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ON PRINCIPAL POSSIBILITY TO TEST THE INFLUENCE OF EXTERNAL GRAVITATIONAL FIELD ON ELEMENTARY PARTICLES LIFETIME

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1. Intrinsic time and world time

The general relativity theory (GRT) assumes that the influence of gravitation reduces to space-time curvature is described by the curvature tensor $g_{\alpha\beta}\ (\alpha,\beta = 0,1,2,3,...)$, and the gravitational field is connected with the energy-momentum tensor $T_{\alpha\beta}$ [1]. In case of a weak gravitational field, when $\varphi/c^2 << 1$, we may proceed from curved space to flat (quasiflat) space, where the contribution of gravitational field is determined by the potential $\varphi(x)$ [2]. Hereby the value φ is equivalent to the Newton gravitational potential [3]. Especially simple expressions are obtained in case of stationary (time-independent) gravitational field. In this case, the non-diagonal terms of curvature tensor are equal to zero, and

$$g_{oo} = 1 + 2\varphi / c^2 \qquad (1).$$

Here, the connection between the intrinsic time (τ), i.e. time, when gravitation is accounted for, and the time (x_o) (the world time), when gravitation is not accounted for, is determined by expression (2):

$$\tau = x_o \sqrt{g_{oo}} / c = inv \qquad (2).$$

Using the expression (2) for relatively weak fields, when. $\varphi / c^2 << 1$, we obtain

$$\tau = x_o (1 + \varphi / c^2) / c \qquad (3).$$

For the photon frequency ω in external gravitational field we have

$$\omega = \omega_0 (1 - \varphi / c^2) \qquad (4),$$

where ω_0 is photon frequency in the absence of gravitational field.

Since the consideration is performed in a flat space, the velocities of time flow in intrinsic reference frames for a rest and a moving object (respectively x_o and x_o') are connected by the following expression:

$$x_o' = \gamma x_o \qquad (5).$$

If we proceed from reference system with time τ to a system with time $\tau' = \tau\gamma$, taking into account that $x_o = c\tau_o$ the general formula for the intrinsic time of relativistic object in gravitational field takes the following form:

$$\tau' = \tau\gamma(1+\varphi/c^2) \qquad (6).$$

The dependence (6) describes duration relation of identical physical processes for a moving and a stationary objects in the given reference system with a presence of gravitation.

The transformation for photon frequency emitted by a relativistic object in gravitational field is determined in a similar way.

2. Constant gravitational field

Using this approach, the expression that describes the energy of a physical object in the external gravitational field (when the object is moving along the world line) will take the following form:

$$E_o = m_o c^2 g_{oo}\, dx_o/ds = m_o c^2 g_{oo}\, dx_o/\sqrt{[g_{oo}(dx_o)^2 - dl^2]} \qquad (7).$$

Introducing the object velocity (v) into the reference system of the observer, we obtain an expression for the whole energy of the object in the given system:

$$v = dl/d\tau = c\,dl/\sqrt{g_{oo}}\,dx_o \qquad (8),$$

$$E = m_o c^2 g_{oo}/\sqrt{(1-v^2/c^2)} = m_o c^2(1+\varphi/c^2)/\sqrt{(1-v^2/c^2)} \qquad (9).$$

This expression is true provided that $\varphi/c^2 << 1$. The feature dependence of relativistic object obtained for the gravitational field can be checked experimentally.

The lifetime at rest of elementary particles (specifically that of charged pions) at the lepton decay is described by the following expression:

$$\tau_{o\pi} = 1/\Gamma_\pi \qquad (10),$$

where Γ_π has the following form:

$$\Gamma_\pi = G^2 f_\pi^2 \cos^2\Theta m^2_l\, m_\pi (1 - m^2_l / m^2_\pi)^2/8\pi, \qquad (11),$$

where f_π is the constant of pion decay, G is the Fermi constant, Θ is mixing angle, m_l is lepton mass, m_π is pion mass.

As the expression (11) includes the pion mass (see (9) in the case of $v=0$), the change of its mass in an exterior gravitational field will obviously cause a change in the pion lifetime.

3. Periodical processes

The period relation of two identical physical processes (T_0 and T) - for an object at rest and for an object moving with a velocity v in the given reference system correspondingly - is described by the known expression:

$$T/T_o = 1/\sqrt{(1-v^2/c^2)} \qquad (12).$$

The similar period relation of the processes (T_0 and T') which take place without the gravitation source and with this source (with a potential φ) provided that $\varphi/c^2 << 1$, are described by the following formula:

$$T'/T_o = (1+\varphi/c^2) \qquad (13).$$

Uniting (12) and (13), at $\varphi/c^2 << 1$, we obtain the following general dependence:

$$T''/T_o = 1/\sqrt{(1-v^2/c^2)}\,(1+\varphi/c^2) = \gamma + \gamma\varphi/c^2 \qquad (14).$$

The relation (14) is equivalent of (6). If we write down the (14) in the other form:

$$(T''/T_o)\,/\,\gamma = 1 + \varphi/c^2 \qquad (14)^*,$$

then we see that it has no dependence on γ.

When a corresponding precision is attained, the experimental check of the given dependence can serve as an additional important test of GRT postulate about the relation of time and gravitation.

4. Sources of gravitational field on the Earth

In the Table below, characteristics of the most important sources of gravitational field which act on the surface of Earth [4-6] are shown.

Gravitational source	Mass of source (kg)	Distance to centre (m)	Potential $\lvert\varphi\rvert$ on the Earth (m^2/s^2)	Relation $\lvert\varphi\rvert / c^2$
Earth	$5.975 \cdot 10^{24}$	$6.37 \cdot 10^{6}$	$6.3 \cdot 10^{7}$	$7 \cdot 10^{-10}$
Sun	$2 \cdot 10^{30}$	$1.5 \cdot 10^{11}$	$8.9 \cdot 10^{8}$	10^{-8}
Galaxy	$2 \cdot 10^{41}$	$3 \cdot 10^{20}$	$4\text{-}6 \cdot 10^{10}$	$5\text{-}7 \cdot 10^{-7}$
Super-cluster	10^{46}	10^{23}	10^{13}	10^{-4}
Great attractor			$>10^{13}$	$>10^{-4}$

As you can see from the Table, when the lifetime determination precision of relativistic particles attains a value of hundredth fractions of a percent, an experimental detection of the influence of gravitational field of the last two sources on the given index becomes possible. A relativistic charged pion can serve as a suitable trying test object. The measurement of the pion lifetime in flight with a needed precision, comparable with measurement precision of its lifetime at rest [7] will allow to confirm or refute the formula (6).

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О принципиальной возможности определения влияния
слабого внешнего гравитационного поля
на время жизни элементарных частиц

В данной работе, в рамках эйнштейновской теории гравитации, получено выражение, описывающее влияние внешнего гравитационного поля на время жизни элементарных частиц. На примере релятивистского пиона показано, что это влияние может быть обнаружено для ныне известных источников гравитации, если будет достигнута точность измерения времени жизни пиона порядка сотых долей процента.

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On Principal Possibility to Test the Influence
of External Gravitational Field on Elementary Particles Lifetime

In the present work, within the Einstein gravitation theory, an expression is obtained that describes the influence of external weak gravitational field on the lifetime of elementary particles. On the example of relativistic pion it is shown that this influence can be tested for existing external gravitational sources if a lifetime determination precision of about hundredth fractions of percent is attained.

The investigation has been performed at the Scientific Center of Application Research, JINR.

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