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ON THE ACCURACY OF THE SPACE LOCATION AND TENSION OF ELECTRODES IN MULTIWIRE CHAMBERS

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1. Introduction

Determination of permissible error in the space location and required value of electrode tension in multiwire chambers is very important, especially in developing large chambers. This problem is usually solved by means of special software tools like GARFIELD [1]. The latter allows full simulation of multiwire chamber operation. But in the course of designing and making a chamber the problem of determining of permissible errors and tension is often of independent importance. In this case, it is useful to have rather simple formulas, which allows calculation of the required values without special software tools. Such formulas were derived in [2, 3, 4] for chambers without potential wires. In this paper the chamber with anode and potential wires was considered. In addition, a formula for plane film cathode tension was derived.

2. Electrostatic field in the chamber

The field map in the chamber (Fig.1) could be obtained by the mirror-images method using the complex potential theory. In the same way as in [4], the system of chamber electrodes was replaced by an infinite grid of linear charges formed by multiple reflection of the wires with respect to the cathode planes, and the field within the chamber was determined by summing the fields from each charge. It was supposed that the wires are infinitely thin and typical cell dimensions, such as the distance between the

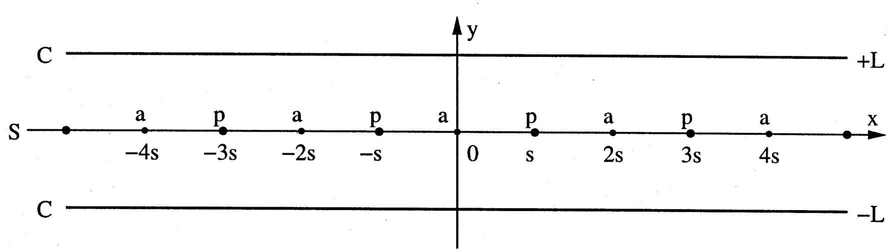

Figure 1: Layout of electrodes. C - cathode planes, S - sensitive (wire) plane, a - anode wires of radius R_a, p - potential wires of radius R_p, L - gap, s - wire spacing.

wires and the cachode plane L and the wire spacing s, are much smaller than the chamber size.

There are two ways to sum the fields. If we first sum over wire numbers and then over image numbers, the potential of the point (x, y) will be:

$$\begin{aligned} U(x,y) &= \frac{q_a}{4\pi\varepsilon_0}\sum_{k=-\infty}^{+\infty}(-1)^k \ln\frac{\sinh^2\frac{\pi L}{2s}(2k+1)+\sin^2\frac{\pi x}{2s}}{\sinh^2\frac{\pi L}{2s}(2k+y/L)+\sin^2\frac{\pi x}{2s}} \\ &- \frac{q_p}{4\pi\varepsilon_0}\sum_{k=-\infty}^{+\infty}(-1)^k \ln\frac{\cosh^2\frac{\pi L}{2s}(2k+1)-\sin^2\frac{\pi x}{2s}}{\cosh^2\frac{\pi L}{2s}(2k+y/L)-\sin^2\frac{\pi x}{2s}}, \end{aligned} \tag{1}$$

where q_a and q_p are the moduli of the linear density of the anode and potential wire charges.

This form of the potential is especially useful when we try to study the field close to the plane of the wires (small y), because the first term of the series with $k = 0$ gives a close approximation (with accuracy $\simeq$ 1%) for most practical problems. So, the capacitance per unit length of the anode wire is:

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{2s}{\pi R_a}\sinh\frac{\pi L}{2s}\right)-\lambda\ln\left(\cosh\frac{\pi L}{2s}\right)}, \tag{2}$$

where λ is the ratio of the moduli of the linear density of the potential and anode wire charges:

$$\lambda = q_p/q_a.$$

If the potential wire and the plane cathodes are under the same voltage, as often happens, then:

$$\lambda = \frac{\ln\left(\cosh\frac{\pi L}{2s}\right)}{\ln\left(\frac{2s}{\pi R_p}\sinh\frac{\pi L}{2s}\right)}. \tag{3}$$

If λ=0, which is equivalent to the absence of potential wires, formula (2) coincides with corresponding formula from [4].

The linear density of the anode wire charge is:

$$q_a = \frac{2\pi\varepsilon_0 U_0}{\ln\left(\frac{2s}{\pi R_a}\sinh\frac{\pi L}{2s}\right)-\lambda\ln\left(\cosh\frac{\pi L}{2s}\right)}, \tag{4}$$

where U_0 is the chamber voltage.

If it is necessary to study field variation in the chamber due to possible deflection of individual wires, it is more convenient to choose another way to sum the fields. First, one should sum over the image fields and then over the wire numbers. The potential of the point (x, y) is:

$$\begin{aligned} U(x,y) &= \frac{q_a}{4\pi\varepsilon_0}\sum_{k=-\infty}^{+\infty}\ln\frac{\cosh^2\frac{\pi}{4L}(2ks+x)-\sin^2\frac{\pi y}{4L}}{\sinh^2\frac{\pi}{4L}(2ks+x)+\sin^2\frac{\pi y}{4L}} \\ &- \frac{q_p}{4\pi\varepsilon_0}\sum_{k=-\infty}^{+\infty}\ln\frac{\cosh^2\frac{\pi}{4L}(s(2k-1)+x)-\sin^2\frac{\pi y}{4L}}{\sinh^2\frac{\pi}{4L}(s(2k-1)+x)+\sin^2\frac{\pi y}{4L}}. \end{aligned} \tag{5}$$

In this case it is impossible to take into account only the term of the series with $k = 0$. For more precise calculation (with accuracy $\simeq$ 1%) we should consider a few first terms.

3. Calculation of permissible error of space location of electrodes

Possible deflection of electrodes results in nonuniform distribution of linear density of the charge of the anode wires, which affects the gas gain factor.

Requirements to the unifomity of the gap (L), wire spacing (s), radius of the anode (R_a) and potential (R_p) wires are determined by differentiation of the expression for the charge derived from (5):

$$\begin{aligned} \frac{\Delta L}{L} &= -\left(\frac{\Delta q_a}{q_a}\right)\frac{2\pi\varepsilon_0}{C}\times \\ &\times\left((1+\lambda^2)(1+2\sum_{k=1}^{+\infty}\frac{\frac{\pi s}{L}k}{\sinh(\frac{\pi s}{L}k)})-2\lambda\sum_{k=-\infty}^{+\infty}\frac{\frac{\pi s}{L}(2k-1)}{\sinh\frac{\pi s}{L}(2k-1)}\right)^{-1}, \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{\Delta s}{s} &= \left(\frac{\Delta q_a}{q_a}\right)\frac{\pi\varepsilon_0}{C}\times \\ &\times\left((1+\lambda^2)\sum_{k=1}^{+\infty}\frac{\frac{\pi s}{L}k}{\sinh(\frac{\pi s}{L}k)}-\lambda\sum_{k=-\infty}^{+\infty}\frac{\frac{\pi s}{L}(2k-1)}{\sinh\frac{\pi s}{L}(2k-1)}\right)^{-1}, \end{aligned} \tag{7}$$

$$\frac{\Delta R_a}{R_a} = \left(\frac{\Delta q_a}{q_a}\right)\frac{2\pi\varepsilon_0}{C}, \tag{8}$$

$$\frac{\Delta R_p}{R_p} = \left(\frac{\Delta q_a}{q_a}\right)\frac{2\pi\varepsilon_0}{C\lambda^2}. \tag{9}$$

If the wire plane is displaced from the center of the gap and deflection is Δy, the linear density of the anode wire charge is changed with respect to formula (4). Relative variation is:

$$\frac{\Delta q_a}{q_a} = \left(\frac{\Delta y}{L}\right)^2 \frac{\pi C}{16\varepsilon_0} \times$$
$$\times \sum_{k=-\infty}^{+\infty} \left(\cosh^{-2} \frac{\pi}{4L}(2ks - R_a) - \lambda \cosh^{-2} \frac{\pi s}{4L}(2k-1)\right). \quad (10)$$

If only one anode wire is displaced relative variation of the linear density of the charge of this wire is:

$$\frac{\Delta q_a}{q_a} = \left(\frac{\Delta x_w}{L}\right)^2 \frac{\pi^2}{4\ln(\frac{4L}{\pi R_a})} \times$$
$$\times \left(\sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{L} k}{\sinh^2 \frac{\pi s}{L} k} - \lambda \sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{2L}(2k-1)}{\sinh^2 \frac{\pi s}{2L}(2k-1)}\right), \quad (11)$$

$$\frac{\Delta q_a}{q_a} = \left(\frac{\Delta y_w}{L}\right)^2 \frac{\pi^2}{4\ln(\frac{4L}{\pi R_a})} \times$$
$$\times \left(\frac{1}{2} + \sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{L} k}{\sinh^2 \frac{\pi s}{L} k} - \lambda \sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{2L}(2k-1)}{\sinh^2 \frac{\pi s}{2L}(2k-1)}\right), \quad (12)$$

where Δx_w and Δy_w are the corresponding deflections of the anode wire.

In Table 1 one can see the values calculated by formulas (6)-(12) for the chambers used in the DIRAC experiment [5]. In the same table the results for chambers without potential wires are shown. Caclulations were carried out on the assumption that permissible nonuniformity of the linear density of the anode wire charge is 1%, which corresponds to displacement of all chamber properties within the limits of 1% along the chamber voltage.

From Table 1 one can conclude that using of potential wire affects on required accuracy of electrode location. In particular the linear density of the anode wire charge in the chamber with potential wires is essentialy less sensitive to variation of the wire spacing within some limits ($\sim 30\%$), slightly depends on possible deflection of the anode wire parallel to the cathode planes and on the potential wire radius. These facts open up some additional possibilities. Rather slight dependence of the amplitude characteristics on the wire spacing may be used in several chambers blocks with common cathodes. In this case it is possible to combine chambers with different wire spacing into one block. It is also possible to make a fan-shaped or ring-shaped chamber. Of course, at the same time the drift function in different parts of the chamber will be different. Therefore, in order to use the time information from this chamber, one should take this difference into account.

4. Calculation of wire tension

Problems of wire tension in proportional chambers were mainly solved in [2, 3, 4]. Below one can see the formula for anode wire tension in a chamber with anode and potential wires, derived under the following assumptions:

1. For the boundary conditions of the differential equation described the string behaviour in a field of external permanent forces:

$$T\frac{d^2y}{dz^2} = -ay,$$

where a is a factor, which depends on field strength in the place of anode wire location, T is the wire tension, the y axis is normal

Table 1: Results of calculations.

	With potential wires	Without potential wires
	s=5mm, L=5mm, R_a=50μm, R_p=100μm	s=10mm, L=5mm, R_a=50μm
C, pF/m	11.6	11.2
$\Delta R_a/R_a$	4.8%	5.0%
$\Delta R_p/R_p$	100%	-
$\Delta L/L$	4.8%	3.1%
$\Delta s/s$	35%	8%
$\Delta y/L$	19%	19%
$\Delta x_w/L$	47%	33%
$\Delta y_w/L$	20%	13%
T, g	26	36
T_c, g/cm	56	87

to the chamber, z runs along the anode wire, we take the anode wire location error. In this case, as shown in [4], the requirement to the wire tension is essentially decreased independently of amount of error:

$$T = \frac{9}{4} T_0,$$

where T_0 is the anode wire tension under the zero boundary conditions.

2. Calculations shows that the largest force affects the wire when only one anode wire is displaced along the y axis.

The required tension of the anode wire of lenght l is:

$$T = 2 \cdot 10^{-6} \frac{U^2 C^2 l^2}{L^2} \left(\frac{1}{4} + \sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{L} k}{\sinh^2 \frac{\pi s}{L} k} - \lambda \sum_{k=1}^{\infty} \frac{\cosh \frac{\pi s}{2L}(2k-1)}{\sinh^2 \frac{\pi s}{2L}(2k-1)} \right), \quad (13)$$

where U is measured in kV, C in pF/m, L , s, l in cm, and T in g. λ is a coefficient defined in (3).

The value of the anode wire tension for one of the DIRAC drift chambers ($l = 80cm$) calculated by formula (13) and for the chamber of the same sizes, but without potential wires, calculated by formula from [4], are presented in Table 1. The chamber voltage in both cases is $U = 4.0kV$.

5. Calculation of the plane film cathode tension

It is possible to find the necessary tension of the film plane cathod in a rectangular wire chamber by solving the equation of a membrane in a field of external permanent forces:

$$T_c \Delta y(x, z) + f = 0,$$

where T_c is the film tension, y is the displacement from the equilibrium position under the action of force f. In our case f is electrostatic force acting on the film surface unit. The boundary conditions for this equation are:

$$\begin{cases} y(0, z) = y(a, z) = 0 \\ y(x, 0) = y(x, b) = 0, \end{cases}$$

where $(a \times b)$ is the film cathod size.

Solving this equation by the Green function method [6] and substituting

$$f = \varepsilon_0 {E_m}^2,$$

where

$$E_m = \frac{q_a(1-\lambda)}{4s\varepsilon_0} \sum_{k=-\infty}^{+\infty} (-1)^{k+1} \tanh \frac{\pi L}{s}(2k-1)$$

is the mean value of the electric field on the film surface, and taking the permissible gap uniformity error found in Section 3. on the assumption of $\frac{\Delta q_a}{q_a} = 1\%$ as a maximal displacement $y(a/2, b/2) = \Delta L_{max}$, we have for the film cathode tension:

$$T_c = \frac{16a^2}{\pi^3} \frac{\varepsilon_0 E_m^2}{(\Delta L_{max})} \sum_{k=0}^{+\infty} \frac{(-1)^{k+1}}{(2k-1)^3} \frac{\sinh^2 \frac{\pi b}{4a}(2k-1)}{\cosh \frac{\pi b}{2a}(2k-1)}. \tag{14}$$

The value of the film tension calculated by formula (14) for one of the DIRAC drift chamber measuring $40 \times 80 cm^2$ is given in Table 1.

Calculation shows that the formula gives the same result after replacement of a by b and the tension mainly depends on the smaller side of the chamber.

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О точности пространственного расположения
и натяжении электродов в многопроволочной камере

Получены формулы для определения допустимых неточностей расположения и натяжения электродов в многопроволочных камерах с чередующимися анодными и потенциальными проволоками. Приведены результаты расчетов для камер, используемых в эксперименте по измерению времени жизни $\pi^+\pi^-$-атомов (эксперимент DIRAC).

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On the Accuracy of the Space Location
and Tension of Electrodes in Multiwire Chambers

Formulas for determining the permissible error of space location and tension of electrodes in multiwire chambers with anode and potential wires are derived. The results of calculations for the chambers used in the experiment on $\pi^+\pi^-$-atom lifetime measurement (experiment DIRAC, PS-212 at CERN) are shown.

The investigation has been performed at the Dzhelepov Laboratory of Nuclear Problems, JINR.

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