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THE q BOSON–FERMION REALIZATIONS OF THE QUANTUM SUPERALGEBRA $U_q(\mathrm{gl}(2/1))$

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1 Introduction

Boson–fermion realizations of a given set of operators via Bose–Fermion creation and annihilation operators are among the main tools of solving various quantum problems. The origin is linked with the Schwinger [1], Dyson [2] and Holstein–Primakoff [3] realizations which are different boson realizations of the algebra sl(2).

Generalizations of the Dyson realization to the Lie algebra sl(n) were derived in [4]. In our paper [5] we formulated the method starting from the Verma modules for obtaining boson realizations and in [6] we obtained explicitly a braid class of realizations which generalized the results from [7, 8].

Later the idea was extended to the Lie superalgebra, and the Dyson type boson–fermion realizations were explicitly given in [9], generalizing the results to sl(2/1) ([10],[11]).

Today these boson–fermion realizations become a standard technique in quantum many–body physics and we can also find several other applications in all fields of quantum physics.

Quantum groups and quantum supergroups or q–deformed Lie algebras and superalgebras imply some specific deformations of the classical Lie algebras and superalgebras. From a mathematical point of view, those are noncommutative associative Hopf algebras and superalgebras. The structure and representation theory of quantum groups were extensively developed by Jimbo [12] and Drinfeld [13]. The first "quantum" version of Holstein–Primakoff was worked out for U_q(sl(2)) [14] and then for U_q((sl(3)) [15]. The Schwinger type realization was written in [16] and [17]. These realizations found immediate applications [18–23].

In our papers [24, 25, 26] we studied the Dyson realizations of the series algebras U_q(sl(2)), U_q(gl(n)), $U_q(B_n)$, $U_q(C_n)$ and $U_q(D_n)$. There is some special case [25] for which the realization of the subalgebra U_q(gl($n-1$)) in the recurrence is trivial. Such special realizations of the quantum algebra U_q(sl(n)) of Dyson type were studied in [27].

The aim of the present paper is to show that there is a possibility of generalizing our method [5] for deriving the boson–fermion realization, too. This will be exemplified by the quantum superalgebra U_q(gl(2/1)). This superalgebra can be applied to physical problems such as strongly correlated electron systems [28, 29, 30]. We explicitly see the recurrence with respect to U_q(gl(1/1)) and consequently we will show that again it is a generalization of the result from [31].

Some preliminary results concerning the general case $U_q(\mathrm{gl}(m/n))$ have already been obtained and prepared for publication.

2 Preliminaries

In this article, we will use the definition of a quantum superalgebra $U_q(\mathrm{gl}(2/1))$ which can be found in [31].

Let q be an independent variable, $\mathcal{A} = C[q, q^{-1}]$ and $\mathcal{C}(q)$ be a division field of $\mathcal{A}$. The superalgebra $U_q(\mathrm{gl}(2/1))$ is the associative superalgebra over $\mathcal{C}(q)$ generated by even generators K_i, K_i^{-1}, $i = 1, 2, 3$, E_{12}, E_{21} and odd generators E_{32}, E_{32} which satisfy the following relations:

$$
\begin{aligned}
&K_i^{\pm 1} K_j^{\pm 1} = K_j^{\pm 1} K_i^{\pm 1}\,, \qquad K_i K_i^{-1} = 1 \\
&K_i E_{jk} = q^{\delta_{ij} - \delta_{ik}} E_{jk} K_i \\
&[E_{12}, E_{32}] = [E_{21}, E_{23}] = 0 \\
&[E_{12}, E_{21}] = \frac{K_1 K_2^{-1} - K_1^{-1} K_2}{q - q^{-1}} \\
&\{E_{23}, E_{32}\} = \frac{K_2 K_3 - K_2^{-1} K_3^{-1}}{q - q^{-1}} \\
&E_{23}^2 = E_{32}^2 = 0 \\
&E_{12} E_{13} - q E_{13} E_{12} = 0 \\
&E_{21} E_{31} - q E_{31} E_{21} = 0
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
E_{13} &= E_{12} E_{23} - q^{-1} E_{23} E_{12} \\
E_{31} &= -E_{21} E_{32} + q^{-1} E_{32} E_{21}
\end{aligned}
$$

The Hopf structure of this superalgebra is defined by the following operations:

1. *Coproduct* Δ

$$
\begin{aligned}
&\Delta(1) = 1 \otimes 1 && \Delta(K_i) = K_i \otimes K_i \\
&\Delta(E_{12}) = E_{12} \otimes K_1 K_2^{-1} + 1 \otimes E_{12} && \Delta(E_{23}) = E_{23} \otimes K_2 K_3 + 1 \otimes E_{23} \\
&\Delta(E_{21}) = E_{21} \otimes 1 + K_1^{-1} K_2 \otimes E_{21} && \Delta(E_{32}) = E_{32} \otimes 1 + K_2^{-1} K_3^{-1} \otimes E_{32}
\end{aligned}
$$

2. *Counit* ε

$$\varepsilon(1) = \varepsilon(K_i) = 1$$
$$\varepsilon(E_{12}) = \varepsilon(E_{23}) = \varepsilon(E_{21}) = \varepsilon(E_{32}) = 0$$

3. *Antipode* S

$$\begin{array}{ll} S(1) = 1 & S(K_i) = K_i^{-1} \\ S(E_{12}) = -E_{12}K_1^{-1}K_2 & S(E_{23}) = -E_{12}K_2^{-1}K_3^{-1} \\ S(E_{21}) = -K_1K_2^{-1}E_{21} & S(E_{32}) = -K_2K_3E_{32} \end{array}$$

We do not use these operations for construction of the realization.

The method of construction used is the same as in the case of the Lie algebras [5] or quantum algebra [26] and is based on using the induced representation. The difference from quantum algebra is that together with q–deformed boson operators [16], [17] we also use fermion operators.

The algebra $\mathcal{H}$ of the q–deformed boson operators is the associative algebra over the field $\mathcal{C}(q)$ generated by the elements of a^+, $a^- = a$, q^x and q^{-x}, satisfying the commutation relations

$$\begin{array}{lll} q^x q^{-x} = q^{-x} q^x = 1\,, & q^x a^+ q^{-x} = q a^+, & q^x a q^{-x} = q^{-1} a, \\ a a^+ - q^{-1} a^+ a = q^x, & a a^+ - q a^+ a = q^{-x}, & \end{array} \qquad (2)$$

The algebra $\mathcal{H}$ has faithful representation on vector space with basic elements $\{|n\rangle$, where $n = 0,\, 1,\, \ldots\}$ of the form

$$q^x|n\rangle = q^n|n\rangle\,, \quad a^+|n\rangle = |n+1\rangle\,, \quad a|n\rangle = [n]|n-1\rangle\,, \qquad (3)$$

where $[n] = \dfrac{q^n - q^{-n}}{q - q^{-1}}$.

Because of odd generators E_{23} and E_{32} we construct realization by means of the algebra $\mathcal{H}$ for even elements, and by fermion elements b^+ and b for odd ones. These fermion elements commute with the elements of $\mathcal{H}$ and together fulfil the relations

$$bb = b^+b^+ = 0\,, \quad bb^+ + b^+b = 1\,. \qquad (4)$$

As in the case of the Lie algebras or quantum groups, our realizations contain elements of quantum sub–superalgebra of $U_q(\mathrm{gl}(2/1))$, namely, quantum superalgebra $U_q(\mathrm{gl}(1/1))$. The element x of this subalgebra commutes with the elements from $\mathcal{H}$, and for the fermion elements $b^{\pm}$ the relation

$$xb^{\pm} = (-1)^{\deg x} b^{\pm} x\,, \tag{5}$$

holds.

Realization of the quantum superalgebra $U_q(\mathrm{gl}(2/1))$ is called the homomorphism ρ of the $U_q(\mathrm{gl}(2/1))$ to associative superalgebra $\mathcal{W}$ generated by $\mathcal{H}$, $b^{\pm}$ and $U_q(\mathrm{gl}(1/1))$.

3 Construction of the realization of $U_q(\mathrm{gl}(2/1))$

First, for construction of the realization we find the induced representation of $U_q(\mathrm{gl}(2/1))$. As subalgebra $\mathcal{A}_0$ of $U_q(\mathrm{gl}(2/1))$ we choose a quantum superalgebra generated by E_{23}, E_{21}, E_{32}, K_i and K_i^{-1}, $i = 1, 2, 3$. Let φ be a representation of $\mathcal{A}_0$ on vector space V. Let λ be the left regular representation on $U_q(\mathrm{gl}(2/1)) \otimes V$, i.e. for x, $y \in U_q(\mathrm{gl}(2/1))$ and $v \in V$ the representation λ is defined by

$$\lambda(x)(y \otimes v) = xy \otimes v\,. \tag{6}$$

Let $\mathcal{I}$ be subspace of $U_q(\mathrm{gl}(2/1)) \otimes V$ generated by the relations

$$xy \otimes v = x \otimes \varphi(y)v\,,$$

for all $x \in U_q(\mathrm{gl}(2/1))$, $y \in \mathcal{A}_0$ and $v \in V$. It is easy to see that the subspace $\mathcal{I}$ is λ-invariant. Therefore, (6) gives the representation on the factor–space $W = [U_q(\mathrm{gl}(2/1)) \otimes V]/\mathcal{I}$.

Let $E_{12}^N E_{13}^M = |N, M\rangle$. Due to the Poincaré–Birkhoff-Witt theorem the space W of the induced representation is generated by the elements $|N, M\rangle \otimes v$ where $N = 0, 1, 2, \ldots$, $M = 0, 1$ and $v \in V$.

To obtain the explicit form of the induced representation, we give some relations. They can be proved by mathematical induction from relations (1).

Lemma 1. For any $n = 0, 1, 2, \ldots$ the following formulae hold:

$$\begin{aligned}
E_{13}E_{12}^n &= q^{-n}E_{12}^n E_{13} \\
E_{23}E_{12}^n &= q^n E_{12}^n E_{23} - q[n]E_{12}^{n-1}E_{13} \\
E_{23}E_{13}^n &= (-q)^n E_{13}^n E_{23} \\
E_{32}E_{13}^n &= (-1)^n E_{13}^n E_{32} + \frac{1-(-1)^n}{2}\, q^{-n} E_{12} E_{13}^{n-1} K_2 K_3
\end{aligned}$$

$$
\begin{aligned}
E_{21}E_{12}^{n} &= E_{12}^{n}E_{21} - \frac{[n]}{q-q^{-1}}\, E_{12}^{n-1}(q^{n-1}K_1K_2^{-1} - q^{-n+1}K_1^{-1}K_2) \\
E_{21}E_{13}^{n} &= E_{13}^{n}E_{21} + \frac{1-(-1)^n}{2}\, E_{13}^{n-1}E_{23}K_1^{-1}K_2 \\
E_{31}E_{12}^{n} &= E_{12}^{n}E_{31} + q^{n-2}[n]E_{12}^{n-1}K_1K_2^{-1}E_{32} \\
E_{31}E_{13}^{n} &= (-1)^n E_{13}^{n}E_{31} + \frac{1-(-1)^n}{2}\, q^{-1}E_{13}^{n-1}\frac{K_1K_3 - K_1^{-1}K_3^{-1}}{q-q^{-1}} \\
E_{32}E_{23}^{n} &= (-1)^n E_{23}^{n}E_{32} + \frac{1-(-1)^n}{2}\, E_{23}^{n-1}\frac{K_2K_3 - K_2^{-1}K_3^{-1}}{q-q^{-1}}
\end{aligned}
$$

We omit the details of the calculations and write the result for the action of the induced representation on the basis elements $|N, M\rangle \otimes v$.

Theorem 1. The formulae

$$
\begin{aligned}
& E_{12}|N,M\rangle \otimes v = |N+1,M\rangle \otimes v \\
& E_{13}|N,M\rangle \otimes v = q^{-N_1}|N,M+1\rangle \otimes v \\
& E_{23}|N,M\rangle \otimes v = -q[N]\,|N-1,M+1\rangle \otimes v + (-1)^M q^{N+M}|N,M\rangle \otimes \varphi(E_{23})v \\
& K_1|N,M\rangle \otimes v = q^{N+M}|N,M\rangle \otimes \varphi(K_1)v \\
& K_2|N,M\rangle \otimes v = q^{-N}|N,M\rangle \otimes \varphi(K_2)v \\
& K_3|N,M\rangle \otimes v = q^{-M}|N,M\rangle \otimes \varphi(K_3)v \\
& E_{32}|N,M\rangle \otimes v = \frac{1-(-1)^M}{2}\, q^{-M}|N+1,M-1\rangle \otimes \varphi(K_2K_3)v+ \\
& \qquad +(-1)^M|N,M\rangle \otimes \varphi(E_{32})v \\
& E_{21}|N,M\rangle \otimes v = -\frac{[N]q^{N+M-1}}{q-q^{-1}}\,|N-1,M\rangle \otimes \varphi(K_1K_2^{-1})v+ \\
& \qquad +\frac{[N]q^{-N-M+1}}{q-q^{-1}}\,|N-1,M\rangle \otimes \varphi(K_1^{-1}K_2)v+ \\
& \qquad +\frac{1-(-1)^M}{2}\,|N,M-1\rangle \otimes \varphi(E_{23}K_1^{-1}K_2)v + |N,M\rangle \otimes \varphi(E_{21})v \\
& E_{31}|N,M\rangle \otimes v = \frac{1-(-1)^M}{2}\, q^{N-1}[N]\,|N,M-1\rangle \otimes \varphi(K_1K_3)v+ \\
& \qquad +(-1)^M q^{N+M-2}[N]\,|N-1,M\rangle \otimes \varphi(K_1K_2^{-1}E_{32})v+ \\
& \qquad +\frac{1-(-1)^M}{2}\,\frac{q^{-1}}{q-q^{-1}}\,|N,M-1\rangle \otimes \Big(\varphi(K_1K_3 - K_1^{-1}K_3^{-})v+ \\
& \qquad +(-1)^M|N,M\rangle \otimes \varphi(E_{31})v
\end{aligned}
$$

give the induced representation of the quantum superalgebra $U_q(\mathrm{gl}(2/1))$.

We construct the realization of quantum superalgebra $U_q(\mathrm{gl}(2/1))$ from the induced representation given in Theorem 1 as follows:
We chose the representation φ, for which $\varphi(E_{21})v = 0$, $\varphi(E_{31})v = 0$, $\varphi(K_1)v = q^{\lambda}v$ and substitute

$$\begin{array}{lll}
q^{\pm N} \to q^{\pm x} & [N]\,|N-1,M\rangle \to a & |N+1,M\rangle \to a^+ \\
q^{\pm M} \to (bb^+ + q^{\pm 1}b^+b) & \dfrac{1-(-1)^M}{2}|N,M-1\rangle \to b & |N,M+1\rangle \to b^+ \\
\varphi(E_{21})v \to 0 & \varphi(E_{31})v \to 0 & \varphi(K_1^{\pm 1})v \to q^{\pm\lambda} \\
\varphi(K_2^{\pm 1})v \to k_2^{\pm 1} & \varphi(K_3^{\pm 1})v \to k_3^{\pm 1} & \\
(-1)^M\varphi(E_{23})v \to e_{23} & (-1)^M\varphi(E_{32})v \to e_{32} &
\end{array}$$

(the last two relations reflect the fact that e_{23} and e_{32} are fermions).

By this substitution we obtain the realization of the quantum superalgebra $U_q(\mathrm{gl}(2/1))$.

Theorem 2. The mapping $\rho : U_q(\mathrm{gl}(2/1)) \to \mathcal{W}$ defined by the formulae

$$\begin{aligned}
&\rho(E_{12}) = a^+ \\
&\rho(E_{13}) = q^{-x}b^+ \\
&\rho(E_{23}) = -qab^+ + q^x(bb^+ + qb^+b)e_{23} \\
&\rho(K_1) = q^{\lambda_1+x}(bb^+ + qb^+b) \\
&\rho(K_2) = q^{-x}k_2 \\
&\rho(K_3) = (bb^+ + q^{-1}b^+b)k_3 \\
&\rho(E_{32}) = q^{-1}a^+bk_2k_3 + e_{32} \\
&\rho(E_{21}) = -\frac{a}{q-q^{-1}}\Big(q^{\lambda_1+x-1}(bb^+ + qb^+b)k_2^{-1} - q^{-\lambda_1-x+1}(bb^+ + q^{-1}b^+b)k_2\Big) \\
&-q^{-\lambda_1}be_{23}k_2 \\
&\rho(E_{31}) = a^+abq^{\lambda_1+x-1}k_3 + aq^{\lambda_1+x-2}(bb^+ + qb^+b)k_2^{-1}e_{32} + q^{-1}b\frac{q^{\lambda_1}k_3 - q^{-\lambda_1}k_3^{-1}}{q-q^{-1}}
\end{aligned}$$

is the realization of the quantum superalgebra $U_q(\mathrm{gl}(2/1))$.
This theorem can be proved by a direct calculation.

4 Conclusion

In this paper we gave the method of construction of the q–boson–fermion realization of quantum superalgebras and applied it to the quantum superalgebra $U_q(\mathrm{gl}(2/1))$. One of the advantages of this method, in comparison with [31], is that we automatically obtain a realization and we do not need to verify the generating relation. The reason is that the representation of q–bosons and fermions on the vector space W with basis $|N, M\rangle$ is faithful.

The other advantage we see in the fact that our realization is expressed by means of polynomials of q–deformed bosons and fermions. On the other hand, we can easily obtain the Dyson realization of quantum superalgebra. For this purpose, it is sufficient to choose a realization of the generators of the algebra $\mathcal{H}$ in the form

$$a^+ = A^+ , \quad a = \frac{[N+1]}{N+1} A , \quad q^x = q^N , \tag{7}$$

where $[A, A^+] = 1$ and $N = A^+ A$. It is easy to verify that the realization of $U_q(\mathrm{gl}(2/1))$ from Theorem 2 with realization (7) of the algebra $\mathcal{H}$ and with a trivial realization of subalgebra $U_q(\mathrm{gl}(1/1))$ leads, after homomorphism of $U_q(\mathrm{gl}(2/1))$, to the realization given in [31]. In this case, the realization is of course expressed by means of a series in operators A^+ and A. Therefore, we prefer our form of realizations.

Finally, our realizations contain, in contrast with those in [31], quantum sub-superalgebras. Various forms of realizations of this sub-superalgebra give various realizations of the quantum superalgebra. In the studied case, this sub-superalgebra is $U_q(\mathrm{gl}(1/1))$, and, therefore, is very simple. We can choose a realization of this superalgebra as

$$\rho(e_{23}) = \rho(e_{32}) = 0 , \quad \rho(k_2) = \rho(k_3^{-1}) = q^{\lambda_2} \quad \text{and} \quad \rho(k_2^{-1}) = \rho(k_3) = q^{-\lambda_2} .$$

In this case, we obtain a realization with one q–deformed boson pair, one fermion pair and two parameters. However, by means of our method we construct other realization of $U_q(\mathrm{gl}(1/1))$, namely, realization of the form

$$\begin{aligned}
\rho(e_{23}) &= b_2^+ \\
\rho(k_2) &= q^{\lambda_2}(b_2 b_2^+ + q b_2^+ b_2) \\
\rho(k_3) &= q^{\lambda_3}(b_2 b_2^+ + q^{-1} b_2^+ b_2) \\
\rho(e_{32}) &= \frac{q^{\lambda_2+\lambda_3} - q^{-\lambda_2-\lambda_3}}{q - q^{-1}} b_2 = [\lambda_2 + \lambda_3] b_2
\end{aligned}$$

where b_2 and b_2^+ are the fermion elements. If we use this realization of the quantum superalgebra in the realization of $U_q(\mathrm{gl}(2/1))$ given in Theorem 2, we obtain realization with one q–deformed boson pair, two fermion pairs and three parameters, which corresponds to the case of the Lie and quantum algebras.

As it is evident from [25, 26], this method of construction of realization is very successful for quantum groups. Therefore, we believe that it will be very useful for construction of realizations of quantum supergroups, too.

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Бурдик Ч., Навратил О. E5-2003-133
q-бозон-фермионные реализации
квантовой супералгебры $U_q(\mathrm{gl}(2/1))$

Показано, что построение реализаций для алгебр и квантовых алгебр может быть также обобщено на квантовые супералгебры. Изучен пример квантовой супералгебры $U_q(\mathrm{gl}(2/1))$ и даны бозон-фермионные реализации для одной пары q-бозонных операторов и двух пар фермионов.

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Burdík Č., Navrátil O. E5-2003-133
The q Boson–Fermion Realizations
of the Quantum Superalgebra $U_q(\mathrm{gl}(2/1))$

We show that our construction of realizations for algebras and quantum algebras can be generalized to quantum superalgebras, too. We study an example of quantum superalgebra $U_q(\mathrm{gl}(2/1))$ and give the boson fermion realization with respect to one pair of q-boson operators and 2 pairs of fermions.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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