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**RADIATIVE CORRECTIONS
TO THE K_{e3}^{\pm} DECAY REVISED**

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1 Motivation

The K_{e3} decay is important since it is the cleanest way to measure the V_{us} matrix element of the CKM matrix. If one uses the current values for V_{ud} , V_{us} , and V_{ub} taken from the PDG then $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ misses unity by 2.2 standard deviations which contradicts the unitarity of the CKM matrix and might indicate physics beyond the Standard Model. The uncertainty brought to the above expression by V_{us} is about the same as uncertainty that comes from V_{ud} . Therefore reducing the error in the V_{us} matrix element would reduce substantially the error in the whole unitarity equation.

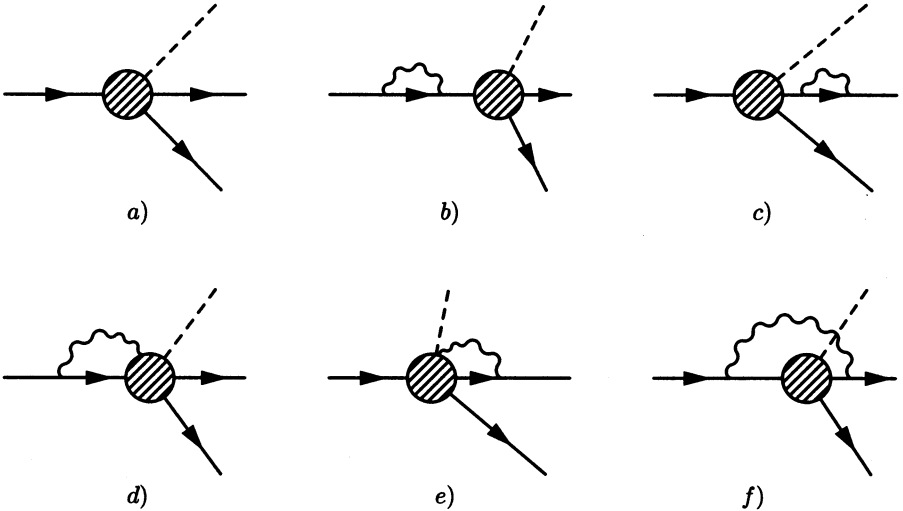


Figure 1: virtual photons

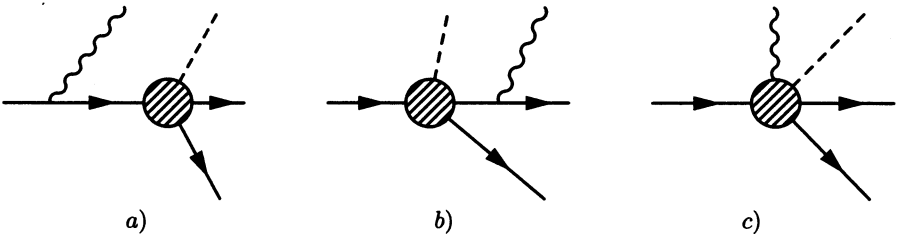


Figure 2: real photons

For corrections due to virtual photons see figure 1, for corrections due to real photons see figure 2.

Reliable radiative corrections, potentially of the order of a few percent are necessary to extract the V_{us} matrix element from the K_{e3} decay width with high precision.

Calculations of the radiative corrections to the K_{e3} decay were performed independently by E.S.Ginsberg and T.Becherrawy in the late 60's [2, 1]. Their results for corrections to the decay rate, Dalitz plot, pion and positron spectra disagree, in some places quite sharply; for example Ginsberg's correction to the decay rate is -0.45% while that of Becherrawy is -2% (corresponding to corrections to the total width Γ of 0.45 and 2 respectively). We have decided to perform a new calculation since results of the experiments will become available soon and to explore the causes of the discrepancies in the previous calculations. Recently a revision of E. Ginsberg's paper, with numerical estimation of the radiation corrections [14] was published.

Our paper is organized as follows. The introduction (Section 2) is devoted to the short review of kinematics of the elastic decay process (without emission of real photon). In Section 3 we put the results concerning the virtual and soft real photons' emission contribution to the differential width. In Section 4 we consider the hard photon emission including both the inner bremsstrahlung (IB) and the structure-dependent (SD) contributions and derive an expression for the differential width by starting with the Born width and adding the known structure functions in the leading logarithmical approximation (the so-called Drell-Yan picture of the process). We give the explicit expressions for the non-leading contributions. In Section 5, we summarize our results and compare them with those in the previously published papers.

Appendix A contains the details of calculations of virtual and real soft photons emission.

Appendix B contains the details of description of hard photon emission both by IB and SD mechanisms. Our approach to study the hard photon emission differs technically from the ones used in papers [1, 2].

Appendix C contains the explicit formulae for description of SD emission including the interference of IB and SD amplitudes.

Appendix D is devoted to analysis of Dalitz-plot distribution and the properties of Drell-Yan conversion mentioned above.

Appendix E contains the list of the formulae used for the numerical integration.

Appendix F contains the details of kinematics of radiative kaon decay and, besides the analysis of relations of our and paper [2] technical approaches.

In tables 1,2 and graphics (fig. (3,4,5,6)) the result of numerical estimation of Born values and the correction to Dalitz-plot distribution and pion and positron spectra are given.

2 Introduction

The lowest order perturbation theory (PT) matrix element of the process $K^+(p) \rightarrow \pi^0(p') + e^+(p_e) + \nu(p_\nu)$ has the form

$$M = \frac{G_F}{\sqrt{2}} V_{us}^* F_\nu(t) \bar{u}(p_\nu) \gamma_\nu (1 + \gamma_5) v(p_e) \quad (1)$$

where $F_\nu(t) = \frac{1}{\sqrt{2}}(p + p')_\nu f_+(t)$. Dalitz plot density which takes into account the radiative corrections (RC) of the lowest order PT is

$$\frac{d^2\Gamma}{dydz} = C a_0(y, z)(1 + \delta(y, z)) \left(1 + \lambda_+ \frac{t}{m_\pi^2}\right)^2 = \frac{d^2\Gamma_0(y, z)}{dydz}(1 + \delta(y, z)), \quad (2)$$

$$\Gamma_0 = \int \frac{d^2\Gamma_0}{dydz} dydz, \quad \Gamma = \int \frac{d^2\Gamma}{dydz} dydz,$$

where the momentum transfer squared between kaon and pion is:

$$t = (p - p')^2 = M_K^2(1 + r_\pi - z) = M_K^2 R(z).$$

We accept here the following form for the strong interactions induced form factor $f_+(t)$:

$$f_+(t) = f_+(0) \left(1 + \lambda_+ \frac{t}{m_\pi^2}\right), \quad (3)$$

according to PDG $\lambda_+ = 0.0276 \pm 0.0021$. From now on we'll use M^2 instead of M_K^2 . We define

$$C = \frac{M^5 G_F^2 |V_{us}|^2}{64\pi^3} |f_+(0)|^2, \quad (4)$$

and

$$a_0(y, z) = (z + y - 1)(1 - y) - r_\pi + O(r_e). \quad (5)$$

Here we follow the notation of [4]:

$$r_e \equiv m_e^2/M^2, \quad r_\pi \equiv m_\pi^2/M^2; \quad (6)$$

where m_e , m_π , and M_K are the masses of electron, pion, and kaon; two convenient kinematical variables are

$$y \equiv 2pp_e/M^2, \quad z \equiv 2pp'/M^2. \quad (7)$$

In the kaon's rest frame, which we'll imply throughout the paper, y and z are the energy fractions of the positron and pion:

$$y = 2E_e/M, \quad z = 2E_\pi/M. \quad (8)$$

The region of y, z -plane where $a_0(y, z) > 0$ we will named as a region D . Later, when dealing with real photons we'll also use

$$x = 2\omega/M. \quad (9)$$

with ω -real photon energy.

The physical region for y and z (further called D -region) is [4]

$$\begin{aligned} 2\sqrt{r_e} &\leq y \leq 1 + r_e - r_\pi, \\ F_1(y) - F_2(y) &\leq z \leq F_1(y) + F_2(y), \\ F_1(y) &= (2 - y)(1 + r_e + r_\pi - y)/[2(1 + r_e - y)], \\ F_2(y) &= \sqrt{y^2 - 4r_e}(1 + r_e - r_\pi - y)/[2(1 + r_e - y)]; \end{aligned} \quad (10)$$

or, equivalently,

$$\begin{aligned}
2\sqrt{r_\pi} &\leq z \leq 1 + r_\pi - r_e, \\
F_3(z) - F_4(z) &\leq y \leq F_3(z) + F_4(z), \\
F_3(z) &= (2-z)(1+r_\pi+r_e-z)/[2(1+r_\pi-z)], \\
F_4(z) &= \sqrt{z^2-4r_\pi}(1+r_\pi-r_e-z)/[2(1+r_\pi-z)].
\end{aligned} \tag{11}$$

For our aims we use the simplified form of physical region (omitting the terms of the order of r_e):

$$2\sqrt{r_e} \leq y \leq 1 - r_\pi, \quad c(y) \leq z \leq 1 + r_\pi \tag{12}$$

with

$$c(y) = 1 - y + \frac{r_\pi}{1-y}$$

or,

$$2\sqrt{r_\pi} \leq z \leq 1 + r_\pi, \quad b_-(z) \leq y \leq b(z) \tag{13}$$

with

$$\begin{aligned}
b_-(z) &= 1 - \frac{1}{2} \left(z + \sqrt{z^2 - 4r_\pi} \right), \\
b(z) &= 1 - \frac{1}{2} \left(z - \sqrt{z^2 - 4r_\pi} \right).
\end{aligned}$$

For definiteness we give here the numerical value for Born total width. It is:

$$\begin{aligned}
\frac{G_F^2 M_K^5 |V_{us} f_+(0)|^2}{64\pi^3} \int dy \int dz a_0(y, z) \left(1 + \lambda_+ \frac{t}{m_\pi^2}\right)^2 \\
= 5.36 |V_{us} f_+(0)|^2 \times 10^{-14} \text{MeV}.
\end{aligned} \tag{14}$$

Comparing this value with PDG result: $(\Gamma_{K_{+e3}})_{exp} = (2.56 \pm 0.03) \times 10^{-15} \text{MeV}$ we conclude

$$(V_{us} f_+(0))_{\alpha=0} = 0.218 \pm 0.002. \tag{15}$$

3 Virtual and soft real photon emission

Taking into account the accuracy level of 0.1% for determination of ρ/ρ_0 we will drop terms of order r_e . We will distinguish 3 kinds of contributions to δ : from emission of virtual, soft real, and hard real photons in the rest frame of kaon: $\delta = \delta_V + \delta_S + \delta_H$. Standard calculation (see Appendix A for details) allows one to obtain the following contributions:

from the soft real photons

$$\delta_S = \frac{\alpha}{\pi} \left\{ (L_e - 2) \ln \frac{2\Delta\epsilon}{\lambda} + \frac{1}{2} L_e - \frac{1}{4} L_e^2 + 1 - \frac{\pi^2}{6} \right\} (1 + O(r_e)), \tag{16}$$

from the virtual photons $\delta_V = \delta_C + \delta_{PLM}$ that make up charged fermion's renormalization, δ_C (throughout this paper we use Feynman gauge):

$$\delta_C = \frac{\alpha}{2\pi} \left\{ \left[-\frac{1}{2} L_\Lambda + \frac{3}{2} \ln r_e + \ln \frac{M^2}{\lambda^2} - \frac{9}{4} \right] + \left[L_\Lambda + \ln \frac{M^2}{\lambda^2} - \frac{3}{4} \right] \right\}, \tag{17}$$

here $L_\Lambda = \ln(\Lambda^2/M^2)$, Λ is ultraviolet momentum cutoff, the first term in the curly braces comes from positron, the second one from kaon; and for the diagram in fig1(f) in the point like meson (PLM) approximation, δ_{PLM} :

$$\delta_{PLM} = -\frac{\alpha}{2\pi} \left\{ -L_\Lambda - \frac{1}{2} \ln^2 r_e - 2L_e + \ln \frac{M^2}{\lambda^2} L_e - 1 + 2 \ln^2 y + 2 \ln y + 2Li_2(1-y) \right\}. \quad (18)$$

When these contributions are grouped all together the dependence on λ (fictitious "photon mass") disappears. According to Sirlin's prescription [7] we set $\Lambda = M_W$. The result can be written in the form:

$$1 + \delta_S + \delta_C + \delta_{PLM} = S_W \left[1 + \frac{\alpha}{\pi} \left[(L_e - 1) \left(\ln \Delta + \frac{3}{4} \right) - \ln \Delta - \frac{\pi^2}{6} + \frac{3}{4} - Li_2(1-y) - \frac{3}{2} \ln y \right] \right], \quad S_W = 1 + \frac{3\alpha}{4\pi} L_W. \quad (19)$$

In the above equations $L_e = 2 \ln y + \ln(1/r_e)$, and $L_W = \ln(M_W^2/M^2)$; M_W is the mass of W^\pm , $\Delta = \Delta\epsilon/E_e$, and $\Delta\epsilon$ is the maximal energy (in the rest frame of kaon) of a real soft photon. We imply $\Delta\epsilon \ll M/2$. For the details of eqs (16), (17), (18), and (19) see Appendix A.

Contribution from soft photon emission from structure-dependent part (such as for example, interaction with resonances and intermediate W^\pm) is small, of the order

$$\frac{\alpha \Delta\epsilon}{\pi M} \ll 1$$

and thus is also neglected.

4 Hard photon emission. Structure function approach

Next we need to calculate contributions from hard photons. We have to distinguish between inner bremsstrahlung (IB) and the structure-dependent (SD) contributions: $\delta_H = \delta_{IB} + \delta_{int} + \delta_{SD}$, where δ_{int} is the interference term between the two. The terms δ_{int} and δ_{SD} are considered in the framework of the chiral perturbation theory (ChPT) to the orders of (p^2) and (p^4) and find their contribution to be at the level of 0.2% (see appendix C).

$$\delta_H = \frac{\alpha}{2\pi a_0(y,z)} \left\{ (L_e - 1) \left(\Psi(y,z) - a_0(y,z) \left(2 \ln \Delta + \frac{3}{2} \right) \right) - 2a_0(y,z) \ln \frac{b(z) - y}{y\Delta} \right\} + \delta^{hard}, \quad (20)$$

with δ^{hard} given below.

Extracting the short-distances contributions in form of replacement $C \rightarrow CS_W$ it is useful to split δ (see eq.(2)) in the form

$$\delta(y, z) = \delta_L + \delta_{NL}, \quad (21)$$

where δ_L is the leading order contribution, it contains the 'large logarithm' L_e and δ_{NL} is the non-leading contribution, it contains the rest of the terms.

δ_L contains terms from δ_C , δ_S , δ_{PLM} , and the contribution from the collinear configuration of hard IB emission (in the collinear configuration the angle between the positron and the emitted photon is small). δ_L turns out to be

$$\delta_L = \frac{\alpha(L_e - 1)}{2\pi a_0(y, z)} \Psi(y, z). \quad (22)$$

First we note that the kinematics of hard photon emission does not coincide with the elastic process (Region D, the strictly allowed boundaries of the Dalitz plot). In hard photon emission an additional region in the y, z plane, namely $y < b_-(z)$ appears. The nature of this phenomenon is the same as the known phenomenon of the radiative tail in the process of hadron production at colliding e^+e^- beams.

The quantity $\Psi(y, z)$ has a different form for Region D and outside it:

$$\Psi(y, z) = \Psi_>(y, z), z > c(y), 2\sqrt{r_e} < y < 1 - r_\pi; \quad (23)$$

and

$$\Psi(y, z) = \Psi_<(y, z), z < c(y), 2\sqrt{r_e} < y < 1 - \sqrt{r_\pi}. \quad (24)$$

$\Psi_<(y, z) = 0$ when $y > 1 - \sqrt{r_\pi}$. Functions $\Psi_<, \Psi_>$ are studied in Appendix D.

δ_{NL} contains contributions from δ_C , δ_{PLM} , from SD part of hard photons and from the interference term of SD and IB parts of hard radiation.

$$\delta_{NL} = \frac{\alpha}{\pi} \eta(z, y), \quad (25)$$

where

$$\frac{\alpha}{\pi} \eta(y, z) = \delta^{hard} + \frac{\alpha}{\pi} \left[\frac{3}{4} - \frac{\pi^2}{6} - Li_2(1 - y) - \frac{3}{2} \ln y - \ln((b(z) - y)/y) \right], \quad (26)$$

and for the case when the variables y, z are inside D region:

$$\delta^{hard} = \frac{\alpha}{2\pi a_0(y, z)} Z_2(y, z); \quad (27)$$

$$Z_2(y, z) = -2Rphot_{1D}(y, z) + Rphot_{2D}(y, z) + \int_0^{b(z)-y} dx \mathcal{J}(x, y, z). \quad (28)$$

Explicit expressions for $Rphot_{1,2}$ and \mathcal{J} are given in appendix B. The Born value and the correction to Dalitz-plot distribution $\Delta(y, z) = \delta(y, z)a_0(y, z)$ is illustrated in tables 1,2.

We see that the leading contribution from virtual and soft photon emission is associated with the so called δ -part of the evolution equation kernel:

$$(\delta_C + \delta_S + \delta_{PLM})^{leading} = \frac{\alpha}{2\pi} (L_e - 1) \int \frac{a_0(t, z)}{a_0(y, z)} P_\delta^{(1)}\left(\frac{y}{t}\right) \frac{dt}{t} \quad (29)$$

where

$$P_\delta^{(1)}(t) = \delta(1-t) \left(2 \ln \Delta + \frac{3}{2} \right). \quad (30)$$

The contribution of hard photon kinematics in the leading order can be found with the method of quasireal electrons [10] as a convolution of the Born approximation with the θ -part of evolution equation kernel $P_\theta(z)$:

$$\delta_H^{leading} \sim \frac{\alpha}{2\pi} (L_e - 1) \int \frac{dt}{t} \frac{a_0(t, z)}{a_0(y, z)} P_\theta^{(1)}\left(\frac{y}{t}\right) \quad (31)$$

where

$$P_\theta^{(1)}(z) = \frac{1+z^2}{1-z} \theta(1-z-\Delta). \quad (32)$$

In such a way the whole leading order contribution can be expressed in terms of convolution of the width in the Born approximation with the whole kernel of the evolution equation:

$$P^{(1)}(z) = \lim_{\Delta \rightarrow 0} \left(P_\delta^{(1)}(z) + P_\theta^{(1)}(z) \right). \quad (33)$$

This approach can be extended to use nonsinglet structure functions $D(t, y)$ [9]:

$$d\Gamma^{LO}(y, z) = \int_{\max\{y, b_-(z)\}}^{b(z)} \frac{dt}{t} d\Gamma_0(t, z) D\left(\frac{y}{t}, L_e\right), \quad t = x + y, \quad (34)$$

$$D(z, L) = \delta(1-z) + \frac{\alpha}{2\pi} L P^{(1)}(z) + \frac{1}{2!} \left(\frac{\alpha L}{2\pi} \right)^2 P^{(2)}(z) + \dots,$$

$$P^{(i)}(z) = \int_z^1 \frac{dx}{x} P^{(1)}(x) P^{(i-1)}\left(\frac{z}{x}\right), \quad i = 2, 3, \dots$$

One can check the validity of the useful relation:

$$\int_0^1 dz P^{(1)}(z) = 0. \quad (35)$$

The above makes it easy to see that in the limit $m_e \rightarrow 0$ terms that contain m_e do not contribute to the total width in correspondence with Kinoshita–Lee–Nauenberg (KLN) theorem [12] as well as with results of E.Ginsberg [2]. Keeping in mind the representation

$$\Psi(y, z) = \int_{\max\{y, b_-(z)\}}^{b(z)} \frac{dt}{t} a_0(t, z) P^{(1)}\left(\frac{y}{t}\right), \quad (36)$$

one can get convinced (see Appendix D) that the leading logarithmical contribution to the total width as well as one to the pion spectrum is zero due to:

$$\int_{2\sqrt{r_\pi}}^{1+r_\pi} dz \int_0^{b(z)} dy \Psi(y, z) = 0. \quad (37)$$

Using the general properties of the evolution equations kernels, eq(35) one can see that KLN cancellation will take place in all orders of the perturbation theory. The spectra in Born approximation are (we omit terms $O(r_e) \sim 10^{-6}$):

For pion

$$\frac{1}{\bar{C}} \frac{d\Gamma_0}{dz} = \phi_0(z), \quad (38)$$

$$\phi_0(z) = \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 \int_{b_-(z)}^{b(z)} dy a_0(y, z) = \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 \frac{1}{6} (z^2 - 4r_\pi)^{3/2},$$

and for positron

$$f(y) = \frac{1}{\bar{C}} \frac{d\Gamma_0}{dy} = f_0(y) \left[1 + \frac{2}{3} \left(\frac{\lambda_+}{r_\pi}\right) \frac{y(1-r_\pi-y)}{1-y} + \frac{1}{6} \left(\frac{\lambda_+}{r_\pi}\right)^2 \frac{y^2(1-r_\pi-y)^2}{(1-y)^2} \right],$$

$$f_0(y) = \frac{y^2(1-r_\pi-y)^2}{2(1-y)}. \quad (39)$$

The corrected pion spectrum in the inclusive set-up of experiment when integrating over the whole region for y ($0 < y < b(z)$) have a form $\phi_0(z) + (\alpha/\pi)\phi_1(z)$ with

$$\phi_1(z) = \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 \left[\int_0^{b_-(z)} dy [\Psi_{<}(y, z) \ln y - a_0(y, z) \ln \frac{b(z)-y}{b_-(z)-y} + \frac{1}{2} \tilde{Z}_2(y, z)] + \int_{b_-(z)}^{b(z)} dy [\Psi_{>}(y, z) \ln y + a_0(y, z) Z_1(y, z) + \frac{1}{2} Z_2(y, z)] \right], \quad (40)$$

the quantities Z_1, \tilde{Z}_2 explained in Appendix E. This function do not depend on $\ln(1/r_e)$. Pion spectrum in the exclusive set-up (y, z in the region D) will depend on L_e . It's expression is given in the Appendix E.

Numerical estimation of pion spectrum is illustrated in figure (3,5).

The inclusive positron spectrum with the correction of the lowest order is $f(y) +$

$(\alpha/\pi)f_1(y)$ with $f(y)$ given above and:

$$\begin{aligned}
f_1(y) &= \frac{1}{2}(L_e - 1)I(y) - \int_{c(y)}^{1+r_\pi} a_0(y, z) \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 \ln((b(z) - y)/y) dz + \\
&\left(\frac{3}{4} - \frac{\pi^2}{6} - \frac{3}{2} \ln y - Li_2(1 - y)\right) f(y) + \frac{1}{2} \int_{c(y)}^{1+r_\pi} Z_2(y, z) \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 dz + \\
&\theta(1 - \sqrt{r_\pi} - y) \int_{2\sqrt{r_\pi}}^{c(y)} dz \left(1 + \frac{\lambda_+}{r_\pi} R(z)\right)^2 \left[\frac{1}{2} \bar{Z}_2 - a_0(y, z) \ln \frac{b(z) - y}{b_-(z) - y}\right], \quad (41)
\end{aligned}$$

with

$$I(y) = j_0(y) + \left(\frac{\lambda_+}{r_\pi}\right) j_1(y) + \left(\frac{\lambda_+}{r_\pi}\right)^2 j_2(y), \quad (42)$$

$$\begin{aligned}
j_0(y) &= \int_y^{1-r_\pi} \frac{dt}{t} \int_{c(t)}^{1+r_\pi} dz a_0(t, z) P^{(1)}\left(\frac{y}{t}\right) = \\
&\left(2 \ln \frac{1 - r_\pi - y}{y} + \frac{3}{2}\right) f_0(y) + \frac{r_\pi^2(1 + y^2)}{2(1 - y)} \ln \frac{1 - y}{r_\pi} + \\
&\frac{1}{12} (1 - r_\pi - y) [1 - 5r_\pi - 2r_\pi^2 + y(4 - 13r_\pi) - 17y^2]; \quad (43)
\end{aligned}$$

explicit expressions for $j_1(y)$ and $j_2(y)$ are given in Appendix D.

Numerical estimation of positron spectrum is illustrated in figure (4,6).

One can check the fulfillment of KLN cancellation of singular in the limit $m_e \rightarrow 0$ terms for the total width correction: $\int_0^{1-r_\pi} I(y) dy = 0$. The expression for $j_0(y)$ agree with A(2) from the 1966 year paper of Eduard Ginsberg [2].

We put here the general expression for differential width of hard photon emission which might be useful for construction of Monte Carlo simulation of real photon emission in K_{e3} :

$$d\Gamma_\gamma^{hard} = d\Gamma_0 \frac{\alpha}{2\pi} \frac{dx}{x} \frac{dO_\gamma}{2\pi a_0(y, z)} T, \quad (44)$$

with

$$x = \frac{2\omega}{M} > \frac{2\Delta\epsilon}{M} = y \frac{\Delta\epsilon}{E_e}, \quad \frac{\Delta\epsilon}{E_e} \ll 1; \quad (45)$$

and dO_γ is an element of photon's solid angle. The quantity T is explained in Appendix B.

For soft photon emission we have

$$d\Gamma_\gamma^{soft} = d\Gamma_0 \frac{\alpha}{2\pi} \frac{dx}{x} \frac{dO_\gamma}{2\pi} \left[-1 - \frac{r_e}{(1 - \beta_e C_e)^2} + \frac{y}{1 - \beta_e C_e}\right], \quad x < y \frac{\Delta\epsilon}{E_e}. \quad (46)$$

Integrating over angles within phase volume of hard photon we obtain the spectral distribution of radiative kaon decay:

$$\frac{d\Gamma}{d\Gamma_0 dx} = \frac{\alpha}{2\pi} \frac{1}{a_0(y, z)} \left[\frac{a_0(x+y, z)}{x(x+y)^2} ((y^2 + (x+y)^2)(L_e - 1) + x^2) - \frac{2}{x} a_0(y, z) - 2 \left(\frac{R(z)}{x+y} - y \right) + \mathcal{J}(x, y, z) \right],$$

$$y\Delta < x < b(z) - y, \quad \Delta = \frac{\Delta\epsilon}{E_e} \ll 1.$$

5 Discussion

Structure-dependent contribution to emission of virtual photons (see Fig 1 d), e)) can be interpreted as a correction to the strong formfactor of $K\pi$ transition, $f_+(t)$. We assume that this formfactor can be extracted from experiment and thus do not consider it. The problem of calculation of RC to K_{e3} and especially the formfactors in the framework of ChPT with virtual photons was considered in a recent paper [14].

As in paper [2] we assume a phenomenological form for the hadronic contribution to the $K - \pi$ vertex, but here we use explicitly the dependence of the form factor in the form

$$f_+(t) = f_+(0) \left(1 + \frac{\lambda_+}{r_\pi} R(z) \right). \quad (47)$$

We assume that the effect of higher order ChPT as well as RC to the formfactors can be taken as a multiplicative scaling factor for $f_+(0)$, which we take from of a recent paper [14].

We assume such an experiment in which only one positron in the final state is present, but place no limits on the number of photons. The ratio of the LO contributions in the first order to the Born contribution is a few percent, and for the second order it is about

$$\left(\frac{\alpha L_e}{2\pi} \right)^2 \leq 0.03\%. \quad (48)$$

Due to non-definite sign structure of the leading logarithm contribution (see eq(22)) there are regions in the kinematically allowed area where $|\Psi(y, z)|$ is close to zero. In these regions the non-leading contributions become dominant.

The contribution of the channels with more than 1 charged lepton in the final state as well as the vacuum polarization effects in higher orders may be taken into account by introducing the singlet contribution to the structure functions. The effect will be at the level of 0.03% and we omit them within the precision of our calculation.

The contribution of the $O(p^4)$ terms [5] turns out to be small. Indeed, one can see that they are of the order $O(\alpha L_e^2, (\bar{p}/\Lambda_c)^2) \leq O(10^{-2}\%)$, $\Lambda_c = 4\pi F_\pi \approx 1.2\text{GeV}$ ($F_\pi = 93\text{MeV}$ is the pion life time constant), where \bar{p} is the characteristic momentum of a final particle in the given reaction, $\bar{p}^2 \leq M^2/16 \sim F_\pi^2$. So the terms of the orders $O(p^4)$ and $O(p^6)$ can be omitted within the accuracy of $O(\frac{\alpha}{\pi} \times 10^{-2}) \leq O(10^{-4})$.

Our main results are given in formulae (2,21,22,26-28) for Dalitz plot distribution ; (38-41) for pion and positron spectra;(47) for hard photon emission;(53) for the value $|V_{us}|$, in the tables and figures. The accuracy of these formulas is determined by the following:

1. we don't account higher order terms in PT, the ones of the order of $(\alpha L_e/\pi)^n, n \geq 2$ which is smaller than 0.03%
2. structure-dependent real hard photon emission contribution to RC we estimate to be at the level of 0.0005.
3. higher order CHPT contributions to the structure dependent part are of the order 0.05% [4, 5]

All the percentages are taken with respect to the Born width. All together we believe the accuracy of the results to be at the level of 0.01. So the final result of our calculation may be written in the form

$$\frac{\Gamma}{\Gamma_0} = (1 + \delta(1 \pm 0.01)) \quad (49)$$

the terms on the RHS are given in (21, 22, 25).

Here is the list of improvements comparing with the older calculations [1, 2]:

1. we eliminate the ultra-violet cutoff dependence by choosing $\Lambda = M_W$,
2. we describe the dependence on the lepton mass logarithm L_e in all orders of the perturbation theory and explain why the correction to the total width does not depend on m_e ,
3. we treat the strong interaction effects by the means of CHPT in its lowest order $O(P^2)$ and show that the next order contribution is small,
4. we give an explicit formulas for the total differential cross section and explicit results for corrections to the Dalitz plot and particle spectra that might be used in experimental analysis.

In the papers of E.Ginsberg the structure-dependent emission was not considered. T.Becheraway, on the other hand, did include it, and this will give rise to differences in the Dalitz plot. In addition, Ginsberg used the proton mass as the momentum cutoff parameter.

We do not consider the evolution of coupling constant effects in the regions of virtual photon momentum modulo square of the quantities of order M_ρ^2 up to M_Z^2 , which can be taken into account [11] (and for details see [14]) by replacing the quantity S_W by the $S_{EW} = 1 + (\alpha/\pi) \ln(M_Z^2/M_\rho^2) = 1.0232$. Taking this replacement into account our result for the correction to the total width is

$$\frac{\Gamma}{\Gamma_0} = 1 + \delta = 1.02, \quad (50)$$

which results in

$$|V_{us}f_+(0)| = 0.214 \pm 0.002. \quad (51)$$

So the correction to the total width is +2% while Ginsberg's result is -0.45% and Becherrawy's is -2%. Neither Ginsberg nor Becherrawy used the factor S_{EW} , and this factor (1.023) accounts for most of the difference between Ginsberg's and our result. Electromagnetic corrections become negative and have an order of 10^{-3} . The effect of the SD part, which E.Ginsberg did not consider, is small, of the order of 0.1%.

We use the value of formfactor $f_+(0) = 0.9842 \pm 0.0084$ calculated in the paper [14] in the framework of ChPT, including virtual photonic loops and terms of order $O(p^6)$ of ChPT. To avoid double counting we use the mesonic contribution to $f_+^{mes}(0) = 1.0002 \pm 0.0022$ and the p^6 terms one $f_+(0)|_{p^6} = -0.016 \pm 0.0008$. Our final result is:

$$|V_{us}| = 0.21715 \pm 0.0055. \quad (52)$$

In estimating the uncertainty we take into account the uncertainties arising from structure-dependent emission ± 0.005 , theoretical errors of order ± 0.0003 , experimental error ± 0.0022 and the ChPT error in p^6 terms 0.0008.

In tables 1,2 we give corrections to the distributions in the Dalitz-plot $d\Gamma/(dydz) \sim a_0(y, z) + (\alpha/\pi)\Delta(y, z)$.

In figures (3-6) we illustrate the corrections to the pion and positron spectra. Here we see qualitative agreement for the positron spectrum and disagreement with the pion spectrum obtained by E.Ginsberg.

6 Acknowledgements

We are grateful to S.I.Eidelman for interest to this problem, and to E. Swanson for constructive critical discussions. One of us (E.K.) is grateful to the Department of Physics and Astronomy of the University of Pittsburgh for the warm hospitality during the last stages of this work, A.Ali for valuable discussion and V.N.Samoilov for support. The work was supported partly by grants RFFI 01-02-17437 and INTAS 00366 and by USDOE DE FG02 91ER40646.

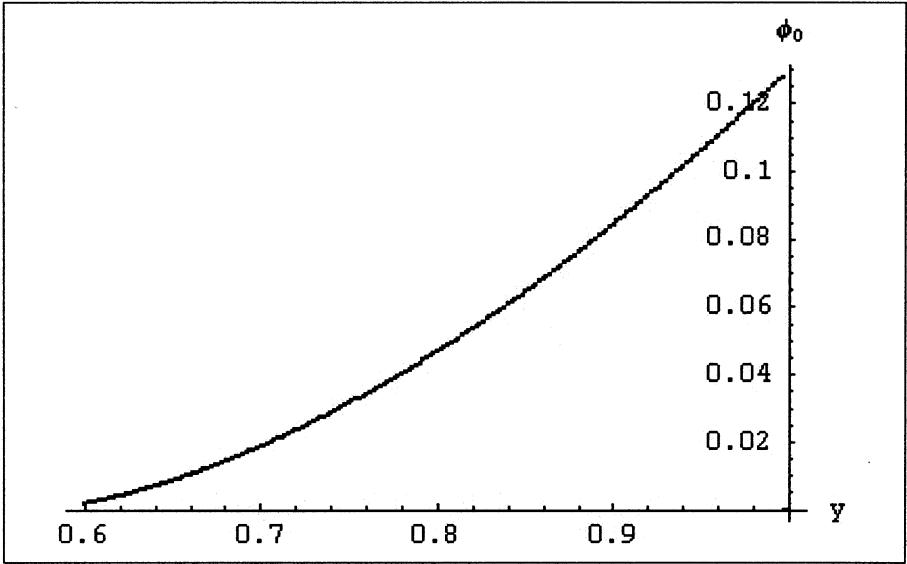


Figure 3: Pion spectrum in Born approximation, $\phi_0(z)$ (see (40)).

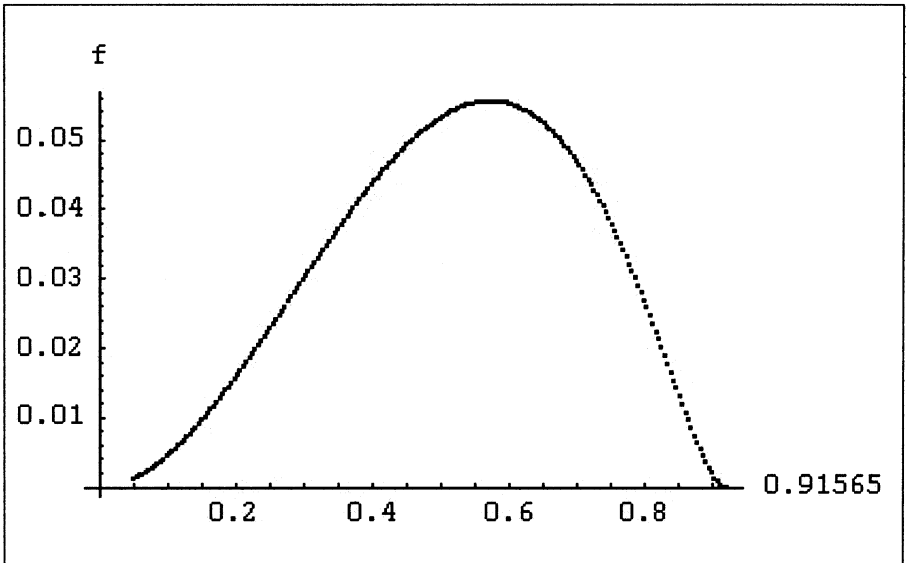


Figure 4: Positron spectrum in Born approximation, $f(y)$ (see (40)).

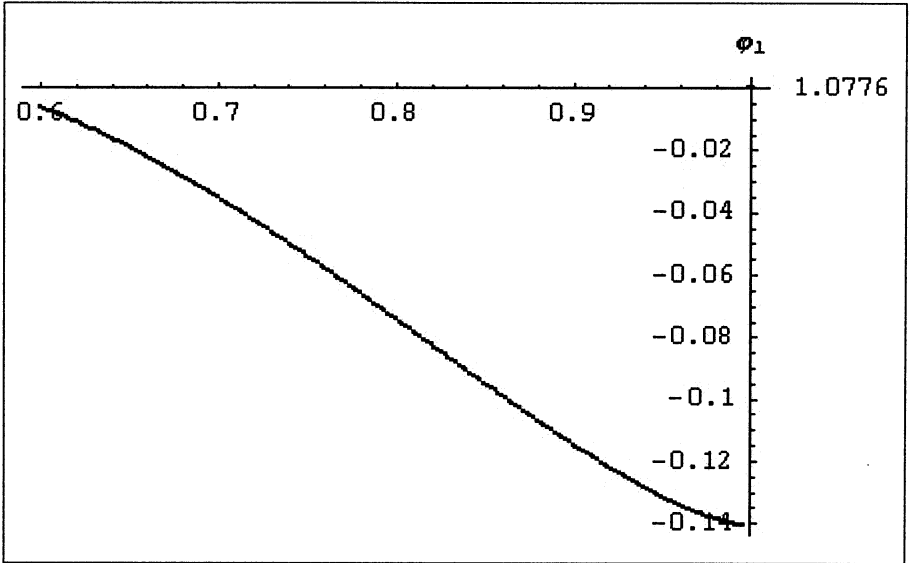


Figure 5: Correction to pion spectrum, $\phi_1(z)$ (see (40)).

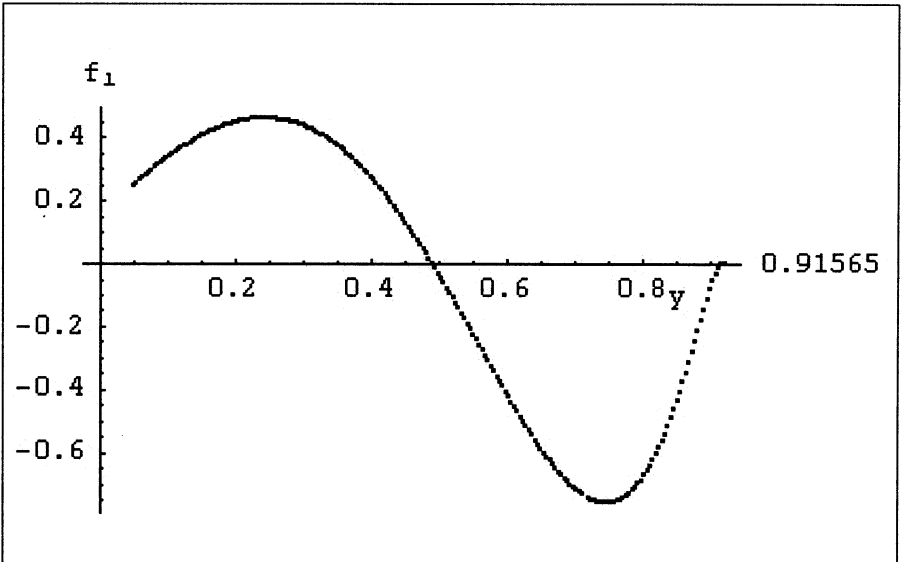


Figure 6: Correction to positron spectrum, $f_1(y)$ (see (41)).

z/y	0.07	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85
1.025	3.83	4.37	3.71	2.01	-0.21	-2.47	-4.31	-5.11	-3.81
0.975		3.76	3.49	2.07	0.05	-2.07	-3.83	-4.61	-3.35
0.925			3.26	2.13	0.32	-1.67	-3.35	-4.11	-2.88
0.875			3.04	2.18	0.58	-1.26	-2.86	-3.60	-2.39
0.825				2.25	0.85	-0.86	-2.37	-3.08	-1.88
0.775					1.14	-0.41	-1.83	-2.51	-1.28
0.725					1.39	-0.04	-1.39	-2.03	-0.72
0.675						0.38	-0.86	-1.48	
0.625							-0.35	-0.89	
0.580								-0.23	

Table 1. Correction to Dalitz plot distribution $\Delta(y, z) = a_0(y, z)\delta(y, z) \times 10^3$ (see eq (2)).

z/y	0.07	0.15	0.25	0.35	0.45	0.55	0.65	0.75	0.85
1.025	0.0084	0.069	0.126	0.163	0.181	0.178	0.156	0.114	0.051
0.975		0.026	0.089	0.131	0.153	0.156	0.139	0.101	0.0437
0.925			0.051	0.099	0.126	0.134	0.121	0.088	0.036
0.875			0.014	0.066	0.099	0.111	0.104	0.076	0.029
0.825				0.0337	0.071	0.089	0.086	0.064	0.021
0.775					0.043	0.066	0.069	0.051	0.014
0.725					0.016	0.044	0.051	0.039	0.006
0.675						0.021	0.034	0.026	
0.625							0.016	0.014	
0.580								0.003	

Table 2. Dalitz plot distribution in Born approximation $a_0(y, z)$.

7 Appendix A

Here we explain how to calculate δ_S , δ_C , δ_{PLM} and how to group them into eq(19).

Contribution from emission of a soft real photon can be written in a standard form in terms of the classical currents:

$$\delta_S = -\frac{4\pi\alpha}{(2\pi)^3} \int \frac{d^3q}{2\omega} \left(\frac{p}{p \cdot q} - \frac{p_e}{p_e \cdot q} \right)^2 \Big|_{\omega = \sqrt{\vec{q}^2 + \lambda^2} < \Delta\epsilon}, \quad (53)$$

where λ is fictitious mass of photon. We use the following formulas:

$$\frac{1}{2\pi} \int \frac{d^3q}{2\omega} \left(\frac{p}{p \cdot q} \right)^2 = \ln \left(\frac{2\Delta\epsilon}{\lambda} \right) - 1;$$

$$\begin{aligned} \frac{1}{2\pi} \int \frac{d^3q}{2\omega} \left(\frac{p_e}{p_e \cdot q} \right)^2 &= \ln \left(\frac{2\Delta\epsilon}{\lambda} \right) - \frac{1}{2} L_e ; \\ \frac{1}{2\pi} \int \frac{d^3q}{2\omega} \frac{2(p \cdot p_e)}{(p \cdot q)(p_e \cdot q)} &= L_e \ln \left(\frac{2\Delta\epsilon}{\lambda} \right) - \frac{\pi^2}{6} - \frac{1}{4} L_e^2 . \end{aligned} \quad (54)$$

From them we obtain the eq(16).

Consider now radiative corrections that arise from emission of virtual photons (excluding SD virtual photons).

Feynman graphs containing self-energy insertion to positron and kaon Green functions (Fig.1,b,c) can be taken into account by introducing the wave function renormalization constants Z_e and Z_K : $M_0 \rightarrow M_0(Z_K Z_e)^{1/2}$. We use the expression for Z_e given in the textbooks [16]; the expression for Z_K is given in the paper [15]. The result is eq(17).

Now consider the Feynman graph in which a virtual photon is emitted by kaon and absorbed by positron or by W -boson in the intermediate state (Fig.1,d,e,f). This long distance contribution is calculated using a phenomenological model with point-like mesons as a relevant degrees of freedom. To calculate the contribution from the region $|k|^2 < \Lambda^2$ (Λ is the ultra violet cutoff) we use the following expressions for loop momenta scalar, vector, and tensor integrals:

$$Re \int \frac{d^4k}{i\pi^2} \frac{1, k^\mu, k^2}{(k^2 - \lambda^2)((k-p)^2 - M^2)((k-p_e)^2 - m_e^2)} = I, I^\mu, J . \quad (55)$$

A standard calculation yields:

$$\begin{aligned} I &= \frac{-1}{yM^2} \left\{ \frac{1}{2} \ln \frac{M^2}{\lambda^2} L_e + \ln^2 y + Li_2(1-y) - \frac{1}{4} \ln^2 r_e \right\} ; \\ I^\mu &= \frac{-1}{yM^2} \left\{ \frac{-y \ln y}{1-y} p^\mu + p_e^\mu \left(\frac{y \ln y}{1-y} + L_e \right) \right\} ; \\ J &= L_\Lambda + \frac{y \ln y}{1-y} + 1 . \end{aligned} \quad (56)$$

where $L_\Lambda = \ln(\Lambda^2/M^2)$ and we omitted terms of the order of $O(m_e^2/M^2)$. As a result we obtain

$$\begin{aligned} \int \frac{d^4k}{i\pi^2} \frac{(1/4)Sp p_\nu(p+p')(-\hat{p}_e + \hat{k})(2\hat{p} - \hat{k})p_e(p+p')}{(k^2 - \lambda^2)((k-p)^2 - M^2)((k-p_e)^2 - m_e^2)} &= 2M^4 a_0(y, z) \times \\ \left\{ -L_\Lambda - \frac{1}{2} \ln^2 r_e - 2L_e + \ln \frac{M^2}{\lambda^2} L_e - 1 + 2 \ln^2 y + 2 \ln y + 2Li_2(1-y) \right\} . \end{aligned} \quad (57)$$

In a series of papers [7] A.Sirlin has conducted a detailed analysis of UV behaviour of amplitudes of processes with hadrons in 1-loop level. He showed that they are UV finite (if considered on the quark level), but the effective cutoff scale on loop momenta is of the order M_W . For this reason we choose

$$L_\Lambda = \ln \frac{M_W^2}{M^2} .$$

The sum $\delta_S + \delta_C + \delta_{PLM}$ yields eq(19)

8 Appendix B

The matrix element of the radiative K_{e3} decay

$$K^+(p) \rightarrow \pi^0(p') + e^+(p_e) + \nu(p_\nu) + \gamma(q) \quad (58)$$

with terms up to $O(p^2)$ in CHPT [3, 4, 5, 6] has the form

$$M^{hard} = \frac{G}{2} f_+ V_{us}^* \sqrt{4\pi\alpha\bar{u}}(p_\nu) Q_\mu^{hard} (1 + \gamma_5) v(p_e) \epsilon^\mu(q) , \quad (59)$$

where

$$\begin{aligned} Q_\mu^{hard} &= Q_\mu^e + Q_\mu^\pi + Q_\mu^{SD} = Q_\mu^{IB} + Q_\mu^{SD} ; \\ Q_\mu^{IB} &= (\hat{p} + \hat{p}') \left[\frac{(-\hat{p}_e - \hat{q} + m_e)\gamma_\mu}{2p_e q} + \frac{p_\mu}{pq} \right] ; \\ Q_\mu^{SD} &= \gamma_\nu R_{\mu\nu} . \end{aligned} \quad (60)$$

where tensor $R_{\mu\nu}$ describes (see eq(4.17) in [4]) the structure-dependent emission (fig 2(c)).

$$R_{\mu\nu} = g_{\mu\nu} - \frac{q_\nu p_\mu}{pq} . \quad (61)$$

Singular at $\chi = 2p_e q \rightarrow 0$ terms which provide contribution containing large logarithm L_e arises only from Q_μ^e . To extract the corresponding terms we introduce four-vector $v = (x/y)p_e - q$, and x is the energy fraction of the photon (9). Note that $v \rightarrow 0$ when $\chi \rightarrow 0$. Separating leading and non-leading terms and letting $f_+(t) = 1$, i.e. neglecting form factor's momentum dependence, we obtain:

$$\delta_H = \frac{d\Gamma^{hard}}{d\Gamma_0} = \frac{\alpha}{2\pi a_0(y, z)} \int \frac{dx}{x} \int \frac{dO_\gamma T}{2\pi} , x > y\Delta . \quad (62)$$

where

$$\begin{aligned} T &= \frac{x^2}{8} \sum_{spins} \left| \bar{u}(p_\nu) (Q_{IB}^{hard} + Q_{SD}^{hard}) (1 + \gamma_5) v(p_e) \right|^2 = \\ &= \frac{y a_0(x+y, z)}{x+y} \left[\frac{y^2 + (x+y)^2}{y^2(1-\beta_e C_e)} - 2 \frac{(1-\beta_e)(x+y)}{y(1-\beta_e C_e)^2} \right] - \frac{y a_0(x+y, z)}{x+y} + \mathcal{P} . \end{aligned} \quad (63)$$

The quantity \mathcal{P} contains some non-leading contributions from the IB part and the ones that arise from the structure-dependent part:

$$\begin{aligned} \mathcal{P} &= \left(\frac{p_e q}{M^2} \left(\frac{p_\nu q}{M^2} + z - \frac{2y}{x+y} (1-x-y) \right) + \frac{p' v y(2-x-y)}{M^2 x+y} \right) \\ &= \left(\frac{x M^2}{4y p_e q} (y^2 + (x+y)^2 - 1) \right) - \frac{M^2 x^2}{8 p_e q} \left(T_v + \frac{2}{x} T_{1v} \right) - \frac{x^2}{8} (T_{RR} + 2T_R) , \end{aligned} \quad (64)$$

with

$$T_v = \frac{1}{4M^4} S_p (\hat{p} + \hat{p}') \hat{p}_\nu (\hat{p} + \hat{p}') \hat{v} ;$$

$$\begin{aligned}
T_{1\nu} &= \frac{1}{4M^6} S p (\hat{p} + \hat{p}') \hat{p}_\nu (\hat{p} + \hat{p}') \hat{v} \hat{p} \hat{p}_e ; \\
T_{RR} &= R_{\mu\lambda} R_{\mu\sigma} \frac{1}{4M^2} S p \hat{p}_\nu \gamma_\lambda \hat{p}_e \gamma_\sigma ; \\
T_R &= R_{\mu\lambda} \frac{1}{4M^2} S p \hat{p}_\nu (\hat{p} + \hat{p}') \left[\frac{p_\mu}{p q} - \frac{(\hat{p}_e + \hat{q}) \gamma_\mu}{\chi} \right] \hat{p}_e \gamma_\lambda . \quad (65)
\end{aligned}$$

To calculate these traces we use the following expressions for the scalar products of the 4-momenta (in units M):

$$\begin{aligned}
p^2 &= 1, \quad q^2 = 0, \quad p_\nu^2 = 0, \quad p'^2 = r_\pi, \quad p_e^2 = 0, \quad p p_e = \frac{y}{2}, \\
p p' &= \frac{z}{2}, \quad p q = \frac{x}{2}, \quad p p_\nu = \frac{1}{2} (2 - y - z - x), \\
p' p_\nu &= \frac{1}{2} (1 - x - y - r_\pi + A_e), \quad p' q = \frac{1}{2} (x - A_e - A_\nu), \\
p' p_e &= \frac{1}{2} (y - R(z) + A_\nu), \quad p_\nu q = \frac{1}{2} A_\nu, \quad p_e q = \frac{1}{2} A_e, \\
p_e p_\nu &= \frac{1}{2} (R(z) - A_e - A_\nu), \quad p v = 0, \quad p_e v = -\frac{1}{2} A_e, \\
q v &= \frac{1}{2} \frac{x}{y} A_e, \quad p' v = \frac{1}{2} \left(\frac{x+y}{y} \bar{A}_\nu + A_e \right), \\
p_\nu v &= -\frac{1}{2y} \left(x A_e + (x+y) \bar{A}_\nu \right), \\
\bar{A}_\nu &= A_\nu - \frac{x}{x+y} R(z).
\end{aligned}$$

Three terms in the rhs of (63) have a completely different behavior.

The first one corresponds to the kinematic region of collinear emission, when photon is emitted along positron's momentum. The relevant phase volume has essentially 3-particles form:

$$\begin{aligned}
(d\phi_4)^{coll} &= \left(\frac{d^3 p_e}{2\epsilon_e} \frac{d^3 q}{2\omega} \frac{d^3 p'}{2\epsilon'} d^4 p_\nu \delta(p_\nu^2) \delta^4(p - p_e - p_\nu - p' - q) \right)^{coll} = \\
M^4 \frac{\pi^2}{64} \beta_\pi z d z y d y x d x d O_\gamma d C_{e\pi} \delta(1 - x - y - z + r_\pi + \frac{x+y}{y} \frac{z y}{2} (1 - \beta_\pi C_{e\pi}) + \frac{2 p_e q}{M^2}) &= \\
\frac{y}{x+y} M^4 \frac{\pi^2}{32} d O_\gamma x d x y d z . \quad (66)
\end{aligned}$$

The limits of photon's energy fraction variation are $y\Delta < x < b(z) - y$. The upper limit is imposed by the Born structure of the width in this kinematical region.

The second term corresponds to the contribution from emission by kaon. The relevant kinematics is isotropic.

The kinematics of radiative kaon decay and the comparison of our and E.Ginsbergs approaches is given in Appendix F.

The third term corresponds to the rest of the contributions which contain neither collinear nor infrared singularities.

Performing the integration over photon's phase volume provided y, z are in the D region we obtain:

$$\int \frac{dx}{x} \int \frac{dO_\gamma}{2\pi} T = \int_{y\Delta}^{b(z)-y} dx \frac{y^2}{x(y+x)^2} a_0(y+x, z) \left[\frac{y^2 + (y+x)^2}{y^2} (L_e - 1) + \frac{x^2}{y^2} \right] \\ - 2 \int_{y\Delta}^{b(z)-y} \frac{dx}{x} \left[a_0(y, z) + x \left(\frac{R(z)}{x+y} - y \right) \right] + \int_0^{b(z)-y} dx \mathcal{J}, \quad (67)$$

we obtain (27) with

$$Rphot_{1D} = \int_0^{b(z)-y} dx \left(\frac{R(z)}{x+y} - y \right) = R(z) \ln \frac{b(z)}{y} - y(b(z) - y); \quad (68)$$

$$Rphot_{2D}(y, z) = \int_0^{b(z)-y} dx \frac{x a_0(y+x, z)}{(y+x)^2} = - (R(z) + y(2-z)) \ln \frac{b(z)}{y} + \\ \frac{1}{2} (b(z) - y) \left(2 \frac{R(z)}{b(z)} + 4 - 2z - b(z) + y \right), \quad (69)$$

and

$$\mathcal{J}(x, y, z) = \frac{1}{x} \int \frac{dO_\gamma}{2\pi} \mathcal{P}. \quad (70)$$

One can check that the sum of RC arising from hard, soft and virtual photons do not depend on the auxiliary parameter Δ .

We note that the leading contribution from hard part of photon spectra can be reproduced using the method of quasi real electrons [10].¹

Now we concentrate on the contribution of the third term in the RHS of (63).

To perform the integration over the phase volume of final states it is convenient to use the following parameterization (see Appendix F):

$$d\phi_4 = \frac{d^3 p' d^3 p_e d^3 p_\nu d^3 q}{2\epsilon' 2E_e 2\epsilon_\nu 2\omega} \delta^4(p - p' - p_e - p_\nu - q) = \beta_\pi \frac{\pi^2}{16} M^4 dy dz x dx \frac{dC_e dC_\pi}{\sqrt{D}}, \quad (71)$$

with

$$D = \beta_\pi^2 (1 - C^2 - C_\pi^2 - C_e^2 + 2CC_\pi C_e), \quad \beta_\pi = \sqrt{1 - \frac{4r_\pi}{z^2}}, \quad (72)$$

$$C = \cos(\vec{p}_e, \vec{p}'), \quad C_e = \cos(\vec{q}, \vec{p}_e), \quad C_\pi = \cos(\vec{q}, \vec{p}').$$

¹the formula (10) in [10] should read

$$d\Gamma_b = \frac{2\epsilon' d^3 \sigma_{0b}}{d^3 p'} \Big|_{\vec{p}' = \vec{p}_3 + \vec{k}} dW_{\vec{p}_3 + \vec{k}}(k) \frac{d^3 p_3}{2\epsilon_3}.$$

The neutrino on-mass shell (NMS) condition provides the relation

$$1 - \beta_\pi C = \frac{2}{yz} \left[x + y + z - 1 - r_\pi - \frac{xz}{2} (1 - \beta_\pi C_\pi) - \frac{xy}{2} (1 - C_e) \right]. \quad (73)$$

For the aim of further integration of \mathcal{P} over angular variables we put it in the form:

$$\mathcal{P} = xP_1 \frac{\tilde{A}_\nu}{A_e} + xP_2 + P_3 A_e + P_4 A_\nu + P_5 A_\nu A_e, \quad (74)$$

$$A_e = \frac{xy}{2} (1 - C_e),$$

$$A_\nu = x - A_e - \frac{xz}{2} (1 - \beta_\pi C_\pi).$$

and

$$P_1 = \frac{y}{2} (1 - x - y); \quad (75)$$

$$P_2 = \frac{R(z)}{x+y} + \frac{1}{2} (z(2x+3y+1) + 2x^2 + 4xy + 3y^2 - 2x - 3y - 2);$$

$$P_3 = 1 - z - y - \frac{1}{2} x(x+y+z);$$

$$P_4 = -1 + x + y + \frac{1}{2} xy; P_5 = -1.$$

Angular integration can be performed explicitly, we have

$$\int \frac{\beta_\pi dC_\pi}{\pi\sqrt{D}} = \frac{y}{\sqrt{A}}, \int \frac{\beta_\pi C_\pi dC_\pi}{\pi\sqrt{D}} = \frac{y(x+y-yt)}{z\beta_\pi A^{3/2}} [2R(z) - (x+y)(2-z) + xyt], \quad (76)$$

with

$$A = (x+y)^2 - 2xyt, \quad t = 1 - C_e. \quad (77)$$

Performing the integration over C_π we have:

$$\begin{aligned} \frac{1}{x} \int \frac{dC_\pi \beta_\pi}{\pi\sqrt{D}} \mathcal{P} &= \frac{2y}{A^{3/2}} \left((y-x) \left(1 - \frac{z}{2} - \frac{R}{x+y} \right) - \frac{1}{2} y(x+y-xt) \right) P_1 + \\ &\quad \frac{y}{A^{1/2}} \left(P_2 + \frac{y}{2} t P_3 \right) + \left(P_4 + \frac{xy}{2} t P_5 \right) \times \\ &\quad \left\{ \frac{y}{A^{1/2}} \left(1 - \frac{z}{2} - \frac{y}{2} t \right) + \frac{y}{A^{3/2}} (x+y-yt) \left(R - (x+y) \left(1 - \frac{z}{2} \right) + \frac{xy}{2} t \right) \right\}. \quad (78) \end{aligned}$$

The following integrals are helpful in integrating the above expression. We define

$$I_n^m = \int_0^2 \frac{dt t^m}{\sqrt{A^n}}, \quad m = 0, 1, 2, 3; \quad n = 1, 3. \quad (79)$$

Then

$$\begin{aligned}
I_1^0 &= \frac{4}{\sigma}, & I_1^1 &= \frac{8(x+y+\sigma)}{3\sigma^2}, \\
I_1^2 &= \frac{16}{15\sigma^3} (3\sigma^2 + 3(x+y)\rho + 5(x+y)^2), \\
I_3^0 &= \frac{4}{\rho\sigma(x+y)}, & I_3^1 &= \frac{8}{\rho\sigma^2}, & I_3^2 &= \frac{16}{3\rho\sigma^3} (2(x+y) + \sigma), \\
I_3^3 &= \frac{32}{5\rho\sigma^4} (\sigma^2 + 2(x+y)\rho + 4(x+y)^2),
\end{aligned} \tag{80}$$

where $\rho = |x - y|$ and $\sigma = x + y + \rho$.

The first term in $d\Gamma^{hard}$ together with the leading contributions from virtual and soft real photons was given in the form required by RG approach eq(36).

The non-leading contributions, δ^{hard} from hard photon emission, include SD emission, IB of point-like mesons as well as the interference terms. It is free from infrared and mass singularities and given above (27) with

$$\mathcal{J}(x, y, z) = P_1 R_1 + P_2 y I_1^0 + P_3 \frac{y^2}{2} I_1^1 + \frac{y}{2} P_4 R_4 + \frac{xy^2}{4} P_5 R_5, \tag{81}$$

and

$$\begin{aligned}
R_1 &= \frac{y}{x+y} (y-x) ((2-z)(x+y) - 2R(z)) I_3^0 - y^2 ((x+y) I_3^0 - x I_3^1), \\
R_4 &= (2-z) I_1^0 - y I_1^1 + (2R(z) - (x+y)(2-z)) ((x+y) I_3^0 - y I_3^1) + \\
&\quad xy ((x+y) I_3^1 - y I_3^2), \\
R_5 &= (2-z) I_1^1 - y I_1^2 + (2R(z) - (x+y)(2-z)) ((x+y) I_3^1 - y I_3^2) + \\
&\quad xy ((x+y) I_3^2 - y I_3^3).
\end{aligned}$$

9 Appendix C

The contribution to δ^{hard} from SD emission have the form:

$$\delta_{SD}^{hard} = \frac{\alpha}{2\pi a_0(y, z)} \int_0^{b(z)} dx J^{SD}(x, y, z), \tag{82}$$

where

$$J^{SD}(x, y, z) = Q_1 R_1 + y Q_2 I_1^0 + \frac{y^2}{2} Q_3 I_1^1 + \frac{y}{2} Q_4 R_4 + \frac{xy^2}{4} Q_5 R_5, \tag{83}$$

with R_i given in Appendix B and

$$\begin{aligned}
Q_1 &= -\frac{1}{4}y(x+y), \\
Q_2 &= \frac{1}{4}[2x(x+2y+z-2) + 3y(y+z-2)], \\
Q_3 &= -\frac{1}{8}[-8 + (z+y)(4+3x) - 2x + 3x^2], \\
Q_4 &= \frac{1}{8}[4y + 4x + 3xy], \quad Q_5 = -\frac{3}{4}.
\end{aligned} \tag{84}$$

The contribution to the total width have a form:

$$\delta^{SD} = \frac{\alpha}{2\pi} \frac{\int \int dydz (1 + \frac{\lambda_{\pm}}{r_{\pi}} R(z))^2 \int_0^N dx J^{SD}(x, y, z)}{\int \int dydz a_0(y, z) (1 + \frac{\lambda_{\pm}}{r_{\pi}} R(z))^2}. \tag{85}$$

Numerical estimation gives:

$$\delta^{SD} = -0.005. \tag{86}$$

10 Appendix D

The function Ψ , defined as

$$\Psi(y, z) = \int_{b_-(z)}^{b(z)} \frac{dt}{t} a_0(t, z) P^{(1)}\left(\frac{y}{t}\right), \tag{87}$$

contains a restriction on the domain of integration, namely t exceed y or equal to it, which is implied by the kernel $P^{(1)}(y/t)$. Explicit calculations give:

$$\begin{aligned}
\Psi_{<}(y, z) &= \int_{b_-(z)}^{b(z)} \frac{dt}{t} a_0(t, z) \frac{y^2 + t^2}{t(t-y)} = [R(z) - y(2-z)] \ln \frac{b(z)}{b_-(z)} + \\
&2a_0(y, z) \ln \frac{b(z) - y}{b_-(z) - y} + \frac{1}{2}(b(z)^2 - b_-(z)^2), \\
2\sqrt{r_e} < y < 1 - \sqrt{r_{\pi}}, \quad \Psi_{<}(y, z) &= 0, y > 1 - \sqrt{r_{\pi}}
\end{aligned} \tag{88}$$

and

$$\begin{aligned}
\Psi_{>}(y, z) &= \int_y^{b(z)} \frac{dt}{t} a_0(t, z) P^{(1)}\left(\frac{y}{t}\right) = a_0(y, z) \left[2 \ln \frac{b(z) - y}{y} + \frac{3}{2} \right] - \\
&\frac{1}{2}(b(z)^2 - y^2) + (b(z) - y)(2 - y - z + b_-(z)) + [R(z) - y(2-z)] \ln \frac{b(z)}{y}.
\end{aligned} \tag{89}$$

One can convince the validity of the relations:

$$j_0(y) = \int_{2\sqrt{r_\pi}}^{c(y)} dz \Psi_{<}(y, z) + \int_{c(y)}^{1+r_\pi} dz \Psi_{>}(y, z) \quad (90)$$

and

$$\int_0^{b_-(z)} dy \Psi_{<}(y, z) + \int_{b_-(z)}^{b(z)} dy \Psi_{>}(y, z) = 0. \quad (91)$$

The last relation demonstrates the KLN cancellation for the pion spectrum obtained by integration of the corrections over y in the interval $0 < y < b(z)$.

The explicit expressions for $j_1(y)$ and $j_2(y)$ are:(for $j_0(y)$ see (42)).

$$\begin{aligned} j_1(y) = & \frac{y^3(1-r_\pi-y)^3}{3(1-y)^2} \left(2 \ln \frac{1-r_\pi-y}{y} + \frac{3}{2} \right) + \\ & \frac{r_\pi^2}{3(1-y)^2} [3(1-y)(1+y^2) + r_\pi(y^3 + 3y - 2)] \ln \frac{1-y}{r_\pi} - \\ & \frac{1-r_\pi-y}{36(1-y)^2} [(1-y)^2 (43y^3 - 15y^2 - 3y - 1 + r_\pi(83y^2 + 26y + 11) + 3r_\pi^3) + \\ & r_\pi^2(31y^3 - 15y^2 - 39y + 47)] , \quad (92) \end{aligned}$$

$$\begin{aligned} j_2(y) = & \frac{y^4(1-r_\pi-y)^4}{12(1-y)^3} \left(2 \ln \frac{1-r_\pi-y}{y} + \frac{3}{2} \right) + \\ & \frac{r_\pi^2}{12(1-y)^3} \ln \frac{1-y}{r_\pi} \left[6(1+y^2)(1-y)^2 - 4r_\pi(1-y)(2y^3 - y^2 + 4y - 3) + \right. \\ & r_\pi^2(y^4 + 6y^2 - 8y + 3) \left. \right] + \frac{1-r_\pi-y}{720(1-y)^3} \left[-(1-y)^3(247y^4 - 88y^3 - 28y^2 - 8y - 3) - \right. \\ & r_\pi(1-y)^3(733y^3 + 341y^2 + 129y + 57) - \\ & r_\pi^2(1-y)(707y^4 - 808y^3 + 212y^2 - 408y + 717) \\ & \left. + r_\pi^3(173y^4 - 72y^3 - 492y^2 + 1048y - 477) - 12r_\pi^4(1-y)^3 \right]. \quad (93) \end{aligned}$$

11 Appendix E

Collection of the relevant formulae.

The Dalitz-plot distribution in the region D :

$$\begin{aligned} \frac{1}{CS_{EW}} \frac{d\Gamma}{dydz} = & \left(1 + \lambda_+ \frac{t}{m_\pi^2} \right)^2 \left(a_0(y, z) + \frac{\alpha}{\pi} \left[\frac{1}{2}(L_e - 1) \Psi_{>}(y, z) + a_0(y, z) Z_1 + \frac{1}{2} Z_2 \right] \right), \\ Z_1 = & \frac{3}{4} - \frac{\pi^2}{6} - \frac{3}{2} \ln y - \ln((b(z) - y)/y) - Li_2(1 - y) . \quad (94) \end{aligned}$$

The function Z_2 is defined in (28). Correction to the total width (we include the contribution of the region outside the region D), $\Gamma = \Gamma_0(1 + \delta)$:

$$1 + \delta = S_{EW} + \frac{\alpha}{\pi} \frac{1}{\int \int dz dy a_0(y, z) \left(1 + \frac{\lambda_{\pm}}{r_{\pi}} R(z)\right)^2} \left[\int_0^{1-r_{\pi}} I(y) \ln y dy + \int_{2\sqrt{r_{\pi}}}^{1+r_{\pi}} dz \left(1 + \frac{\lambda_{\pm}}{r_{\pi}} R(z)\right)^2 \left[\int_0^{b_-(z)} dy [-a_0(y, z) \ln \frac{b(z)-y}{b_-(z)-y} + (1/2)\tilde{Z}_2(y, z)] + \int_{b_-(z)}^{b(z)} dy [a_0(y, z)Z_1 + (1/2)Z_2] \right] \right], \quad (95)$$

with

$$\begin{aligned} \tilde{Z}_2(y, z) &= Rphot_{2A}(y, z) - 2Rphot_{1A}(y, z) + \int_{b_-(z)-y}^{b(z)-y} dx \mathcal{J}(x, y, z); \\ Rphot_{1A}(y, z) &= R(z) \ln \frac{b(z)}{b_-(z)} - y(b(z) - b_-(z)) \\ Rphot_{2A}(y, z) &= \int_{b_-(z)-y}^{b(z)-y} \frac{dx x}{(x+y)^2} a_0(x+y, z) = \\ &= (b(z) - b_-(z)) \left(1 - \frac{z}{2} + 2y\right) - (y(2-z) + R(z)) \ln \frac{b(z)}{b_-(z)}. \end{aligned} \quad (96)$$

The expression in big square brackets in right-hand side of (95) can be put in the form:

$$\int_{2\sqrt{r_{\pi}}}^{1+r_{\pi}} \phi_1(z) dz = \int_{2\sqrt{r_e}}^{1-r_{\pi}} f_1(y) dy = -0.035 \quad (97)$$

which results in $\delta = 0.02$. For the aim of comparison with E. Ginsberg result we must put here

$$\lambda_{\pm} = 0, \quad I(y) = j_0(y), \quad M_W = M_p, \quad (98)$$

as was mentioned above we have reasonable agreement with E. Ginsberg results. For the inclusive set-up of experiment (energy fraction of positron is not measured) we have for pion energy spectrum given above (40). When we restrict ourselves only by the region D the spectrum becomes dependent on $\ln(1/r_e)$:

$$\frac{1}{CS_{EW}} \frac{d\Gamma}{dz} = \left\{ \phi_0(z) + \frac{\alpha}{\pi} [(1/2)P(z)(\ln(1/r_e) - 1) + \int_{b_-(z)}^{b(z)} dy [\Psi_{>}(y, z) \ln y + a_0(y, z)Z_1 + \frac{1}{2}Z_2]] \right\} \left(1 + \frac{\lambda_{\pm}}{r_{\pi}} R(z)\right)^2, \quad (99)$$

with

$$P(z) = \frac{1}{6}b_-(z)^2(3b(z) + b_-(z)) \ln \frac{b(z)}{b_-(z)} + \frac{1}{3}(b(z) - b_-(z))^3 \ln \frac{b(z) - b_-(z)}{b(z)} - \frac{1}{6}b_-(z)(b(z) - b_-(z))(3b_-(z) + b(z)). \quad (100)$$

12 Appendix F

Our approach to study the radiative kaon decay has an advantage compared to the one used by E. Ginsberg – it has a simple interpretation of electron mass singularities based on Drell-Yan picture. The [2] approach to study noncollinear kinematics is more transparent than ours one. We remind the reader of some some topics of [2] paper. One can introduce the missing mass square variable

$$l = (p_\nu + k)^2/M^2 = A_\nu = (M - E_\pi - E_e)^2/M^2 - (\vec{p}_\pi + \vec{p}_e)^2/M^2. \quad (101)$$

the limits of this quantity variation at fixed y, z are pu by the last term: for collinear or anticollinear kinematics of pion and positron 3-momenta. Being expressed in terms of y, z they are (we consider the general point of Dalitz-plot and omit positron mass dependence):

$$0 < l < b_-(z)(b(z) - y), \quad (102)$$

for the y, z in the D region and

$$b(z)(b_-(z) - y) < l < b_-(z)(b(z) - y), \quad (103)$$

for the case when they are in the region A outside D :

$$0 < y < b_-(z), 2\sqrt{r_\pi} < z < 1 + r_\pi. \quad (104)$$

For our approach with separating the case of soft and hard photon emission we must modify the lower bound for l in the region D . It can be done using another representation of l :

$$l = x[1 - (y/2)(1 - C_e) - (z/2)(1 - \beta C_\pi)], \quad (105)$$

with C_e, C_π -the cosine of the angles between photon 3-momentum and positron and pion ones, $\beta = \sqrt{1 - m^2/E_\pi^2}$ -is the pion velocity. Maximum of this quantity is $b(z)$. Taking this into account we obtain for the region oh hard photon

$$x > 2\Delta\varepsilon/M = y\Delta, \Delta = \Delta\varepsilon/E_e \ll 1, \quad (106)$$

for the region D :

$$y\Delta < x < b(z) - y, \quad yb(z)\Delta < l < b_-(z)(b(z) - y); \quad (107)$$

and for region A :

$$b_-(z) - y < x < b(z) - y, \quad b(z)(b_-(z) - y) < l < b_-(z)(b(z) - y). \quad (108)$$

In particular for the collinear case we must choice $C_e = 1; C_\pi = -1$, which corresponds to $x + y < b(z)$. Let infer this condition using the NMS condition :

$$(P_k - p_e - p_\pi - k)^2/M^2 = R(z) - x - y + (xy/2)(1 - C_e) + (xz/2)(1 - \beta C_\pi) + (yz/2)(1 - \beta C_{e\pi}) = 0. \quad (109)$$

In collinear case we have $C_e = 1; C_\pi = C_{e\pi}$. From NMS condition we obtain $1 - \beta C_\pi = (2/z(x + y))(x + y - R(z))$. Using this value we obtain $l_{coll} = R(z)x/(x + y)$. Using further the relation $R(z) = b(z)b_-(z)$ we obtain again $x < b(z) - y$ in the case of emission along positron.

Comparing the phase volumes in general case calculated in our approach with using NMS condition with [2] approach we obtain the relation:

$$\int x dx \int \frac{dO_\gamma}{4\pi} = \int dl \int d\gamma, \int d\gamma = \int \frac{d^3 p_\nu}{E_\nu} \frac{d^3 k}{k_0} \frac{\delta^4(P - p_\nu - k)}{2\pi}. \quad (110)$$

The non-leading contribution arising from hard photon emission considered above:

$$I_{IB} = \int \frac{dx}{x} \int \frac{dO_\gamma}{4\pi} \mathcal{P}_{IB}, \quad (111)$$

with

$$\begin{aligned} \mathcal{P}_{IB} &= xG_1 \frac{\bar{A}_\nu}{A_e} + xG_2 + G_3 A_e + G_4 A_\nu + G_5 A_e A_\nu, \\ G_1 &= \frac{y}{4}(2 - y - x), \\ G_2 &= \frac{R(z)}{2(x + y)} + \frac{x^2}{2} + \frac{1}{2}x(z + 2y) + \frac{1}{4}(2z + 3y(y + z)) - 1; \\ G_3 &= -\frac{1}{8}x^2 - \frac{1}{8}x(2 + z + y) - \frac{1}{2}(y + z); \quad G_4 = \frac{1}{8}x(4 + y) + \frac{1}{8}y - 1; \\ G_5 &= -\frac{1}{4} \end{aligned} \quad (112)$$

(note that $G_i + Q_i = P_i$, see appendix B and C) can be transformed to the form:

$$\begin{aligned} I_{IB} &= (1/4) \int dl [4 - 2y - 4z - (1/4)R(z) + (1/4)l + y \ln \frac{(R(z) - l)^2}{l} - \\ &2 \ln \frac{y^2 R(z)^2}{l(l + y(2 - z))} + [z + (3/2)y(y + z) - 2 + (1/4)l(4 + y)]I_{10} - \\ &(1/2)I_{1-1} - ((1/2)l + y + z)I_{2-1} + I_z]. \end{aligned} \quad (113)$$

Here we use the list of integrals obtained in the paper of [2]:

$$\begin{aligned}
I_{mn} &= \int d\gamma \frac{1}{(kP_K/M^2)^m (kp_e/M^2)^n}; & (114) \\
I_{10} &= \frac{2}{s} \ln \frac{2-y-z+s}{2-y-z-s}; & I_{20} = 4/l; I_{00} = 1; \\
I_{-1,0} &= (2-y-z)/2; & I_{11} = \frac{4}{yl} \ln \frac{y^2}{l}; \\
I_{01} &= \frac{2}{R(z)-l} \ln \frac{(R(z)-l)^2}{lr_e}; \\
I_{1-1} &= \frac{R(z)(2-y-z) - (2+y-z)l}{s^2} + \\
&\frac{2l(y(2-y-z) - 2R(z) + 2l)}{s^3} \ln \frac{2-y-z+s}{2-y-z-s}; \\
I_{2-1} &= \frac{2(y(2-y-z) + 2l - 2R(z))}{s^2} + \\
&\frac{R(z)(2-y-z) - (2+y-z)l}{s^3} \ln \frac{2-y-z+s}{2-y-z-s}, \\
s &= \sqrt{(2-y-z)^2 - 4l}.
\end{aligned}$$

Besides we need two additional ones:

$$\begin{aligned}
I_e &= \int d\gamma \frac{1}{(kp_e/M^2)(2(kP_K/M^2) + y)} = \frac{2}{yR(z)} \ln \frac{y^2 R(z)^2}{l(l+y(2-z))r_e}; & (115) \\
I_z &= \int d\gamma \frac{1}{(kP_K/M^2)(2(kP_K/M^2) + y)} = \frac{4}{ys} \ln \frac{2l + ys + y(2-y-z)}{2l + ys - y(2-y-z)}.
\end{aligned}$$

One can see the cancellation of mass singularities (terms containing $\ln(1/r_e)$) in the expression I_{IB} .

Numerical calculations in agreement (within few percent) of this and the given above expressions.

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Радиационные поправки к распаду K_{e3}^{\pm}

Рассматриваются поправки в низшем порядке теории возмущений к процессу $K^{\pm} \rightarrow \pi^0 e^{\pm} \nu$, известному как K_{e3}^{\pm} -распад. Данный процесс является ключевым в определении V_{us} -элемента в матрице КKM. Учет радиационных поправок необходим для точного вычисления значения V_{us} . Вычисления подобного рода были проведены в 60-е годы [1,2], но они не согласуются между собой. Расчет в работе [2] зависит от ультрафиолетового обрезания. Необходимость получения точного значения V_{us} и противоречия между вышеупомянутыми работами являются мотивацией данной публикации.

В соответствии с рекомендацией А. Сирлина параметр ультрафиолетового обрезания выбран равным массе W -бозона. Установлен общий вид зависимости от малых лептонных масс. Дано простое объяснение сокращений расходимостей в лептонных членах пионного спектра. Точность результатов оценивается на уровне 1 %.

Величина полной ширины распада $\Gamma = \Gamma_0 (1 + \delta)$, где $\delta = 0,0200 \pm 0,0002$. При этом использовалось значение формфактора, равное $f_+(0) = 0,9842 \pm 0,0084$. Значение элемента матрицы КKM оказывается равным $|V_{us}| = 0,2172 \pm 0,0055$.

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Radiative Corrections to the K_{e3}^{\pm} Decay Revised

We consider the lowest order radiative corrections for the decay $K^{\pm} \rightarrow \pi^0 e^{\pm} \nu$, usually referred as K_{e3}^{\pm} decay. This decay is the best way to extract the value of the V_{us} element of the CKM matrix. The radiative corrections become crucial if one wants to get a precise value of V_{us} . The existing calculations were performed in the late 1960s [1,2] and are in disagreement. The calculation in [2] turns out to be ultraviolet cutoff sensitive. The necessity of precise knowledge of V_{us} and the contradiction between the existing results constitute the motivation of our paper.

We remove the ultraviolet cutoff dependence by using A. Sirlin's prescription; we set it equal to the W mass. We establish the whole character of small lepton mass dependence based on the renormalization group approach. In this way we can provide a simple explanation of Kinoshita–Lee–Nauenberg cancellation of singularities in the lepton mass terms in the total width and pion spectrum. We give an explicit evaluation of the structure-dependent photon emission based on ChPT in the lowest order. We estimate the accuracy of our results to be at the level of 1 %. The corrected total width is $\Gamma = \Gamma_0 (1 + \delta)$ with $\delta = 0.0200 \pm 0.0002$. The form factor value $f_+(0) = 0.9842 \pm 0.0084$ calculated in [14] leads to $|V_{us}| = 0.2172 \pm 0.0055$.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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