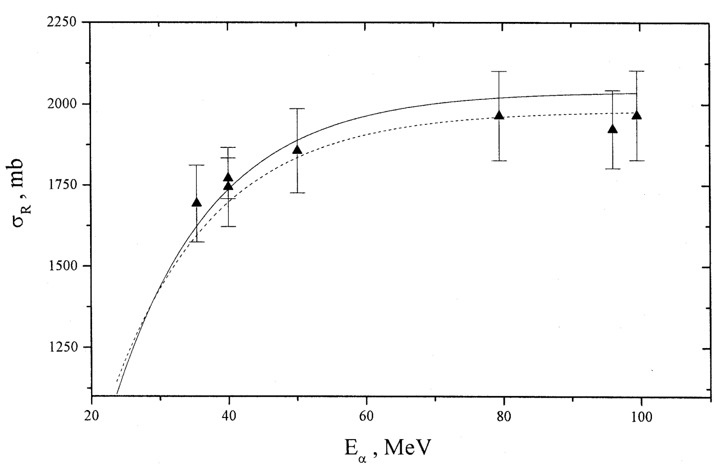

Fig. 2. Total reaction cross sections for α-particles at nucleus ^{90}Zr. Experimental data (open triangles) are taken from [10] and other published sources. Theoretical values of total reaction cross sections are shown for macroscopic OP by solid line, for semi-microscopic model by dash line.

We investigated possibilities for "transparency effect" description for nuclei with A=90 [7,16] using the received global parameters. A pronounced minimum at energy ~ 23 MeV [7,16] is observed in excitation function at scattering of α-particles at nucleus ^{90}Zr at angle of 176^0 in c.m. Figure 3 presents a satisfactory description of indicated effect on an example of ^{90}Zr using received global OP parameters.

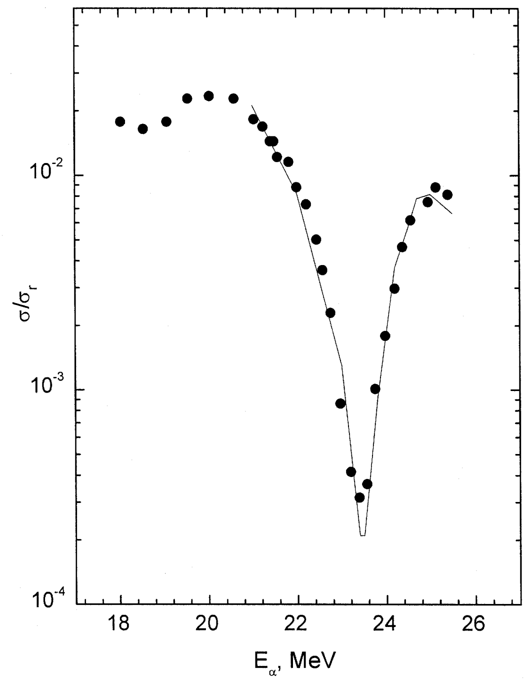

Fig.3. "Transparency" effect at ^{90}Zr with energies of α-particles 18 - 26 MeV. Experimental data - points [7] and theoretical values on macroscopic OM- solid curve.

After selection by the method of the least squares the linear dependencies of OP parameters from E_α (firm lines at fig.4) were received in the following form:

$$V=160.229-0.260\bullet E_\alpha,\ W=8.109+0.138\bullet E_\alpha, \quad (6)$$

$$a_v=0.621+0.0026\bullet E_\alpha,\ a_w=0.544+0.0012\bullet E_\alpha, \quad (7)$$

where values V, W, E_α are in MeV, and diffuseness parameters a_v and a_w – in fm. Radial parameters r_v and r_w showed weak change with energy E_α: r_v =1.2456±0.0004 fm и r_w =1.5704±0.0018 fm. For comparison dash lines on Fig. 4 present global dependencies at energies E_α > 80 MeV from [3].

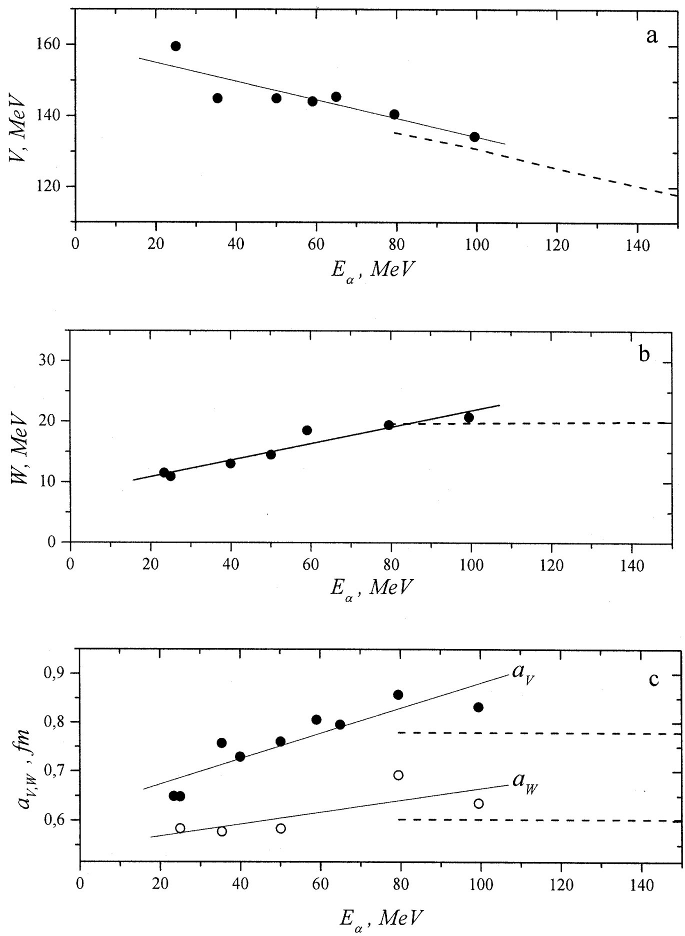

Fig. 4. Energy dependence of macroscopic OP-parameters: *a* – depth of real part; *b* – depth of imaginary part; *c* - radii r_v and r_w; *d* - diffusenesses a_v and a_w. Solid lines – global dependencies received in the present work; dash lines – in [3].

3.1.2. *Mass dependence*

There were studied mass dependence of OP parameters for α-particles with energy ~ 50 MeV at nuclei ^{12}C, ^{24}Mg, ^{28}Si, $^{48,50}Ti$, ^{58}Ni, $^{68,70}Zn$, $^{90,94}Zr$, $^{120,124}Sn$ in the energy range up to 80 MeV on the basis of experimental data [10, 12, 13], received in the Institute of Nuclear Physics NNC RK.

Used algorithm of the OP parameters adjustment was the same as at investigation of energy dependence. The received values of optimal parameters of macroscopic OP are presented in table 3.

Table 3. Optimal values of OP parameters for different nuclei at $E_\alpha = 50.5$ MeV.

Nucleus	*V*, *MeV*	r_V, *fm*	a_V, *fm*	*W*, *MeV*	r_W, *fm*	a_W, *fm*	χ^2/N	$J_r/4A$, MeV•fm^3
^{12}C	108.40	1.228	0.747	20.064	1.551	0.688	28.40	357
^{24}Mg	111.85	1.256	0.776	24.138	1.578	0.591	25.94	330
^{28}Si	118.80	1.247	0.808	19.018	1.570	0.716	14.81	350
^{48}Ti	123.62	1.246	0.806	24.925	1.571	0.559	14.16	329
^{50}Ti	128.24	1.245	0.766	18.804	1.570	0.643	3.82	330
^{58}Ni	140.78	1.247	0.760	17.492	1.571	0.650	4.12	356
^{68}Zn	163.73	1.246	0.776	29.685	1.572	0.509	16.42	408
^{70}Zn	130.97	1.245	0.824	23.106	1.580	0.601	17.61	332
^{90}Zr	144.13	1.245	0.805	18.550	1.570	0.510	6.49	351
^{94}Zr	135.93	1.246	0.782	16.623	1.573	0.628	8.30	327
^{120}Sn	147.88	1.245	0.750	15.841	1.570	0.580	2.66	343
^{124}Sn	142.79	1.245	0.763	15.144	1.570	0.658	2.65	331

Fig. 5 shows satisfactory description of experimental ADDS of elastic scattering for α-particles with energy 50.5 MeV at nuclei with A=12-124. It is seen from the value analysis χ^2/N (table 3) that they increase with decrease of mass number *A*. This evidences that the adopted geometry or dependence of OP parameters is worse with larger a target nucleus differs in mass from ^{90}Zr, in particular at nucleus ^{12}C. A number of experimental and theoretic investigations of scatterings at light nuclei [19] show that there is an anomalous growth of cross sections at the large angles in ADDS of elastic scattering at nucleus ^{12}C in investigated energy range. This effect cannot be explained within the framework of standard OM as it is caused by other mechanisms including mechanism of alpha-cluster exchange between incident particle and target nucleus. So, during the analysis of experimental ADDS we paid a special attention to quality of data analysis for angles up to 90^0, where the potential scattering is the main mechanism.

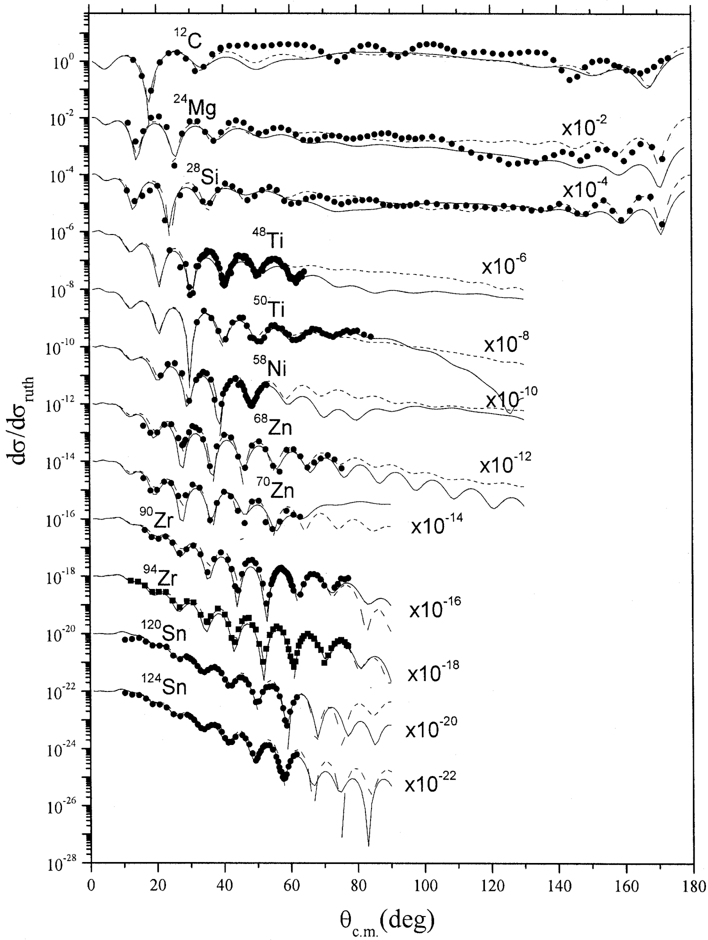

Fig. 5. Angular distributions of differential cross-sections of elastic scattering for α–particles with $E_\alpha \sim 50$ MeV at nuclei A=12-124. Points – experimental data, solid lines – macroscopic OM, dash lines – semi-microscopic folding-model.

Changing tendency of experimental values on total reaction cross sections σ_R from mass number A in investigated field of energies of α-particles is well described by the received number of parameters for macroscopic OP (fig.6). Since no measured data on σ_R at E_α ~ 50 MeV we have used for testing its mass dependence the experimental values σ_R obtained in [10] and other published sources at energies 40 MeV [20] and 69.6 MeV [21]. Fig. 6 shows that the theoretical dependencies of σ_R are in satisfactory correspondence with the mentioned experimental values.

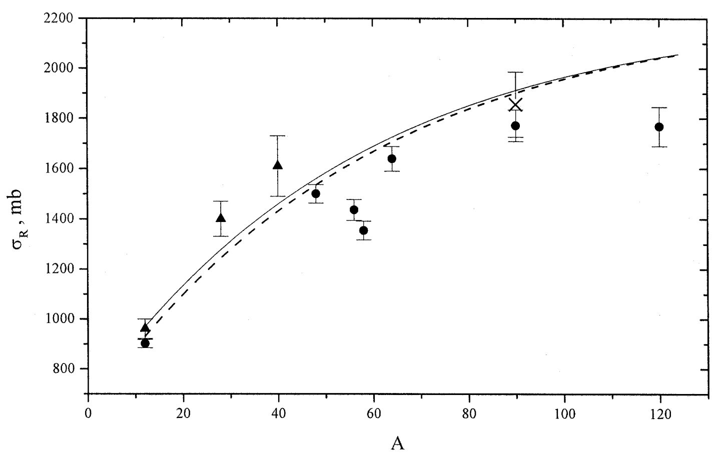

Fig. 6. Total reaction cross sections for α–particles with E_α ~ 50 MeV at nuclei A=12-124. Experimental data: crosses denote sections for 50.1 MeV [10], circles – 40 MeV [20], triangles – 69.6 MeV [21]. Theoretical values of total reaction cross sections are shown for macroscopic OP by the solid line, for semi-microscopic model – by the dash line.

Use as a selection criterion of OP value of volume integral J_R^P showed (table3) that its value changes weakly for different mass numbers at E_α ~ 50 MeV being in the interval J_R^P ~ (330-360) MeV•fm^3.

The received (based on data from table 3) global mass dependence of OP parameters at $E_{\alpha} = 50.5$ MeV is approximated by linear dependence (fig. 7). They can be analytically presented in the following form:

$$V=93.838+5.177\bullet Z\bullet A^{-1/3},\ W=28.122-2.202\bullet A^{1/3}, \quad (8)$$

$$r_v=1.241+0.00066\bullet A^{1/3},\ r_w=1.563+0.0011\bullet A^{1/3}, \quad (9)$$

$$a_v=0.840-0.0141\bullet A^{1/3},\ a_w=0.705-0.0222\bullet A^{1/3}. \quad (10)$$

3.2. GLOBAL SEMI-MICROSCOPIC DATA ANALYSIS

Semi-microscopic optical potential $U(R)$ is built within the framework of folding-model on the basis of full M3Y-effective interaction and nucleon densities calculated under the method of density matrix functional [6]. Interaction potential of two collided nuclei in the first order on effective forces can be presented as:

$$U(R) = U^E(R) + U^D(R), \quad (11)$$

where $U^D(R)$ – "direct" potential of double folding model [1]:

$$U^D(R) = \iint \rho^{(1)}(r_1) V^D(s)\ \rho^{(2)}(r_2) dr_1\, dr_2 . \quad (12)$$

The term $V^D(s)$ in expression (11) is a direct component of effective interaction $(s = r_2 - r_1 + R)$, $\rho^{(i)}(r_i)$ are densities of collided nuclei (i =1, 2). Such calculation scheme of "exchange" potential $U^E(R)$ is described in [6]. The effects of one-nuclide exchange, which are described in formality of density matrix [22] give the main contribution in it:

$$U^{EX}(R) = \iint \rho^{(1)}(r_1, r_1 + s) V_{EX}(s)\ \rho^{(2)}(r_2, r_2 - s)\ exp(ik(R)\, s/\eta)\ dr_1\, dr_2 , \quad (13)$$

where $V_{EX}(s)$ – exchange part of effective nucleon-nucleon forces, $\rho^{(i)}(r, r')$, $(i=1, 2)$ – density matrix of collided nuclei with mass numbers A_1 and A_2, $k(R)$ – local impulse of relative movement of nuclei determined by the ratio

$$k^2(R) = (2m\eta/h^2)[E - U(R) - V_c(R)]. \quad (14)$$

Here $\eta = A_1 A_2 /(A_1 + A_2)$, $s = r_2 - r_1 + R$, E – energy in the center-of-mass system of and $V_c(R)$ – Coulomb potential. Thus, net potential depends upon energy because of effects of one-nucleon exchange. Initial data for calculation of potentials are effective nucleon-nucleon forces and proton and neutron densities of collided nuclei.

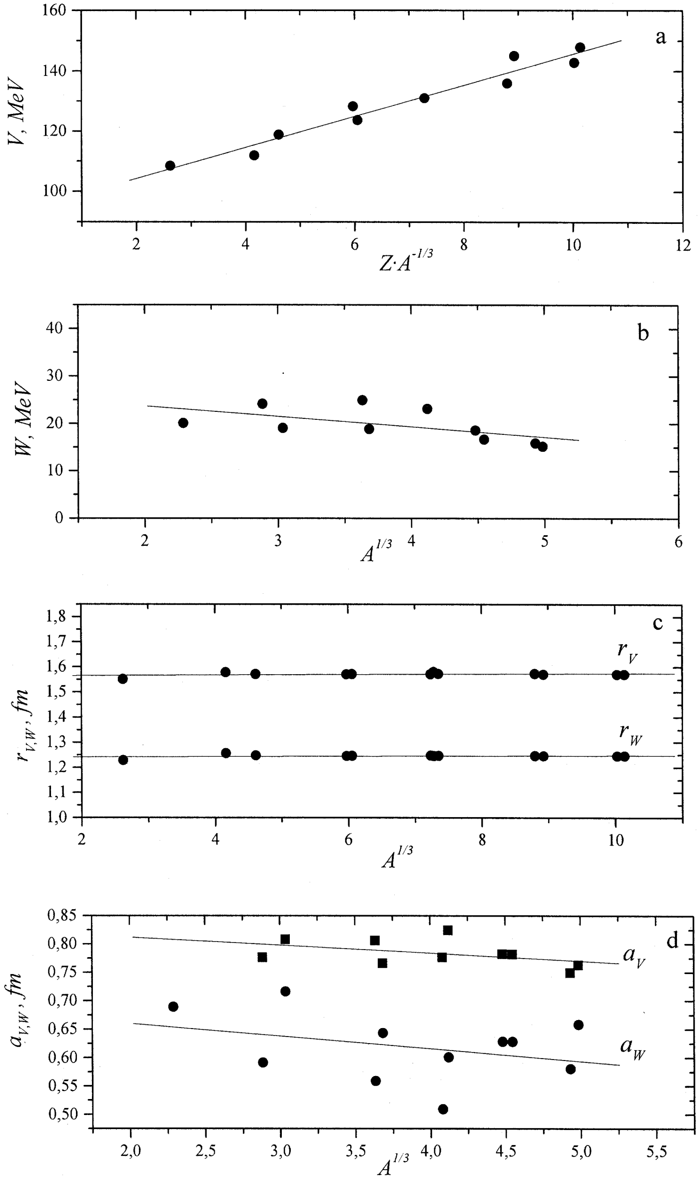

Fig. 7. Mass dependence of parameters OP: *a* – depth of real part; *b* – depth of imaginary part; *c* - radiuses r_v and r_w ; *d* - diffusenesses a_v and a_w. Firm lines – global dependencies received in the present work.

Full optical potential SFM besides real part should include the imaginary part responsible for incident particle absorption in inelastic channels. In our case we built potential of absorption depended on calculated real part in the form [23]:

$$W(R) = i\,[N_w U(R) - \varphi_w\ R\, dU(R)/dR]\,, \qquad (15)$$

where $U(R)$ – potential of double folding (8) and N_w and φ_w – parameters that characterize volume and surface parts of absorption potential, correspondingly. A surface term imitating the contribution of dynamic polarized potential is introduced in the real part of the potential [24]. Full optical potential SFM has the following form:

$$U_t(R) = U(R) - \varphi_v\ dU(R)/dR + i\,[N_w U(R) - \varphi_w dU(R)/dR\,], \qquad (16)$$

where φ_v , N_w, φ_w are varying parameters.

At calculation of ADDS of inelastic scattering the form-factor of inelastic transition has a form $\varphi_L \dfrac{dU_t(R)}{dR}$ [25].

In frameworks of SFM we have carried out global (energy and mass) analysis of the same experimental data (as well as at investigation a macroscopic OM) in explored region of α-particles energies for the nuclei with A=12-124.

Theoretical cross sections of elastic scattering are calculated under the modified version of the program ECIS-88 [26] (ECIS-PM), in which semimicroscopic potentials are constructed as in the formula (16). The adjustment theoretical ADDS of elastic scattering and values σ_R to experimental data was carried out by a variation of parameters φ_v , N_w, φ_w. One should mention that at the joint analysis of differential and total reaction cross sections the SFM parameters are determined uniquely.

In the present work the nucleon densities for α-particles were calculated in Gaussian representation with root-mean-square radius 1.57 fm [3], and for nucleus - targets the densities were calculated by the density-functional method [22].

In fig. 1 and 2 (dashed lines) the results of calculations ADDS for elastic scattering and total reaction cross sections on SFM on the ^{90}Zr nucleus at energies of α-particles 21- 99.5 MeV are presented. Good agreement of theoretical calculations with experimental data is obtained. The optimum values of the obtained SFM parameters are presented in the table 4; errors are 1%. Tendencies of the SFM parameters change with energy E_α are presented in fig. 8.

Table 4. Parameters of semimicroscopic potential of α-particles interaction, volume integrals J_V in (•10^3 MeV•fm^3) and root-mean-square radii r_{sfm} in fm for folding potentials on ^{90}Zr at different energies of α-particles.

E_α, MeV	φ_v	N_W	φ_w	J_V	$<r^2_{sfm}>^{1/2}$
21	0.010	0.10	0.010	-129.3	4.989
23.4	0.020	0.10	0.010	-128.7	4.990
25	0.020	0.10	0.010	-128.4	4.990
31	0.017	0.18	0	-127.3	4.990
35.4	0.050	0.10	0	-126.5	4.991
40	0.045	0.12	0.012	-125.7	4.992
50.1	0.052	0.11	0.010	-123.9	4.993
59.1	0	0.25	0.015	-122.3	4.994
65	0	0.18	0.025	-121.3	4.994
79.5	0	0.26	0.030	-118.9	4.996
99.5	-0.020	0.35	0.032	-115.7	4.999

From tab. 4 one can see that φ_v parameter increases with energy increase of scattering α-particles in energy range up to 50 MeV, further its value is equal to 0 and, further, becomes negative at energy 99.5 MeV. The N_W parameter increase slowly with energy. The φ_w parameter decreases to 0 at energy 31 and 34.5 MeV, and further increases for 1.6 times at energies 40-99.5 MeV. Due to non-linearity of φ_v-parameter we have described it with Gaussian dependence from E_α (see fig. 8a):

$$\varphi_v = -0.0108 + 0.0612 \bullet exp(-0.003 \bullet (E_\alpha - 41.485)^2) \ . \qquad (17)$$

Analytical approximation of N_W and φ_w parameters by the linear dependence (fig. 8 b,c) from energy E_α by the least square method is in the following form:

$$N_W = 0.0234 + 0.0030 \bullet E_\alpha \ ; \quad \varphi_w = -0.0036 + 0.0004 \bullet E_\alpha \ . \qquad (18)$$

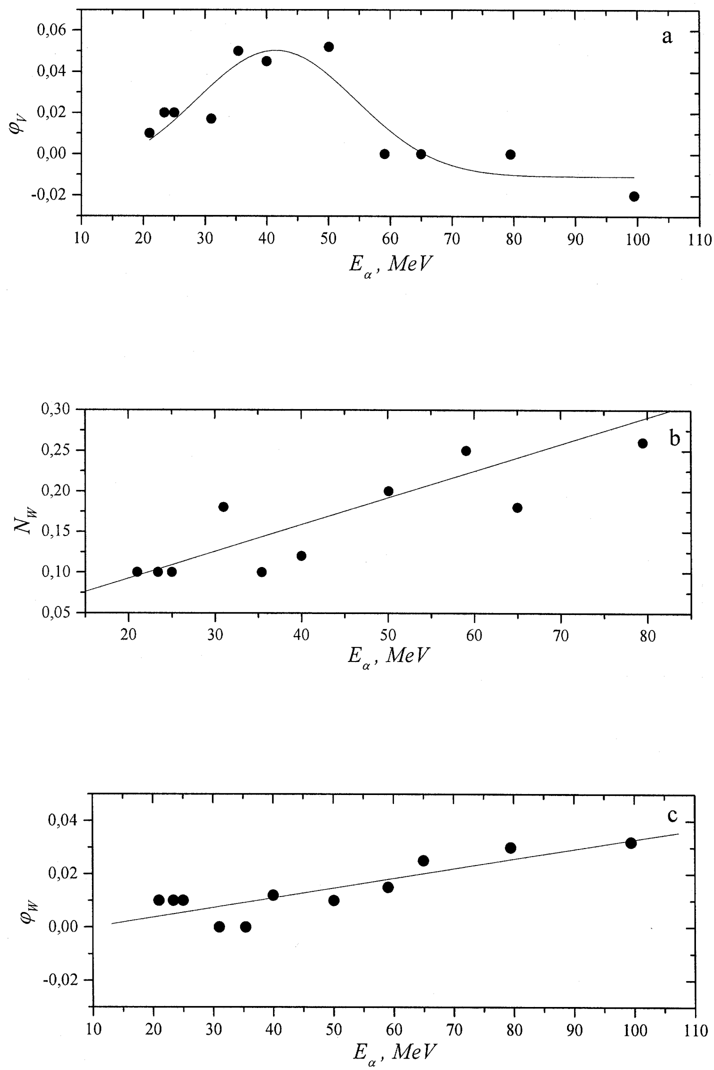

Fig. 8. Energy dependence for parameters of SFM potential: points - optimum values, solid lines – approximation a) by the Gaussian dependence for the parameter φ_v ; b), c) by a linear dependence for the parameters N_W and φ_w .

From table 4 one can see that the volume integral J_v is decreases on an absolute value with increase of E_α at fixed A. The magnitudes of root-mean-square radii (tab. 4) of folding-potentials with increase of α-particles energy for given A =90 vary insignificantly.

At examination of mass dependence of parameters SFM for α-particles with energy 50.5 MeV a good agreement between theoretical and experimental ADDS of elastic scattering and total reaction cross sections on the nuclei with A=12-124 (fig. 5-6) is obtained. It once again confirms reliability of the theoretical sections calculated using discussed above semimicroscopic approach. From fig. 5 it is possible to see, that the more light nuclei - target, the theoretical sections to experimental ADDS are worse adjusted. Thus, in a region of angles up to 60 degrees the theoretical sections, as a rule, well describe the experimental data. We have revealed that the values of adjusted SFM parameters in the angular range to 30 deg. influence slightly to the theoretical curve since at small angles the section value, mainly, is determined by the contribution from the real part of the total full microscopic potential (16). In the angular diapason 30-60 deg. the experimental elastic scattering ADDS and values for total reaction cross-sections are described well only with adjustment of all SFM parameters. The discrepancy of theoretical and experimental ADDS at larger angles, in particular on the ^{12}C nucleus, apparently means that other reaction mechanisms begin to play an essential role.

In table 5 the root-mean-square radii for a density distribution of protons, neutrons and matter, and also their difference Δr_{np} for nuclei-targets are presented. It shows, that for the nuclei ^{12}C, ^{24}Mg and ^{28}Si the magnitude of densities distribution radii for protons exceed the same for neutron, and for the remaining nuclei (A=48 -124) – visa versa, except for the nucleus ^{58}Ni.

In the table 6 the integrated characteristics of 50.5 MeV α-particles interaction potentials for isotopes with A=12-124 in the frameworks SFM is presented: J_V - volume integrals, $<r^2_{sfm}>^{1/2}$ - root-mean-square radii of the folding-potentials.

Table 5. Root-mean-square radii (in fm) of neutron, proton and matter density distributions, and also the difference $\Delta r_{np}=\langle r_n^2\rangle^{1/2} - \langle r_p^2\rangle^{1/2}$ at E_α= 50.5 MeV.

Nucleus	$\langle r^2{}_n\rangle^{1/2}$ *fm*	$\langle r^2{}_p\rangle^{1/2}$ *fm*	$\langle r^2{}_m\rangle^{1/2}$ *fm*	Δr_{np} *fm*
^{12}C	2.39	2.41	2.40	-0.02
^{24}Mg	2.84	2.86	2.85	-0.02
^{28}Si	2.95	2.98	2.97	-0.03
^{48}Ti	3.41	3.37	3.39	0.04
^{50}Ti	3.45	3.38	3.42	0.07
^{58}Ni	3.67	3.67	3.67	0.00
^{68}Zn	3.87	3.81	3.84	0.06
^{70}Zn	3.97	3.86	3.93	0.11
^{90}Zr	4.26	4.19	4.23	0.07
^{94}Zr	4.37	4.24	4.31	0.13
^{120}Sn	4.71	4.59	4.66	0.12
^{124}Sn	4.77	4.61	4.70	0.16

From table 6 one can see, that the volume integral J_V increases by its absolute value with increase of mass number at fixed energy E_α of an incident α-particle. The magnitudes of root-mean-square radii (tab. 6) of folding-potentials also are increased with increase of A at energy E_α=50.5 MeV.

It is established, that between the SFM parameters there are no noticeable correlation and the submitted sets (table 6) of parameters for each of the nuclei–targets are optimal. Errors for SFM parameters are 1 %. Parameter φ_v influences phase shifts in ADDS to the right or to the left depending on its sign. The tendencies of the parameter φ_v modification with increase of a mass number of a nucleus - target are as follows (fig.9): for nuclei-targets with A = 12-50 values of this parameter are equal to zero, i.e. there is no phase shift. For nucleus-targets with A= 58-70 values of the parameter φ_v are negative, i.e. without introduction of the parameter φ_v the theoretical curve would be moved to the right from the experimental points. For moderate heavy nuclei (A=90-124) the magnitude φ_v is, on the contrary, positive and shifts the curve to the left. If observe the dependence of an imaginary part of a potential, one could conclude that the parameter N_w, gradually decreases with growth of mass number, and the value of parameter φ_W , first, grows with increase of A, reaches a maximum at A=70, and then monotonically decreases. Since the dependence of φ_v-parameter on mass number at fixed energy E_α =50.5 MeV is quite complex, fig.9a presents interpolation of its optimal values. Analytical approximation of the parameters N_w and φ_W by a linear dependence of mass number A (fig. 9 b,c) by the least square method has the following form:

$$N_W = 0.3469 - 0.0021 \bullet A \;;\quad \varphi_w = -0.01030 + 0.00001 \bullet A \,. \qquad (19)$$

Table 6. SFM parameters, volume integral J_V in ($\bullet 10^3$ MeV$\bullet$fm^3) and root-mean-square radii r_{sfm} in fm for folding-potentials for the nuclei with A=12-124 at E_α=50.5 MeV.

Nucleus	φ_v	N_W	φ_w	J_V	$<r^2_{sfm}>^{1/2}$
^{12}C	0	0.37	0	-17.36	3.45
^{24}Mg	0	0.27	0.012	-30.82	3.77
^{28}Si	0	0.26	0.010	-41.16	3.99
^{48}Ti	0	0.25	0	-60.40	4.27
^{50}Ti	0	0.26	0.021	-62.75	4.30
^{58}Ni	-0.011	0.22	0.012	-80.13	4.50
^{68}Zn	-0.015	0.21	0.022	-93.95	4.69
^{70}Zn	-0.010	0.24	0.023	-96.69	4.73
^{90}Zr	0.052	0.11	0.010	-123.9	4.99
^{94}Zr	0.053	0.13	0.010	-129.6	5.07
^{120}Sn	0.051	0.104	0.007	-165.2	5.39
^{124}Sn	0.051	0.13	0.007	-170.5	5.44

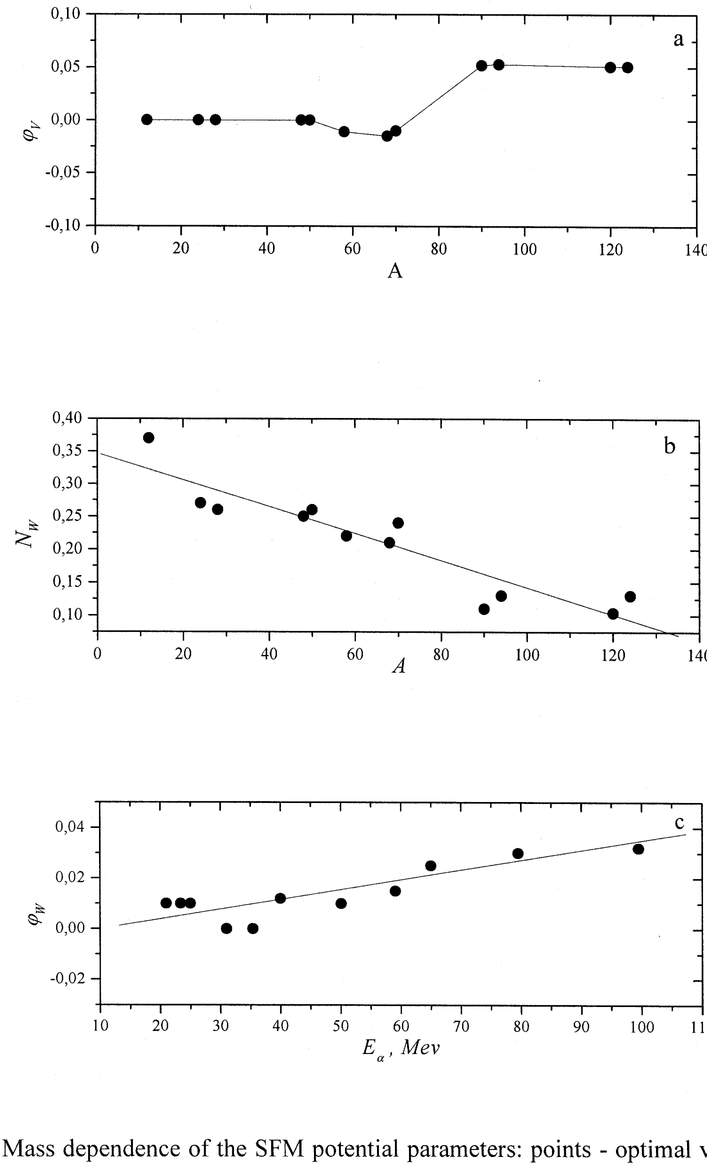

Fig.9. Mass dependence of the SFM potential parameters: points - optimal values, solid lines - approximation by a linear fit.

The absence of experimental ADDS of scattering and values of total reaction cross section on the nuclei with A>124 at E_α =50.5 MeV has not allowed to trace a course of dependence of SFM parameters from a mass number for nucleus – targets beyond this region.

4. RESULTS AND DISCUSSION

Within the framework of a macroscopic OM and semimicroscopic model a good description of differential and total reaction cross section, tendencies of a volume integral of a real part magnitude change, effect of "transparency" (for the nuclei with A=90) with α-particles involved in the energy range up to 80 MeV at the investigated nuclei were obtained. We have verified how the obtained global potential describes experimental ADDS of inelastic scattering for α-particles. At testing of global dependencies of the parameters for macroscopic OP and SFM approaches, quite satisfactory description of ADDS for inelastic scattering with excitation of low lying collective states of the nuclei is obtained. In fig. 10 on an example of the nucleus ^{124}Sn the analysis of ADDS for inelastic scattering of 50.5 MeV α-particles is carried out within the framework of a method of coupled channels (MCC) by ECIS-PM and distorted waves (DWBA) by the code DWUK4 [27].

From the performed analysis it was established, that the macroscopic OM and the SFM approach provide us with good enough shape and magnitude of differential and total cross sections at nuclei with A=12-124 in forward area of angles at all energies of α-particles. For large angles and light nuclei the correspondence was not reached, what can be probably explained by the contribution to elastic scattering of exchange mechanisms resulted in back scattering. With mass number A increase the correspondence of the theory and experiment is essentially improved.

On the basis of the systematic analysis of experimental data we have received energy and mass components of global dependencies of macroscopic OP for α-particles in the range of energies up to 80 MeV. After cross-linking and adjustment of dependencies (6)-(7) and (8) - (10) the following values for coefficients in the formulas (3) - (5) particular are obtained (table 7).

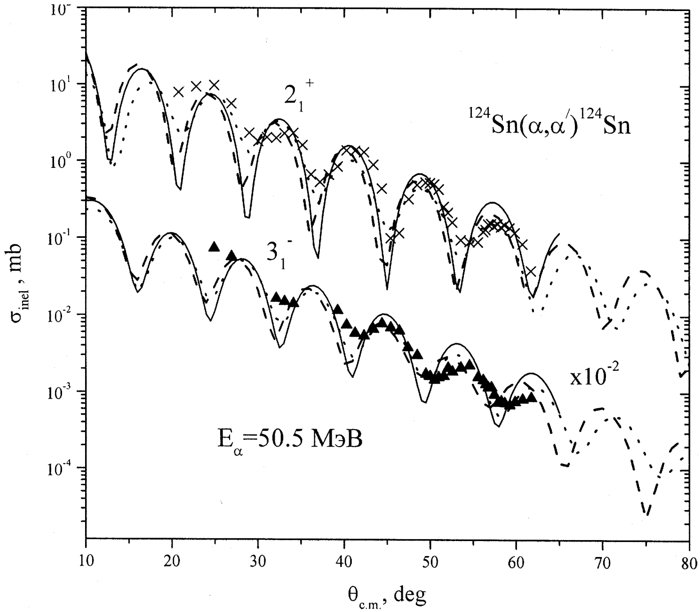

Fig. 10. Angular distribution of inelastic scattering of α-particles on the nuclei ^{124}Sn (points - experiment, solid curve – semimicroscopic folding model, dot line - macroscopic analysis on MCC for 2_1^+ and 3_1^- states, dash line - DWBA) at E_α = (50.5±0.5) MeV.

Table 7. Coefficients for global parameters of macroscopic OP for α-particles with energy up to 80 MeV.

a_0	110.500±5.041	b_0	19.578±4.527	c_0	1.241±0.006
a_1	5.177±0.398	b_1	-2.202±1.077	c_1	0.00066±0.00080
a_2	-0.260±0.064	b_2	0.138±0.018	c_2	0
d_0	1.564±0.007	e_0	0.697±0.051	f_0	0.643±0.096
d_1	0.00109±0.0009	e_1	-0.014±0.010	f_1	-0.022±0.021
d_2	0	e_2	0.0026±0.0005	f_2	0.0012±0.0006

5. CONCLUSION

In the present paper the global (energy and mass) dependencies of parameters for macroscopic OM and semimicroscopic folding-model for α-particles in the energy range up to 80 MeV for a wide class of the nuclei are obtained for the first time. For their build-up the joint analysis of a set of experimental data (ADDS of scattering, total reaction cross sections, tendency of the volume integral changes, etc.) is carried out, what will allow to restrict ambiguities in determination of parameters in the indicated energy region. One should mention, that identification and localization of the potential parameters for complex particles, in an acceptable approximation, can be carried out with using the above-stated criteria. Unity of our approach with the same used in [3, 4] and possibility to join together the data and reconstruct the global dependencies below and higher 80 MeV considerably strengthens value of the present work.

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REFERENCES

1. G.R. Satchler , *Direct Nuclear Reactions* (Oxford University Press, Oxford, 1983).
P.E. Hodson, Rep. of Progress in Physics. **34**, 756 (1971).

2. R.Barret, D.Jackson, Razmery i Structura jader (Naukova dumka, Kiev, 1981).
R.C. Barrett and D.F. Jackson, *Nuclear Sizes and Structure* (Oxford University Press, Oxford, 1977).

3. M. Nolte, H. Machner, J. Bojowald , Phys. Rev. C **36**, 1312 (1987).

4. L.W. Put and A.M.J. Paans, Nucl. Phys. A **291**, 93 (1977).

5. R.M.Drisko, G.R.Satchler, R.M.Bassel, Phys.Lett. **5**, 347 (1963).

6. O.M. Knyazkov, I.N. Kukhtina, S.A. Fayans, Physics of Particles and Nuclei. **30**, 870 (1999).
O.M. Knyazkov, I.N. Kukhtina, S.A. Fayans, Physics of Particles and Nuclei. **28**, 1061 (1997).

7. M.Wit, J.Schiele, and K.A. Eberhard, Phys.Rev. C **12**, 1447 (1975).

8. E.J. Martens and A. M. Bernstein, Nucl. Phys. A **117**, 241 (1968).

9. D. Rychel, R. Gyufko, B. van Kruchten, *et al.*, Z. Phys. A **326**, 455 (1987).

B.J. Lund, N.P.T. Bateman, S. Utku, *et al.*, Phys. Rev. C **51**, 635 (1995).
10. A.Duisebaev, K.A.Kuterbekov, I.N.Kukhtina i dr., Preprint № P6-2001-223, JINR (Dubna 2001); (prinjata k pechati v zhurnal Yadernaja fizika, 2002).
11. C.R. Bingham, M.L. Halbert, R.H. Bassel, Phys. Rev. **148**, 1174 (1966).
12. N.Burtebaev, A. Duisebaev, G.N.Ivanov i dr., Preprint 88-01, INP (Alma-Ata, 1988);
N.N.Pavlova, C.J.Aisina, K.A.Kuterbekov i dr., Preprint INP (Alma-Ata, 1990);
K.A.Kuterbekov, C.J.Aisina, N.N.Pavlova, i dr., Preprint INP (Alma-Ata, 1991).
13 K.A.Kuterbekov, A. Duisebaev, N.Burtebaev, Izv. RAN. Ser. Fiz.59, 112 (1995)
14. A. Duisebaev, B.A. Duisebaev, K.M.Ismailov i dr., Izv. Ministerstva Obrazovanija i Nauki Respubliki Kazakhstan, NAN RK. Ser. Fiz.-mat., №2, 103 (2002).
15. F. Perey. SPI-GENOA an optical model search code (unpubl.). NBI100 version by B.S. Nillson. (1976).
16. F. Michel, G. Reidemeister, S. Ohkubo, International conference "Clustering Phenomena in Nuclear Physics", June 14-17, 2000, St.-Peterburg. 2000. P.30.
17. F.G. Perey, Phys. Rev. 131, 745 (1963).
18. P.E. Hodgson. Opticheskaja model. Moskva. Atomizdat, 1966, 232s.
J.S. Nodvik, D.S. Saxon, Phys. Rev. 117, 1539 (1960).
19. J.C. Correli, E. Bleuler and J. Tandem, Phys.Rev. 1959. V.116. P.1184.
I.B.Teplov, N.S.Zelenskaja, B.M.Lebedev i dr., Fiz.Elem.Chastits At.Yadra. 1977. T.8,Vyp.4. S.769-816 .
20. A.K. Ghaudhuri, D.N. Basu, B. Sinha, Nucl. Phys. A 439, 415 (1985).
21. S.A. Fayans, O.M. Knyazkov, I.N. Kuchtina et al., Phys. Lett. B357, 509 (1995).
22. D.B.Bolotov,O.M.Knyazkov, I.N. Kukhtina i dr., Yad. Fiz. 63,1631 (2000).
23. I. Tanihata, T. Kobayashi, O. Yamakawa et al., Phys. Lett. B 206, 592 (1988).
24. C.M. Perey and F.G. Perey, At.Data and Nucl.Data Tables 17, 1 (1976).
25. R.B. Firesrone, Table of Isotopes, Eighth Edition (A. Wiley-Interscience Publication JOHN WILEY & SONS, New York, 1999, Update).
26. A. Ingemarson, J. Nyberg, P.U. Renberg et al., Nucl.Phys. A 676, 3 (2000).
27. P.D. Kunz, Computer program DWUCK4 (unpubl.).
28. J. Raynal, ECIS-88 (unpubl).

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Кутербеков К. А. и др. E7-2002-220

Глобальная зависимость параметров оптического потенциала для альфа-частиц с энергией до 80 МэВ

Получены глобальные (энергетическая и массовая) зависимости оптического потенциала для α-частиц с энергией до 80 МэВ. Для описания макроскопического потенциала использовался формфактор Вудса–Саксона. Впервые были исследованы энергетические и массовые зависимости полумикроскопических параметров α-частиц. Хорошее описание упругого и неупругого дифференциального и полного сечений реакций для различных ядер, использующее данные глобальные параметры, было получено в пределах макроскопического и полумикроскопического подходов.

Работа выполнена в Лаборатории ядерных реакций им. Г. Н. Флерова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2002

Kuterbekov K. A. et al. E7-2002-220

Global Dependence of Optical Potential Parameters for Alpha-Particles with Energies up to 80 MeV

Global (energy and mass) dependences of optical potential for α-particles with energies up to 80 MeV have been received. A Woods–Saxon form factor for macroscopic potential has been used. Energy and mass dependences of the semi-microscopic α-particle potential parameters have been investigated for the first time. In general, a good description of elastic and inelastic differential and total reactions cross sections for different nuclei using the revealed global parameters has been received within the framework of macroscopic and semi-microscopic approaches.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

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