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PION DAMPING WIDTH FROM $SU(2) \times SU(2)$ NAMBU–JONA-LASINIO MODEL

1 Introduction

In the atmosphere of growing interest to the quark gluon plasma, observations of the dileptons (electron-positron pairs) production in relativistic heavy-ion collisions draw more and more attention of physicists making efforts to find out more about hadron physics in extreme conditions of temperature and density. Experimental data have already been coming, and some recent analyses on the dilepton production in Pb+Au (158 GeV/u) collisions are available, e.g., from CERES collaboration [1, 2].

Insofar as the description of hot and dense media directly from QCD is not yet available, the modelling of such processes is an urgent problem. Recently, an attempt was made to account for the observed dilepton production rate [1, 2], using the simple Bjorken scenario of the space-time evolution [3, 4] of an ultrarelativistic heavy-ion collision and the vector dominance model expression for the pion electromagnetic form factor [4]. In [4], it was concluded that the experimental dilepton spectrum could not be explained without assuming a modification of the ρ meson mass and width in medium.

Besides the ρ meson mass and width modification, the same is expected for pions. Pions dominate in heavy-ion collisions and it is very important to study how their properties change with increasing temperature and density, especially when approaching a phase transition in a hot matter. The behaviour of the pion in extreme conditions has already been investigated in models of the NJL type [5, 6, 7, 8]; however, the mean-field approximation, which is common in NJL [9, 10], does not take into account collisions in the pion gas. However, as we show in our paper, collisions can give a significant contribution to the pion width in extreme conditions.

For light particles (like pions) the density increases with temperature approximately as T^3; thus, one expects that the density is large near the supposed phase transition temperature T_c, and collisions of particles occur at a much higher rate than in cold matter. Collisions lead to shorter lifetimes (or larger widths) of hadronic states at extreme environmental conditions. In terms of hadron correlators, an additional imaginary contribution to the hadron self-energy operator comes form collision integrals [11], thereby broadening all particles in the hadron gas. The resulting width is closely related to the process of returning a disturbed many-particle system to an equilibrium state: damping. Further, the width thus formed is called the damping width.

There are several approaches to calculate the collision integrals. In our work, we follow the prescription given by Kadanoff and Baym [11]. This way, the contribution to the hadron self-energy from collision integrals has a straightforward interpretation: its reciprocal is the average time between two consecutive collisions, the lifetime. To estimate the average lifetime of a hadron state, one needs to know cross-sections for different collision processes averaged over the density of particles with the Bose amplification and Pauli suppression taken into account.

One can try to find the modified self-energy of a particle using a self-consistent functional formalism, as it is described, e.g., by Hees and Knoll in [12]. However, in [12], the pion damping width was treated as an external parameter and was not fixed. Our purpose is to find an estimate for the pion damping width from a simple quark model and investigate possible implications of the pion broadening. We calculate cross sections in the framework of the bosonized NJL model with the infrared cut-off [8]. The meson spectral functions are

chosen to be of the Breit-Wigner form. Then we iterate the equation for the width until a self-consistent solution is found.

The structure of our paper is as follows. In Section 2, we introduce the pion damping width. In Section 3, the cross section of the process $\pi\pi \to \pi\pi$ is derived. The numerical results are given in Sect. 4. The discussion is given in the last Section.

2   Pion lifetime in hot matter

In the vacuum, the lifetime of a particular state is determined by the probability of its decay into other states. Thus, a stable particle gains no width in the vacuum at all, whereas in dense medium, a particle can strike another one and change its initial state. So the lifetime of a particular state is determined by its collision rate. There are also inverse processes that can restore the decayed state due to the interaction of particles in medium, and then one can find this state again. This process prolongs the lifetime of the state. Therefore, the average lifetime of a particular state is determined by the direct and inverse processes.

The in-medium hadron properties are usually investigated in terms of correlators. A two-point correlator can then be expressed through a spectral function which is just the correlator's imaginary part. For a stable particle, whose correlator has only one pole, the spectral function is simply a delta function, while for a state whose energy is spread, the spectral function is a continuous function of energy and momentum.

As discussed in [11], the probability that, after putting a particle with momentum p to the gas at the time t, removing the particle with the same momentum at the time t', one can find the gas at the same

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state as in the beginning decays as $e^{-\Gamma(t'-t)}$ where Γ is a constant determining the decay rate and is usually called the width. If one chooses the spectral function in the Breit-Wigner form

$$A(s) = \frac{M\Gamma}{(s - M^2)^2 + M^2\Gamma^2}, \tag{1}$$

where M is the mass of the state, the probability indeed decays as $e^{-\Gamma(t'-t)}$ (see [11]). Here, the width is a measure of the dispersion of the state energy. In general, the width is a function of energy and momentum, and one would finally come to a set of functional equations for the width, which are difficult to solve. In our work, we approximate the pion width by a constant and obtain a simple equation to be solved by iterations. However, the width is supposed to depend on the temperature and density.

The pion damping width (or lifetime τ) can be calculated by the following formula from [11]:

$$\Gamma(p) = \tau^{-1}(p) = \Sigma^>(p) - \Sigma^<(p), \tag{2}$$

where

$$\Sigma^<(p) = \int\limits_{p_1}\int\limits_{p_3}\int\limits_{p_4} (2\pi)^4 \delta_{p_1,p;p_3,p_4} |\mathcal{T}|^2 G^>(p_1) G^<(p_3) G^<(p_4), \tag{3}$$

$$\Sigma^>(p) = \int\limits_{p_1}\int\limits_{p_3}\int\limits_{p_4} (2\pi)^4 \delta_{p_1,p;p_3,p_4} |\mathcal{T}|^2 G^<(p_1) G^>(p_3) G^>(p_4), \tag{4}$$

in accordance with the notation given in [11]. Here, $\mathcal{T}$ is the process amplitude, $G_i^>(p) = [1 + n_i(\vec{p}, s_i, T)] A_i(p^2); \quad G_i^<(p) = n_i(\vec{p}, s, T) A_i(p^2)$, with n_i being the boson occupation numbers

$$n_i(\vec{p}, s_i, T) = \left[\exp\left(\frac{\sqrt{\vec{p}^2 + s_i}}{T}\right) - 1\right]^{-1} \tag{5}$$

and $A_i(p^2)$ the spectral function of the ith state; we also use the notation $\int\limits_{p} = \int \frac{d^4p}{(2\pi)^4}$ where $\delta_{p_1,p_2;p_3,p_4} = \delta(p_1 + p_2 - p_3 - p_4)$. The

integration is performed in the four-dimensional momentum space over the momenta p_1, p_2, p_3, p_4, with the indices 1 and 2 corresponding to the initial states, and 3 and 4 to the final ones.

For the inverse lifetime of a pion, one thus obtains

$$\tau^{-1} = \Gamma = \int \frac{d^3\vec{p}}{(2\pi)^3} \int ds_1 v_{\text{rel}} A_\pi(s_1)$$
$$\times [n_\pi(\vec{p_1}, s_1, T) \sigma^{\text{dir}*}(s; s_1, s_2)$$
$$- (1 + n_\pi(\vec{p_1}, s_1, T)) \sigma^{\text{inv}*}(s; s_1, s_2)], \qquad (6)$$

where v_{rel} is the relative velocity of particles 1 and 2, and $\sigma^*(s; s_1, s_2)$ is the averaged cross section with the off-shellness of the final state taken into account

$$\sigma^*(s; s_1; s_2) = \int ds_3 \int ds_4 A_\pi(s_3) A_\pi(s_4) \sigma(s; s_1, s_2, s_3, s_4). \qquad (7)$$

What one needs then is the cross sections of the processes under investigation. The amplitudes of these processes can be calculated in an effective model of pion interaction. In the next section, we calculate the amplitudes and cross sections in the framework of the NJL model with the infrared cut-off [8].

3 Pion-pion scattering amplitude and cross section from NJL model

3.1 The $\pi\pi \to \pi\pi$ amplitude

Let us consider scattering of π^0 on a pion from the medium: $\pi^0\pi^0 \to \pi^0\pi^0$, $\pi^0\pi^0 \to \pi^+\pi^-$, $\pi^0\pi^\pm \to \pi^0\pi^\pm$. We allow also for the lightest scalar isoscalar resonance, the σ-meson, as an intermediate state $(\pi\pi \to \sigma \to \pi\pi)$ because of its importance shown in various investigations of the pion-pion interaction [7, 9, 13, 14]. Here, we use an

$SU(2) \times SU(2)$ chiral quark model of the NJL type [9, 10] where, using the bosonization procedure, one obtains the Lagrangian for pions and σ-mesons. The part that contains three- and four-particle vertices has the form

$$\mathcal{L}_{\text{int}} = 2mg_\sigma \sigma^3 + 2mg_\pi \sqrt{Z} \sigma(2\pi^+\pi^- + (\pi^0)^2) - g_\pi^2 \sigma^2 \pi^2$$
$$- \frac{g_\pi^2 Z}{2}(4\pi^+\pi^-\pi^+\pi^- + 4\pi^+\pi^-(\pi^0)^2 + (\pi^0)^4) - \frac{g_\sigma^2}{2}\sigma^4. \quad (8)$$

Here m is the constituent quark mass ($m = g_\pi f_\pi$; $f_\pi = 93$ MeV is the pion weak decay constant in the vacuum). The constants g_π and g_σ describe the interaction of the pion and σ-meson with quarks, respectively. They are related to each other by the equation

$$g_\pi = g_\sigma \sqrt{Z}. \quad (9)$$

The constant Z originates from $\pi - a_1$ transitions and is equal to

$$Z = \left(1 - \frac{6m^2}{M_{a_1}^2}\right)^{-1}, \quad (10)$$

where $M_{a_1}^2 = 1250$ MeV is the mass of a_1 meson. The values of the constituent quark mass and the constants g_π and g_σ were calculated in [8]. In the vacuum we have: $m = 242$ MeV, $g_\pi = 2.61$, and $g_\sigma = 2.18$. Their values at finite temperatures and zero chemical potential were calculated in [8]; here we use these results as input data in our calculations.

In our approach, the total amplitude of $\pi^0\pi^0 \to \pi^0\pi^0$ consists of a contact term and three resonant contributions in the scalar channel (see Figs. 1a,b,c,d)

$$\mathcal{T}_{\pi^0\pi^0 \to \pi^0\pi^0} = -24g_\pi^2 + 3\mathcal{T}_\sigma(s) + 3\mathcal{T}_\sigma(t) + 3\mathcal{T}_\sigma(u), \quad (11)$$

where

$$\mathcal{T}_\sigma(x) = \frac{4g_\pi^2 m^2}{M_\sigma^2 - x - i M_\sigma \Gamma_\sigma}, \quad (12)$$

and s, t, u are kinematic invariants:

$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2, \tag{13}$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2, \tag{14}$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2, \tag{15}$$

for which the following identity is satisfied:

$$s + t + u = s_1 + s_2 + s_3 + s_4; \quad s_i = p_i^2. \tag{16}$$

The momentum p_1 corresponds to the impacting pion, while p_2 relates to the pion rested in the heat bath. The momenta p_3 and p_4 are those for the particles produced after collision.

For the charged pions in the final state, the amplitudes contain two terms

$$\mathcal{T}_{\pi^0\pi^0 \to \pi^+\pi^-} = -8g_\pi^2 + 2\mathcal{T}_\sigma(s) \tag{17}$$

for the process $\pi^0\pi^0 \to \pi^+\pi^-$ (Figs. 1a and 1b);

$$\mathcal{T}_{\pi^0\pi^\pm \to \pi^0\pi^\pm} = -8g_\pi^2 + 2\mathcal{T}_\sigma(t) \tag{18}$$

for $\pi^0\pi^\pm \to \pi^0\pi^\pm$ (Figs. 1a and 1c).

In medium, model parameters depends on temperature and density. In our work, as only mesons are considered, we restrict ourselves to the case of zero chemical potential for which the temperature dependence of the model parameter has been obtained in [8]. We use these results in [8] as input data in our calculations.

3.2   Cross sections

The differential cross section for a pion-pion scattering is determined by the equation

$$\frac{d\sigma^{\mathrm{dir}}}{dt} = \frac{|\mathcal{T}|^2(1 + n_3)(1 + n_4)}{64\pi s |\vec{p}_{1\mathrm{cm}}|^2} \tag{19}$$

for the direct and

$$\frac{d\sigma^{\text{inv}}}{dt} = \frac{|\mathcal{T}|^2 n_3 n_4}{64\pi s |\vec{p}_{1\text{cm}}|^2} \qquad (20)$$

for the inverse processes. Here $p_{1\text{cm}}$ is the momentum of pion 1 that hits pion 2 which is at rest in the heat bath frame; n_3 and n_4 are the occupation numbers of produced pions.

Having integrated over t in the interval defined by the lower and upper limits

$$t_{\text{min}} = \left(\frac{s_1 - s_2 - s_3 + s_4}{2\sqrt{s}}\right)^2 - (p_{1,\text{cm}} + p_{3,cm})^2, \qquad (21)$$

$$t_{\text{max}} = \left(\frac{s_1 - s_2 - s_3 + s_4}{2\sqrt{s}}\right)^2 - (p_{1,\text{cm}} - p_{3,cm})^2. \qquad (22)$$

we obtain the total cross section. The momenta of pions in the center of mass frame are determined by the equation

$$p_i = \sqrt{E_i^2 - s_i}, \qquad (i = 1, 3), \qquad (23)$$

where the energies E_i are defined in the center of mass frame

$$E_{1,\text{cm}} = \frac{s + s_1 - s_2}{2\sqrt{s}}, \qquad (24)$$

$$E_{3,\text{cm}} = \frac{s + s_3 - s_4}{2\sqrt{s}}. \qquad (25)$$

To calculate the occupation numbers n_i in (19) and (20), one needs to know the energies of pion 3 and pion 4 in the heat bath frame

$$E_3 = \frac{s_2 + s_3 - t}{2\sqrt{s_2}}, \qquad E_4 = \frac{s_2 + s_4 - t}{2\sqrt{s_2}}. \qquad (26)$$

In the case of pion-pion collisions, four processes contribute to the total cross section

$$\sigma(\pi\pi \to \pi\pi) = \sigma(\pi^0\pi^0 \to \pi^0\pi^0) + \sigma(\pi^0\pi^0 \to \pi^+\pi^-)$$
$$+ 2\,\sigma(\pi^0\pi^+ \to \pi^0\pi^+). \qquad (27)$$

The process $\pi^0\pi^+ \to \pi^0\pi^+$ occurs at the same rate as $\pi^0\pi^- \to \pi^0\pi^-$, so we do not calculate them separately; we just put the factor 2 at the last term in (27). To obtain the pion damping width, we substitute the obtained cross sections into equations (6) and (7).

4   Numerical results

Using the definition of the pion lifetime given in Sect. 2 and the cross sections determined in Sect. 3, we evaluate numerically the pion damping width at the temperatures ranging from 0 to 180 MeV. The upper limit on the temperature scale corresponds to the expected transition from the phase with broken chiral symmetry to the symmetric phase. The resulting curves are shown on Fig. 2.

All calculations are performed for a neutral pion rested in the heat bath frame. The pion self-energy is approximated by a constant.

As one can see from the figures, the pion state broadens noticeably in a hot matter, as compared to the vacuum state, already at $T \approx 60$ MeV. At $T = 160$, the pion-pion scattering accounts for about 80 MeV in the total width; this is the maximum value. Then the curve in Fig. 2 turns down; this behaviour of the damping width is caused by a noticeable decrease of the constant g_π after $T = 160$ MeV. The cross section is proportional to g_π^4 whose value reduces by half at $T = 180$ MeV, comparing to $T = 160$ MeV. For the temperatures from 160 MeV to 180 MeV, this weakening of the pion-pion interaction overpowers the expected increase in collision integrals.

Besides the pion-pion scattering, we estimated also some other possible contributions to the pion width. They come from the following processes: $\pi\pi \to \sigma\sigma$, $\pi\sigma \to \pi\sigma$, $\pi\pi \to \sigma$, and $\pi\pi \to \bar{q}q$. All these contributions were calculated separately, i.e, with other modes

switched off. (This gives the upper limit for a solution of the equation for width, because the collision integrals decrease if the spectral functions broaden.) They turned out to be small. At $T = 180$ MeV (which is a little below $T_c = 186$ MeV [8]), $\pi\sigma \to \pi\sigma$ gives about 5 MeV; $\pi\pi \to \sigma\sigma$ about 1 MeV, and $\pi\pi \to \sigma$ even less than 1 MeV. The decays to quarks contribute to the pion width less than 7 MeV. The smallness of these contributions was the reason to discard them in our calculations whose purpose is to make a qualitative estimate for the pion damping width.

5   Discussion and conclusion

In the framework of the $SU(2) \times SU(2)$ NJL model, the pion damping width was calculated for the range of temperatures from 0 to 180 MeV. The definition for the damping width, given in [11], was used. A self-consistent method where the pion width does not depend on energy and momentum was used to estimate the contributions to the pion damping width from pion-pion scattering in a hot and dense matter. Upper limits for alternative contributions were estimated qualitatively. It was found that the pion-pion scattering is dominant, and other processes that give small contributions can be discarded.

In our investigation we came to the conclusion that, in a hot gas, the pion spectral function significantly broadens, while nearing the phase transition point. This may have many implications for various processes in a hot and dense matter where pions are involved. In particular, this can affect the dilepton production through pion-pion annihilation in heavy-ion collisions [4].

Of course, a more systematic and formal approach, like one suggested by Hees and Knoll [12], would be more preferable for a study
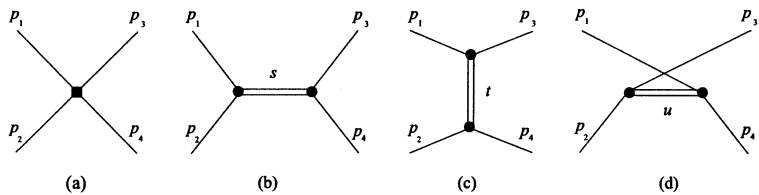
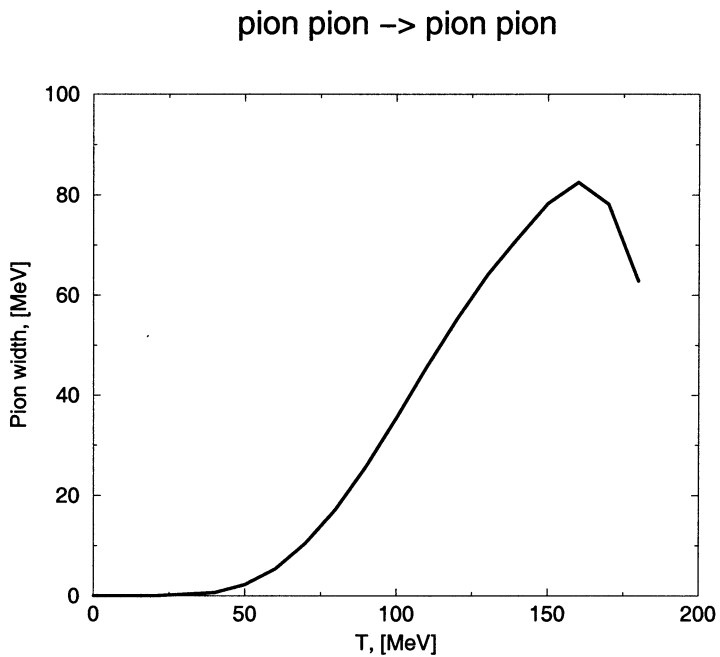Figure 1: Diagrams contributing to the $\pi\pi \to \pi\pi$ amplitude.

Figure 2: Pion damping width as a function of T.

of processes occurring in a hot matter. Nevertheless, our approach, which stems from Quantum Statistical Mechanic, has a clear physical interpretation, and it allowed us to make a prediction for the behaviour of the pion damping width in a hot matter. Further, similar calculations can be done for other particles, e.g., for the σ and ρ mesons. A particular interest is about the σ meson width at those temperatures at which the direct decay $\sigma \to \pi\pi$ is suppressed and contributions from collision integrals to the σ width become noticeable. Moreover, the diagrams with the σ-pole play an important role in different processes occurring in a hot matter.

References

[1] CERES Collab., G. Agakichiev et al., Phys. Lett. B **422**, 405 (1998).

[2] CERES Collab., B. Lenkeit et al., Nucl. Phys. A **661**, 23 (1999).

[3] J. D. Bjorken, Phys. Rev. D **27**, 140 (1983).

[4] H.-J. Schulze and D. Blaschke, Phys. Lett. B **386**, 429 (1996).

[5] M. Asakawa, K. Yazaki, Nucl. Phys. A **504**, 668 (1989).

[6] D. Ebert, Yu. L. Kalinovsky, L. Munchov, M. K. Volkov, Int. J. Mod. Phys. A **8**, 1295 (1993).

[7] T. Hatsuda, T. Kunihiro, Phys. Rep. **247**, 221 (1994).

[8] D. B. Blaschke, G. R. G. Burau, M. K. Volkov, and V. L. Yudichev, Eur. Phys. J. A **11**, 319 (2001).

[9] M. K. Volkov, Sov. J. Part. Nucl. **17**, 186 (1986).

[10] D. Ebert, H. Reinhardt, and M. K. Volkov, Progr. Part. Nucl. Phys. **35**, 1 (1994).

[11] L. P. Kadanoff, G. Baym, *Quantum Statistical Mechanics*, (New York, 1962).

[12] H. van Hees and J. Knoll, Nucl. Phys. A **683**, 369 (2000).

[13] M. K. Volkov, A.E. Radzhabov, N.L. Russakovich, hep-ph/0203170 (submitted to Yad. Fiz.).

[14] E. Quack, P. Zhuang, Yu. L. Kalinovsky, S. P. Klevansky, J. Hüfner, Phys. Lett. B **348**, 1 (1995).

Блашке Д., Волков М. К., Юдичев В. Л.
Демпфирующая ширина пиона
из $SU(2) \times SU(2)$ модели Намбу–Иона-Лазинио

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В рамках модели Намбу–Иона-Лазинио мы исследуем модификацию демпфирующей ширины пиона в горячем пионном газе для температур в диапазоне от 0 до 180 МэВ. Обнаружено, что пион становится заметно широким резонансом уже при $T > 60$ МэВ. Вблизи кирального фазового перехода ($T \sim 180$ МэВ) ширина пиона насыщается и составляет 70 МэВ. Основной вклад в ширину дают пион-пионные столкновения. Другие вклады оказываются пренебрежимо малыми.

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Blaschke D., Volkov M. K., Yudichev V. L.
Pion Damping Width
from $SU(2) \times SU(2)$ Nambu–Jona-Lasinio Model

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Within the framework of the Nambu–Jona-Lasinio model, we investigate the modification of the pion damping width in a hot pion gas for the temperatures ranging from 0 to 180 MeV. The pion is found to broaden noticeably at $T > 60$ MeV. Near the chiral phase transition $T \sim 180$ MeV, the pion width is saturated and amounts to 70 MeV. The main contribution to the width comes from pion-pion collisions. Other contributions are found negligibly small.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Макет *Т. Е. Попеко*