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PROCESS $\gamma^*\gamma \to \sigma$ AT LARGE VIRTUALITY OF γ^*

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A recent analysis of the experimental data on the pseudoscalar mesons production in the transition process $\gamma^*\gamma \to P$ [1] (where γ^*, γ are photons with the large and small virtuality, respectively, and P is a pseudoscalar meson) revealed satisfactory agreement with the pertrubative QCD (pQCD) predictions [2, 3, 4, 5, 6] for the asymptotic behaviour of the $\gamma^*\gamma \to P$ form factor at large negative virtuality of one of the photons. However, for a correct description of the asymptotic behaviour of the transition form factor it is not enough to apply only the pQCD technique, and it is necessary to use the nonpertrubative sector for determining a distribution amplitude (DA) of quarks in a meson. In the lowest order of pQCD, the light-cone Operator Product Expansion (OPE) predicts the high Q^2 behaviour of the form factor as follows [2, 3]:

$$F_P(q_1^2, q_2^2, p^2 = 0)|_{Q^2\to\infty} = J_P(\omega)\frac{f_P}{Q^2} + O\left(\frac{\alpha_s}{\pi}\right) + O\left(\frac{1}{Q^4}\right), \qquad (1)$$

with the asymptotic coefficient given by

$$J_P(\omega) = \frac{4}{3}\int_0^1 \frac{dx}{1-\omega^2(2x-1)^2}\varphi_P^A(x), \qquad (2)$$

where $\varphi_P^A(x)$ is the leading-twist meson light-cone DA normalized by

$$\int_0^1 dx\, \varphi_P^A(x) = 1; \qquad (3)$$

α_s is the strong coupling constant; Q^2 and ω are, respectively, the total virtuality of the photons and the asymmetry in their distribution

$$Q^2 = -(q_1^2 + q_2^2) \geqslant 0, \quad \text{and} \quad \omega = (q_1^2 - q_2^2)/(q_1^2 + q_2^2), \quad |\omega| \leqslant 1, \qquad (4)$$

where q_1,q_2 are the momenta of photons; f_P is the meson weak decay constant (for the pion $f_\pi = 93$ MeV).

Unfortunately, the determination of $\varphi_P^A(x)$ is a rather nontrivial problem and can not be performed in the framework of only pQCD. At asymptotically high Q^2, DA is $\varphi_P^{A,\text{asympt}}(x) = 6x(1-x)$ and $J_P^{\text{asympt}}(|\omega| = 1) = 2$. The fit of the CLEO data[1] for the pion corresponds to $J_\pi^{\text{CLEO}}(|\omega| \approx 1) = 1.6 \pm 0.3$, indicating that already at moderately high momenta ($Q^2 = 8$ GeV^2) this value is not too far from its asymptotic limit.

In our previous work [7], it has been shown that the calculation of a process $\gamma^*\gamma \to P$ amplitude in the framework of a chiral quark model of the NJL type [8] is free from difficulties connected with the necessity of determining DA. Our results for the form factor asymptotics agreed with experiment [1], and with the predictions made in QCD sum rules [4, 5, 6] and the instanton induced quark model (IQM) [9]. Also, both the NJL and IQM model allow one to derive the meson DA.

Here, we continue the investigation started in [7] and consider the scalar isoscalar meson production through the process $\gamma^*\gamma \to \sigma$ where σ is associated with the lightest scalar isoscalar state $f_0(400-1200)$ [10], and the photon γ^* is off-shell. Direct observation of σ is hardly possible; however, in the low-mass region it can show itself as a resonance in the pion pair production [11].

In our paper, all the calculations are performed in the framework of the $SU(2) \times SU(2)$ NJL model. We derive a gauge-invariant expression for the amplitude of $\gamma^*\gamma \to \sigma$ and determine the form factor of the process for which the asymptotic behaviour is studied at large virtualities of γ^*. Notice that the asymptotic behaviour of the $\gamma^*\gamma \to \sigma$ and $\gamma^*\gamma \to \pi$ form factors is similar.

The interaction of mesons with u and d quarks is described by the following quark-meson Lagrangian:

$$L(\bar{q}, q, A, \sigma, \pi) = \bar{q}(x)(i\hat{\partial} - M - eQ\hat{A}(x) + g(\sigma(x) + i\gamma_5 \boldsymbol{\tau}\boldsymbol{\pi}(x)))q(x), \tag{5}$$

where $\bar{q}$ and q are the u and d quark fields

$$q(x) = \begin{pmatrix} u(x) \\ d(x) \end{pmatrix}; \tag{6}$$

M is the constituent mass matrix, $M = \mathrm{diag}(m_u, m_d)$ $(m_u \approx m_d \approx m)$; $\hat{A} = A_\mu \gamma^\mu$ is the photon field; σ stands for the scalar isoscalar meson; $\boldsymbol{\pi}$ is the pion triplet,$\boldsymbol{\tau}$ are the Pauli matrices ; e is e.m. charge $e^2/4\pi = 1/137$; $Q = 1/6(1+3\tau_3)$ is the charge operator; and g is the constant of a quark-meson interaction [1] defined by the relation $g^{-2} = 4I_2$ where

$$I_2 = \frac{3}{(2\pi)^4} \int \frac{d_E^4 k\, \theta(\Lambda^2 - k^2)}{(k^2 + m^2)^2}. \tag{7}$$

The subscript E in $d_E^4 k$ means that the integration is performed for momenta in the Euclidean metric. The cut-off Λ eliminates the UV divergence.

Using the Goldberger-Treiman relation $g = m/f_\pi$ and the relation $g_\rho = \sqrt{6}g$ [8], where $g_\rho \approx 6.1$ is the constant describing the decay $\rho \to 2\pi$, we determine the constituent quark mass $m = 234$ MeV and $\Lambda = 1050$ MeV.

[1]Following [7], we do not take into account $\pi - a_1$ transitions. Therefore, $g_\pi = g_\sigma = g$.

Let us note that the $\gamma^*\gamma \to \sigma$ amplitude also describes a two photon decay of σ. The amplitudes for radiative decays of scalar mesons (S) were obtained in [12]. The amplitude of the process $S \to \gamma\gamma$ has the form

$$A = e^2 T^{\mu\nu}_{S\to\gamma\gamma}\epsilon_\mu(q_1)\epsilon_\nu(q_2), \tag{8}$$

where ϵ are polarization vectors; q_1 and q_2 are photon momenta.

Up to the one-loop, the process is described by the diagrams in Fig. 1 that give for the tensor $T^{\mu\nu}_{S\to\gamma\gamma}$

$$T^{\mu\nu}_{S\to\gamma\gamma} = T^{(1)\mu\nu}_{S\to\gamma\gamma} + T^{(2)\mu\nu}_{S\to\gamma\gamma}, \tag{9}$$

where

$$T^{(1)\mu\nu}_{S\to\gamma\gamma} = C_S \left(q_1^\nu q_2^\mu - g^{\mu\nu}(q_1\cdot q_2)\right) J_m(q_1,q_2), \tag{10}$$

$$T^{(2)\mu\nu}_{S\to\gamma\gamma} = C_S B^{\mu\nu}(q_1,q_2), \tag{11}$$

and the transversality condition $\epsilon_\mu(q_1)q_1^\mu = \epsilon_\nu(q_2)q_2^\nu = 0$ is taken into account. Here

$$J_m(q_1,q_2) = -\frac{i}{(2\pi)^4}\int d^4k \frac{1}{D(0)D(q_1)D(-q_2)}, \tag{12}$$

$$B^{\mu\nu}(q_1,q_2) = -\frac{i}{(2\pi)^4}\int d^4k \frac{g^{\mu\nu}(m^2-k^2)+4k^\mu k^\nu}{D(0)D(q_1)D(-q_2)}, \tag{13}$$

$$D(q) = m^2 - (k+q)^2. \tag{14}$$

The constant C_S is to be specified for each type of scalar mesons. In the case of σ, it is

$$C_\sigma = \frac{40}{3} gm. \tag{15}$$

The term $T^{(1)\mu\nu}_{\sigma\to\gamma\gamma}$ has a gauge-invariant form. The Pauli-Villars regularization is necessary to restore the gauge invariance of $T^{(2)\mu\nu}_{\sigma\to\gamma\gamma}$ which leads to the following form of $B^{\mu\nu}$ (see Appendix for details) for $q_1^2 = q_2^2 = 0$:

$$B^{\mu\nu}(q_1,q_2) = 2\left(\frac{q_1^\nu q_2^\mu}{(q_1\cdot q_2)} - g^{\mu\nu}\right) m^2 (J_m(0,0) - J_m(q_1,q_2)). \tag{16}$$

In our paper, we apply the Pauli-Villars regularization with one subtraction for an arbitrary function $Y(m^2)$ as follows

$$Y^{\mathrm{PV}}(m^2) = Y(m^2) - Y(M^2). \tag{17}$$

Formally, as all integrals we calculate here converge, the regularization can be lifted off by reaching the $M^2 \to \infty$ limit. In our particular case, the regularized expression will, however, differ from the non-regularized one, the constant term, which violates gauge invariance, is thereby cancelled.

In the case of σ production through $\gamma^*\gamma \to \sigma$, the amplitude can also be divided in two parts

$$T^{\mu\nu}_{\gamma^*\gamma\to\sigma} = T^{(1)\mu\nu}_{\gamma^*\gamma\to\sigma} + T^{(2)\mu\nu}_{\gamma^*\gamma\to\sigma}, \tag{18}$$

where the first term is again gauge-invariant

$$T^{(1)\mu\nu}_{\gamma^*\gamma\to\sigma} = C_\sigma \left(q_1^\nu q_2^\mu - g^{\mu\nu}(q_1 \cdot q_2)\right) J_m(q_1, q_2), \tag{19}$$

whereas the second one becomes gauge invariant after the Pauli-Villars regularization is implemented and then lifted off. Moreover, as one of the photons is off-shell, $B^{\mu\nu}$ acquires an additional term proportional to q_1^2 (see Appendix):

$$\begin{aligned} T^{(2)\mu\nu}_{\gamma^*\gamma\to\sigma} &= C_\sigma \left(\frac{q_1^\nu q_2^\mu}{(q_1 \cdot q_2)} - g^{\mu\nu}\right) \Bigg[2m^2 \left(J_m(0,0) - J_m(q_1, q_2)\right) - \\ &- \frac{q_1^2}{2(q_1 \cdot q_2)} \frac{i}{(2\pi)^4} \int d^4k \frac{1}{D(q_1)} \left(\frac{1}{D(-q_2)} - \frac{1}{D(0)}\right)\Bigg]. \end{aligned} \tag{20}$$

After the gauge invariance of the amplitude is restored, we define the process form factor $F(q_1, q_2)$:

$$T^{\mu\nu}_{\gamma^*\gamma\to\sigma} = (q_1^\nu q_2^\mu - (q_1 \cdot q_2) g^{\mu\nu}) F(q_1, q_2), \tag{21}$$

where

$$\begin{aligned} F(q_1, q_2) = \frac{40 f_\pi g^2}{3} \Bigg[J_m(q_1, q_2) - \frac{2m^2}{(q_1 \cdot q_2)} (J_m(q_1, q_2) - J_m(0,0)) - \\ - \frac{q_1^2}{2(q_1 \cdot q_2)^2} \frac{i}{(2\pi)^4} \int d^4k \frac{1}{D(q_1)} \left(\frac{1}{D(-q_2)} - \frac{1}{D(0)}\right)\Bigg]. \end{aligned} \tag{22}$$

After the change of variable $k = k' - (q_1 - q_2)/2$ (in the following we omit the prime symbol), the form factor takes the form

$$\begin{aligned} F(q_1, q_2) &= \frac{5 f_\pi g^2}{6\pi^2} \int \frac{d^4k}{i\pi^2} \times \\ &\times \Bigg[\frac{1 - 2m^2/(q_1 \cdot q_2)}{D(-p/2) D(p/2) D(-(q_1 - q_2)/2)} + \frac{2m^2}{(q_1 \cdot q_2)} \frac{1}{D(0)^3} + \\ &+ \frac{q_1^2}{2(q_1 \cdot q_2)^2} \frac{1}{D(p/2)} \left(\frac{1}{D(-p/2)} - \frac{1}{D(-(q_1 - q_2)/2)} \right)\Bigg], \end{aligned} \tag{23}$$

where $p = q_1 + q_2$. It is convenient to consider $F(q_1, q_2)$ in terms of p^2 (notice that $p^2 = M_\sigma^2$) , q_1^2, and q_2^2. Further, we restrict ourselves to the case of $q_2^2 = 0$; thus, we have two independent variables p^2 and q_1^2. The kinematics of the process under investigation corresponds to large negative q_1^2, and we introduce, for convenience, a positive magnitude Q^2 defined as $Q^2 = -q_1^2$. After that, the form factor $F(q_1, q_2)$ can be considered as depending on p^2 and Q^2

$$\tilde{F}(p^2, Q^2) = F(q_1, q_2)|_{q_2^2=0; q_1^2<0}, \tag{24}$$

$$\begin{aligned} \tilde{F}(p^2, Q^2) &= \frac{5 f_\pi g^2}{6\pi^2} \left[\left(1 - \frac{4m^2}{p^2+Q^2}\right) K_\Lambda(p^2, Q^2) + \frac{4m^2}{p^2+Q^2} L_\Lambda - \right. \\ & \left. - \frac{2Q^2}{(p^2+Q^2)^2} \left(M_\Lambda(p^2) - N_\Lambda(p^2, Q^2)\right) \right], \end{aligned} \tag{25}$$

where

$$K_\Lambda(p^2, Q^2) = \int^\Lambda \frac{d^4k}{i\pi^2} \frac{1}{D(-p/2)D(p/2)D(-(q_1-q_2)/2)}, \tag{26}$$

$$L_\Lambda = \int^\Lambda \frac{d^4k}{i\pi^2} \frac{1}{D(0)^3}, \tag{27}$$

$$M_\Lambda(p^2) = \int^\Lambda \frac{d^4k}{i\pi^2} \frac{1}{D(-p/2)D(p/2)}, \tag{28}$$

$$N_\Lambda(p^2, Q^2) = \int^\Lambda \frac{d^4k}{i\pi^2} \frac{1}{D(-(q_1-q_2)/2)D(p/2)}. \tag{29}$$

In all integrals, we implement the UV cut-off Λ that constrains quark momenta to the domain where chiral symmetry is spontaneously broken and the bosonization of quarks takes place(see (7)).

An analogous method has been used in [7]. One can compare (25) with the pion form factor obtained in [7] for $q_1^2 \neq 0$, $q_2^2 \neq 0$

$$F_\pi(p^2, q_1^2, q_2^2) = C_\pi \int \frac{d^4k}{i\pi^2} \frac{1}{D(-p/2)D(p/2)D(-(q_1-q_2)/2)}, \tag{30}$$

where C_π is a constant. In [7], the following expression was obtained for $K_\Lambda(p^2, Q^2)$ after introducing Feynman parameters, integrating over the angles, and changing the variables $k^2 = u$:

$$K_\Lambda(p^2,Q^2) = \int\limits_0^{\Lambda^2} \frac{du\,u}{m^2+u-\frac{p^2}{4}} \times \tag{31}$$

$$\times \int\limits_0^1 dx \left[\frac{1}{\sqrt{b^2-a_+}\left(b+\sqrt{b^2-a_+}\right)} + \frac{1}{\sqrt{b^2-a_-}\left(b+\sqrt{b^2-a_-}\right)} \right].$$

Here

$$\begin{aligned} b &= m^2+u+\frac{1}{2}xQ^2-\frac{1}{4}\left(1-2x\right)p^2, \\ a_\pm &= 2uxQ^2\left(x\pm(1-x)\right)-\left(1-2x\right)up^2. \end{aligned} \tag{32}$$

Analogously, one can write the integrals $N_\Lambda(p^2,Q^2)$,$M_\Lambda(p^2)$ as follows:

$$N_\Lambda(p^2,Q^2) = 2\int\limits_0^{\Lambda^2} du\,u \int\limits_0^1 dx \frac{1}{\sqrt{b^2-a_+}\left(b+\sqrt{b^2-a_+}\right)}, \tag{33}$$

$$M_\Lambda(p^2) = 2\int\limits_0^{\Lambda^2} \frac{du\,u}{b_0\left(b_0+\sqrt{b_0^2-a_{+0}}\right)}. \tag{34}$$

Here

$$b_0 = b|_{x\to 0} = m^2+u-\frac{1}{4}p^2, \quad a_{+0} = a_+|_{x\to 0} = -up^2. \tag{35}$$

Now we can calculate the asymptotics for $\tilde{F}(p^2,Q^2)$ when $Q^2 \to \infty$. Using the approximation described in [7] and expanding in a series of $1/Q^2$, we obtain for $K_\Lambda(p^2,Q^2)$, L_Λ , $M_\Lambda(p^2)$ and $N_\Lambda(p^2,Q^2)$ at $p^2=0$

$$K_\Lambda(p^2=0,Q^2) = \frac{1}{Q^2}\int\limits_0^{\Lambda^2/m^2} \frac{du}{1+u}\ln\left(1+2u\right) + O\left(\frac{1}{Q^4}\right), \tag{36}$$

$$L_\Lambda = \frac{1}{m^2}\int\limits_0^{\Lambda^2/m^2} \frac{du\,u}{(1+u)^3}, \tag{37}$$

$$M_\Lambda(p^2=0) = \int\limits_0^{\Lambda^2/m^2} \frac{du\,u}{(1+u)^2}, \tag{38}$$

$$N_\Lambda(p^2=0,Q^2) = \frac{2m^2}{Q^2}\int\limits_0^{\Lambda^2/m^2} du\,\ln\left(1+u\right) + O\left(\frac{1}{Q^4}\right). \tag{39}$$

Note that the last term is of the order of $1/Q^4$. As a result, the first three terms give us the following asymptotics for the form factor $\tilde{F}(p^2 = 0, Q^2)$:

$$\tilde{F}(p^2 = 0, Q^2) \quad = \quad J_S \frac{f_\pi}{Q^2} + O\left(\frac{1}{Q^4}\right), \tag{40}$$

$$J_S \quad \approx \quad 3.28 + 0.96 - 2.22 = 2.02. \tag{41}$$

We also calculate numerically $Q^2\tilde{F}(p^2, Q^2)$ for intermediate values of Q^2 and for a different choice of p^2. The curves drawn for $p^2 = 0$, $p^2 = M_\sigma^2 = (0.45\ \mathrm{GeV})^2$ are shown in Fig. 2. The value $M_\sigma = 450$ MeV is consistent with recent experimental [13] and theoretical [14] data. A comparison of the obtained results for the function $Q^2\tilde{F}(p^2, Q^2)$ with an analogous function for the pion [7] shows similarity in their asymptotic behaviour. However, some differences take place at intermediate values of Q^2. Indeed, the pion function grows monotonically in the whole region of Q^2. At the beginning it grows rapidly and after $Q^2 > 2\mathrm{GeV}^2$ it slowly approaches the asymptotic value. In the case of the σ meson the function $Q^2\tilde{F}(p^2, Q^2)$ also experiences a rapidly growth up to approximately $2\mathrm{GeV}^2$; after that it is slowly decreasing to an asymptotic value. Asymptotic values for these functions are practically equal to each other.

In this work, we have shown that in the framework of the $SU(2) \times SU(2)$ chiral quark model of the NJL type, it is possible to describe the behaviour of the $\gamma^*\gamma \to \sigma$ form factor in a wide region of Q^2. The exact expression for the form factor of the process is obtained, and its asymptotic behaviour is investigated. A comparison of the π (see [7]) and σ form factors reveals similarity in their asymptotic behaviour, which may be understood as a consequence of chiral symmetry.

Our results can be useful in investigations of the processes where pion pairs in the S-wave are produced in two photon collisions [15]. Indeed, in these processes, as a rule, the σ-pole diagram plays the dominant role [8, 11]. Some data on the pair pion production is already available [16]. Also, one can find a discussion on this topic in [17].

Notice also that the information concerning the amplitudes $\gamma^*\gamma \to \pi$, $\gamma^*\gamma \to \sigma$ can allow us to calculate corrections to the muon anomalous magnetic moment from the processes $\gamma^*\gamma \to \pi \to \gamma^*\gamma^*$,$\gamma^*\gamma \to \sigma \to \gamma^*\gamma^*$, where γ^* interact with the muon. Last year, this topic has been discussed in various papers (see [18]).

Further, we plan to calculate the transition form factor for two off-shell photons with arbitrary virtualities in the framework of both the NJL and IQM model, and it allow us also to define DA.

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Appendix

Let us rewrite the expression for $J_m(q_1, q_2)$ (12), using the Feynman parameterization

$$J_m(q_1,q_2) = -\frac{i}{(2\pi)^4}\int d^4k \frac{1}{D(0)D(q_1)D(-q_2)} = \tag{42}$$

$$= -\frac{2i}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \int \frac{d^4k}{(m^2-k^2-2k(q_1x-q_2y)-q_1^2x)^3}.$$

Shifting the variable $k \to k-(q_1x-q_2y)$ and integrating over momenta and y, we obtain

$$J_m(q_1,q_2) = -\frac{1}{(4\pi)^2}\frac{1}{2(q_1\cdot q_2)}\int\limits_0^1 \frac{dx}{x}\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right). \tag{43}$$

Let us rewrite the expression for $B^{\mu\nu}$ (see (13)), using the Feynman parameters

$$B^{\mu\nu}(q_1,q_2) = -\frac{i}{(2\pi)^4}\int d^4k \frac{g^{\mu\nu}(m^2-k^2)+4k^\mu k^\nu}{D(0)D(q_1)D(-q_2)} = \tag{44}$$

$$= -\frac{2i}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \int d^4k \frac{g^{\mu\nu}(m^2-k^2)+4k^\mu k^\nu}{(m^2-k^2-2k(q_1x-q_2y)-q_1^2x)^3}.$$

Again, after the change of variables, we have for $B^{\mu\nu}$

$$B^{\mu\nu}(q_1,q_2) = -\frac{2i}{(2\pi)^4}\int\limits_0^1 dx \int\limits_0^{1-x} dy \int d^4k \times \tag{45}$$

$$\times \frac{A^{\mu\nu}}{(m^2-k^2-q_1^2x(1-x)-2(q_1\cdot q_2)xy)^3},$$

$$\begin{aligned} A^{\mu\nu} &= g^{\mu\nu}(m^2-k^2+2k\cdot(q_1x-q_2y)-q_1^2x^2+2(q_1\cdot q_2)xy)+ \\ &+4(k^\mu k^\nu - k^\mu(q_1x-q_2y)^\nu-(q_1x-q_2y)^\mu k^\nu + \\ &+(q_1x-q_2y)^\mu(q_1x-q_2y)^\nu). \end{aligned} \tag{46}$$

The integration over momenta gives

$$B^{\mu\nu}(q_1,q_2) = \frac{g^{\mu\nu}}{(4\pi)^2}\int\limits_0^1 dx \int\limits_0^{1-x} dy \frac{m^2-q_1^2x^2+2(q_1\cdot q_2)xy}{m^2-q_1^2x(1-x)-2(q_1\cdot q_2)xy} -$$

$$-\frac{4q_1^\nu q_2^\mu}{(4\pi)^2}\int\limits_0^1 dx \int\limits_0^{1-x} dy \frac{xy}{m^2-q_1^2x(1-x)-2(q_1\cdot q_2)xy}. \tag{47}$$

Let us denote the denominator in (47) as Z

$$Z = m^2 - q_1^2 x(1-x) - 2(q_1 \cdot q_2)xy. \tag{48}$$

Using this equation, we substitute the mass squared m^2 in the numerator of (47)

$$B^{\mu\nu}(q_1, q_2) = \frac{g^{\mu\nu}}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{Z + 4(q_1 \cdot q_2)xy + q_1^2 x(1-2x)}{Z} - \\ - \frac{q_1^\nu q_2^\mu}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{4xy}{Z}. \tag{49}$$

Now it is easy to separate the gauge-invariant and noninvariant parts in (49)

$$\begin{aligned} B^{\mu\nu}(q_1, q_2) &= \frac{g^{\mu\nu}(q_1 \cdot q_2) - q_1^\nu q_2^\mu}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{4xy}{Z} \\ &+ \frac{g^{\mu\nu}}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{Z + q_1^2 x(1-2x)}{Z}. \end{aligned} \tag{50}$$

The first term in (50) is gauge-invariant, while the other is noninvariant and can also be divided in two terms

$$\frac{1}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy + \frac{q_1^2}{(4\pi)^2} \int_0^1 dx \int_0^{1-x} dy \frac{x(1-2x)}{Z}. \tag{51}$$

The first term in (51) is a constant and is cancelled by the Pauli-Villars regularization. Integrating over y in the second term, we obtain

$$-\frac{1}{(4\pi)^2} \frac{q_1^2}{2(q_1 \cdot q_2)} \int_0^1 dx (1-2x) \ln\left(\frac{m^2 - (q_1^2 + 2(q_1 \cdot q_2))x(1-x)}{m^2 - q_1^2 x(1-x)}\right). \tag{52}$$

After the substitution $1 - 2x = z$, one can see that this part equals zero

$$-\frac{1}{(4\pi)^2} \frac{q_1^2}{4(q_1 \cdot q_2)} \int_{-1}^1 dz z \ln\left(\frac{4m^2 + (q_1^2 + 2(q_1 \cdot q_2)(1-z^2)}{4m^2 + q_1^2(1-z^2)}\right) = 0. \tag{53}$$

Therefore, the expression for $B^{\mu\nu}$ becomes gauge invariant after the Pauli-Villars regularization is implemented and then lifted off

$$B^{\mu\nu}(q_1,q_2) = (q_1^\nu q_2^\mu - (q_1\cdot q_2)g^{\mu\nu})(I(m^2)-I(M^2))|_{M^2\to\infty}, \quad (54)$$

$$I(m^2) = -\frac{1}{(4\pi)^2}\int_0^1 dx \int_0^{1-x} dy \frac{4xy}{Z}. \quad (55)$$

Integrating over y in $I(m^2)$ we obtain

$$I(m^2) = \frac{1}{(4\pi)^2}\frac{1}{(q_1\cdot q_2)} + \quad (56)$$
$$+\frac{1}{(4\pi)^2}\frac{m^2}{(q_1\cdot q_2)^2}\int_0^1 \frac{dx}{x}\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right) -$$
$$-\frac{1}{(4\pi)^2}\frac{q_1^2}{(q_1\cdot q_2)^2}\int_0^1 dx(1-x)\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right).$$

Now, let us show that $I(m^2)$ can be expressed through the integral

$$\frac{i}{(2\pi)^4}\int d^4k \frac{D(0)-D(-q_2)}{D(0)D(q_1)D(-q_2)} =$$
$$= \frac{2i}{(2\pi)^4}\int_0^1 dx \int_0^{1-x} dy \int d^4k \frac{(k\cdot q_2)}{(m^2-k^2-2k(q_1x-q_2y)-q_1^2x)^3} =$$
$$= \frac{1}{(4\pi)^2}\int_0^1 dx \ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right). \quad (57)$$

Noting that the following identity is satisfied:

$$\frac{1}{(4\pi)^2}\int_0^1 dx(1-x)\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right) = \quad (58)$$
$$\frac{1}{2(4\pi)^2}\int_0^1 dx\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right),$$

we obtain for (57)

$$\frac{i}{(2\pi)^4}\int d^4k \frac{D(0)-D(-q_2)}{D(0)D(q_1)D(-q_2)} = \tag{59}$$
$$\frac{2}{(4\pi)^2}\int_0^1 dx(1-x)\ln\left(\frac{m^2-p^2x(1-x)}{m^2-q_1^2x(1-x)}\right).$$

Using (43) and (60), we rewrite $I(m^2)$ as follows:

$$\begin{aligned} I(m^2) &= \frac{1}{(4\pi)^2}\frac{1}{(q_1\cdot q_2)} - \frac{2m^2}{(q_1\cdot q_2)}J_m(q_1,q_2) - \\ &- \frac{q_1^2}{2(q_1\cdot q_2)^2}\frac{i}{(2\pi)^4}\int d^4k \frac{D(0)-D(-q_2)}{D(0)D(q_1)D(-q_2)}. \end{aligned} \tag{60}$$

Formally, the first term in (60) can be rewritten as

$$\frac{2m^2}{(q_1\cdot q_2)}J_m(0,0) \tag{61}$$

because

$$J_m(0,0) = -\frac{i}{(2\pi)^4}\int \frac{d^4k}{D(0)^3} = \frac{1}{2(4\pi)^2m^2}. \tag{62}$$

Finally, the regularized expression for $B^{\mu\nu}(q_1,q_2)$ has the form

$$\begin{aligned} B^{\mu\nu}(q_1,q_2) &= (q_1^\nu q_2^\mu - (q_1\cdot q_2)g^{\mu\nu}) \times \\ &\times \left(-\frac{2m^2}{(q_1\cdot q_2)}J_m(q_1,q_2) + \frac{2m^2}{(q_1\cdot q_2)}J_m(0,0) - \right. \\ &\left. - \frac{q_1^2}{2(q_1\cdot q_2)^2}\frac{i}{(2\pi)^4}\int d^4k \frac{D(0)-D(-q_2)}{D(0)D(q_1)D(-q_2)}\right). \end{aligned} \tag{63}$$

Equation (63) is expressed in terms of formal integrals. This gives us an advantage of further implementing a regularization different from the Pauli-Villars scheme.

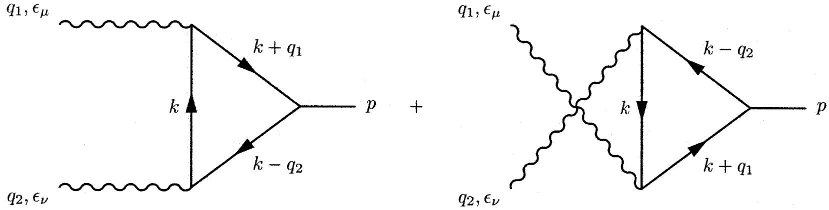

Figure 1: Diagrams contributing to the amplitude of the process $\gamma^*\gamma \to \sigma$

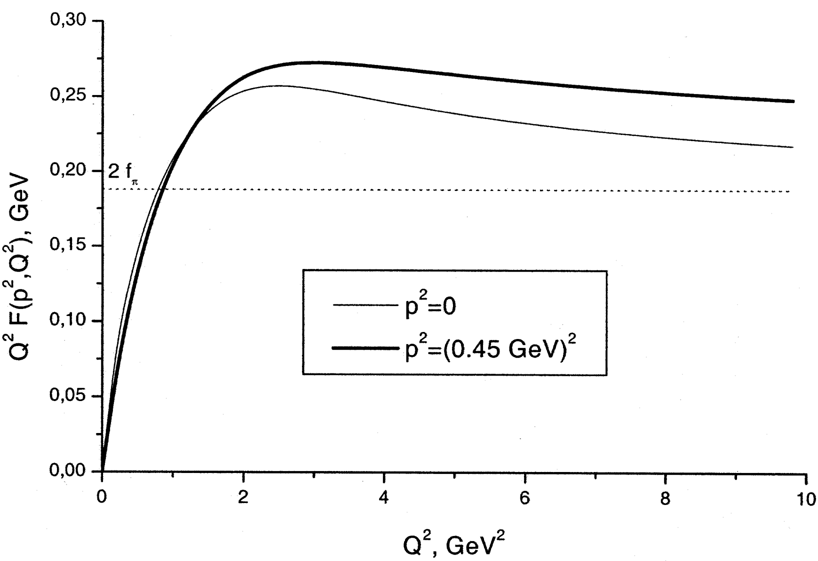

Figure 2: The form factor of the process $\gamma^*\gamma \to \sigma$ multiplied by Q^2 at Q^2 from 0 to 10 GeV^2. The thick curve is for $p^2 = (0.45 \text{ GeV})^2$ and the thin is for $p^2 = 0$. We compare these curves with the theoretical limit for the pion transition form factor: $2f_\pi$

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Процесс $\gamma^*\gamma \to \sigma$ при большой виртуальности γ^*

Процесс $\gamma^*\gamma \to \sigma$ изучен в рамках киральной $SU(2)\times SU(2)$-модели Намбу–Иона-Лазинио. Формфактор процесса получен для произвольной виртуальности γ^* в евклидовой кинематической области. Асимптотическое поведение формфактора сходно с асимптотическим поведением формфактора процесса $\gamma^*\gamma \to \pi$.

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Process $\gamma^*\gamma \to \sigma$ at Large Virtuality of γ^*

The process $\gamma^*\gamma \to \sigma$ is investigated in the framework of the $SU(2)\times SU(2)$ chiral Nambu–Jona-Lasinio model. The form factor of the process is derived for arbitrary virtuality of γ^* in the Euclidean kinematic domain. The asymptotic behaviour of this form factor resembles the asymptotic behaviour of the $\gamma^*\gamma \to \pi$ form factor.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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