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STANDARD MODEL TREATMENT OF THE RADIATIVE CORRECTIONS TO THE NEUTRON β-DECAY

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I. INTRODUCTION

Nowadays, it has been well realized that a thorough and all-round study of the neutron $\beta-$decay conduces to gain an insight into physical gist of the semiweak processes and into the elementary particle physics in general. That is why for the past decade a great deal of efforts has been directed to measure with a high accuracy (better than $\sim 1\%$) the main characteristics of the $\beta-$decay of free neutrons: the lifetime τ [1], the asymmetry factors (as neutrons are polarized) of the electron momentum distribution A [2] and the antineutrino momentum distribution B [3], the recoil proton distribution and the electron-antineutrino correlation coefficient a [4], the coefficient D of triple correlation of the electron momentum, the antineutrino momentum and the neutron spin [5]. Further experiments are believed to come to fruition before long [6].

In treating the experimental data, the task is posed to inquire into the effective 4-fermion interaction [7-9]

$$\mathcal{L}_{WF}(x) = \frac{G_F|V_{ud}|}{\sqrt{2}}(\bar{\psi}_e(x)\gamma_\alpha(1-\gamma^5)\psi_\nu(x))\times$$
$$\times \sum_{\mathbf{P}_n,\sigma_n,\mathbf{P}_p,\sigma_p} \bar{\Psi}_p(P_p,\sigma_p,x)\Big\{(\gamma^\alpha g_V(q^2) + g_{WM}(q^2)\sigma^{\alpha\nu}q_\nu) -$$
$$-(\gamma^\alpha g_A(q^2) + g_{IP}(q^2)q^\alpha)\gamma^5\Big\}\Psi_n(P_n,\sigma_n,x), \qquad q = P_n - P_p, \tag{1.1}$$

the quantities $|V_{ud}|$, g_V, g_A,... herein to be specified with the same accuracy which has been attained in the experimental measurements. This effective Lagrangian (1.1) is generally considered as descending from the Standard Model, the nowaday elementary particle theory (see, for instance, Ref. [8]). In the expression (1.1), $\psi_e(x)$, $\psi_\nu(x)$ stand for the electron (positron), (anti)neutrino fields, and $\Psi_N(P_N,\sigma_N,x)$, N=n,p, represent the nucleon states with the momenta P_N and polarizations σ_N. The system of units h=c=1 is adapted, and γ^5, $\sigma^{\mu\nu}$ are defined by $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$, $\sigma^{\mu\nu}$=$(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu)/2$. G_F is the Fermi constant and $|V_{ud}|$ is the Cabibbo-Kobayashi-Maskawa (CKM) [10] quark-mixing matrix element. By confronting the experimental data with the results of the appropriate calculations, the $|V_{ud}|$, g_V, g_A... values are to be fixed so strictly that we should be in position to fathom the principles of the elementary particle theory. In particular, the CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \tag{1.2}$$

should be verified as strictly as possible [10].

So far as the transferred momentum q is very small when compared to the nucleon mass, $|q|/M_N{\sim}0.0005$, Eq. (1.1) provides the bulk amplitude $\mathcal{M}^0$ of the neutron $\beta-$decay with presuming $M_N{\to}\infty$, neglecting the terms with g_{WM}, g_{IP}, and replacing the functions $g_V(q^2)$, $g_A(q^2)$ by their values at $q^2{=}0$: $g_V(0){=}1$, $g_A(0)$ [7–9]. Finiteness of the nucleon mass causes the sizable, about 1%, corrections to the calculated decay characteristics [11] that have been taken into consideration in experimental data processing in Refs. [1–3].

As we strive to acquire the quantities $|V_{ud}|$, g_V, g_A,... with an accuracy better than 1%, the electromagnetic corrections are to be allowed for in treating the neutron $\beta-$decay. Therefore the effective Lagrangian (1.1) is to be accomplished by the interactions of electrons and nucleons with electromagnetic field A

$$\mathcal{L}_{e\gamma}(x) = -e\bar{\psi}_e(x)\gamma^\mu\psi_e(x)\cdot A_\mu(x)\,, \tag{1.3}$$

$$\mathcal{L}_{N\gamma}(x) = -e\sum_{N,P_N,\sigma_N}\bar{\Psi}_N(P_N,\sigma_N,x)f_N^\mu(q)\Psi_N(P_N,\sigma_N,x)\cdot A_\mu(x)\,, \tag{1.4}$$

where $f_N^\mu(q)$ are the nucleon electromagnetic form-factors. These interactions give rise to the electromagnetic corrections to the bulk amplitude $\mathcal{M}^0$.

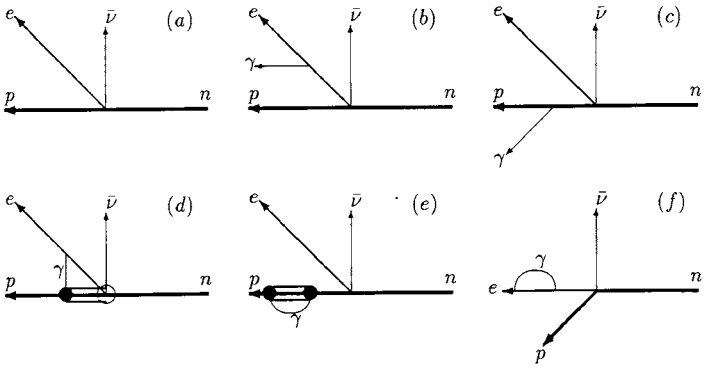
If the effective Lagrangian

$$\mathcal{L}_{eff} = \mathcal{L}_{WF} + \mathcal{L}_{e\gamma} + \mathcal{L}_{N\gamma} \tag{1.5}$$

could consistently describe the radiative $\beta-$decay of neutrons

$$n \Longrightarrow p + e^- + \bar{\nu} + \gamma\,, \tag{1.6}$$

the actual transition amplitude $\mathcal{M}$ of order α would merely be presented by the set of ordinary Feynman diagrams originating immediately from the interactions (1.1), (1.3), (1.4)



(1.7)

where the triplex lines represent various baryonic states, the blobs depict the formfactors $f_N^{\mu}(q)$ in (1.4) and the empty circle stands for the matrix element of the interaction (1.1) with allowance for $q-$dependence. So, upon straightforward unsophisticated calculating, the amplitude $\mathcal{M}$ and, subsequently, the observables τ, A, B, a,... would directly be obtained in terms of the quantities $G_F, |V_{ud}|$, g_V, g_A,... residing into $\mathcal{L}_{WF}$ (1.1). Then, accordingly the aim proclaimed, it would quite natural appear that these desirable quantities should be ascertained by confronting the experimental values of τ, A, B, a,... with their values calculated in the aforesaid way. But, alas, this plain calculation shows up to be contradictory because the ultra violet (UV) divergences (the terms multiple to $\ln \Lambda/M_N$, $\Lambda \to \infty$) inhere in the contributions from the one-loop diagrams $(d),(e),(f)$ in (1.7). So far the treatment is solely based upon the Lagrangian (1.5) itself, there is no way to cope with this failure. To deal with well-defined quantities in practical evaluating the observables τ, A, B, a,..., the extra UV cut-off $\Lambda = M_V \approx 100\,\mathrm{GeV}$ could be set up, supplementing the calculation based on the local interaction (1.5), see, for instance, Refs. [12–21]. Yet, this recipe is rather untenable, and we would never be able to repose full confidence in the results obtained in this way. Thus, the description of the radiative decay (1.6) with the effective interaction (1.5) is not self-contained.

Although the 4-fermion local theory is quite sufficient for the calculations in the lowest order, without the radiative corrections, it is not satisfactory because of its violation of unitarity and its nonrenormalizability, which prevents us from dealing with electroweak high order effects in a convincing way. A stringent self-contained treatment of the neutron $\beta-$decay ought to be founded upon the Standard Model of elementary particle physics. The Standard Model Lagrangian $\mathcal{L}^{SM}$ [8] embodies the nowaday knowledge of the strong and electroweak interactions of the leptons and the quarks,

$$\mathcal{L}^{SM} = \mathcal{L}^{EW} + \mathcal{L}^{qq}_{str}. \tag{1.8}$$

There are several review articles and books available which thoroughly describe the structure of $\mathcal{L}^{EW}$, $\mathcal{L}^{qq}_{str}$. In the work presented, we pursue the way paved in Refs. [8,22–25].

In Sec. II, we concisely recapitulate the structure of the basic electroweak Lagrangian $\mathcal{L}_{EW}$ and the respective renormalization procedure in view of the current calculation of the radiative corrections to the neutron $\beta-$decay in the one-loop approach, with intent to attain an accuracy about 0.1%. By introducing the nucleon weak transition current and electromagnetic formfactors, the needful parameterizing of the effects caused by nucleon compositeness is set forth in Sec. III.

In Secs. IV-X, we acquire successively, term by term, the total decay amplitude of order α. In particular, the influence of nucleon structure on the calculated radiative corrections is estimated in Secs. VI, IX. The radiative corrections to the electron momentum distribution and to neutron lifetime are acquired in Sec. XI. In the last Sec., we fairly well try and compare our results with the long-known noteworthy assertions of the former investigations of the radiative corrections to the neutron $\beta-$decay. We purposely defer this needful discussion till the final stage of the work to have at our disposal all the desirable persuasive arguments to be offered for substantiating our inferences. Upon realizing what is the accuracy actually attainable in the nowaday calculations, we brief a feasible way to acquire the quantities G_F, $|V_{ud}|$, g_V, g_A,... residing in Eq. (1.1) as precisely as possible from appropriate experimental data processing.

II. ELECTROWEAK INTERACTIONS IN DESCRIPTION OF THE NEUTRON $\beta-$DECAY.

The basic electroweak Lagrangian to start with,

$$\mathcal{L}^{EW}(A_\mu, Z_\mu, W^\pm_\mu, H, \psi_f, e, M_Z, M_W, M_H, m_f, \xi)\,, \tag{2.1}$$

is expressed amenably to Refs. [22–25] in terms of the bare physical fields and parameters. A_μ, Z_μ, $W^\pm_\mu$, H, ψ_f stand for the electromagnetic, $Z-$boson, $W^\pm-$boson, Higgs-boson and generic fermion fields, and the quantities $e{=}\sqrt{4\pi\alpha}$, M_Z, M_W, M_H, m_f are the unit of charge and the masses of the $Z-$boson, $W-$boson, Higgs-boson, and fermions, respectively; ξ represents generically the gauge parameters. Taking the line of [22–24], we choose the Feynman gauge, $\xi = 1$. The physical fields A_μ, Z_μ, $W^\pm_\mu$ are related to the isotriplet of vector fields W^a_μ, $a{=}1,2,3$, and to the isosinglet vector field B_μ by the equations [22–25]

$$Z_\mu = c_W W^3_\mu + s_W B_\mu, \quad A_\mu = -s_W W^3_\mu + c_W B_\mu, \quad W^\pm_\mu = \frac{1}{\sqrt{2}}(W^1_\mu \mp i W^2_\mu)\,. \tag{2.2}$$

Chosen e, $M_{Z,W}$, m_f as input parameters,

$$c_W = \frac{M_W}{M_Z}\,, \quad s^2_W = 1 - c^2_W \tag{2.3}$$

are nothing but merely shorthand notations to simplify formulae. The gauge coupling constants are given by

$$g_2 = \frac{e}{s_W}, \quad g_1 = \frac{e}{c_W}\,, \tag{2.4}$$

and the masses of physical particles are written as

$$M_W = \frac{1}{2} g_2 \mathcal{V} \, , M_Z = \frac{1}{2} \sqrt{g_1^2 + g_2^2} \mathcal{V} \, , m_f = \frac{f_f \mathcal{V}}{\sqrt{2}} \, , \tag{2.5}$$

where $\mathcal{V}$ is the vacuum expectation value of the Higgs field, and f_f stand for the Yukawa couplings of fermions to the Higgs field. $\mathcal{L}^{EW}$ (2.1) has been constructed in Refs. [22-24] so that the bilinear terms, i.e. the inverse propagator terms, take eventually the simplest form:

$$\begin{aligned} \mathcal{L}_0^{EW} = & \bar{\psi}_f (i\gamma^\mu \partial_\mu - m_f)\psi_f + W_\mu^+ g^{\mu\nu}(\Box + M_W^2) W_\nu^- + \\ & \frac{1}{2} Z_\mu g^{\mu\nu}(\Box + M_Z^2) Z_\nu + \frac{1}{2}(\Box + {m_\gamma}^2) A_\mu g^{\mu\nu} A_\nu \, . \end{aligned} \tag{2.6}$$

The propagators of free fields are consequently

W, Z $$D_{\alpha\beta}^{Z,W}(x) = \delta_{\alpha\beta} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\exp(-ikx)}{k^2 - M_{Z,W}^2 + i0} \, , \tag{2.7}$$

$A\gamma$ $$D_{\alpha\beta}^{A\gamma}(x) = \delta_{\alpha\beta} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{\exp(-ikx)}{k^2 - m_\gamma^2 + i0} \, , \tag{2.8}$$

$f \equiv e, \nu, u, d$ $$G^f(x) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \exp(-ipx) \frac{\not{p} + m_f}{p^2 - m_f^2 + i0} \, . \tag{2.9}$$

The fictitious photon mass m_γ is included in (2.6), (2.8) to treat the integrals which involve the photon propagator $D^{A\gamma}$. It is to mention that in the ensuing calculation we shall have to deal not only with the infinitesimal photon mass $m_\gamma = \lambda \to 0$, but also with $m_\gamma = M_S$ specified so as $M_N^2 \ll M_S^2 \ll M_W^2$.

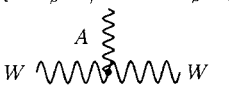
To treat thereafter the neutron $\beta-$decay in the one-loop approach, the electroweak interactions of lepton, quark, $W-$, $Z-$boson and electromagnetic fields are to be specified [22-25]:

$$\mathcal{L}_{int}^{EW} = \mathcal{L}^{WWZ} + \mathcal{L}^{WWA} + \mathcal{L}^{WJJ} + \mathcal{L}^{ZJJ} + \mathcal{L}^{AJJ} \, , \tag{2.10}$$

$$\mathcal{L}^{WWZ} = i \frac{g_2^2}{\sqrt{g_1^2 + g_2^2}} \Big(g^{\alpha\gamma} g^{\delta\beta} - g^{\alpha\delta} g^{\gamma\beta} \Big) [\partial_\alpha W_\beta^+ W_\gamma^- Z_\delta + \partial_\alpha W_\beta^- Z_\gamma W_\delta^+ + \partial_\alpha Z_\beta W_\gamma^+ W_\delta^-] = \tag{2.11}$$

$$= \Gamma_{\mu\nu\lambda}^{WWZ} W^{+\mu} W^{-\nu} Z^\lambda \, ,$$

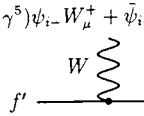
$$\mathcal{L}^{WWA} = ie \Big(g^{\alpha\gamma} g^{\delta\beta} - g^{\alpha\delta} g^{\gamma\beta} \Big) [\partial_\alpha W_\beta^+ W_\gamma^- A_\delta + \partial_\alpha W_\beta^- A_\gamma W_\delta^+ + \partial_\alpha A_\beta W_\gamma^+ W_\delta^-] = \tag{2.12}$$



$$= \Gamma_{\mu\nu\lambda}^{WWA} W^{+\mu} W^{-\nu} A^\lambda \, ,$$

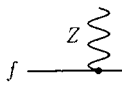
$$\mathcal{L}^{Wff'} = \frac{g_2}{2\sqrt{2}}\left(\bar{\psi}_{i+}V_{+-}T_i^+\gamma^\mu(1-\gamma^5)\psi_{i-}W_\mu^+ + \bar{\psi}_{i-}V_{-+}T_i^-\gamma^\mu(1-\gamma^5)\psi_{i+}W_\mu^-\right) = \tag{2.13}$$

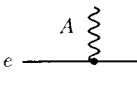
$$= \bar{\psi}_f\Gamma_\mu^{Wff'}\psi_{f'}W^{\mu\pm}\,,$$



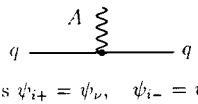
$$\mathcal{L}^{Zff} = \frac{1}{2}\sqrt{g_1^2+g_2^2}\left(\bar{\psi}_{i+}\gamma^\mu(\frac{1-\gamma^5}{2} - 2Q_{i+}\frac{g_1^2}{g_1^2+g_2^2})\psi_{i+} - \right.$$

$$\left. -\bar{\psi}_{i-}\gamma^\mu(\frac{1-\gamma^5}{2} + 2Q_{i-}\frac{g_1^2}{g_1^2+g_2^2})\psi_{i-}\right)Z_\mu = \bar{\psi}_f\Gamma_\mu^{Zff}\psi_f Z^\mu \tag{2.14}$$





$$\mathcal{L}^{Aee} = -e\bar{\psi}_e\gamma^\mu\psi_e A_\mu\,, \tag{2.15}$$



$$\mathcal{L}^{Aqq} = ee_q\bar{\psi}_q\gamma^\mu\psi_q A_\mu \equiv ee_q\bar{q}\gamma^\mu q A_\mu\,. \tag{2.16}$$

As usual, for leptons $\psi_{i+} = \psi_\nu$, $\psi_{i-} = \psi_e$, $V_{+-} = 1$, $Q_{i+} = 0$, $Q_{i-} = -1$, and in the case of u, d quarks $\psi_{i+} = \psi_u \equiv u$, $\psi_{i-} = \psi_d \equiv d$, $V_{+-} = V_{ud}$, $Q_{i+} = e_u = 2/3$, $Q_{i-} = e_d = -1/3$. The operator T^+ increases, T^- decreases weak isospin projection by one unite: $T^+\psi_e = \psi_\nu$, $T^-\psi_\nu = \psi_e$, $T^+\psi_d = \psi_u$, $T^-\psi_u = \psi_d$, $T^-\psi_e = T^-\psi_d = T^+\psi_\nu = T^+\psi_u = 0$. In the interactions (2.11)-(2.16) and in the analogous expressions hereupon, the $\mathcal{N}$products of the field operators

$$W_\mu^+(x) = \sum_{\mathbf{q}}\left(c_\mu(\mathbf{q})w_\mu^+(\mathbf{q})\mathrm{e}^{-iqx} + c_\mu^+(\mathbf{q})w_\mu^-(\mathbf{q})\mathrm{e}^{iqx}\right)\,, \tag{2.17}$$

$$\psi_f(x) = \sum_{\mathbf{p},r}\left(a_f(\mathbf{p},r)u_f(\mathbf{p},r)\mathrm{e}^{-ipx} + b_f^+(\mathbf{p},r)u_f(-\mathbf{p},-r)\mathrm{e}^{ipx}\right)\,, \tag{2.18}$$

and so on, are implied. Here f specifies a sort of fermions and r stands for other quantum numbers: spin, isospin, their projections.

In calculating the neutron $\beta-$decay amplitude in the one-loop approach, we leave out the effects of Higgs-fermion interactions, since they are of the order of the Higgs coupling to fermions $\sim m_f/M_W$ [8,22-25]. Also only the first generations of leptons (e, ν_e) and quarks $(u-, d-$quarks) come into the forthcoming consideration.

The transition amplitude $\mathcal{M}$ of the process (1.6), when calculated in the one-loop approach according to (2.6)-(2.16) directly in terms of the bare fields and parameters, is UV divergent, and renormalization is necessary. The multiplicative renormalization of the Lagrangian (2.6)-(2.16) is

performed amenably to the non-minimal on-mass-shell (OMS) renormalization scheme [22-24,8], with the renormalization constants and renormalized quantities defined in such a way that

$$\begin{aligned} W_\mu^a &\Longrightarrow (z_2^W)^{1/2} W_\mu^a , \qquad B_\mu \Longrightarrow (z_2^B)^{1/2} B_\mu , \\ \psi_f^{L,R} &\Longrightarrow (z_{L,R}^f)^{1/2} \psi_f^{L,R}, \qquad \psi_f^{L,R} = \frac{1 \mp \gamma^5}{2} \psi_f , \\ m_f^2 &\Longrightarrow m_f^2 + \delta m_f^2, \quad M_{W,Z}^2 \Longrightarrow M_{W,Z}^2 + \delta M_{W,Z}^2 , \\ g_2 &\Longrightarrow z_1^W (z_2^W)^{-3/2} g_2, \qquad g_1 \Longrightarrow z_1^B (z_2^B)^{-3/2} g_1 . \end{aligned} \tag{2.19}$$

Expanding the renormalization constants

$$z = 1 + \delta z , \tag{2.20}$$

we obtain

$$\mathcal{L}^{EW} = \mathcal{L}_{tree}^{EW} + \mathcal{L}_{ct}^{EW} , \tag{2.21}$$

where the expression for $\mathcal{L}_{tree}^{EW}$ in terms of renormalized quantities is identical with the original one, (2.6)-(2.16), but now it contains the renormalized physical parameters and fields. The counter term Lagrangian

$$\mathcal{L}_{ct}^{EW}(A_\mu,\ Z_\mu,\ W_\mu^\pm,\ H,\ \psi_f,\ e,\ M_W,\ M_Z,\ m_f;\ \delta z_{1,2}^{W,B},\ \delta z_{L,R}^f,\ \delta M_{W,Z}^2,\ \delta m_f^2) \tag{2.22}$$

is determined by the quantities $\delta z_{1,2}^{W,B}$, $\delta z_{L,R}^f$, $\delta M_{W,Z}^2$, δm_f^2 in (2.19). The linear combinations of the field renormalization constants $\delta z_2^{W,B}$ and the coupling renormalization constants $\delta z_1^{W,B}$ are introduced [23,24]

$$\begin{pmatrix} \delta z_m^\gamma \\ \delta z_m^Z \end{pmatrix} = \begin{pmatrix} s_W^2 & c_W^2 \\ c_W^2 & s_W^2 \end{pmatrix} \cdot \begin{pmatrix} \delta z_m^W \\ \delta z_m^B \end{pmatrix}, \quad \delta z_m^{\gamma Z} = c_W s_W (\delta z_m^W - \delta z_m^B) = \frac{c_W s_W}{c_W^2 - s_W^2} (\delta z_m^Z - \delta z_m^\gamma) , \tag{2.23}$$
$$m = 1, 2 .$$

Accordingly the OMS renormalization scheme [22–25], the fine structure constant $\alpha = e^2/4\pi = 1/137.036$ (defined in the Thomson limit) is used as an expansion parameter, and all the renormalization constants and the renormalized quantities in Eqs. (2.19)-(2.23) are fixed on the mass-shell of gauge bosons, fermions and Higgs bosons. With this condition, the renormalized masses are identical to the pole positions of the propagators, i.e. the physical masses. All the residues in the diagonal propagators are normalized to 1, and the residues in the non-diagonal parts of propagators are chosen to be equal to 0 in order to forbid mixing for on-mass-shell particles, so

as no additional renormalization of wave functions is required, besides what given by Eqs. (2.19). Thus, the OMS renormalization scheme does preserve physical meaning of the original quantities in the electroweak Lagrangian $\mathcal{L}^{EW}$ (2.6)-(2.16).

The formulated OMS renormalization conditions [22–24] allow us to obtain explicitly $\delta z^{W,B}_{1,2}$, $\delta z^{f}_{L,R}$, $\delta M^2_{W,Z}$, δm^2_f (2.19) in terms of the unrenormalized self-energies of gauge bosons, $\Sigma^{W,Z}(M^2_{W,Z})$, $\Sigma^{A}(0)$, $\Sigma^{Z\gamma}(0)$, and fermions $\Sigma^{f}(m_f)$, and their derivatives $\partial\Sigma^{A,Z,W}(k^2)/\partial k^2$, $\partial\Sigma^{f}(\not{p})/\partial\not{p}$, which are calculated in the one-loop approximation amenably to the Lagrangian (2.6)-(2.16). In particular, the fermion self-energies are given in the usual way by the graphs

$$\Sigma^{f}(\not{p}) = \qquad \begin{array}{c} W, Z, A\gamma \\ \sim\sim\sim \\ f \end{array} \qquad , \tag{2.24}$$

where the wavy line renders the propagators of $W-, Z-$bosons, $D^{W,Z}$ (2.7), and photons, $D^{A\gamma}$ (2.8), with the fictitious mass m_γ which hereafter takes not only the infinitesimal value $m_\gamma{=}\lambda{\to}0$, but also the value $m_\gamma{=}M_S$ specified so as $M^2_N{\ll}M^2_S{\ll}M^2_W$.

Upon calculating the radiative corrections with the fields, masses and coupling constants renormalized amenably to the OMS renormalization scheme, not only the UV divergencies occurring in the loop expansion (of propagators as well as $S-$matrix elements) are absorbed in the infinite parts of the renormalization constants, $\delta z^{W,B}_{1,2}$, $\delta z^{f}_{L,R}$, $\delta M^2_{W,Z}$, δm^2_f, but also the finite parts of the radiative corrections are fixed. These lead to physically observable consequences.

The essential ingredients to obtain radiative corrections are the three-particle vertex functions. First we are to acquire the electroweak radiative corrections to the bare $e\nu W-$vertex

$$\Gamma^{e\nu W}_{\alpha} = \frac{e}{2\sqrt{2}s_W}\gamma_\alpha(1-\gamma^5) = \qquad \begin{array}{ccc} p_e,\sigma_e & \xleftarrow{\quad e \quad} & \nu \xrightarrow{\quad} -p_\nu,-\sigma_\nu \\ & W^-\sim\sim q & \end{array} \tag{2.25}$$

in $\mathcal{L}^{WJJ'}$ (2.13).

The renormalized corrected $e\nu W-$vertex $\hat{\Gamma}^{e\nu W}_{\alpha}(p_e,-p_\nu,q)$ is determined by the matrix element

$$\langle\phi^+_e(p_e,\sigma_e)|\mathcal{S}^{EW}|\phi_\nu(-p_\nu,-\sigma_\nu),W^{-\,\alpha}(q)\rangle =$$

$$= i(2\pi)^4\delta(q-p_\nu-p_e)(\bar{u}_e(p_e,\sigma_e)\hat{\Gamma}^{e\nu W}_{\alpha}(p_e,-p_\nu,q)w^{-\,\alpha}(q)u_\nu(-p_\nu,-\sigma_\nu)) \tag{2.26}$$

of the $\mathcal{S}^{EW}-$operator

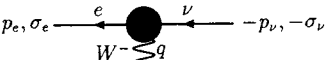
$$\mathcal{S}^{EW} = \mathcal{T}\exp[i\int \mathrm{d}^4x\mathcal{L}^{EW}_{int}(x)]\,, \tag{2.27}$$

with $\mathcal{L}^{EW}_{int}(x)$ given by (2.10). Here $\mathcal{T}$ represents ordinary time ordering, $\phi_\nu(-p_\nu,-\sigma_\nu)$ stands for a neutrino with the momentum $-p_\nu$ and the polarization $-\sigma_\nu$ in an initial state, and $\phi_e(p_e,\sigma_e)$

stands for an electron with the momentum p_e and the polarization σ_e in a final state, $u_{e,\nu}$ indicate the Dirac spinors of leptons. In the transition from the initial to the final state, a W^-−boson with the momentum $q = p_e + p_\nu$ and the polarization α is absorbed (or W^+ emitted).

Pursuant to the aforecited OMS renormalization scheme [22–25], we obtain in the one-loop order, $O(\alpha)$,

$$(\bar{u}_e(p_e,\sigma_e)\hat{\Gamma}^{e\nu W}_{\alpha}(p_e,-p_\nu,q)w^{-\alpha}(q)u_\nu(-p_\nu,-\sigma_\nu)) =$$



(2.28)

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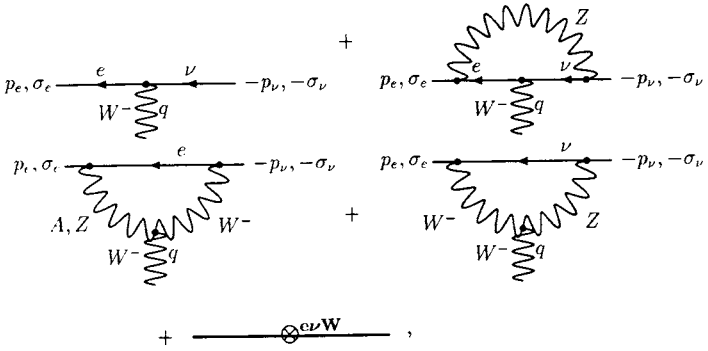


where the last diagram represents the relevant counter term

$$\Gamma^{e\nu W}_{ct\,\alpha} = \Gamma^{e\nu W}_{\alpha}\delta z^{e\nu W}\,, \tag{2.29}$$

$$\delta z^{e\nu W} = \Big(\frac{1}{2}\delta z^{e}_{L} + \frac{1}{2}\delta z^{\nu}_{L} + \delta z^{W}_{1} - \delta z^{W}_{2}\Big)\,, \tag{2.30}$$

as one can infer from Eqs. (2.11)-(2.16), (2.19)-(2.23). Here $\delta z^{e,\nu}_{L}$ render the renormalization of the electron and neutrino wave functions, and the difference $\delta z^{W}_{1} - \delta z^{W}_{2}$ is expressed through the $z\gamma$−transition self-energy [23,24]

$$\delta z^{W}_{1} - \delta z^{W}_{2} = \frac{-1}{M^2_Z s_W c_W}\Sigma^{Z\gamma}(0) = \frac{-\alpha}{4\pi}\frac{2}{s^2_W}\Delta(M_W)\,. \tag{2.31}$$

Neglecting all the terms of $O(m_e/M_{Z,W})$, $O(p^2_{e,\nu}/M^2_{W,Z})$ and presuming the fictitious photon mass in Eq. (2.9) $m_\gamma{=}\lambda{\to}0$, we obtain in the one-loop order, $O(\alpha)$,

$$\delta z^{e\nu W} = -\frac{\alpha}{4\pi}\Big\{2\ln\frac{\lambda}{m} + \ln\frac{M_Z}{m} + \frac{9}{4} - \frac{5}{s^2_W}\ln c_W + \frac{1}{s^2_W} + \frac{10c^2_W+1}{4c^2_W s^2_W}\Big(\Delta(M_Z) - \frac{1}{2}\Big)\Big\}\,. \tag{2.32}$$

In (2.31), (2.32) and thereafter, the quantities $\Delta(M_i)$ stand for the UV divergent singular terms for given masses M_i. Within the method of dimensional regularization (see, for instance, [8,25]), $\Delta(M_i)$ are known to be given as

$$\Delta(M) = \frac{2}{4-D} - \gamma - \ln\frac{M^2}{4\pi\mu^2}\,, \tag{2.33}$$

where D, γ, μ are the space-time dimension, the Euler constant and the mass scale, respectively. Let us behold that amenably to the old-established momentum-space cut-off, $\Delta(M_i)$ could merely be presented as

$$\Delta(M) = \frac{1}{2} + 2\ln\frac{\Lambda}{M}\,, \tag{2.34}$$

with the momentum-space cut-off parameter Λ [7–9]. It goes as a matter of course that neither D, γ, μ, nor Λ will occur in the corrected renormalized vertexes, propagators and self-energy parts of fermions and gauge bosons. The corrected renormalized $e\nu W-$vertex (2.28) results as

$$\hat{\Gamma}^{e\nu W}_{\alpha} = \Gamma^{e\nu W}_{\alpha}\Big\{1 + \frac{\alpha}{4\pi}\Big(2\ln\frac{m}{\lambda} + \ln\frac{m}{M_Z} - \frac{9}{4} + \frac{3}{s^2_W} + \frac{6c^2_W - s^2_W}{s^4_W}\ln c_W\Big)\Big\}. \tag{2.35}$$

As seen, the renormalized corrected $e\nu W-$vertexes is multiple to the bare one, and quarks are not involved in (2.35), within the applied one-loop approach. The infrared divergence, $\sim\ln\lambda/m$, occurring in (2.35) is known to disappear out of the eventual result for $\beta-$decay probability [7–9,26].

To acquire the neutron-proton-$W-$boson vertex function $\hat{\Gamma}^{pnW}_{\alpha}$ we shall hereafter have to deal with the renormalized corrected $udW-$vertex $\hat{\Gamma}^{udW}_{S\,\alpha}$ for the pure quark transition $d\to u+W^-$ in the quark system described by the electroweak Lagrangian (2.10)-(2.16), with the fictitious photon mass $m_\gamma = M_S$ $(M^2_N \ll M^2_S \ll M^2_W)$ adopted. In this case, the calculation involves the "massive photon" propagator

$$D^{As}_{\alpha\beta}(x) = \delta_{\alpha\beta}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\frac{\exp(-ikx)}{k^2 - M^2_S + i0}\,. \tag{2.36}$$

In particular, the wavy line in (2.24) renders $D^{As}_{\alpha\beta}(x)$ (2.36). What is to emphasize is that this subsidiary mass M_S is negligible as compared to the heavy boson mass M_W, though the nucleon mass M_N is, in turn, negligible as compared with M_S.

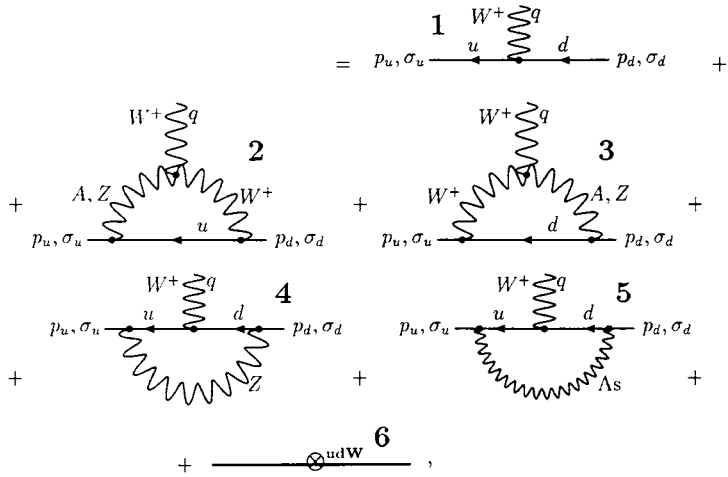
In much the same way as in the leptonic case, the corrected renormalized vertex $\hat{\Gamma}^{udW}_{S\,\alpha}$ is introduced by the matrix element

$$\langle\phi^+_u(p_u,\sigma_u)|\mathcal{S}^{EW}|\phi_d(p_d,\sigma_d), W^{+\,\alpha}(q)\rangle =$$
$$= i(2\pi)^4\delta(q + p_d - p_u)(\bar{u}_u(p_u,\sigma_u)\hat{\Gamma}^{udW}_{S\,\alpha}(p_u,p_d,q)w^{+\,\alpha}(q)u_d(p_d,\sigma_d)) \tag{2.37}$$

to describe the transition of an initial $d-$quark with the momentum p_d and polarization σ_d into a final $u-$quark with the momentum p_u and polarization σ_u, when a W^+-boson with the momentum

$q=p_u-p_d$ and polarization α is absorbed (or W^- emitted). The quantities $u_{u,d}$ indicate the Dirac spinors of quarks. Following the above expounded OMS renormalization scheme [22–25], we acquire from the Lagrangian (2.6)-(2.16), with $m_\gamma{=}M_S$ assumed, in the one-loop order, $O(\alpha)$,

$$(\bar{u}_u(p_u,\sigma_u)\hat{\Gamma}^{udW}_{S\,\alpha}(p_u,p_d,q)w^{+\,\alpha}(q)u_d(p_d,\sigma_d)) = \tag{2.38}$$



where the wavy line with the tag As stands for the "massive photon" propagator D^{As} (2.36). The first graph in (2.38) depicts the bare $udW-$vertex

$$\Gamma^{udW}_{\alpha} = |V_{ud}|\frac{e}{2\sqrt{2}s_W}\gamma_\alpha(1-\gamma^5) \tag{2.39}$$

originating from $\mathcal{L}^{WJJ'}$ (2.13), and the last one accordingly Eqs. (2.19)-(2.23) represents the counter term

$$\hat{\Gamma}^{udW}_{S\,\alpha\,ct} = \Big(\frac{1}{2}\delta z^u_L + \frac{1}{2}\delta z^d_L + \delta z^W_1 - \delta z^W_2\Big)\cdot\Gamma^{udW}_{\alpha}\,, \tag{2.40}$$

where $\delta z^{u,d}_L$ render the renormalization of the quark wave functions, and the difference $\delta z^W_1 - \delta z^W_2$ is given by (2.31). Omitting the terms $O(p^2_{u,d}/M^2_{W,Z,S})\,, O(M^2_S/M^2_{W,Z})$, we obtain the corrected renormalized vertex

$$\hat{\Gamma}^{udW}_{S\,\alpha} = \Gamma^{udW}_{\alpha}\cdot\Gamma(W)\,, \tag{2.41}$$

$$\Gamma(W) = \left\{1 + \frac{\alpha}{4\pi}\Big(\ln\frac{M_S}{M_Z} + \frac{3}{s^2_W} + \frac{6c^2_W - s^2_W}{s^4_W}\ln(c_W)\Big)\right\}, \tag{2.42}$$

multiple to the bare vertex (2.39). Of course, there occurs no infrared divergence in $\hat{\Gamma}^{udW}_{S\,\alpha}$ (2.42).

So, we have acquired the renormalized corrected $e\nu W-$ and $udW-$vertices which are needed to calculate the neutron $\beta-$decay amplitude.

III. TREATMENT OF NUCLEON STRUCTURE IN DESCRIBING THE NEUTRON $\beta-$DECAY

Up to now, we have dealt with the pure electroweak interactions $\mathcal{L}_{int}^{EW}$ (2.10)-(2.16). As the nucleon is a complex system of strong interacting quarks, the neutron $\beta-$decay (1.6) can never be reduced to the pure transition

$$d \Longrightarrow u + e^{-} + \bar{\nu} + \gamma \,. \tag{3.1}$$

We are to allow for the nucleon compositeness, such as excited states and formfactors associated with the nucleon intrinsic structure caused by the strong quark-quark interactions. Therefore, $\mathcal{L}_{int}^{EW}$ (1.8) is to be completed by $\mathcal{L}_{str}^{qq}$ to describe the transition (3.1) in a system of strong interacting quarks,

$$\mathcal{L}_{int}(x) = \mathcal{L}_{int}^{EW}(x) + \mathcal{L}_{str}^{qq}(x) \,. \tag{3.2}$$

Ignored the strong quark-quark interactions $\mathcal{L}_{str}^{qq}(x)$, the baryon is a free quark system described (in terms of quark occupation numbers) by the Heisenberg wave function $\Phi_{0\,B}^{q}(P_B, \sigma_B)$ with the given total momentum P_B, and the spin $\boldsymbol{\sigma}_B$ and polarization σ_{Bz} indicated as σ_B. So far as interactions vanish at infinity,

$$\mathcal{L}_{int}(x) \longrightarrow 0 \,, \quad \text{when} \quad x^0 \longrightarrow \mp\infty \,, \tag{3.3}$$

the baryon wave function in the interaction representation is written in the ordinary form:

$$\Phi_B^q(P_B, \sigma_B, x^0) = \mathcal{S}_{str}(x^0, \mp\infty)\Phi_B^q(P_B, \sigma_B, \mp\infty) = \mathcal{S}_{str}(x^0, \mp\infty)\Phi_{0B}^q(P_B, \sigma_B) \,, \tag{3.4}$$

$$\mathcal{S}_{str}(x^0, -\infty) = \mathcal{T}\exp\Big(i\int\limits_{-\infty}^{x^0} \mathrm{d}x^0 \int \mathrm{d}\mathbf{x}\mathcal{L}_{str}^{qq}(x)\Big) \,, \qquad \mathcal{S}(t,t')\cdot\mathcal{S}(t',t_0) = \mathcal{S}(t,t_0) \,.$$

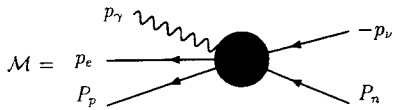
The operator

$$\mathcal{S}_{str}(x_1^0, x_2^0) = \mathcal{T}\exp\Big(i\int\limits_{x_1^0}^{x_2^0} \mathrm{d}x^0 \int \mathrm{d}\mathbf{x}\mathcal{L}_{str}^{qq}(x)\Big) \tag{3.5}$$

transforms a state of the quark system at a time-point x_1^0 to a state at a time-point x_2^0 :

$$\Phi_B^q(P_B, \sigma_B, x_2^0) = \mathcal{S}_{str}(x_2^0, x_1^0)\Phi_B^q(P_B, \sigma_B, x_1^0) \,. \tag{3.6}$$

The transition amplitude

$\mathcal{M} =$ 

(3.7)

to describe the neutron $\beta-$decay (1.6) is determined by the matrix element of $\mathcal{S}_{int}$

$$\mathcal{M}\cdot i(2\pi)^4\delta(P_n-P_p-p_e-p_\nu-p_\gamma)=$$

$$\langle\Phi^{q\,+}_{0p}(P_p,\sigma_p),\phi^+_e(p_e,\sigma-e),A(p_\gamma)|\mathcal{S}_{int}|\Phi^q_{0n}(P_n,\sigma_n),\phi_\nu(-p_\nu,-\sigma_e)\rangle\,, \tag{3.8}$$

$$\mathcal{S}_{int}\equiv\mathcal{S}_{int}(\infty,-\infty)=\mathcal{T}\exp\Big(i\int \mathrm{d}^4x\mathcal{L}_{int}(x)\Big)=\mathcal{T}\exp\Big(i\int \mathrm{d}^4x[\mathcal{L}^{EW}_{int}(x)+\mathcal{L}^{qq}_{str}(x)]\Big)\,. \tag{3.9}$$

For now, there sees no option, but to parameterize the effects of strong interactions in treating the neutron $\beta-$decay. We do not intend neither to specify an actual form of $\mathcal{L}^{qq}_{int}(x)$, nor to procure an explicit expression of the baryon wave function $\Phi^q_B(P_B,\sigma_B)$ in (3.4)-(3.9), but we posit an appropriate parameterization of matrix elements of the electroweak interactions $\mathcal{L}^{EW}_{int}$ (2.10)-(2.16) between the baryon wave functions $\Phi^q_B(P_B,\sigma_B)$. In this respect, by introducing the ordinary nucleon weak transition current

$$\mathcal{J}^\beta_{np}(k)=\gamma^\beta g_V(k^2)+g_{WM}(k^2)\sigma^{\beta\nu}k_\nu-(\gamma^\beta g_A(k^2)+g_{IP}(k^2)k^\beta)\gamma^5\,, \tag{3.10}$$

the matrix element of $\mathcal{L}^{WJJ'}$ (2.13)

$$\Lambda^{npW}_{0\alpha}(k)=\int \mathrm{d}^4y\langle\Phi^{q\,+}_p(P_p,\sigma_p)|\bar\psi_q(y)\Gamma^{udW}_\chi(k)T^+_q\psi_q(y)W^{+\,\chi}(y)|\Phi^q_n(P_n,\sigma_n),W^+_\alpha(k)\rangle\,, \tag{3.11}$$

$$k=P_p-P_n\,,$$

is rewritten in terms of the nucleon field operators,

$$\Psi_N(y)=\sum_{P_N,\sigma_N}\Big(U_N(\mathbf{P}_N,\sigma_N)a_N(\mathbf{P}_N,\sigma_N)\exp[-iP_Ny]+$$

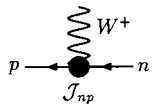
$$U_N(-\mathbf{P}_N,-\sigma_N)b^+_N(\mathbf{P}_N,\sigma_N)\exp[iP_Ny]\Big)\,, \tag{3.12}$$

and the nucleon wave functions $\Phi^N_{n,p}(P_{n,p},\sigma_{n,p})$ describing the single-nucleon states with the given $P_{n,p},\sigma_{n,p}$. What results is

$$\Lambda^{npW}_{0\alpha}(k)=\int \mathrm{d}^4y\langle\Phi^{N\,+}_p(P_p,\sigma_p)|\bar\Psi_N(y)\Gamma^{npW}_\chi(k)T^+_N\Psi_N(y)W^{+\,\chi}(y)|\Phi^N_n(P_n,\sigma_n),W^+_\alpha(k)\rangle=$$

$$=(2\pi)^4\delta(P_n-P_p+k)\bar U_p(P_p,\sigma_p)\Gamma^{npW}_\alpha(k)T^+_NU_n(P_p,\sigma_n)w^+_\alpha(k)\,, \tag{3.13}$$

where

$$\Gamma^{npW}_\alpha(k)=\frac{e|V_{ud}|}{2\sqrt{2}s_W}\mathcal{J}_{np\,\alpha}(k)=$$ 

, (3.14)

the operator T_N^+ transforms the neutron into the proton, $U_{n,p}$ indicate the Dirac spinors of nucleons. So, the matrix element $\Lambda_0^{npW}(k)$, originally written in terms of the quark states, results to be expressed through the nucleon states and the electroweak formfactors g_V, g_A, g_{WM}, g_{IP}. Hereafter we shall also have to deal with the general case of weak transitions between the single-baryonic states $\Phi_s^B(P_s,\sigma_s)$ including, besides the neutron and proton, various excited states of the nucleon. Alike Eqs. (3.13), (3.14), the matrix elements to describe these processes are written in terms of the baryonic field operators $\Psi_s^B(x)$ and the appropriate generalized transition currents

$$\Lambda_{0\alpha}^{rsW} = \int \mathrm{d}^4y \langle \Phi_r^{B+}(P_r,\sigma_r)|\bar{\Psi}_r(y)\Gamma_\chi^{rsW}(k)T_B^+\Psi_s(y)W^{+\chi}(y)|\Phi_s^B(P_s,\sigma_s),W_\alpha^+(k)\rangle\,, \tag{3.15}$$

where

$$\Gamma_\alpha^{rsW}(k) = \frac{e|V_{ud}|}{2\sqrt{2}s_W}\mathcal{J}_{rs\alpha}(k) = \quad r \longrightarrow \bullet \longleftarrow s \quad (W^+,\ \mathcal{J}_{rs}) \tag{3.16}$$

and T_B^+ increases baryon charge by one unite.

In much the same way, the matrix element of $\mathcal{L}^{Aqq}$ (2.16) transforms as follows

$$\int \mathrm{d}^4x \langle \Phi_B^{q+}(P_B,\sigma_B)|\mathcal{L}^{Aqq}(x)|\Phi_{B'}^q(P_{B'},\sigma_{B'}),A^\alpha(k)\rangle = \quad B' \longrightarrow \bullet \longleftarrow B \quad (A,\ f^{BB'})$$
$$= -e(2\pi)^4\delta(P_{B'}-P_B-k)\Big(\bar{U}_B(P_B,\sigma_B)f_\alpha^{BB'}(k)U_{B'}(P_{B'},\sigma_{B'})\Big)A^\alpha(k)\,, \tag{3.17}$$

where the formfactors $f_\alpha^{BB'}(k)$ to describe the electromagnetic transitions of baryons $B'\to B$ are of the usual form [7–9]

$$f_\alpha^{NN}(k) = f_1^{NN}(k^2)\gamma_\alpha + f_2^{NN}(k^2)k^\beta\sigma_{\alpha\beta} \tag{3.18}$$

in the case of neutron and proton (N=n,p) interactions with electromagnetic field A^α. At the momentum transferred $k^2\lesssim M_N$, the quantity g_{WM} is given through the nucleon anomalous magnetic moments,

$$g_{WM} \approx \frac{\mu_n-\mu_p}{2M_p} \approx -\frac{3.7}{2M_p}\,, \tag{3.19}$$

the assessment

$$g_{IP}(k^2) \approx \frac{2M_p\,g_A(k^2)}{k^2-m_\pi^2} \sim \frac{2M_p\,g_A(0)}{m_\rho^2-m_\pi^2} \sim \frac{8g_A(0)}{2M_p}\,, \tag{3.20}$$

is appropriate, and the estimations

$$f_1^{pp}(k^2) \approx \frac{-m_\rho^2}{k^2-m_\rho^2}\,,\quad f_2^{pp}(k^2) \approx \Big(\frac{1.79}{2M_p}\Big)\frac{-m_\rho^2}{k^2-m_\rho^2}\,,\quad f_1^{nn}=0\,,\quad f_2^{nn}(k^2) = \Big(\frac{1.93}{2M_n}\Big)\frac{m_\rho^2}{k^2-m_\rho^2} \tag{3.21}$$

hold true within the vector-dominant model (see, for instance, Refs. [7–9]). Here m_π, m_ρ are conceived to be of the order of the $\pi-$ and $\rho-$meson masses. Evidently, at $k^2 \ll M_N^2$, Eqs. (3.10), (3.18) are reduced to

$$\mathcal{J}^{\beta}_{np}(0) = \gamma^{\beta} - \gamma^{\beta} g_A \gamma^5 , \tag{3.22}$$

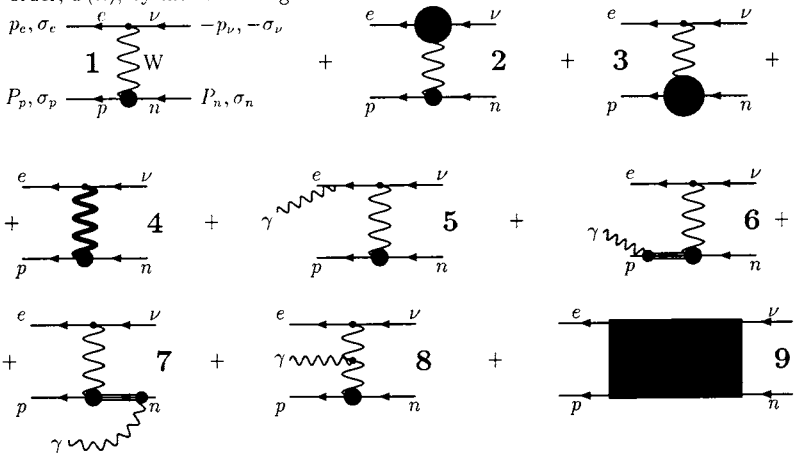
$$\Gamma^{npW}_{\alpha}(0) = \frac{e|V_{ud}|}{2\sqrt{2}s_W}\gamma_\alpha(1 - g_A(0)\gamma^5) , \tag{3.23}$$

$$f^{pp}_{\alpha}(0) = \gamma_\alpha , \quad f^{nn}(0) = 0 , \tag{3.24}$$

and the nucleon is treated as being a point-like particle, except for the residence of g_A in the nucleon weak transition current (3.22).

IV. TRANSITION AMPLITUDE.

As dictated by $\mathcal{L}_{int}$ (3.2), the transition amplitude $\mathcal{M}$ (3.7)-(3.9) is represented in the one-loop order, $O(\alpha)$, by the set of diagrams



(4.1)

with the contents heretofore given by (2.10)-(2.16), (2.24), (3.23), (2.28), (2.38), (3.14) (3.16) and also currently explicated hereafter, as far as used. At the lowest order in $\mathcal{L}^{EW}_{int}$ (2.10)-(2.16), that is without radiative corrections, the uncorrected Born amplitude $\mathcal{M}^0$ presented by the first graph in (4.1) is determined by

$$\mathcal{M}^0 \cdot i(2\pi)^4\delta(P_n - P_p - p_e - p_\nu) = \Big(\frac{e}{2\sqrt{2}s_W}\Big)^2 |V_{ud}| \int \frac{\mathrm{d}^4k}{(2\pi)^4} \int \mathrm{d}^4x \times$$
$$\times \langle \phi^+_e(p_e,\sigma_e)|\bar{\psi}_e(x)\gamma^\alpha(1-\gamma^5)\psi_\nu(x)|\phi_\nu(-p_\nu,-\sigma_\nu)\rangle (i)\frac{g_{\alpha\beta}}{k^2 - M_W^2} \times \tag{4.2}$$
$$\times \int \mathrm{d}^4y e^{-ik(x-y)} \langle \Phi^{q\,+}_{0p}(P_p,\sigma_p)|\mathcal{T}\Big\{\bar{\psi}_q(y)\gamma^\beta(1-\gamma^5)T^+_q\psi_q(y)\cdot\mathcal{S}_{str}\Big\}|\Phi^q_{0n}(P_n,\sigma_n)\rangle ,$$

where the strong interactions intrude via $\mathcal{S}_{str}\equiv\mathcal{S}_{str}(\infty,-\infty)$ (3.5). With allowance for the relations

$$\mathcal{S}_{str}=\mathcal{S}_{str}(\infty,y^0)\mathcal{S}_{str}(y^0,-\infty)\,,\quad \Phi^q_N(P_N,\sigma_N,y^0)=\mathcal{S}_{str}(y^0,-\infty)\Phi^q_{0N}(P_N,\sigma_N)\,, \tag{4.3}$$

the last integral in (4.2) is reduced as follows

$$\int \mathrm{d}^4y e^{iky}\langle\Phi^{q\,+}_{0p}(P_p,\sigma_p)\mathcal{S}_{str}(\infty,y^0)|\bar\psi_q(y)\gamma^\beta(1-\gamma^5)T^+_q\psi_q(y)|\mathcal{S}_{str}(y^0,-\infty)\Phi^q_{0n}(P_n,\sigma_n)\rangle=$$
$$\int \mathrm{d}^4y e^{iky}\langle\Phi^{q\,+}_{p}(P_p,\sigma_p)|\bar\psi_q(y)\gamma^\beta(1-\gamma^5)T^+_q\psi_q(y)|\Phi^q_{n}(P_n,\sigma_n)\rangle\,. \tag{4.4}$$

Applying to the expressions (3.10)-(3.14), the Born amplitude proves to be

$$\mathcal{M}^0=\bar u_e(p_e,\sigma_e)\Gamma^{e\nu W}_\alpha u_\nu(-p_\nu,-\sigma_\nu)\times$$
$$\times\int \mathrm{d}^4y e^{iqy}\langle\Phi^{q\,+}_{p}(P_p,\sigma_p)|\bar\psi_u(y)\,\Gamma^{udW}_\beta(q)\,\psi_d(y)|\Phi^q_n(P_n,\sigma_n)\rangle\cdot D^W_{\alpha\beta}(q)=$$
$$=\bar u_\iota(p_\iota,\sigma_\iota)\Gamma^{e\nu W}_\alpha u_\nu(-p_\nu,-\sigma_\nu)\cdot\bar U_p(P_p,\sigma_p)\Gamma^{npW}_\beta(q)U_n(P_n,\sigma_n)\cdot D^W_{\alpha\beta}(q)\,, \tag{4.5}$$
$$\Gamma^{e\nu W}_\alpha=\frac{e}{2\sqrt{2}s_W}\gamma_\alpha(1-\gamma^5)\,,\quad \Gamma^{npW}_\alpha(q)=|V_{ud}|\frac{e}{2\sqrt{2}s_W}\mathcal{J}^\alpha_{np}(q)\,,$$
$$q=P_n-P_p-p_e-p_\nu$$

As $q^2\ll M_p^2\ll M_W^2$, the quantities $\Gamma^{npW}_\alpha(q)$, $\mathcal{J}^\alpha_{np}(q)$ are replaced by (3.23), (3.22), and

$$D^W_{\alpha\beta}(q)=\frac{g_{\alpha\beta}}{q^2-M_W^2}=\frac{-g_{\alpha\beta}}{M_W^2}\,. \tag{4.6}$$

With allowance for the radiative corrections, the bare, uncorrected vertexes $\Gamma^{e\nu W}_\alpha$, $\Gamma^{npW}_\alpha(q)$ and $W-$propagator $D^W_{\alpha\beta}(q)$ in $\mathcal{M}^0$ (4.5), depicted by the point, blob and thin wavy line in the graph **1** in Eq. (4.1), will give place to the corrected renormalized quantities $\hat\Gamma^{e\nu W}_\alpha$, $\hat\Gamma^{npW}_\alpha(q)$, $\hat D^W_{\alpha\beta}(q)$, what counts is that the terms presented by the graphs **2, 3, 4** emerge in $\mathcal{M}$ (4.1) in the one-loop order, $O(\alpha)$; $\hat\Gamma^{e\nu W}_\alpha$, $\hat\Gamma^{npW}_\alpha(q)$, $\hat D^W_{\alpha\beta}(q)$ are depicted by the shaded circle, the shaded circle with heavy core, and the heavy wavy line in the graphs **2, 3, 4**, respectively.

The terms presented by the graphs **5, 6, 7, 8** describe the real $\gamma-$radiation, and the graphs of the type **9**, usually called the "box-diagrams", render generically all the irreducible four-particle processes.

The contribution of the graph **2** is merely acquired from (4.5) by replacement of $\Gamma^{e\nu W}_\alpha$ in (4.5) by $\hat\Gamma^{e\nu W}_\alpha$ (2.28), (2.35).

The corrected renormalized vertex $\hat\Gamma^{npW}_\alpha$ in the graph **3** in (4.1) describes the $n\to p$ transition by absorbing a $W^{+\alpha}(q)$ boson with the polarization α and the momentum q (or emitting $W^{-\alpha}(q)$). The contribution of the graph **3** originates from (4.5) by replacing $\Gamma^{npW}_\alpha\Longrightarrow\hat\Gamma^{npW}_{N\alpha}$. So, the calculation of $\hat\Gamma^{npW}_\alpha(q)$ is in order.

V. THE RADIATIVE CORRECTIONS TO THE pnW-VERTEX WITHOUT INVOLVING STRONG QUARK-QUARK INTERACTIONS.

In the third order in the quark part of $\mathcal{L}_{int}^{EW}$ (2.10), the vertex $\hat{\Gamma}_{\alpha}^{npW}(q)$ is defined by the matrix element which involves besides the electroweak interactions, $\mathcal{L}^{Zqq}$, $\mathcal{L}^{Wqq}$, $\mathcal{L}^{Aqq}$, $\mathcal{L}^{ZWW}$, $\mathcal{L}^{AWW}$ (2.11)-(2.16), the strong quark-quark interactions $\mathcal{L}_{str}^{qq}$ as well, via $\mathcal{S}_{str}\equiv\mathcal{S}_{str}(\infty,-\infty)$ (3.5) :

$$
\begin{aligned}
i(2\pi)^4\delta(P_n-P_p+q)\,(\bar{U}_p(P_p,\sigma_p)\hat{\Gamma}_{\alpha}^{pnW}(P_n,P_p,q)w^{+\,\alpha}(q)U_n(P_n,\sigma_n)) = i\Lambda_{0\alpha}^{npW}(q)+ \\
(-i)\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\langle\Phi_{0p}^{q+}(P_p,\sigma_p)|\mathcal{T}\Big\{\Big(\mathcal{L}^{Wqq}(x_1)\mathcal{L}^{Wqq}(x_2)\mathcal{L}^{Wqq}(x_3)+ \\
\mathcal{L}^{Wqq}(x_1)\mathcal{L}^{Zqq}(x_2)\mathcal{L}^{ZWW}(x_3)+\mathcal{L}^{Wqq}(x_1)\mathcal{L}^{Aqq}(x_2)\mathcal{L}^{AWW}(x_3)+ \\
\mathcal{L}^{Wqq}(x_1)\mathcal{L}^{Zqq}(x_2)\mathcal{L}^{Zqq}(x_3)+\mathcal{L}^{Wqq}(x_1)\mathcal{L}^{Aqq}(x_2)\mathcal{L}^{Aqq}(x_3)\Big)\cdot\mathcal{S}_{str}\Big\}|\Phi_{0n}^{q}(P_n,\sigma_n),W_{\alpha}^{+}(q)\rangle\equiv \\
\equiv i\Lambda_{0\alpha}^{npW}(q)+\Lambda_{\alpha}^{WWW}(q)+\Lambda_{\alpha}^{WWZ}(q)+\Lambda_{\alpha}^{WAW}(q)+\Lambda_{\alpha}^{WZZ}(q)+\Lambda_{\alpha}^{WAA}(q)\,.
\end{aligned}
\tag{5.1}
$$

The processes of different kinds contribute to $\hat{\Gamma}_{\alpha}^{pnW}(P_n,P_p,q)$ (5.1).

All the terms but last in the integrand in (5.1) prove to incorporate the propagators of heavy gauge bosons $D_{\alpha\beta}^{W,Z}$ (2.7). So, in the r.h.s. of (5.1), Λ^{WWW}, Λ^{WWZ}, Λ^{WAW}, Λ^{WZZ} render the processes where the quark-quark electroweak interactions are due to the heavy gauge bosons exchange that corresponds to large momenta transferred, $q^2{\sim}M_{Z,W}^2{\gg}M_N^2$, and therefore the short-range, $\sim 1/M_{W,Z}$, quark-quark electroweak interactions cause these processes. By emitting or absorbing a virtual heavy gauge boson, large momenta $q^2{\sim}M_{W,Z}^2$ is transferred to the quarks constituting the nucleon. As quark momenta inside the nucleon are relatively small, $q^2{\lesssim}M_N^2$, quarks possess large momenta, $q^2{\sim}M_{Z,W}^2{\gg}M_N^2$, in the intermediate states between emission and absorption of heavy gauge bosons in the vertexes $\mathcal{L}^{Wqq}(x_1)$, $\mathcal{L}^{Zqq}(x_2)$ in (5.1). What is the underlying inherent principle of the Standard Model to emphasize at this very stage is that the strong quark-quark interactions die out when quarks possess the large momenta $q^2{\gg}M_N^2$. Consequently, given the fact that quarks have got such a large momenta, the strong quark-quark interactions die out, i.e. $\mathcal{L}_{str}^{qq}$ vanishes, in these intermediate states, and we deal with free quarks [8,9,22–25]. In this respect, on rewriting (with allowance for Eqs. (2.11)-(2.16), (3.4)-(3.6), (4.3)) the quantities Λ^{WWZ}, Λ^{WAW} in the form

$$
\begin{aligned}
\Lambda_{\alpha}^{WWZ}(q) = -\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\langle\Phi_p^{q+}(P_p,\sigma_p)|\mathcal{T}\Big\{(\bar{q}(x_1)\Gamma_{\delta}^{Wqq}(x_1)T_q^{+}q(x_1))\cdot\mathcal{S}_{str}(x_1^0,x_2^0)\times \\
(\bar{q}(x_2)\Gamma_{\beta}^{Zqq}(x_2)q(x_2))\Gamma_{\chi\nu\lambda}^{WWZ}(x_3)W^{+\times}(x_3)\Big\}|\Phi_n^{q}(P_n,\sigma_n),W_{\alpha}^{+}(q)\rangle\cdot D_{\beta\lambda}^{Z}(x_1-x_3)\cdot D_{\nu\delta}^{W}(x_3-x_2)\,, \\
\Lambda_{\alpha}^{WAW}(q) = -\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\langle\Phi_p^{q+}(P_p,\sigma_p)|\mathcal{T}\Big\{(\bar{q}(x_1)\Gamma_{\delta}^{Wqq}(x_1)T_q^{+}q(x_1))\cdot\mathcal{S}_{str}(x_1^0,x_2^0)\times
\end{aligned}
\tag{5.2}
$$

$$(\tilde{q}(x_2)e_q\gamma^\beta q(x_2))\Gamma^{WAW}_{\chi\nu\lambda}(x_3)W^{+\chi}(x_3)\Big\}|\Phi^q_n(P_n,\sigma_n),W^+_\alpha(q)\rangle\cdot D^A_{\beta\lambda}(x_1-x_3)\cdot D^W_{\nu\delta}(x_3-x_2)\,, \quad (5.3)$$

we presume $\mathcal{S}_{str}(x^0_1,x^0_2)$=1 herein, so far as $\mathcal{L}^{qq}_{str}(x)$=0 at $x^0_1\leq x^0\leq x^0_2$ in (3.5). Then, without involving the strong quark-quark interactions, the sum $\Lambda^{WWZ}+\Lambda^{WAW}$ transforms to the matrix element of the $\mathcal{T}-$product of quark field operators presented by the diagrams **2** and **3** in (2.38) between the neutron and proton wave functions (3.4),

$$\Lambda^{WWZ}_\alpha(q)+\Lambda^{WAW}_\alpha(q)=$$

$$=i\int \mathrm{d}^4y e^{iqy}\langle\Phi^{q+}_p(P_p,\sigma_p)|\bar{\psi}_u(y)\,\Gamma^{udW}_\chi(y)W^{+\chi}(y)\psi_d(y)|\Phi^q_n(P_n,\sigma_n),W^+_\alpha(q)\rangle\cdot\Gamma(WZA)\,. \quad (5.4)$$

Here, the bare vertex Γ^{udW}_α is given by (2.39) and

$$\Gamma(WZA)=\frac{3\alpha}{4\pi}\Big\{\frac{1}{2s^2_W}[(1-2e_us^2_W)+(1+2e_ds^2_W)](\Delta(M_Z)-\frac{1}{2})+(e_u-e_d)(\Delta(M_W)-\frac{1}{2})\Big\}+$$

$$+\frac{\alpha}{4\pi}\Big(4(e_u-e_d)+\Big(4+6\frac{c^2_W}{s^2_W}\ln c_W\Big)[\frac{1}{s^2_W}+(e_d-e_u)s^2_W]\Big)\,, \quad (5.5)$$

accordingly a direct evaluation of the contribution from the diagrams **2** and **3** in (2.38). With making use of Eqs. (3.10)-(3.14), (3.22), (3.23), the expression (5.4) results as

$$\Lambda^{WWZ}_\alpha(q)+\Lambda^{WAW}_\alpha(q)=$$

$$=i(2\pi)^4\delta(P_n-P_p+q)\Big(\bar{U}_p(P_p,\sigma_n)\Gamma^{npW}_\alpha(q)w^{+\alpha}(q)U_n(P_n,\sigma_n)\Big)\Gamma(WZA)\,. \quad (5.6)$$

Certainly, $\Gamma^{npW}_\beta(0)$ (3.23) resides herein at $q^2\ll M^2_N$.

Recalling Eqs. (2.19)-(2.24), we acquire in much the same way

$$\Lambda^{WWW}_\alpha(q)=i\int \mathrm{d}^4y e^{iqy}\langle\Phi^{q+}_p(P_p,\sigma_p)|\bar{\psi}_u(y)\,\Gamma^{udW}_\chi W^{+\chi}(y)\,\psi_d(y)|\Phi^q_n(P_n,\sigma_n),W^+_\alpha(q)\rangle\times$$

$$\times\Big(\delta z^W_1-\delta z^W_2+\frac{1}{2}\delta z^u_L(M_W)+\frac{1}{2}\delta z^d_L(M_W)\Big)=$$

$$i(2\pi)^4\delta(P_n-P_p+q)\Big(\bar{U}_p(P_p,\sigma_n)\Gamma^{npW}_\alpha(q)w^+_\alpha(q)U_n(P_n,\sigma_n)\Big)\times$$

$$\times\Big(\delta z^W_1-\delta z^W_2+\frac{1}{2}\delta z^u_L(M_W)+\frac{1}{2}\delta z^d_L(M_W)\Big)\,, \quad (5.7)$$

where the difference $\delta z^W_1-\delta z^W_2$ is given by (2.31), and the quantities

$$\frac{1}{2}\delta z^u_L(M_W)=\frac{1}{2}\delta z^d_L(M_W)=\frac{-1}{4s^2_W}\Big(\Delta(M_W)-\frac{1}{2}\Big)\frac{\alpha}{4\pi} \quad (5.8)$$

specify renormalization of the $u-,d-$quark wave functions caused by the quark self-energies (2.24) with a virtual $W-$boson.

Amenably to Eqs. (3.6), (4.3), (2.19)-(2.24), the quantity Λ^{WZZ} is presented likewise Λ^{WWZ}, Λ^{WAW}, Λ^{WWW} (5.2)-(5.8) in the form

$$\Lambda_{\alpha}^{WZZ}(q) = -\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 \langle \Phi_p^{q\,+}(P_p,\sigma_p)|\mathcal{T}\Big\{\Big(\bar{q}(x_2)\Gamma_{\chi}^{Zqq}(x_2)q(x_2)\Big)\mathcal{S}_{str}(x_2^0,x_1^0)$$
$$\Big(\bar{q}(x_1)\Gamma_{\mu}^{Wqq}(x_1)W^{+\mu}(x_1)T_q^{+}(x_1)q(x_1)\Big)\mathcal{S}_{str}(x_1^0,x_3^0)\times$$
$$\times\bar{q}(x_3)\Gamma_{\beta}^{Zqq}(x_3)q(x_3)\Big\}|\Phi_n^q(P_n,\sigma_n),W_{\alpha}^{+}(q)\rangle D_{\chi\beta}^{Z}(x_2-x_3)\,, \qquad (5.9)$$

where we can presume the strong quark-quark interactions die out in the intermediate states,

$$\mathcal{S}_{str}(x_1^0,x_3^0)=\mathcal{S}_{str}(x_1^0,x_3^0)=1\,,$$

alike in Eqs. (5.2), (5.3). Then, in much the same way as Λ^{WWZ}, Λ^{WAW}, Λ^{WWW} have transformed to (5.4), (5.6), (5.7), the quantity Λ^{WZZ} (5.9) transforms as follows

$$\Lambda_{\alpha}^{WZZ}(q) = i\int \mathrm{d}^4y\epsilon^{iqy}\langle\Phi_p^{q\,+}(P_p,\sigma_p)|\bar{\psi}_u(y)\,\Gamma_{\chi}^{udW}W^{+\chi}(y)\,\psi_d(y)|\Phi_n^q(P_n,\sigma_n),W_{\alpha}^{+}(q)\rangle\times$$
$$\times\Big\{\Gamma(WZ)+\frac{1}{2}\delta z_L^u(M_Z)+\frac{1}{2}\delta z_L^d(M_Z)\Big\}=$$
$$i(2\pi)^4\delta(P_n-P_p+q)\Big(\bar{U}_p(P_p,\sigma_p)\Gamma_{\alpha}^{npW}(q)w_{\alpha}^{+}(q)U_n(P_n,\sigma_n)\Big)\times$$
$$\times\Big\{\Gamma(WZ)+\frac{1}{2}\delta z_L^u(M_Z)+\frac{1}{2}\delta z_L^d(M_Z)\Big\}\,, \qquad (5.10)$$

where the value of $\Gamma(WZ)$ is presented by the diagram **4** in (2.38) what counts is

$$\Gamma(WZ) = -\frac{\alpha}{4\pi}\frac{1}{4s_W^2c_W^2}[1+2s_W^2(e_d-e_u)-4e_de_us_W^4]\Big(\Delta(M_Z)-\frac{1}{2}\Big)\,, \qquad (5.11)$$

and, amenably to Eqs. (2.19)-(2.23), the renormalization constants of the $u-,d-$quark wave functions

$$\frac{1}{2}\delta z_L^d(M_Z) = -\frac{\alpha}{4\pi}\frac{1}{8c_W^2s_W^2}(1+2e_ds_w^2)^2\Big(\Delta(M_Z)-\frac{1}{2}\Big)\,, \qquad (5.12)$$

$$\frac{1}{2}\delta z_L^u(M_Z) = -\frac{\alpha}{4\pi}\frac{1}{8c_W^2s_W^2}(1-2e_us_w^2)^2\Big(\Delta(M_Z)-\frac{1}{2}\Big) \qquad (5.13)$$

are caused by the self-energies (2.24) with a virtual $Z-$boson.

For the consistent treatment of the issue of strong interactions, we rewrite the last term Λ^{WAA} in (5.1) as follows

$$\Lambda_{\alpha}^{WAA} = \Lambda_{s\alpha}^{WAA}+\Lambda_{l\alpha}^{WAA} =$$
$$= -e^2\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\langle\Phi_{0p}^{q\,+}(P_p,\sigma_p)|\mathcal{T}\Big\{\Big(\mathcal{L}^{Wqq}(x_1)\Big(\bar{q}(x_2)e_q\gamma^{\nu}q(x_2)\times$$
$$\Big(D_{\mu\nu}^{As}(x_2-x_3)+D_{\mu\nu}^{Al}(x_2-x_3)\Big)\times \qquad (5.14)$$
$$\bar{q}(x_3)e_q\gamma^{\mu}q(x_3)\Big)\cdot\mathcal{S}_{str}\Big\}|\Phi_{0n}^q(P_n,\sigma_n),W_{\alpha}^{+}(q)\rangle\,,$$

the propagator $D^{A\lambda}(x_2-x_3)$ (2.8) of a virtual photon is split herein into two parts

$$D^{A\lambda}_{\mu\nu}(x_2-x_3)=g_{\mu\nu}\int\frac{\mathrm{d}^4k}{(2\pi)^4}\Big(\frac{1}{k^2-M_S^2+i0}+\frac{-M_S^2}{(k^2-\lambda^2+i0)(k^2-M_S^2+i0)}\Big)e^{-ik(x_2-x_3)}= \quad (5.15)$$
$$= D^{As}_{\mu\nu}(x_2-x_3)+D^{Al}_{\mu\nu}(x_2-x_3)\,,$$

with introducing the subsidiary matching parameter M_S, chosen so that $M_p^2\ll M_S^2\ll M_W^2$ [27,28]. The quantity $D^{As}(x)$, involving only the integration over large momenta $k^2\gtrsim M_S^2$, is natural to be treated as the propagator of a "massive photon" with the mass M_S.

The corrected renormalized vertex $\hat{\Gamma}^{npW}_{\alpha}$ in Eq. (5.1) is written as the sum

$$\hat{\Gamma}^{npW}_{\alpha}=\hat{\Gamma}^{npW}_{s\,\alpha}+\hat{\Gamma}^{npW}_{l\,\alpha}\,, \quad (5.16)$$

where the quantities $\hat{\Gamma}^{npW}_{s\,\alpha}$ and $\hat{\Gamma}^{npW}_{l\,\alpha}$ are determined as follows

$$i(2\pi)^4\delta(P_n-P_p+q)\Big(\bar{U}_p(P_p,\sigma_p)\hat{\Gamma}^{npW}_{s\,\beta}(q)w^{+\beta}(q)U_n(P_n,\sigma_n)\Big)=$$
$$\Lambda^{npW}_{0\beta}+\Lambda^{WWZ}_{\beta}+\Lambda^{WAW}_{\beta}+\Lambda^{WWW}_{\beta}+\Lambda^{WZZ}_{\beta}+\Lambda^{WAA}_{s\beta}\,, \quad (5.17)$$

$$i(2\pi)^4\delta(P_n-P_p+q)\Big(\bar{U}_p(P_p,\sigma_p)\hat{\Gamma}^{npW}_{l\,\beta}(q)w^{+\beta}(q)U_n(P_n,\sigma_n)\Big)=\Lambda^{WAA}_{l\beta}\,. \quad (5.18)$$

So, $\hat{\Gamma}^{npW}_{s}$ is due to the electroweak quark-quark interactions mediated by $W-,Z-$bosons and "massive photons", whereas $\hat{\Gamma}^{npW}_{l}$ is due to "soft photons". The quantity Λ^{WAA}_{s} in (5.14), which involves the propagator D^{As} of a "massive photon" (2.36), (5.15), describes the processes where quarks interact exchanging virtual "massive photons". Consequently, the large momenta, $k^2\sim M_S^2\gg M_p^2$, are transferred to the quark system by the electromagnetic interactions thereby. Therefore quarks possess the large momenta in the intermediate states between emission and absorption of a "massive photon", alike in the processes described by Λ^{WWZ}, Λ^{WAW}, Λ^{WWW} Λ^{WZZ}, where the quark-quark electroweak interactions are mediated by $W-,Z-$bosons. In this respect, the strong quark-quark interactions in these intermediate states can be ignored in treating Λ^{WAA}_{s}. Consequently, Λ^{WAA}_{s} in (5.14) can be written in much the same way as Λ^{WZZ} (5.9) in the form

$$\Lambda^{WAA}_{s\,\alpha}(q)=-e^2\int\mathrm{d}^4x_1\int\mathrm{d}^4x_2\int\mathrm{d}^4x_3\langle\Phi^{q+}_p(P_p,\sigma_p)|\mathcal{T}\Big\{\Big(\bar{q}(x_2)e_q\gamma_\chi q(x_2)\Big)\mathcal{S}_{str}(x_2^0,x_1^0)\times$$
$$\times\Big(\bar{q}(x_1)\Gamma^{Wqq}_{\mu}(x_1)W^{+\mu}(x_1)T^{+}_{q}(x_1)q(x_1)\Big)\mathcal{S}_{str}(x_1^0,x_3^0)\times$$
$$\times\Big(\bar{q}(x_3)e_q\gamma_\beta q(x_3)\Big)\Big\}|\Phi^{q}_n(P_n,\sigma_n),W^{+}_{\alpha}(q)\rangle\;D^{As}_{\chi\beta}(x_2-x_3)\,, \quad (5.19)$$

with accepting $\mathcal{S}(x_1^0,x_2^0)=\mathcal{S}(x_1^0,x_3^0)=1$ (3.5) herein. So, we are again to treat, alike in Eqs. (5.4), (5.10), (5.7), the matrix element of the $\mathcal{T}-$product of the pure quark field operators presented by the graphs **5** in the expression (2.38) between the neutron and proton wave functions (3.4).

In much the same way as in calculating Λ^{WZZ}, Λ^{WWW}, Λ^{WWZ}, Λ^{WAW} (5.4), (5.7), (5.10), we acquire

$$\Lambda^{WAA}_{s\alpha}(q) = i\int \mathrm{d}^4y e^{iqy}\langle\Phi^{q\,+}_p(P_p,\sigma_p)|\bar\psi_u(y)\,\Gamma^{udW}_\chi W^{+\chi}\psi_d(y)|\Phi^q_n(P_n,\sigma_n),W^+_\alpha(q)\rangle\times$$
$$\times\Big\{\Gamma(WAS)+\frac{1}{2}\delta z^u_L(M_S)+\frac{1}{2}\delta z^d_L(M_S)\Big\}=$$
$$i(2\pi)^4\delta(P_n-P_p-q)\Big(\bar U_p(P_p,\sigma_p)\Gamma^{npW}_\alpha(q)w^+_\alpha(q)U_n(P_n,\sigma_n)\Big)\times$$
$$\times\Big\{\Gamma(WAS)+\frac{1}{2}\delta z^u_L(M_S)+\frac{1}{2}\delta z^d_L(M_S)\Big\}. \qquad (5.20)$$

Here

$$\Gamma(WAS)=\frac{\alpha}{4\pi}e_ue_d\Big(\Delta(M_S)-\frac{1}{2}\Big), \qquad (5.21)$$

and

$$\frac{1}{2}\delta z^u_L(M_S)+\frac{1}{2}\delta z^d_L(M_S)=-\frac{\alpha}{4\pi}\frac{e_u^2+e_d^2}{2}\Big(\Delta(M_S)-\frac{1}{2}\Big) \qquad (5.22)$$

provides the renormalization (2.19) of the $u-,d-$quark wave functions caused by the $u-,d-$quark self-energies (2.24) where the wavy line stands for the "massive photon" propagator D^{As} (2.36), (5.15).

Summarizing the results (5.6), (5.7), (5.10), (5.20), the quantity $\hat\Gamma^{npW}_{s\alpha}$ in Eqs. (5.16), (5.17) proves to be

$$\hat\Gamma^{npW}_{s\alpha}=\Gamma^{npW}_\alpha\cdot\Gamma^W, \qquad (5.23)$$

where Γ^W and Γ^{npW}_α are given by (2.42) and (3.14), (3.23), (3.22), (4.5).

VI. EFFECT OF STRONG INTERACTIONS ON THE npW-VERTEX.

As only the momenta $k^2\lesssim M_s^2$ contribute into D^{Al} (5.15), only these comparatively small momenta are transformed to quarks by emitting or absorbing virtual photons in the processes described by the quantity Λ^{WAA}_l in (5.1), (5.14), (5.18). Possessing the comparatively small momenta, $k^2\lesssim M_s^2$, quarks can be considered to constitute the baryon in the intermediate state between emitting and absorbing a virtual "soft photon". Then, with allowance for Eqs. (3.6), (4.3), Λ^{WAA}_l can be transformed as follows

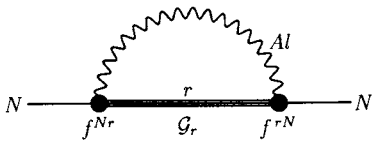
$$\Lambda^{WAA}_{l\alpha}(q) = -e^2 \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 \langle \Phi^{q\,+}_p(P_p,\sigma_p)| \mathcal{T}\Big\{ \Big(\bar{q}(x_2) e_q \gamma_\chi q(x_2)\Big) \mathcal{S}_{str}(x^0_2, x^0_1) \times$$
$$\times \Big(\bar{q}(x_1) \Gamma^{Wqq}_\mu(x_1) W^{+\mu}(x_1) T^+_q(x_1) q(x_1)\Big) \mathcal{S}_{str}(x^0_1, x^0_3) \times$$
$$\times \Big(\bar{q}(x_3) e_q \gamma_\beta q(x_3)\Big)\Big\} |\Phi^q_n(P_n,\sigma_n), W^+_\alpha(q)\rangle \; D^{Al}_{\chi\beta}(x_2 - x_3) = \qquad (6.1)$$
$$= -e^2 \int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 \sum_{r,s} \mathcal{T}\Big\{ \langle \Phi^{q\,+}_p(P_p,\sigma_p)| \Big(\bar{q}(x_2) e_q \gamma_\chi q(x_2)\Big) |\Phi^q_r(P_r,\sigma_r)\rangle \times$$
$$\times \langle \Phi^{q\,+}_r(P_r,\sigma_r)| \Big(\bar{q}(x_1) \Gamma^{Wqq}_\mu(x_1) W^{+\mu}(x_1) T^+_q(x_1) q(x_1)\Big) |\Phi^q_s(P_s,\sigma_s), W^+_\alpha(q)\rangle \times$$
$$\times \langle \Phi^{q\,+}_s(P_s,\sigma_s)| \Big(\bar{q}(x_3) e_q \gamma_\beta q(x_3)\Big) |\Phi^q_n(P_n,\sigma_n)\rangle\Big\} D^{Al}_{\chi\beta}(x_2 - x_3) .$$

Here the sum runs over the intermediate quark states with relatively small momenta $P^2_{r,s} \lesssim M^2_S$ described by the baryonic wave functions $\Phi^q_{r,s}(P_{r,s},\sigma_{r,s})$ (3.4). Of course, the proton and neutron intermediate states are included therein too. The matrix elements of the $\mathcal{T}-$products of quark operators between $\Phi^q_{r,s}(P_{r,s},\sigma_{r,s})$ (3.4) are defined by Eqs. (3.11)-(3.18) in terms of the matrix elements of the $\mathcal{T}-$products of the baryon field operators Ψ_r between the baryon wave functions $\Phi^B_{r,s}$, with the baryon formfactors Γ^{rsW}_β, f^{rs}_α (3.14), (3.16), (3.17) introduced thereby. Defined ordinarily the baryon field propagator

$$\mathcal{G}_{rs}(x-y) = -i\langle 0|\Psi_r(x)\bar{\Psi}_s(y)|0\rangle = \delta_{rs} \frac{1}{(2\pi)^4} \int \mathrm{d}^4p \, \mathcal{G}_r(p) \, \mathrm{e}^{-p(x-y)} , \qquad (6.2)$$

and the baryon self-energy (2.24) with the virtual "soft photon" (5.15)

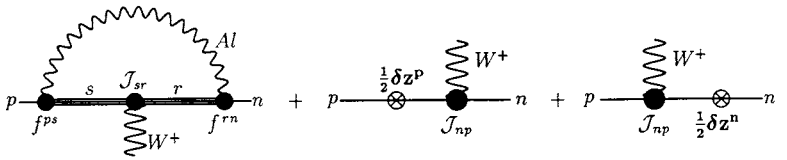
$$\Sigma_{Nl}(P_N) = -e^2 \sum_r \int \frac{\mathrm{d}^4k}{i(2\pi)^4} f^{Nr}_\alpha(k) D^{Al}_{\alpha\beta}(k) f^{rN}_\beta(k) \, \mathcal{G}_r(P_N - k) =$$
$$= -e^2 \sum_r \int \frac{\mathrm{d}^4k}{i(2\pi)^4} f^{Nr}_\alpha(k) \, \mathcal{G}_r(P_N - k) f^{rN}_\beta(k) \frac{-M^2_S g^{\alpha\beta}}{(k^2 - \lambda^2 + i0)(k^2 - M^2_S + i0)} = \qquad (6.3)$$

$=$ 

,

the corrected renormalized vertex $\hat{\Gamma}^{npW}_{l\alpha}$ in Eq. (5.18) proves to be

$$\hat{\Gamma}^{npW}_{l\alpha} = \Gamma_\alpha(WAl) + \Gamma^{npW}_\alpha \cdot [\frac{1}{2}\delta z^p + \frac{1}{2}\delta z^n] =$$



, (6.4)

where the first graph represents the quantity

$$\bar{U}_p(P_p,\sigma_p)\Gamma_\alpha(WAl)U_n(P_n,\sigma_n)w^{+\,\alpha}(q) =$$
$$\frac{e^3|V_{ud}|}{2\sqrt{2}s_W}\bar{U}_p(P_p,\sigma_p)\sum_{r,s}\int\frac{\mathrm{d}^4k}{i(2\pi)^4}f^{pr}_\mu(k)\mathcal{G}_r(P_p-k)\times$$
$$\times\mathcal{J}^\alpha_{rs}(q)(T^+_B)_{rs}\mathcal{G}_s(P_n-k)f^{sn}_\nu(k)D^{Al}_{\mu\nu}(k)U_n(P_n,\sigma_n)w^+_\alpha(q)\,, \tag{6.5}$$

and the finite renormalization constants $\frac{1}{2}\delta z^{n,p}$ of the neutron and proton wave functions come from

$$\delta z^N = -\frac{\partial\Sigma_{Nl}(P)}{\partial\not{P}}\,|_{\not{P}=M_N}\,. \tag{6.6}$$

In (6.3)-(6.5), the wavy lines tagged by Al represent the "soft photon" propagator D^{Al} (5.15), the triplex lines generically render various baryonic states (including the nucleon), and the blobs stand for the $NB\gamma-, BN\gamma-, BB'\gamma-, BB'W-$vertices with the appropriate formfactors (3.14)-(3.21). Apparently, as only the integration over the momenta $k^2\lesssim M_S^2$ contributes to (6.3)-(6.6), no UV divergence emerges therein.

The prevailing part of (6.3)-(6.6) is obtained by retaining in the sum over r,s only the single nucleon intermediate states $r,s=N$ with the propagator

$$G_N(P_N) = \frac{\not{P}+M_N}{P_N^2-M_N^2+i0}\,, \tag{6.7}$$

and also presuming (3.22)-(3.24). Then, the quantity $\Gamma_\alpha(WAl)$ (6.5) evidently vanishes, as f^{nn}=0 is utilized, and we arrive at

$$\hat{\Gamma}^{pnW}_{l\alpha} = \left(\frac{1}{2}\delta z^p_0+\frac{1}{2}\delta z^n_0\right)\Gamma^{npW}_\alpha\,, \tag{6.8}$$

with the finite renormalization constants (6.6) of the neutron and proton wave functions

$$\delta z^p_0 = -\frac{\alpha}{4\pi}\Big(2\ln\frac{M_S}{M_p}+\frac{9}{2}-4\ln\frac{M_p}{\lambda}\Big)\,,\quad \delta z^n_0 = 0\,. \tag{6.9}$$

To estimate the effect of nucleon structure on δz^N, we first retain only the single nucleon intermediate state with G_N (6.7) in (6.3)-(6.6), yet specify the nucleon formfactors into (6.3)-(6.6) by Eqs. (3.18), (3.21) which are plausible at the momenta k^2 transferred by a virtual "soft photon". Then, after a due calculation, laborious but rather plain, we arrive at the estimation

$$\delta\tilde{z}^p = -\frac{\alpha}{2\pi}\Big\{-2\ln\frac{M_p}{\lambda}+\frac{9}{4}-J(r)+\frac{r}{2}\frac{\partial}{\partial r}J(r)\Big\}+\frac{\alpha}{2\pi}\frac{1.79^2}{2}\Big\{I(0)-I(r)-\frac{r}{2}\frac{\partial}{\partial r}I(r)\Big\}, \tag{6.10}$$

$$\delta\tilde{z}^n = \frac{\alpha}{2\pi}\frac{1.93^2}{2}\Big\{I(0)-I(r)-\frac{r}{2}\frac{\partial}{\partial r}I(r)\Big\}, \tag{6.11}$$

where r=m_ρ/M_p is to set, and

$$J(r) = \int_0^1 \frac{dx}{x^2 + r^2(1-x)} [r^2(\frac{x^2}{2} - x) + x(2x - 2 + x^2)] ,$$

$$I(r) = \int_0^1 \frac{xdx}{8(x^2 + r^2(1-x))} [r^4(x+6) + 2r^2(3x^2 - 6x - 8) - 8x^2(x-3)].$$

For the intermediate states in (6.3) with $r \neq N$, the quantities f_α^{pr} , f_α^{rn} describe the transitions between these nucleon excited states r and the proton and neutron states p, n , respectively. These intermediate states are naturally to be treated as the well-known excited states of the proton, such as the $\Delta_{33}-$isobar, Roper-resonance, and so on. To realize the effect of the exited states on δz^N (6.6), (6.3), we consider the contribution into (6.6) due to an intermediate $\Delta_{33}-$isobar, the simplest proton excited state, the internal structure of which is much the same as the structure of the nucleon ground state. In the nucleon as well as in the $\Delta_{33}-$resonance, all three quarks occupy the state $1S_{1/2}$. Therefore, the amplitude $f_\alpha^{p\Delta_{33}}$ in (6.3), (6.6) does not differ substantially from f_α^{pp}. Also along these lines, the very distinction of $\mathcal{G}_{\Delta_{33}}$ (6.2) from G_p (6.7), which is of vital importance for the current estimation, actually results in replacing $M_p \longrightarrow M_\Delta$ (see, for instance, Refs. [30]). What is of crucial value in evaluating (6.3), (6.6) with $r \neq N$ is that

$$M_r^2 - M_p^2 \sim M_p^2 , \quad d = \frac{M_{\Delta_{33}}^2 - M_p^2}{M_{\Delta_{33}}} \approx \frac{1}{2} . \tag{6.12}$$

Then, by assuming the formfactors (3.22)-(3.24), the direct estimation of the contribution to (6.6) from the term with the Δ_{33} intermediate state gives

$$\delta z_\Delta^p = -\frac{\alpha}{2\pi} \{ J_\Delta(M_S/M_\Delta) - J_\Delta(0) \} , \tag{6.13}$$

$$J_\Delta(r) = \int_0^1 \frac{\mathrm{d}x}{x^2 + dx(1-x) + r^2(1-x)} \{ (x - \frac{x^2}{2})[2x + d(1-2x) - r^2] - 2x(1-x^2) \} .$$

The relations $M_S^2 \gg M_N^2, m_p^2, M_\Delta^2 - M_N^2$ were utilized in obtaining (6.9)-(6.11), (6.13). Let us behold that δz_Δ^p , $\delta \tilde{z}^n$ are free of the infrared divergencies, unlike δz^p , $\delta \tilde{z}^p$.

Now it is only a matter of straightforward numerical evaluation to become convinced that the difference

$$[(\delta z_\Delta^p + \delta \tilde{z}^p + \delta \tilde{z}^n) - \delta z_0^p] \lesssim 0.1 \cdot \delta z_0^p \tag{6.14}$$

constitutes less than $\sim$10% to the main quantity δz_0^p (6.9).

Except for the $\Delta_{33}-$isobar, the structure of the nucleon excited states and the structure of the ground state of the nucleon are disparate. Therefore, the values of f_α^{pr} with $r \neq p, \Delta_{33}$ are anyway substantially smaller than the f_α^{pp} value. Consequently, the contribution of these excited states into (6.3), (6.6) is still far smaller than (6.13).

The quantity (6.5) is exclusively caused by the small formfactors f^{nn}, f^{sn} (3.17), (3.21). It incorporates also two baryonic intermediate states. In this respect, the contribution of (6.5) into (6.4) is realized to be still far smaller than (6.10), (6.11), (6.13). All the more so, we may abandon the contribution of simultaneous allowance for the nucleon formfactors and the nucleon excited states.

Thus, with an accuracy better than $\sim$10%, Eq. (6.8) holds true, the quantity (6.5) is negligible, and the renormalization constants of the neutron and proton wave functions are given by (6.9). As the whole radiative corrections constitute a few per cent to the uncorrected $\beta-$decay probability, we commit an error $\lesssim$0.1% but never more, making use of (6.8), (6.9) in the further calculations.

Finally, adding (5.23) and (6.8), the corrected renormalized $npW-$vertex proves (with the aforesaid accuracy) to be multiple to the uncorrected vertex (3.14):

$$\hat{\Gamma}^{npW}_{\alpha}(P_n,P_p,q)=\hat{\Gamma}^{npW}_{s\,\alpha}(P_n,P_p,q)+\hat{\Gamma}^{npW}_{l\alpha}(P_n,P_p,q)= \\ =\Gamma^{npW}_{\alpha}(q)\left\{1+\frac{\alpha}{4\pi}\left(\ln\frac{M_p}{M_Z}-2\ln\frac{\lambda}{M_p}-\frac{9}{4}+\frac{3}{s^2_W}+\frac{6c^2_W-s^2_W}{s^4_W}\ln(c_W)\right)\right\}. \qquad (6.15)$$

This quantity is just what is depicted by the shaded circle with heavy core in the graph **3** in the amplitude (4.1).

VII. THE RADIATIVE CORRECTIONS TO THE $W-$BOSON PROPAGATOR.

Next, the propagator $D^W(q)$ (2.7) of the bare $W-$boson in (4.5) gives place to the corrected regularized $W-$boson propagator $\hat{D}^W(q)$ [22-25,31] ,

$$D^W(q)=\frac{1}{q^2-M^2_W+i0}\Longrightarrow\hat{D}^W(q)=\frac{1}{q^2-M^2_W+\hat{\Sigma}(q^2)}\approx \\ \approx\left(-\frac{1}{M^2_W}\right)\frac{1}{1-\frac{\hat{\Sigma}(0)}{M^2_W}}\,,\quad \text{for } q^2\ll M^2_W\,, \qquad (7.1)$$

as represented by the graph **4** in the expression $\mathcal{M}$ (4.1) where the heavy wavy line stands for $\hat{D}^W$. The renormalized $W-$boson self-energy $\hat{\Sigma}(0)$ is rather not amenable to a precise reliable evaluation because it includes light quarks contributions in the momentum region where strong interaction effects cannot be ignored [24]. Fortunately, one can acquire from the analysis of the $\mu-$meson decay [24,31] that

$$\frac{G_\mu}{\sqrt{2}}=\frac{\alpha\pi(1+\delta_v)}{2M^2_W s^2_W\left(1-\frac{\hat{\Sigma}^W(0)}{M^2_W}\right)}\,,\quad G_\mu=1.1663\cdot10^{-5}\,\mathrm{GeV}^{-2}\,,\quad \delta_v\approx0.006,\ . \qquad (7.2)$$

The estimation $\hat{\Sigma}^W(0)/M_W^2 \approx 0.066$ was ascertained in Refs. [23,24].

It is expedient to redefine $\mathcal{M}_0$ as the sum of the amplitudes **1** and **4** in expression $\mathcal{M}$ (4.1), writing hereupon $\mathcal{M}_0$ as

$$\mathcal{M}^0 = \left(\frac{e}{2\sqrt{2}s_W}\right)^2 \hat{D}^W(q)|V_{ud}|(\bar{u}_e(p_e)\gamma_\alpha(1-\gamma^5)u_\nu(-p_\nu)) \cdot (\bar{U}_p(P_p)\gamma^\alpha(1-\gamma^5 g_A)U_n(P_n)). \quad (7.3)$$

Accordingly (7.1), (7.2), the coefficient in (7.3) reads

$$\left(\frac{e}{2\sqrt{2}s_W}\right)^2 \hat{D}^W(q) = -\frac{G_\mu}{\sqrt{2}}(1-\delta_v) = -\frac{G}{\sqrt{2}}. \quad (7.4)$$

The contributions from all the diagrams in (4.1) but **4** are themselves of the order $\alpha/4\pi$, even without allowance for replacing $D^W \to \hat{D}^W$. Therefore, in treating the α-order radiative corrections caused by the processes depicted by these graphs, it stands to reason to set

$$\left(\frac{e}{2\sqrt{2}s_W}\right)^2 \frac{1}{M_W^2} = \frac{G}{\sqrt{2}}, \quad (7.5)$$

which is put to use henceforward.

VIII. THE RADIATIVE CORRECTIONS DUE TO THE IRREDUCIBLE npeν–VERTEX (THE "BOX DIAGRAMS").

By now, we have considered the terms in $\mathcal{M}$ (4.1) which stem from the Born amplitude $\mathcal{M}_0$ (4.5) by replacing the vertices $\Gamma_\alpha^{e\nu W}$, Γ_α^{npW} and the $W-$boson propagator D^W with the corrected renormalized quantities $\hat{\Gamma}_\alpha^{e\nu W}$, $\hat{\Gamma}_\alpha^{npW}$, $\hat{D}^W$. Besides these terms, which are due to the aforesaid modification of the separate blocks in the graph **1** (4.1), the total amplitude $\mathcal{M}$ (3.7), (3.8) incorporates also the part represented by the graphs **9** in (4.1) which are of the second order in both the lepton and quark electroweak interactions (2.13)-(2.16). The matrix element

$$i(2\pi)^4\delta(P_n - P_p - p_e - p_\nu)\mathcal{M}_{2\gamma} =$$
$$\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 \int \mathrm{d}^4x_4 \langle \Phi_{0p}^{q+}(P_p,\sigma_p), \phi_e^+(p_e,\sigma_e)|$$
$$\mathcal{T}\Big\{\Big(\mathcal{L}^{Zqq}(x_1)\mathcal{L}^{Wqq}(x_2)\mathcal{L}^{We\nu}(x_3)\mathcal{L}^{Zee}(x_4) + \mathcal{L}^{Zqq}(x_1)\mathcal{L}^{Wqq}(x_2)\mathcal{L}^{We\nu}(x_3)\mathcal{L}^{Z\nu\nu}(x_4) +$$
$$\mathcal{L}^{Aqq}(x_1)\mathcal{L}^{Aee}(x_2)\mathcal{L}^{Wqq}(x_3)\mathcal{L}^{We\nu}(x_4)\Big)\cdot \mathcal{S}_{str}\Big\}|\Phi_{0n}^q(P_n,\sigma_n), \phi_\nu(-p_\nu,-\sigma_\nu)\rangle =$$
$$= \Lambda^{ZW} + \Lambda^{AW}, \quad (8.1)$$

defines this part of the amplitude $\mathcal{M}_{2\gamma}$, usually referred to as the contribution from the "box-type" diagrams. It comprises the terms of different nature, the strong quark-quark interactions $\mathcal{L}_{str}^{qq}$

entangled herein through $\mathcal{S}_{str}\equiv\mathcal{S}_{str}(\infty,-\infty)$. The second term, Λ^{AW}, in r.h.s. of (8.1) involves the interactions of quarks $\mathcal{L}^{Aqq}$ (2.15) and electrons $\mathcal{L}^{Aee}$ (2.16) with electromagnetic field. Inasmuch as Λ^{AW} describes the processes in which a photon is exchanged between an electron and a quark, the expression of Λ^{AW} includes the virtual photon propagator $D^{A\lambda}$ (2.8). Then, by disparting $D^{A\lambda}$ into the "massive" D^{As} and "soft" D^{Al} photon propagators, pursuant to Eq. (5.15), Λ^{AW} is split into two parts corresponding to large, $k^2\gtrsim M_S^2$, and comparatively small, $k^2\lesssim M_S^2$, momenta transferred from leptons to quarks by a virtual photon, much in the same way as in the case of Eq. (5.14). So, with allowance for Eqs. (3.4), (3.6), (4.3), the quantity Λ^{AW} in (8.1) is written in the form

$$\Lambda^{AW}=\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\int \mathrm{d}^4x_4\langle\Phi_p^{q+}(P_p,\sigma_p),\phi_e^+(p_e,\sigma_e)|$$
$$\mathcal{T}\Big\{e(\bar\psi_q(x_1)\gamma^\alpha e_q\psi_q(x_1))\mathcal{S}_{str}(x_1^0,x_2^0)(\bar\psi_u(x_2)\Gamma_\mu^{udW}(x_2)T_q^+\psi_d(x_2))\times \tag{8.2}$$
$$\times(\bar\psi_e(x_3)\Gamma_\rho^{e\nu W}(x_3)\psi_\nu(x_3))(-e)(\bar\psi_e(x_4)\gamma^\beta\psi_e(x_4))\Big\}$$
$$|\Phi_n^q(P_n,\sigma_n),\phi_\nu(-p_\nu,-\sigma_\nu)\rangle\Big(D_{\alpha\beta}^{As}(x_1-x_4)+D_{\alpha\beta}^{Al}(x_1-x_4)\Big)D_{\mu\rho}^W(x_2-x_3)=\Lambda^{AWs}+\Lambda^{AWl}.$$

In (8.1), the term Λ^{ZW} including the electroweak interactions of heavy bosons with quarks and leptons, $\mathcal{L}^{Zqq},\mathcal{L}^{Wqq},\mathcal{L}^{Zee},\mathcal{L}^{Z\nu\nu},\mathcal{L}^{We\nu}$, is due to the $Z-$boson exchange between quarks and leptons. It contains the propagators $D^{W,Z}$ (2.7) of virtual heavy gauge bosons. This case evidently corresponds to the large momenta, $q^2\gtrsim M_S^2\gg M_p^2$, transferred from leptons to quarks. Recalling Eqs. (3.4), (3.6), (4.3), we find out

$$\Lambda^{ZW}=\int \mathrm{d}^4x_1\int \mathrm{d}^4x_2\int \mathrm{d}^4x_3\int \mathrm{d}^4x_4\langle\Phi_p^{q+}(P_p,\sigma_p),\phi_e^+(p_e,\sigma_e)|$$
$$\mathcal{T}\Big\{(\bar\psi_q(x_1)\Gamma_\mu^{Zqq}(x_1)\psi_q(x_1))\mathcal{S}_{str}(x_1^0,x_2^0)(\bar\psi_u(x_2)\Gamma_\lambda^{udW}(x_2)T_q^+\psi_d(x_2))\times \tag{8.3}$$
$$\times(\bar\psi_e(x_3)\Gamma_\alpha^{e\nu W}(x_3)\psi_\nu(x_3))[(\bar\psi_e(x_4)\Gamma_\beta^{Zee}(x_4)\psi_e(x_4))+(\bar\psi_\nu(x_4)\Gamma_\beta^{Z\nu\nu}(x_4)\psi_\nu(x_4))]\Big\}$$
$$|\Phi_n^q(P_n,\sigma_n),\phi_\nu(-p_\nu,-\sigma_\nu)\rangle D_{\mu\beta}^Z(x_1-x_4)D_{\lambda\alpha}^W(x_2-x_3).$$

The amplitude $\mathcal{M}_{2\gamma}$ is written as the sum

$$\mathcal{M}_{2\gamma}=\mathcal{M}_{2\gamma s}+\mathcal{M}_{2\gamma l}, \tag{8.4}$$

where the quantities $\mathcal{M}_{2\gamma s}$, $\mathcal{M}_{2\gamma l}$ are defined as follows

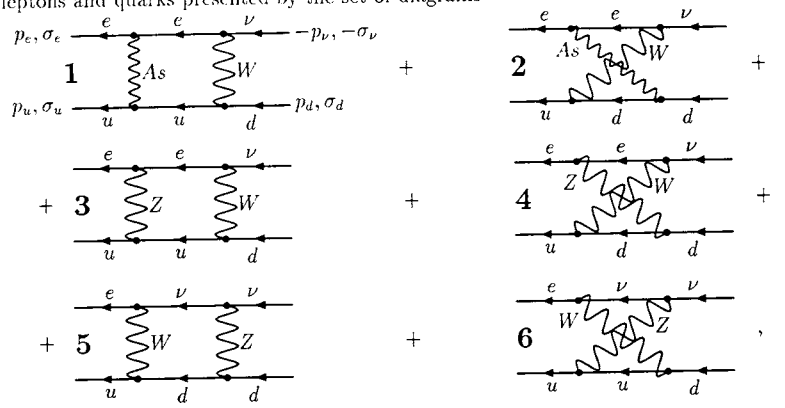
$$i(2\pi)^4\delta(P_n-P_p-p_e-p_\nu)\mathcal{M}_{2\gamma s}=\Lambda^{ZW}+\Lambda^{AWs}, \tag{8.5}$$
$$i(2\pi)^4\delta(P_n-P_p-p_e-p_\nu)\mathcal{M}_{2\gamma l}=\Lambda^{AWl}. \tag{8.6}$$

Quark momenta inside the nucleon are known to be relatively small, $k^2 \lesssim M_p^2$. Large momenta, $k^2 \gtrsim M_S^2 \gg M_p^2$, $k^2 \gtrsim M_{Z,W}^2 \gg M_p^2$, are transferred by virtual gauge bosons and "massive" photons to the quark system in the processes described by Λ^{ZW}, Λ^{AWs} (8.5). Therefore, quarks have got the large momenta $k^2 \gg M_p^2$ in the intermediate states between emission and absorption of gauge bosons and "massive" photons at the time-points x_1^0 and x_2^0 in such processes. At this point, we invoke again the Standard Model assumption that the strong quark-quark interactions $\mathcal{L}_{str}^{qq}$ vanish provided quarks possess the momenta $k^2 \gg M_p^2$. Consequently, the operator $\mathcal{S}_{str}(x_1^0, x_2^0)$ (3.5) in Λ^{ZW}, Λ^{AWs} turns out to be unit, $\mathcal{S}_{str}(x_1^0, x_2^0)$=1. Then, by straightforward calculating Λ^{ZW}, Λ^{AWs} (8.2), (8.3), we obtain (8.5)

$$\Lambda^{ZW} + \Lambda^{AWs} = i(2\pi)^4 \mathcal{M}_{2\gamma s}\, \delta(P_n - P_p - p_e - p_\nu) =$$

$$= \int d^4x \langle \Phi_p^{q\,+}(P_p, \sigma_p), \phi_e(p_e, \sigma_e) | \Big(\bar{\psi}_e(x) \bar{\psi}_q(x)\, \hat{\Gamma}^{e\nu ud}\, \psi_q(x) \psi_\nu(x) \Big) | \Phi_n^q(P_n, \sigma_n), \phi_\nu(-p_\nu, -\sigma_\nu) \rangle\,, \quad (8.7)$$

with the operator $\Big(\bar{\psi}_e(x) \bar{\psi}_q(x)\, \hat{\Gamma}^{e\nu ud}\, \psi_q(x) \psi_\nu(x) \Big)$ to describe the pure electroweak transitions of leptons and quarks presented by the set of diagrams



(8.8)

where, in particular, the wavy line with the tag As depicts the "massive photon" propagator D^{As} (2.36), (5.15). Recalling Eqs. (4.2), (4.5), (4.6), (3.14), (3.23), we eventually find the amplitude

$$\mathcal{M}_{2\gamma s} = -\mathcal{M}^0 \frac{\alpha}{4\pi} \left\{ \left(1 + \frac{5c_W^4}{s_W^4} \right) \ln (c_W) - 6 \ln \frac{M_W}{M_S} \right\} \quad (8.9)$$

being multiple to the Born amplitude $\mathcal{M}_0$ (4.5). It is to emphasize once again the relations

$$m_f \ll M_p\,, \quad M_n - M_p \ll M_N\,, \quad |p_f|^2 \ll M_N^2\,, \quad f \equiv e, \nu, u, d\,, \quad |\mathbf{P}_N|^2 \ll M_N^2\,, \quad M_p^2 \ll M_S^2 \ll M_W^2\,,$$
$$\frac{M_n - M_p}{M_p} \ln \frac{M_n - M_p}{M_p} \sim 0\,, \quad \frac{M_S}{M_W} \ln \frac{M_W}{M_S} \sim 0 \quad (8.10)$$

were used in obtaining (8.7)-(8.9), as well as far and wide over the work.

The second term in (8.9) is due to the contributions of the first and second diagrams in (8.8). In view of the discussion given in the last section, it is to take cognizance that if we had a neutral initial particle instead of a $d-$quark and a final particle with the charge $+1$ instead of an $u-$quark, the contribution of the second diagram in (8.8) would apparently vanish and the coefficient in front of $\ln M_W/M_S$ would be equal to 8 instead of 6.

IX. THE IRREDUCIBLE npe$\nu-$VERTEX WITH ALLOWANCE FOR NUCLEON COMPOSITENESS.

Unlike the case of Λ^{AWs}, in the processes described by Λ^{AWl} (8.6), (8.2), quarks and leptons exchange a virtual $W-$boson and a virtual "soft photon" (5.15). The amplitude $\mathcal{M}_{2\gamma l}$ (8.6) includes the "soft photon" propagator D^{Al} (5.15). This case corresponds to the comparatively small momenta, $k^2 \lesssim M_S^2$, transferred from leptons to quarks. Therefore, the intermediate quark system, between quark interactions with a $W-$boson and a "soft photon", possesses the relatively small momenta, and we deal with the intermediate baryonic states B, the ground or excited states of the nucleon. With allowance for (3.4), (4.3), Λ^{AWl} (8.6) is written as the sum over these baryonic states

$$\Lambda_l^{AW} = i(2\pi)^4 \mathcal{M}_{2\gamma l}\delta(P_n - P_p - p_e - p_\nu) =$$
$$\int \mathrm{d}^4x_1 \int \mathrm{d}^4x_2 \int \mathrm{d}^4x_3 \int \mathrm{d}^4x_4 \sum_B \langle \Phi_p^{q+}(P_p,\sigma_p), \phi_e^+(p_e,\sigma_e)|$$
$$\mathcal{T}\Big\{(\bar\psi_e(x_4)\Gamma_\rho^{e\nu W}\psi_\nu(x_4))(-e)(\bar\psi_e(x_2)\gamma^\beta\psi_e(x_2))\Big(\bar\psi_u(x_3)\Gamma_\mu^{duW}\psi_d(x_3)\Big)|\Phi_B^q(P_B,\sigma_B)\rangle$$
$$\times\langle\Phi_B^{q+}(P_B,\sigma_B)|\Big(\bar\psi_q(x_1)\gamma^\alpha e e_q\psi_q(x_1)\Big)\Big\}|$$
$$\Phi_n^q(P_n,\sigma_n),\psi_\nu(-p_\nu,-\sigma_\nu)\rangle D_{\mu\rho}^W(x_3-x_4)D_{\alpha\beta}^{Al}(x_1-x_2)\,. \qquad (9.1)$$

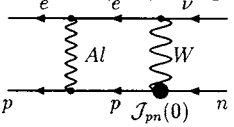
Recalling Eqs. (3.11)-(3.18), (6.2), the amplitude $\mathcal{M}_{2\gamma l}$ is presented as the sum of the contributions of two diagrams

(9.2)

$$= \int \frac{\mathrm{d}k^4}{(2\pi)^4 i}(\bar u_e(p_e)(-e)\gamma^\beta G_e(p_e-k)\Gamma^{e\nu W\,\alpha}u_\nu(-p_\nu))\frac{-M_S^2}{(k^2-\lambda^2+i0)(k^2-M_S^2+i0)}\times$$

$$\frac{1}{k^2-M_W^2+i0}\sum_B\Big\{(\bar U_p(P_p)ef_\beta^{pB}(k)\mathcal{G}_B(P_p+k)\Gamma_\alpha^{BnW}(k)U_n(P_n))+$$
$$+(\bar U_p(P_p)\Gamma_\alpha^{pBW}(k)\mathcal{G}_B(P_p-k)ef_\beta^{Bn}(k)U_n(P_n))\Big\}\,,$$

where the wavy line tagged by Al stands for the "soft photon" propagator D^{Al} (5.15) and the triplex line represents generically the propagator of a quark system in the intermediate states. The forthcoming estimations will prove that omitting all the nucleon excited states and describing the nucleon formfactors and nucleon transition current by Eqs. (3.22)-(3.24), we commit no more than a few per cent error in evaluating $\mathcal{M}_{2\gamma l}$, in much the same way as in evaluating $\hat\Gamma_{l\alpha}^{npW}$ (6.8). In this approach, liable for providing the dominant part of $\mathcal{M}_{2\gamma l}$, the contribution of the second term in (9.2) disappears, as f^{nn}=0 is adopted, and the contribution of the first term gets simplified, utilizing (6.2), (3.22)-(3.24). Then, with allowance for (8.10), straightforward calculation gives

$\mathcal{M}^0_{2\gamma l}=$ 

$=$ (9.3)

$$=\Big(\frac{e}{2\sqrt{2}s_W}\Big)^2\frac{|V_{ud}|}{(2\pi)^4}\frac{-1}{M_W^2}\Big\{\frac{1}{2}I_1(2M_p\varepsilon,\lambda)\mathcal{P}_0^{\alpha\beta}h^0_{\beta\alpha}-\frac{1}{2}I_1(2M_pk_\delta,\lambda)\mathcal{P}_1^{\beta\delta\alpha}h^0_{\beta\alpha}-$$
$$-\mathcal{P}_1^{\beta\delta\alpha}h^{1\nu}_{\beta\alpha}(I_1(k_\delta k_\nu,\lambda)-I_1(k_\delta k_\nu,M_S))\Big\}\,,$$

where

$$I_1(\mathcal{C},\mu)=\int \mathrm{d}^4k\,\varphi(k,P_p,p_e)\frac{\mathcal{C}}{k^2-\mu^2+i0}\,,$$
$$\varphi(k,P_p,p_e)=\frac{i}{[(p_e-k)^2-m^2+i0][(P_p+k)^2-M_p^2+i0]}\,, \tag{9.4}$$
$$\mathcal{P}_0^{\beta\alpha}=\bar u_e(p_e)\Big(\gamma^\beta\big(\frac{\hat p_e+m}{\varepsilon}\big)\gamma^\alpha(1-\gamma^5)\Big)u_\nu(-p_\nu)\,, \tag{9.5}$$
$$\mathcal{P}_1^{\beta\delta\alpha}=\bar u_e(p_e)\gamma^\beta\gamma^\delta\gamma^\alpha(1-\gamma^5)u_\nu(-p_\nu)\,,$$
$$h^0_{\alpha\beta}=\bar U_p(P_p)\gamma_\beta\Big(\frac{\hat P_p}{M_p}+1\Big)\gamma_\alpha(1-\gamma^5g_A)U_n(P_n)\,,$$
$$h^{1\nu}_{\beta\alpha}=\bar U_p(P_p)\gamma_\beta\gamma^\nu\gamma_\alpha(1-g_A\gamma^5)U_n(P_n)\,,\quad \hat p\equiv p_\alpha\gamma^\alpha\,,$$

and $\varepsilon=\sqrt{m^2+\mathbf{p}_e^2}$, $v=|\mathbf{p}_e|/\varepsilon$ are the electron energy and velocity.

Now, the point is to acquire what comes out of allowance for the nucleon compositeness: formfactors and excited states. In what follows, we shall treat concisely these two effects separately, one after other. We start with retaining only the pure single proton intermediate state, B=p, in the first term in (9.2) and approximating thereby the nucleon formfactors by Eq. (3.10), (3.18)-(3.21). Then, with allowance for (8.10), we obtain the respective contribution to (9.2)

$$\mathcal{M}^1_{2\gamma l} = \quad = \left(\frac{e}{2\sqrt{2}s_W}\right)^2 \frac{|V_{ud}|}{(2\pi)^4}\frac{-1}{M_W^2}\times$$

$$\times\left\{\frac{1}{2}I_1(2M_p\varepsilon,\lambda)\mathcal{P}_0^{\alpha\beta}h^0_{\beta\alpha} - \Big(I_1(2k_\delta M_p,\lambda) - \delta_{0\delta}I_1(2k_\delta M_p, m_\rho)\Big)\frac{1}{2}\mathcal{P}_1^{\beta\delta\alpha}h^0_{\beta\alpha} - \right.$$

$$-\Big((I_1(k_\delta k_\nu,\lambda) - I_1(k_\delta k_\nu, M_S)) - (I_1(k_\delta k_\nu, m_\rho) - I_1(k_\delta k_\nu, M_S))\Big)\mathcal{P}_1^{\beta\delta\alpha}h^{1\nu}_{\beta\alpha} + \qquad (9.6)$$

$$\left. +[I_1(k_\delta k_\nu k_\rho/M_p,\lambda) - I_1(k_\delta k_\nu k_\rho/M_p, m_\rho)]\mathcal{P}_1^{\beta\delta\alpha}M_p h^{2\nu\rho}_{\alpha\beta}\right\},$$

where, in addition to (9.5), we have defined

$$h^{2\mu\nu}_{\beta\alpha} = \bar{U}_p(P_p)\Big(\gamma_\beta\gamma^\mu(g_{WM}\sigma^\nu_\alpha - g_{IP}\delta_{\alpha\nu}\gamma^5) + \frac{1.79}{2M_p}\sigma^\mu_\beta\gamma^\nu\gamma_\alpha(1-g_A\gamma^5)\Big)U_n(P_n) \sim \frac{1}{M_p}. \qquad (9.7)$$

In (9.3), (9.6), the terms involving $h^0_{\beta\alpha}$ $h^1_{\beta\alpha}$ (9.5) are associated with the electric formfactor, whereas $h^{2\nu\mu}_{\beta\alpha}$ (9.7) is due to the magnetic formfactors and electroweak formfactors (3.10)-(3.21). Hereafter, the calculation of the $\alpha-$order decay probability will call for the real part of $\mathcal{M}_{2\gamma}$, as $\mathcal{M}^0$ is real, see Eq. (11.1) below. All the integrals I_1 but $I_1(2M_p\varepsilon,\lambda)$ are real, and their expressions prove to be rather plain,

$$I_1(2M_pk_\delta,\lambda) = \frac{p_{e\delta}I_1}{\varepsilon} + \delta_{0\delta}I_{10}, \quad I_1 = \frac{\pi^2}{v}\ln(x),$$
$$I_{10} = \pi^2[2\ln(\frac{m}{M_p}) - \frac{1}{v}\ln(x)], \quad x = \frac{1-v}{1+v}, \qquad (9.8)$$
$$I_1(k_\delta k_\nu,\lambda) - I_1(k_\delta k_\nu, M_S) = -g_{\delta\nu}(I_2 - \delta_{0\delta}I_{20}),$$
$$I_2 = \frac{\pi^2}{4}(\frac{3}{2} + 2\ln\frac{M_S}{M_p}), \quad I_{20} = \frac{\pi^2}{2}.$$

The careful calculation of $\mathrm{Re}I_1(2M_p\varepsilon,\lambda)$ was expounded in Ref. [21] with the result

$$\mathrm{Re}I_1(2M_p\varepsilon,\lambda) = \mathcal{I}(P_p,p_e,\varepsilon) =$$
$$= -\frac{\pi^2}{v}[\ln(x)\ln(\lambda/m) - \frac{1}{4}(\ln(x))^2 + F(1/x-1) - \frac{M_p\pi^2}{A}\cdot\frac{v\,\varepsilon}{t_2-t_1}], \qquad (9.9)$$

where

$$t_{1,2} = -\frac{m^2 - M_p^2 \pm 2\cdot\sqrt{(Pp_e)^2 - M_p^2m^2}}{m^2 + M_p^2 + 2(Pp_e)}, \quad 4A{=}m^2 + M_p^2 + 2p_eP_p,$$

and F is the Spence-function [32]. This quantity (9.9) determines the first, most important term in the amplitudes (9.3) and (9.6). Let us behold that the "Coulomb correction" is incorporated therein in the natural way, via the last term in $\mathcal{I}(P_p,p_e,\varepsilon)$ (9.9).

The second term in (9.6) comes out of the second term in (9.3) by subtracting $I_1(2M_pk_\delta, m_\rho)$ from $I_1(2M_pk_\delta, \lambda)$. For the mass $\mu \gtrsim m_\rho$, the estimation is obtained

$$I_1(2M_pk_\alpha, \mu) \approx -\pi^2\delta_{0\alpha}\Big[\frac{(r^2-4)^{3/2}}{12r}\ln\Big(\frac{r\sqrt{r^2-4}}{2}+\frac{r^2}{2}-1\Big)+\Big(1-\frac{r^2}{6}\Big)\ln r+\frac{1}{6}\Big], \qquad (9.10)$$

where $r=\frac{\mu}{M_p}$. At $\mu=m_\rho$, $r\approx 1$, we have got

$$I_1(2M_pk_\alpha, m_\rho) \approx -\pi^2\delta_{0\alpha}. \qquad (9.11)$$

This value is to be compared to

$$\delta_{0\alpha}I_{10} \approx \pi^2 \ln\frac{m}{M_p}\delta_{0\alpha} \approx -15\pi^2\delta_{0\alpha} \qquad (9.12)$$

in $I_1(2M_pk_\alpha, \lambda)$. As seen, $I_1(2M_pk_\alpha, m_\rho)$ can be omitted in (9.6) with an error smaller than 6%.

Taking into consideration (8.10), the differences which determine the third terms in (9.3) and in (9.6) are reduced to

$$I_1(k_\delta k_\nu, \lambda) - I_1(k_\delta k_\nu, M_S) = -g_{\alpha\beta}\frac{\pi^2}{2}\Big[\Big(\frac{3}{4}+\ln\frac{M_S}{M_p}\Big)-\delta_{0\alpha}\Big], \qquad (9.13)$$

$$I_1(k_\delta k_\nu, m_\rho) - I_1(k_\delta k_\nu, M_S) = \frac{\pi^2}{10}\delta_{\delta\nu}\delta_{0\delta} - \frac{\pi^2}{2}g_{\delta\nu}\Big(\ln\frac{M_S}{M_p}-\frac{3}{4}-I(m_\rho)\Big), \qquad (9.14)$$

$$I(\mu) = \int_0^1 x\mathrm{d}x\int_0^1 \mathrm{d}y\ln[x^2(y-1)^2+r_\mu^2(1-x)], \quad r_\mu=\frac{\mu}{M_p}. \qquad (9.15)$$

Next, it is only a matter of straightforward numerical evaluation to become convinced that the quantity (9.14) makes up no more than $\sim$10% to (9.13). So, with this accuracy, the third term in $\mathcal{M}^1_{2\gamma l}$ (9.6) is seen to coincide with the third term in $\mathcal{M}^0_{2\gamma l}$ (9.3).

In the last term in (9.6), the factor $\mathcal{P}_1^{\beta\delta\alpha}M_ph^{2\nu\rho}_{\beta\alpha}$ is realized to be of the same order, as the factors $\mathcal{P}_1^{\beta\delta\alpha}h^0_{\beta\alpha}$ and $\mathcal{P}_1^{\beta\delta\alpha}h^{1\nu}_{\beta\alpha}$ in (9.3), (9.6). Upon a labor-consuming but rather unsophisticated evaluation of the corresponding integrals $I_1(k_\delta k_\nu k_\rho/M_p, \mu)$, we arrive at the estimation of the difference

$$I_1(k_\delta k_\nu k_\rho/M_p, \lambda) - I_1(k_\delta k_\nu k_\rho/M_p, m_\rho) \approx$$
$$\approx -\frac{\pi^2 r_\mu^2}{6}\Big(-1+2r^2\int_0^1\frac{\mathrm{d}z z^2}{z^2-z(2-r^2)+1}\Big) \approx -\frac{\pi^2}{10}, \quad r=\frac{m_\rho}{M_p}, \qquad (9.16)$$

which constitutes $\lesssim$1% to the integrals $I_1(2M_p\varepsilon, \lambda)$, $I_1(2M_pk_\delta, \lambda)$, $I_1(k_\delta k_\nu, \lambda) - I_1(k_\delta k_\nu, \lambda)$, determining $\mathcal{M}^0_{2\gamma l}$ (9.3). So, the last term in $\mathcal{M}^1_{2\gamma l}$ (9.6) is seen to constitute no more than $\sim$1% to $\mathcal{M}^0_{2\gamma l}$ (9.3) and can be abandoned with this accuracy.

Thus, we have realized the difference $\mathcal{M}^1_{2\gamma l} - \mathcal{M}^0_{2\gamma l}$ caused by allowance for the nucleon formfactors (3.10)-(3.20) amounts to less than $\sim$10% to the dominant quantity $\mathcal{M}^0_{2\gamma l}$ (9.3). Consequently,

with committing an error less than ~10%, the formfactors (3.10)-(3.20) can be replaced by (3.22)-(3.24) so that $\mathcal{M}^1_{2\gamma l}$ reduces to $\mathcal{M}^0_{2\gamma l}$. All the more so, we can neglect, at least with the same accuracy, the contribution from the second term in (9.2) which is due to nothing but the neutron formfactor $f_2^{nn}\sim(M_n - M_p)/M_p \sim 0$ (3.21) exclusively, even in the simplest case corresponding to the pure neutron intermediate state, $B{=}n$.

Now, we are to consider the terms with $B{\neq}N$ in the sum in (9.2) which present the processes involving the virtual excited states of the nucleon, depicted by the triplex lines in the diagrams (9.2). These intermediate states are naturally to be treated as the well-known nucleon excited states, such as the $\Delta_{33}-$isobar, the Roper-resonance and so on, with the propagators $\mathcal{G}_B$ (6.2) (depending on the masses M_B, $M_N{<}M_B{\ll}M_S$) instead of the nucleon propagator G_N (6.7). For the current estimation, it is of a drastic value that the quantities m^2, $(M_n{-}M_p)^2$ are actually negligible as compared to the differences $M_B^2{-}M_N^2$,

$$\frac{(M_n - M_p)^2}{M_B^2 - M_N^2} \sim 0\,, \qquad \frac{m^2}{M_B^2 - M_N^2} \sim 0\,. \tag{9.17}$$

Indeed, even in the case of the $\Delta_{33}-$isobar, the lowest nucleon excited state, we have got $M_\Delta - M_p{\approx}300\,\mathrm{MeV}$. All the more so, Eqs. (9.17) hold true for any other nucleon excited state $B{\neq}\Delta_{33}$. Moreover, the important relation is obviously valid

$$M_B^2 - M_N^2 \sim M_N^2\,. \tag{9.18}$$

In the processes involving these intermediate states $B{\neq}N$, the quantities (3.10)-(3.20) describe the weak and electromagnetic transitions between the excited and ground states of the nucleon. For purpose of the current estimation, we take up the processes with a $\Delta_{33}-$isobar, $B{=}\Delta_{33}$, the simplest exited state of the nucleon, the internal structure of which is much the same as that of the nucleon ground state. In the nucleon as well as in the $\Delta_{33}-$isobar, all three quarks occupy the same state $1S_{1/2}$. Therefore, it is plausible in the current estimation to presume the amplitudes $f_\mu^{N\Delta}(k)$, $\mathcal{J}_\alpha^{n\Delta}(k)$ do not differ substantially from $f_\mu^{NN}(k)$, $\mathcal{J}_\alpha^{np}(k)$ (3.10)-(3.21). Also along these lines, as G_p gives place to $\mathcal{G}_B$ in the amplitude $\mathcal{M}^0$ (9.3), the very modification which is of vital importance for the qualitative assessment actually consists in replacing

$$M_p \Longrightarrow M_\Delta \tag{9.19}$$

in the proton propagator. Then the respective contribution into the amplitude (9.2) reduces to

$$\mathcal{M}^{\Delta}_{2\gamma l} = \quad = \int \frac{\mathrm{d}k^4}{(2\pi)^4 i}(\bar{u}_e(p_e)(-e)\gamma^\beta G_e(p_e-k)\Gamma^{e\nu W\,\alpha}u_\nu(-p_\nu))\times \tag{9.20}$$

$$\frac{-M_S^2}{(k^2-\lambda^2+i0)(k^2-M_S^2+i0)(k^2-M_W^2+i0)}\Big\{(\bar{U}_p(P_p)e\gamma_\beta\mathcal{G}_\Delta(P_p+k)\Gamma^{pnW}_\alpha(k)U_n(P_n))\Big\}.$$

The estimation of $\mathcal{M}^{\Delta}_{2\gamma l}$ (9.20) is procured by replacing

$$I_1 \Longrightarrow I_{1\Delta} \tag{9.21}$$

in $\mathcal{M}^{0}_{2\gamma l}$ (9.3), where $I_{1\Delta}$ comes out of I_1 (3.24) with changing the proton mass M_p by the $\Delta_{33}-$isobar mass M_Δ in the function $\varphi(k,P_p,p_e)$ (9.4). What is of crucial importance for the current evaluation is that

$$(P_\Delta+k)^2-M_p^2 = M_\Delta^2-M_p^2 \sim M_p^2 \gg (M_n-M_p)^2 \tag{9.22}$$

at k=0 in the denominators of the integrands in $I_{1\Delta}$, instead of zero in the integrands of I_1 (3.24), i.e. with M_p in place of M_Δ. In particular, that is why there occurs no infrared divergence in the integral $I_{1\Delta}(2M_p\varepsilon,\lambda)$, as opposed to $I_1(2M_p\varepsilon,\lambda)$. As $\mathcal{M}^{\Delta}_{2\gamma l}$ is expressed in terms of $I_{1\Delta}$ alike $\mathcal{M}^{0}_{2\gamma l}$ is expressed in terms of I_1, the integrals $I_{1\Delta}(\mathcal{C},\mu)$ are to be evaluated with $\mu=\lambda, M_S$ and confronted to the respective integrals $I_1(\mathcal{C},\mu)$ in order to assess the $\mathcal{M}^{\Delta}_{2\gamma l}$ (9.20) value as compared with the value of $\mathcal{M}^{0}_{2\gamma l}$ (9.3). The most important integrals $I_{1\Delta}(2M_p\varepsilon,\mu)$ which determine the dominant part of $\mathcal{M}^{\Delta}_{2\gamma l}$ (as $I_1(2M_p\varepsilon,\mu)$ do in the case of $\mathcal{M}^{0}_{2\gamma l}$) are given by

$$I_{1\Delta}(2M_p\varepsilon,\mu) \approx 2M_p\varepsilon\pi^2\int\limits_0^1 \mathrm{d}x x\int\limits_0^1 dy\frac{1}{y^2x^2M_p^2+yx(M_\Delta^2-M_p^2)+\mu^2(1-x)+x^2m^2}. \tag{9.23}$$

With allowance for Eqs. (9.17)-(9.22), we acquire the estimation of the integral $I_{1\Delta}(2M_p\varepsilon,\lambda)$ in $\mathcal{M}^{\Delta}_{2\gamma l}$ (9.20)

$$I_{1\Delta}(2M_p\varepsilon,\lambda) \approx \frac{4\pi^2M_p\varepsilon}{(M_\Delta^2-M_p^2)}\ln\Big(\frac{(M_\Delta^2-M_p^2)}{mM_p}\Big) \sim 0. \tag{9.24}$$

instead of the integral $I_1(2M_p\varepsilon,\lambda)$ (9.9), mostly determining the evaluation of $\mathcal{M}^{0}_{2\gamma l}$ (9.3). Likewise, the estimation of (9.23) at μ=M_S gives

$$I_{1\Delta}(2M_p\varepsilon,M_S) \approx \frac{4\pi^2\varepsilon}{M_S}\ln\Big(\frac{(M_\Delta^2-M_p^2)}{mM_p}\Big) \sim 0. \tag{9.25}$$

In much the same way, it is straightforward to become convinced that all the remaining integrals $I_{1\Delta}(\mathcal{C},\mu)$ in $\mathcal{M}^{\Delta}_{2\gamma l}$ (9.20) prove also to be negligible as compared to the respective integrals $I_1(\mathcal{C},\mu)$ in $\mathcal{M}^{0}_{2\gamma l}$ (9.3), and consequently $\mathcal{M}^{\Delta}_{2\gamma l}$ (9.20) results to be rather negligible as compared with $\mathcal{M}^{0}_{2\gamma l}$ (9.3).

Except for the aforesaid Δ_{33}−resonance case, the structure of excited states of the nucleon differs drastically from the structure of the nucleon ground state. Therefore, the values of all the amplitudes $\mathcal{J}^{\alpha}_{nB}$, f^{pB}_{μ} with $B\neq N$ and $B\neq\Delta_{33}$ are substantially smaller than $\mathcal{J}^{\mu}_{pn}\sim\mathcal{J}^{\mu}_{\Delta n}$, $f^{pp}_{\mu}\sim f^{\Delta p}$. Given this fact, it stands to reason that the contribution to $\mathcal{M}_{2\gamma l}$ (9.2) due to these intermediate states can not exceed anyway the contribution from the intermediate Δ_{33}−isobar state considered above. So, all the corrections to $\mathcal{M}^{0}_{2\gamma l}$ (9.3) caused by the terms involving the intermediate excited states with $B\neq p$ in Eq. (9.2) prove to be negligible, as a matter of fact. All the more so, we can abandon the contributions to $\mathcal{M}_{2\gamma l}$ (9.2) which are due to simultaneous allowance for the excited states, $B\neq p$, and the formfactors $f^{BB'}_{\mu}$, $\mathcal{J}^{\mu}_{rs}$ (3.16), (3.17), respecting the above estimations associated with Eqs. (9.10)-(9.16) and the relevant discussion thereat.

Thus, summing up, we have ascertained the amplitude $\mathcal{M}_{2\gamma l}$ (9.2) can be reduced to $\mathcal{M}^{0}_{2\gamma l}$ (9.3) with the accuracy better than $\sim$10%. On substituting (9.5), (9.9) in (9.3), $\mathcal{M}_{2\gamma l}$ is finally put into the explicit form:

$$\begin{aligned}\mathcal{M}_{2\gamma l}=\Big(\frac{e}{2\sqrt{2}s_W}\Big)^2|V_{ud}|\frac{1}{M_W^2}\cdot\frac{\alpha}{4\pi}\Big\{\Big(\bar{u}_e(p_e)\gamma^{\beta}(\not{p}_e+m)\gamma^{\alpha}(1-\gamma^5)u_{\nu}(-p_{\nu})\frac{1}{2\varepsilon M_p v}\times\\ \times[\ln(x)\ln\frac{\lambda}{m}-\frac{1}{4}(\ln(x))^2+F(1/x-1)-\frac{M_p\pi^2}{A}\cdot\frac{v\varepsilon}{t_2-t_1}]-\\ -\bar{u}_e(p_e)\gamma^{\beta}\gamma^{\delta}\gamma^{\alpha}(1-\gamma^5)u_{\nu}(-p_{\nu})\frac{1}{2M_p}[-\frac{p_{e\delta}}{v\varepsilon}\ln(x)+\delta_{0\delta}(\frac{1}{v}\ln(x)-2\ln\frac{m}{M_p})]\Big)\times\\ \times(\bar{U}_p(P_p)\gamma_{\beta}(\not{P}_p+M_p)\gamma_{\alpha}(1-\gamma^5 g_A)U_n(P_n))-(\bar{u}_e(p_e)\gamma^{\beta}\gamma^{\delta}\gamma^{\alpha}(1-\gamma^5)u_{\nu}(-p_{\nu}))\times\\ \times(\bar{U}_p(P_p)\gamma_{\beta}\gamma^{\nu}\gamma_{\alpha}(1-g_A\gamma^5)U_n(P_n))g_{\delta\nu}\Big(\frac{3}{8}+\frac{1}{2}(\ln\frac{M_W}{M_p}-\frac{M_W^2}{M_W^2-M_S^2}\ln\frac{M_W}{M_S})-\delta_{0\delta}\frac{1}{2}\Big)\Big\}.\end{aligned}\tag{9.26}$$

What is the inherent feature of $\mathcal{M}_{2\gamma l}$ (9.2), (9.26) to be emphasized is that this amplitude shows up to be not multiple to the uncorrected Born amplitude $\mathcal{M}^0$ (4.5), even though $\mathcal{M}_{2\gamma l}$ (9.26) has ensued from the general expression (9.2) on leaving aside the effects of nucleon structure. In this regard, $\mathcal{M}_{2\gamma l}$ on principle differs from the above considered quantities $\mathcal{M}_{2\gamma s}$ (8.9), $\hat{\Gamma}^{npW}_{\alpha}$ (6.15), $\hat{\Gamma}^{e\nu W}$ (2.35), which all have turned out to be proportional to the corresponding uncorrected quantities $\mathcal{M}^0$, Γ^{npW}_{α}, $\Gamma^{e\nu W}$ (4.5), (3.23), (3.14).

X. REAL γ—RADIATION

In the first α—order, the real γ—emission accompanying the neutron β—decay is presented by the diagrams $\mathbf{5}-\mathbf{8}$ in the amplitude $\mathcal{M}$ (4.1). The triplex lines in the graphs $\mathbf{6},\mathbf{7}$ represent the conceivable excited states of the nucleon. As $m, M_n - M_p \ll M_p \ll M_W$, the contributions from the diagrams $\mathbf{6}-\mathbf{8}$ are negligible as compared to the one coming out of the diagram $\mathbf{5}$, which renders the common bremsstrahlung of a final electron. The corresponding amplitude of the real γ—radiation with the momentum k and the polarization $\epsilon^{(r)}$

$$\mathcal{M}^{(r)}_{1\gamma}(k) = \Big(\frac{e}{2\sqrt{2}s_W}\Big)^2 |V_{ud}|\Big(\frac{-1}{M_W^2}\Big) e\epsilon^{(r)}_\alpha(\bar{u}_e(p_e)\gamma^\alpha \frac{(\not{p}_e + \not{k} + m)}{(p_e+k)^2 - m^2}\gamma^\lambda(1-\gamma^5)u_\nu(-p_\nu)) \times \qquad (10.1)$$
$$\times(\bar{U}_p(P_p)\gamma_\lambda(1 - g_A\gamma^5)U_n(P_n)), \qquad (a, r = 1,2,3),$$

is seen to be not proportional to the uncorrected quantity $\mathcal{M}^0$ (4.5), alike $\mathcal{M}_{2\gamma l}$, yet against $\mathcal{M}_{2\gamma s}$, $\hat{\Gamma}^{npW}_\alpha$, $\hat{\Gamma}^{e\nu W}$ which all are multiple to the uncorrected quantities $\mathcal{M}^0$, Γ^{npW}_α, $\Gamma^{e\nu W}$.

XI. THE RADIATIVE CORRECTIONS TO THE ELECTRON MOMENTUM DISTRIBUTION AND THE NEUTRON LIFETIME

With allowance for the radiative corrections of order α, the absolute square of the transition amplitude $\mathcal{M}$ (4.1) proves expedient to be written in the form

$$|\mathcal{M}|^2 = |\mathcal{M}^R + \mathcal{M}_{2\gamma l} + \mathcal{M}^{(r)}_{1\gamma}|^2 \approx |\mathcal{M}^R|^2 + |\mathcal{M}^{(r)}_{1\gamma}|^2 + 2\mathrm{Re}[\mathcal{M}^0\mathcal{M}_{2\gamma l}], \qquad (11.1)$$

where

$$\mathcal{M}^R = (\bar{u}_e(p_e)\hat{\Gamma}^{e\nu_\mu}_\alpha u_\nu(-p_\nu))\cdot(\bar{U}_p(P_p)\hat{\Gamma}^{npW}_\beta U_n(P_n))\hat{D}^W_{\alpha\beta}(p_\nu + p_e) + \mathcal{M}_{2\gamma s} \approx$$
$$\approx \mathcal{M}^0\Big\{1 - \frac{\alpha}{4\pi}\Big(2\ln\frac{M_Z}{M_p} + 4\ln\frac{\lambda}{m} + \frac{9}{2} - \ln\frac{M_p}{m} - \frac{6}{s^2_W} - 6\ln\frac{M_Z}{M_S} - \frac{3+4c^2_W}{s^4_W}\ln(c_W)\Big)\Big\} \qquad (11.2)$$

comprises all the terms proportional to the Born amplitude $\mathcal{M}^0$ (4.5).

As a final state after the neutron β—decay involves a proton, an electron, an antineutrino and γ—rays, the probability of the polarized neutron β—decay, upon summarizing the absolute square $|\mathcal{M}|^2$ of the transition amplitude over the polarizations of all the particles in the final state, is obviously put into the following well-known general form

$$d\mathbf{W}(\mathbf{p_e}, \mathbf{P}, \mathbf{p}_\nu, \mathbf{k}, \boldsymbol{\xi}) = (2\pi)^4\delta(M_n - E_P - \omega_\nu - \varepsilon - \omega)\delta(\mathbf{P}+\mathbf{p_e}+\mathbf{p}_\nu+\mathbf{k}) \times$$
$$\frac{1}{2M_n}\sum_{if}|\mathcal{M}_{if}|^2\frac{d\mathbf{P}d\mathbf{p_e}d\mathbf{p}_\nu d\mathbf{k}}{(2\pi)^{12}\,2E_p\,2\varepsilon\,2\omega_\nu\,2\omega} =$$
$$w(\mathbf{p_e}, \mathbf{P}, \mathbf{p}_\nu, \mathbf{k}, \boldsymbol{\xi})\,d\mathbf{P}d\mathbf{p_e}d\mathbf{p}_\nu d\mathbf{k}\delta(M_n - E_P - \omega_\nu - \varepsilon - \omega)\delta(\mathbf{P}+\mathbf{p_e}+\mathbf{p}_\nu+\mathbf{k}), \qquad (11.3)$$

where $\boldsymbol{\xi}$ stands for the polarization vector of a resting neutron, and $p_e{=}(\varepsilon,\mathbf{p_e})$, $P{=}(E_P,\mathbf{P})$, $p_\nu{=}(\omega_\nu,\mathbf{p}_\nu)$, $k{=}(\omega,\mathbf{k})$ are the electron, proton, antineutrino and $\gamma-$ray four-momenta, respectively. The familiar expression (11.3) renders the momentum distribution of electrons, protons, antineutrinos and $\gamma-$rays in the final state.

In the work presented, our purpose is to calculate the $\beta-$decay probability integrated over the final proton, antineutrino and photon momenta and summarized over the polarizations of all the final particles,

$$\mathrm{d}\mathbf{W}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) = \mathrm{d}\mathbf{W}^R(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) + \mathrm{d}\mathbf{W}_{1\gamma}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) + \mathrm{d}\mathbf{W}_{2\gamma l}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi})\,, \tag{11.4}$$

where $\mathrm{d}\mathbf{W}^R$, $\mathrm{d}\mathbf{W}_{1\gamma}$, $\mathrm{d}\mathbf{W}_{2\gamma l}$ are due to $|\mathcal{M}^R|^2$, $|\mathcal{M}^{(r)}_{1\gamma}|^2$, $2\mathrm{Re}[\mathcal{M}^0\mathcal{M}_{2\gamma l}]$, (11.1) , respectively.

Although the calculation of the distribution (11.4) turns out to be cumbersome and labour-consuming, it runs along a plain and unsophisticated way, as a matter of fact. So, we shall not expound this calculation at full length, in details, but only set forth the main stages in evaluating $\mathrm{d}\mathbf{W}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi})$ (11.4).

As being due to $|\mathcal{M}^R|^2$, the quantity

$$\mathrm{d}\mathbf{W}^R(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) \approx \mathrm{d}\mathbf{W}^0(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi})\Big\{1 - \frac{\alpha}{2\pi}\Big(2\ln\frac{M_W}{M_p} + 4\ln\frac{\lambda}{m} + \frac{9}{2} - \ln\frac{M_p}{m} - \\ - \frac{6}{s_W^2} - 6\ln\frac{M_Z}{M_S} - \frac{5+2c_W^2}{s_W^4}\ln(c_W)\Big)\Big\} \tag{11.5}$$

is apparently proportional to the uncorrected decay probability

$$\mathrm{d}\mathbf{W}^0(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) = \mathrm{d}\mathbf{w}(\varepsilon,\mathbf{p}_e)(1+3g_A^2+\mathbf{v}\boldsymbol{\xi}2g_A(1-g_A))\,,$$
$$\mathrm{d}\mathbf{w}(\varepsilon,\mathbf{p}_e) = \frac{G^2}{2\pi^3}\varepsilon|\mathbf{p}_e|k_m^2\mathrm{d}\varepsilon\frac{\mathrm{d}\mathbf{n}}{4\pi}\,,\quad \mathbf{n}=\mathbf{p}_e/|\mathbf{p}_e|\,,\quad \mathbf{v}=\mathbf{p}_e/\varepsilon\,,\quad k_m = M_n - M_p - \varepsilon\,. \tag{11.6}$$

The contribution of the real γ-radiation $\mathrm{d}\mathbf{W}_{1\gamma}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi})$ [20] stems from $|\mathcal{M}_{1\gamma}|^2$

$$\mathrm{d}\mathbf{W}_{1\gamma}(\varepsilon,\mathbf{p}_e) = \mathrm{d}\mathbf{w}(\varepsilon,\mathbf{p}_e)\int_0^{k_m}\mathrm{d}k\Big((1+3g_A^2)W_{0\gamma}(\varepsilon,k) + \mathbf{v}\boldsymbol{\xi}2g_A(1-g_A)W_{\xi\gamma}(\varepsilon,k)\Big) = \\ = \mathrm{d}\mathbf{w}(\varepsilon,\mathbf{p}_e)\{(1+3g_A^2)[\tilde{B}(\varepsilon)+\tilde{C}_0'(\varepsilon)] + \mathbf{v}\boldsymbol{\xi}2g_A(1-g_A)[\tilde{B}(\varepsilon)+\tilde{C}_\xi'(\varepsilon)]\}\,, \tag{11.7}$$

where

$$\tilde{B} = \frac{2\alpha}{\pi}([\frac{1}{v}\ln\frac{p_e+\varepsilon}{m} - 1]\cdot\ln(\frac{2k_m}{\lambda}) - \frac{\mathcal{K}(\varepsilon)}{2v})\,,$$
$$\tilde{C}_0' = \frac{2\alpha}{\pi}\Big\{[\frac{1}{v}\ln\frac{p_e+\varepsilon}{m} - 1]\Big(\frac{k_m}{3\varepsilon} - \frac{3}{2}\Big) + \frac{k_m^2}{24v\varepsilon^2}\ln\frac{p_e+\varepsilon}{m}\Big\}, \tag{11.8}$$
$$\tilde{C}_\xi' = \frac{2\alpha}{\pi}[\frac{1}{v}\ln\frac{p_e+\varepsilon}{m} - 1]\cdot\Big(\frac{k_m}{\varepsilon v^2}(\frac{1}{3}+\frac{k_m}{24\varepsilon}) - \frac{3}{2}\Big)\,,$$
$$\mathcal{K} = \frac{1}{2}(F(x) - F(1/x) - \ln(1/x)\cdot\ln(\frac{1-v^2}{4})) - v + \frac{1}{2}\ln(x) + F(v) - F(-v)\,.$$

The contribution from $2\mathrm{Re}\mathcal{M}^0\mathcal{M}_{2\gamma l}$ is

$$d\mathbf{W}_{2\gamma l}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) = d\mathbf{w}(\varepsilon,\mathbf{p}_e)\{[1+3g_A^2+\mathbf{v}\boldsymbol{\xi}2g_A(1-g_A)]B_{2\gamma}(\varepsilon)+ \\ +C_{02\gamma}(g_A,\varepsilon)+\mathbf{v}\boldsymbol{\xi}C_{\xi 2\gamma}(g_A,\varepsilon)\}. \tag{11.9}$$

Here

$$B_{2\gamma}(\varepsilon)=\frac{\alpha}{2\pi^3}[(2\mathcal{I}_1(\varepsilon)-I_1)-I_{10}],$$

$$C_{02\gamma}(g_A,\varepsilon)=\frac{\alpha}{2\pi^3}\{-I_1v^2[1+3(g_A)^2]+2I_{2s}[5+12g_A+15g_A^2]-2I_{20}[2+3g_A+3g_A^2]\},$$

$$C_{\xi 2\gamma}(g_A,\varepsilon)=\frac{\alpha}{2\pi^3}\{-I_12g_A(1-g_A)+2I_{2s}[3+4g_A-7g_A^2]-2I_{20}[1+g_A-2g_A^2]\}, \tag{11.10}$$

$$I_{2s}=\frac{\pi^2}{4}[3/2+2\Big(\ln\frac{M_W}{M_p}-\frac{M_W^2}{M_W^2-M_S^2}\ln\frac{M_W}{M_S}\Big)],$$

$$\mathcal{I}_1(\varepsilon)=-\frac{\pi^2}{v}[\ln(x)\ln(\lambda/m)-\frac{1}{4}(\ln(x))^2+F(1/x-1)-\frac{v\pi^2}{\tilde{v}(\varepsilon)}],$$

$$\tilde{v}(\varepsilon)=\frac{1}{2}\left(\sqrt{(v+\frac{mk_m}{M_p\varepsilon})^2+2v\frac{k_m}{\varepsilon}(\frac{m}{M_p})^2}+\sqrt{(v-\frac{mk_m}{M_p\varepsilon})^2-2v\frac{k_m}{\varepsilon}(\frac{m}{M_p})^2}\right), \tag{11.11}$$

where the quantities I_1, I_{10}, I_2, I_{20} are given in (9.8). It is to recall once more that all the results are obtained utilizing the relations (8.10). Let us behold the last term in $\mathcal{I}_1(\varepsilon)$ could naturally be associated with the contribution of the Coulomb interaction between an electron and a proton in the final state.

Eventually, the electron momentum distribution in the $\beta-$decay of a polarized neutron results to be

$$d\mathbf{W}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) = d\mathbf{w}(\varepsilon,\mathbf{p}_e)\{W_0(g_A,\varepsilon)+\boldsymbol{v}\boldsymbol{\xi}W_\xi(g_A,\varepsilon)\}, \tag{11.12}$$

$$W_0(g_A,\varepsilon)=(1+3g_A^2)[1+\tilde{C}_0(\varepsilon)+\mathcal{B}(\varepsilon)]+C_0(g_A,\varepsilon)$$

$$W_\xi(g_A,\varepsilon)=2g_A(1-g_A)[1+\tilde{C}_\xi(\varepsilon)+\mathcal{B}(\varepsilon)]+C_\xi(g_A,\varepsilon)$$

$$C_0=\frac{\alpha}{2\pi}[2\ln(\frac{\varepsilon+p}{m})v(1+3g_A^2)+\frac{33g_A^2}{4}+6g_A+\frac{7}{4}+ \\ +\ln(\frac{M_W}{M_p})(3+12g_A+9g_A^2)-\frac{M_W^2}{M_W^2-M_S^2}\ln\frac{M_W}{M_S}(5+12g_A+15g_A^2)],$$

$$C_\xi=\frac{\alpha}{2\pi}[\frac{4g_A(1-g_A)}{v}\ln(\frac{\varepsilon+p}{m})+\frac{5}{4}+2g_A-\frac{13}{4}{g_A}^2+ \\ \ln(\frac{M_W}{M_p})3(1-{g_A}^2)-\frac{M_W^2}{M_W^2-M_S^2}\ln\frac{M_W}{M_S}(3+4g_A-7g_A^2)],$$

$$\mathcal{B}=\frac{2\alpha}{\pi}\cdot[\frac{1}{v}\ln\frac{p_e+\varepsilon}{m}-1]\cdot\ln(\frac{2k_m}{m}),\quad \tilde{C}_0=\tilde{C}_0'+\tilde{C}_1,\quad \tilde{C}_\xi=\tilde{C}_\xi'+\tilde{C}_1,$$

$$\tilde{C}_1=\frac{2\alpha}{\pi}[\frac{\mathcal{J}}{2v}-\frac{\mathcal{K}}{2v}+\frac{1}{4}\Big(3\ln(M_p/m)-9/2+\frac{6}{s_W^2}+6\frac{M_Z^2}{M_Z^2-M_S^2}\ln\frac{M_Z}{M_S}+\frac{5+2c_W^2}{s_W^4}\ln(c_W)\Big)],$$

$$\mathcal{J}(\varepsilon)=\frac{1}{4}(\ln(x))^2-F(1/x-1)+\frac{\pi^2v}{\tilde{v}(\varepsilon)},$$

which can also be rewritten as

$$\mathrm{d}\mathbf{W}(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi}) = \mathrm{d}\mathbf{W}^0(\varepsilon,\mathbf{p}_e,\boldsymbol{\xi})\cdot[1+\mathcal{B}(\varepsilon)+\tilde{C}_1(\varepsilon)] + \mathrm{d}\mathbf{w}(\varepsilon,\mathbf{p}_e)\times \tag{11.13}$$
$$\times\Big((1+3g_A^2)\tilde{C}_0'(\varepsilon) + 2\boldsymbol{v}\boldsymbol{\xi}g_A(1-g_A)\tilde{C}_\xi'(\varepsilon) + C_0(g_A,\varepsilon) + \boldsymbol{v}\boldsymbol{\xi}C_\xi(g_A,\varepsilon)\Big).$$

We purposely retain the factors $M_{W,Z}^2/(M_{W,Z}^2 - M_S^2)$ in front of $\ln M_{W,Z}/M_S$ in order to clarify that nothing out-of-the-way will occur even in the case $M_S \to M_{W,Z}$ and the dependence on M_S is very smooth.

The total decay probability W, reverse of the lifetime τ, and the asymmetry factor of electron momentum distribution $A(\varepsilon)$ are acquired from (11.12) in the familiar way:

$$W = \frac{1}{\tau} = \frac{G^2}{2\pi^3}\int\limits_m^{M_n-M_p} \mathrm{d}\varepsilon\,\varepsilon|\mathbf{p}_e|k_m^2\,W_0(g_A,\varepsilon)\,, \tag{11.14}$$

$$A(g_A,\varepsilon) = \frac{W_\xi(g_A,\varepsilon)}{W_0(g_A,\varepsilon)}\,. \tag{11.15}$$

The radiative corrections cause the relative modification of the total decay probability W

$$\frac{\int\limits_m^{M_n-M_p}\mathrm{d}\varepsilon\,\varepsilon|\mathbf{p}_e|k_m^2 W_0(g_A,\varepsilon)}{(1+3g_A^2)\int\limits_m^{M_n-M_p}\mathrm{d}\varepsilon\,\varepsilon|\mathbf{p}_e|k_m^2} - 1 = \delta W\,. \tag{11.16}$$

The uncorrected asymmetry factor of the electron angular distribution A_0 is replaced by the quantity $A(\varepsilon)$ accounting for the radiative corrections,

$$A_0 = \frac{2g_A(1-g_A)}{1+3g_A^2} \Longrightarrow \frac{W_\xi(g_A,\varepsilon)}{W_0(g_A,\varepsilon)} = A(\varepsilon,g_A)\,. \tag{11.17}$$

So, the quantities δW (11.16) and

$$\frac{A(\varepsilon,g_A)-A_0}{A_0} = \delta A(\varepsilon) \tag{11.18}$$

render the effect of radiative corrections on the total decay probability W (11.14) and asymmetry coefficient A (11.15).

The results of numerical evaluation of δW, δA are discussed in the next section.

XII. DISCUSSION OF THE RESULTS.

Before setting forth the numerical evaluation, several valuable features of the ultimate result (11.12) deserve to be spotlighted.

Surely, upon adding the contributions from the processes involving virtual and real infrared photons, the fictitious infinitesimal photon mass λ has disappeared from the final expression (11.12), amenably to the received removal of the infrared divergency [26,7–9].

Let us behold that if we got a neutral initial particle in place of a $d-$quark and a final particle with the charge $+1$ in place of an $u-$quark in the expression (8.8), the coefficient 6 in front of $\ln M_Z/M_S$ in $\tilde{C}_1$ would be replaced by 8, following what was observed at the end of Sec. 8. Subsequently, if g_A were therewith equal to 1, the subsidiary parameter M_S would be cancelled in the final result (11.12). Being generically represented by the first diagram in (8.8), this conceivable case might be associated with the *neutron→proton* transition involving exchange of a $W-$boson and a "massive photon" between leptons and quarks, with the weak nucleon transition current being pure left. As one can see, the description of the neutron $\beta-$decay would not involve the parameter M_S in this case.

The form of dependence of (11.12) on the UV cut-off , i.e. on $\ln M_W/M_p$, asks for a special attention. First, it is readily seen straight away that the portion of (11.12) multiple to $\ln M_W/M_p$ would strictly vanish, if there were $g_A{=}{-}g_V{=}{-}1$, that is if the nucleon weak transition current were pure right, $(V+A)$, instead of the actual current (3.10), (3.22). This fact is associated with the general theorem ascertained in Refs. [33].

As one might infer from Refs. [34,35,28], the amplitude $\mathcal{M}$ and the probability $\mathrm{d}\mathbf{W}$ of any semileptonic decay ought generically to be of the form

$$\mathcal{M}{\approx}\mathcal{M}^0[1+\frac{3\alpha}{2\pi}\tilde{q}\ln(\frac{M_W}{M_p})]\cdot[1+\mathcal{O}_1(\alpha)]\,, \tag{12.1}$$

$$\mathrm{d}\mathbf{W}{\approx}\mathrm{d}\mathbf{W}^0[1+\frac{3\alpha}{2\pi}\cdot 2\tilde{q}\ln(\frac{M_W}{M_p})]\cdot[1+\mathcal{O}_2(\alpha)]\,, \tag{12.2}$$

up to the terms of order α. Here, $\mathcal{M}^0$ and $\mathrm{d}\mathbf{W}^0$ render the uncorrected (Born) values of $\mathcal{M}, \mathrm{d}\mathbf{W}$, and

$$\tilde{q}=\frac{2\bar{Q}+1}{2}=-(Q_{1in}Q_{2in}+Q_{1out}Q_{2out})\,, \tag{12.3}$$

where $\bar{Q}$ is the average charge of the isodoublet involved in the decay [34,35], and

$$Q_{1in}Q_{2in}\,,\ \ Q_{1out}Q_{2out}$$

are the products of charges of incoming and outgoing particles, respectively [28]. In the case of the neutron $\beta-$decay, i. e. for the (n,p) doublet, $\bar{Q}=1/2$, and

$$Q_{1in}Q_{2in}=0\,,\ \ Q_{1out}Q_{2out}=-1\,,$$

so that $\tilde{q}$=1. So, the distribution (11.12) ought to have taken the form

$$\mathrm{d}\mathbf{W}(g_A,\varepsilon,\alpha)\approx\mathrm{d}\mathbf{W}^0(g_A,\varepsilon)\cdot[1+\frac{3\alpha}{2\pi}\cdot 2\ln(\frac{M_W}{M_p})]\cdot[1+\mathcal{O}_P(\alpha)]\,, \tag{12.4}$$

with the quantity $\mathcal{O}_P(\alpha)$ independent of g_V, g_A, $\ln M_W/M_p$. Apparently, it is not the case: the expression (11.12) can never be reduced to the form (12.4). Yet though one might think we encounter some puzzling mismatch, there is no real contradiction between the assertions of Refs. [34,35.28] and our straightforward consistent calculation based on the electroweak Lagrangian (2.10)-(2.16) and the parameterization (3.10), (3.22), (3.23) of the nucleon weak transition current. To perceive the matter, we rewrite the actually used current $\mathcal{J}^{\beta}_{np}$ (3.22) and the distributions $\mathrm{d}\mathbf{W}^0$ (11.6), $\mathrm{d}\mathbf{W}$ (11.12) in terms of the amplitudes

$$g_L=\frac{g_V+g_A}{2}\,,\quad g_R=\frac{g_V-g_A}{2} \tag{12.5}$$

introduced instead of the original ones g_V=1, $g_A\neq g_V$:

$$\mathcal{J}^{\beta}_{np}(0)=\gamma^{\beta}[(1-\gamma^5)g_L+(1+\gamma^5)g_R]\,, \tag{12.6}$$

$$\mathrm{d}\mathbf{W}^0=\mathrm{d}\mathbf{w}\cdot[4(g_L^2+g_R^2-g_Lg_R)+\boldsymbol{v\xi}4g_R(g_L-g_R)]\,, \tag{12.7}$$

$$\begin{aligned}\mathrm{d}\mathbf{W}\approx\mathrm{d}\mathbf{w}\Big\{&4g_L^2\Big(1+\frac{3\alpha}{2\pi}\ln\frac{M_W}{M_p}\Big)^2(1+\mathcal{O}_L(\alpha))+\\&+4g_R^2(1+\mathcal{O}_R(\alpha))-4g_Lg_R\Big(1+\frac{3\alpha}{2\pi}\ln\frac{M_W}{M_p}\Big)(1+\mathcal{O}_{LR}(\alpha))+\\&+\boldsymbol{v\xi}\Big(-4g_R^2\big(1+\mathcal{O}^{\xi}_R(\alpha)\big)+4g_Lg_R\Big(1+\frac{3\alpha}{2\pi}\ln\frac{M_W}{M_p}\Big)(1+\mathcal{O}^{\xi}_{RL}(\alpha))\Big)\Big\}\,,\end{aligned} \tag{12.8}$$

$$\mathcal{O}_L(\alpha)=\mathcal{O}_0(\alpha)+\frac{\alpha}{2\pi}\Big(4+2\ln\frac{M_W}{M_S}\Big)\,,\quad \mathcal{O}_R(\alpha)=\mathcal{O}_0(\alpha)+\frac{\alpha}{2\pi}\Big(1+8\ln\frac{M_W}{M_S}\Big)\,,$$

$$\mathcal{O}_{RL}(\alpha)=\mathcal{O}_0(\alpha)+\frac{\alpha}{2\pi}\Big(\frac{13}{4}-5\ln\frac{M_W}{M_S}\Big)\,,\quad \mathcal{O}^{\xi}_R(\alpha)=\mathcal{O}_{\xi}(\alpha)+\frac{\alpha}{2\pi}\Big(1-2\ln\frac{M_W}{M_S}\Big)\,,$$

$$\mathcal{O}^{\xi}_{RL}(\alpha)=\mathcal{O}_{\xi}(\alpha)+\frac{\alpha}{2\pi}\Big(\frac{9}{4}-5\ln\frac{M_W}{M_S}\Big)\,,\quad \mathcal{O}_0(\alpha)=\tilde{C}_0+\mathcal{B}+\frac{\alpha}{\pi}v\ln\frac{\varepsilon+p_e}{m}\,,$$

$$\mathcal{O}_{\xi}(\alpha)=\tilde{C}_{\xi}+\mathcal{B}+\frac{\alpha}{\pi}\frac{1}{v}\ln\frac{\varepsilon+p_e}{m}\,.$$

It stands to reason that the values $g_L\neq 1$, $g_R\neq 0$ reflect the mixture of the left and right hadronic currents on account of the effect of nucleon structure. In confronting (12.7) and (12.8), one grasps that the amplitude g_L gets the renormalization factor which corresponds to that in (12.1) accordingly to Refs. [8,34,35,28], whereas the modification of g_R does not depend on the cut-off $\ln M_W/M_p$ at all, in accordance with Ref. [33] as was discussed above. If there were the pure left hadronic current, i.e. g_L=1, g_R=0, the relation (12.4) between the uncorrected (12.7) and

corrected (12.8) distributions would apparently hold true as prescribed by Refs. [8,34,35,28]. In the case of the pure right hadronic current, i.e. g_L=0, g_R=1, the final result (12.8) would not depend on $\ln M_W/M_p$ at all.

Inquiring carefully into the calculations carried out in Refs. [34,35,28], we realize that the semileptonic decays considered therein are actually described by the interactions which correspond to the case g_L=1, g_R=0, i.e. a pure left hadronic current. It is to emphasize that the assertions (12.1), (12.2) of Refs. [34,35,28] hold true for any decays induced by a pure left (g_L=1, g_R=0, g_V=g_A=1) hadronic current, in particular for the semileptonic decays which can be reduced to the pure $d\to u$ transitions of free quarks . Thus, Eqs. (12.1), (12.2) are valid to describe the manifold semileptonic decays such as $\pi\to\mu\bar{\nu}_\mu\gamma$, $\pi\to e\bar{\nu}_e\gamma$, $K\to\mu\nu_\mu\gamma$, $\tau\to\pi\nu_\tau\gamma$, $\tau\to K\nu_\tau\gamma$ and so on (see, for instance, [29,36] in addition to [34,35,28]). The Eqs. (12.1), (12.2) might although be pertinent to treat the transitions caused by the pure axial (g_L=g_R=$-g_A$=-1, g_V=0) hadronic current, such as $\Sigma^\pm\to\Lambda^0 e^\pm\nu(\bar{\nu})\gamma$, or by the pure vector current (g_L=g_R=g_V=1, g_A=0), such as the super-allowed $0^+\to 0^+$ nuclear transitions. But all the aforesaid is not our case, it is not relevant to describe the neutron $\beta-$decay.

Evidently, as the total amplitude $\mathcal{M}$ (3.8) is not multiple to $\mathcal{M}^0$ (4.5), the distribution (11.12) can never be transformed to an expression multiple to (11.6), unlike the results asserted in several calculations [13–19,37] which were entailed by the original work [38] where the decay probability was reduced, to all intents and purposes, to the "model independent" part merely proportional to $\mathrm{d}\mathbf{W}^0$ (11.6), that is explicitly not our case.

The original investigation [38] had been undertaken before the Standard Model of elementary particle physics was brought to completion in the nowaday form [8,22–25]. Then, for the lack of the renormalizable Electroweak Weinberg-Salam Theory, there was seen no way to treat the neutron $\beta-$decay with self-contained allowance for the radiative corrections. The purpose of the ingenious work [38] was to circumvent the problem of UV divergence and sidestep the consideration of the electromagnetic corrections in the UV region, by appropriate separating the whole electromagnetic corrections of order α into two conceivable parts, a "model independent" (MI) and a "model dependent" (MD), of different purports. The first one, MI, was chosen and sorted out so that it should evidently be UV-finite and could merely be obtained by multiplying the uncorrected (Born) decay probability $\mathrm{d}\mathbf{W}^0$ (11.6) by a single universal function $g(\varepsilon, M_n - M_p, m)$, see Eqs. (20) in Ref. [38], which was calculated within the effective 4-fermion-interaction approach (1.1)-(1.5), without taking into consideration the electroweak and strong interactions as prescribed by the Standard

Model. In Ref. [38], this MI part was presumed to describe the electromagnetic effects on the neutron $\beta-$decay. All the left-over radiative corrections were conceived to be incorporated into the second, MD part, assuming the electroweak and strong interactions conspire somehow to give the finite corrections to the quantities g_V , g_A , V_{ud} which reside in the uncorrected, Born decay probability (11.6), see Eqs. (19), (20) in Ref. [38]. Thus, in all the calculations, such as [13-19,37], presuming the approach launched by the work [38], the corrected decay probability merely shows up to be reduced to the uncorrected one multiplied by the function $g(\varepsilon, M_n - M_p, m)$, with the whole effect of the remained MD part absorbed into the quantities g_V , g_A , V_{ud} which thereby would get the new values g'_V , g'_A , V'_{ud} instead of the original ones : the CKM matrix element V_{ud} in (2.13) and the amplitudes g_V , g_A specifying the nucleon weak transition current (3.10), (3.22). Thus, the experimental data would be described in terms of these "new" quantities g'_V , g'_A , V'_{ud}. However, any explicit and definite, quantitative one-to-one correspondence between these two sets of parameters, g_V , g_A , V_{ud} and g'_V , g'_A , V'_{ud}, would never be asserted in Refs. [38,13-19,37]. Yet the guide tenet is to ascertain, as precise as possible, the very genuine values of g_V , g_A , V_{ud} from experimental data processing. In particular, we are in need of the stringent $|V_{ud}|$ value in order to verify strictly the validity of the CKM identity (1.2) [10]. So, the aforesaid calculations [38,13-19,37] making use of the very handy, but rather untenable simplifications cannot be said to be eligible for now, in so far as an accuracy $\sim$1% or even better goes.

In our treatment, the amplitude $\mathcal{M}$ (3.8), (4.1) and, subsequently, the distribution $\mathrm{d}\mathbf{W}$ (11.12) comprise all the $\alpha-$order radiative corrections, without disparting the Coulomb term and separating the MI and the MD parts. Adopting M_S=10 GeV , $(M_p^2 \ll M_S^2 \ll M_W^2)$ and taking all the input parameters in (11.12) from Ref. [39], we obtain the corrections (11.16) and (11.18)

$$\delta W = 8.7\% \,, \qquad \delta A = -2\% \tag{12.9}$$

to the uncorrected W^0 and A_0 values. As a matter of fact, the correction δA (11.18), (12.9) is independent of ε. Practically, the same values of δW and δA were acquired in the previous calculation [20] based immediately on the effective Lagrangian of the local 4-fermion theory of weak interactions, with the UV-cut-off Λ=M_W assumed. Apparently, our results (12.9) pronouncedly differ from the respective MI-values

$$\delta W_{MI} \approx 5.4\% \,, \qquad \delta A_{MI} \approx 0 \tag{12.10}$$

asserted in Refs. [38,13-19,37]. Consequently, the values of $|V_{ud\,MI}|$ and $g_{A\,MI}$ ascertained from experimental data processing with utilizing δW_{MI}, δA_{MI} (12.10) will alter, when they are ob-

tained with δW, δA (12.9). The modifications are of the noticeable magnitude: $\delta g_A \approx 0.47\%$, $\delta|V_{ud}| = -1.7\%$. For instance, the values $g_A = 1.2739$, $|V_{ud}| = 0.9713$ given in [1,2,39] will be modified to $g_A \approx 1.28$, $|V_{ud}| \approx 0.96$, provided the same value of the quantity G is used.

Now we are to discuss what is the precision attainable in the actual calculations nowadays, a pivotal question that matters a lot.

As from the first we have been calculating the radiative corrections in the one-loop order, $O(\alpha)$, the relative uncertainty $\sim\alpha\sim 10^{-2}$ resides in the evaluated radiative corrections (12.9), from the very beginning.

We further recall that the terms of relative order

$$\frac{M_n - M_p}{M_p}\,, \quad \frac{M_n - M_p}{M_p}\cdot \ln\frac{M_n - M_p}{M_p}\,,$$

and smaller have been neglected far and wide, with a relative error $\lesssim 10^{-3}$ entrained thereby.

Yet a far more substantial task than the aforesaid ones is to inquire into the ambiguities caused by entanglement of the strong quark-quark interactions in the neutron β-decay.

The final result (11.12) involves the matching parameter M_S , $(M_p^2 \ll M_S^2 \ll M_W^2)$ posited to treat separately quark systems with large, $k^2 \gtrsim M_S^2$, and comparatively small, $k^2 \lesssim M_S^2$, momenta. The dependence of the results δW , δA (12.9) on the M_S value shows up to be very faint : we have got $\delta W = 8.6\%$ at $M_S = 5\,\mathrm{GeV}$ and $\delta W = 8.8\%$ at $M_S = 30\,\mathrm{GeV}$, and δA is practically independent of M_S at all. So, the uncertainties because of the M_S involvement in (11.12) are about 0.1% in δW and practically zero in δA (12.9).

Further, M_S is chosen so that $M_p^2 \ll M_S^2$, and we took for granted the generally accepted standpoint of the Standard Model that the strong quark-quark interactions die out when a quark system possesses momenta $k^2 \gtrsim M_S^2 \gg M_p$. At relatively small momenta $k^2 \lesssim M_S^2$, a quark system was considered to form various baryonic states, including the nucleon. Let us recall all the actual calculations have been carried out assuming Eqs.(3.22)-(3.23) and retaining only the single nucleon intermediate state (6.7) in the expressions (6.3), (6.4), (6.6), (9.2), what counts is the final result (11.12), (12.9). In calculating the radiative corrections, we did not intend to allow for nucleon compositeness rigorously, but (in sections VI and IX) we only tried and estimated how those basic calculations alter when including the nucleon excited states (6.2) and the formfactors (3.10), (3.17)-(3.21) into the expressions (6.3), (6.6), (9.2). As was found out in sections VI and IX, the different terms in the amplitude $\mathcal{M}$ (3.8), (4.1) (and subsequently in the distribution $\mathrm{d}\mathbf{W}$ (11.12)) are affected by allowance for compositeness of the nucleon to a different extent. As a matter

of fact, there is no modification in the first, prevailing term in (9.3) which is determined by the integral $I(2M_p\varepsilon, \lambda)$ (9.9). It includes, in particular, the Coulomb correction. The direct evaluation shows that this major term causes the share of about $\delta W_I \approx 5\%$ in the whole correction $\delta W \approx 8.7\%$ (12.9). All the other left-over terms in the decay amplitude $\mathcal{M}$ provide the remnant portion $\delta W - \delta W_I \approx 4\%$ of δW and the whole value $\delta A = -2\%$ (12.9). The effect of nucleon compositeness on these terms was estimated (in sections IV, IX) to constitute no more than $\sim 10\%$ to their whole value. For now, there sees no real reliable way to calculate precisely these corrections-to-corrections in treating the neutron $\beta-$decay. With the ascertained estimations, they are abandoned in the actual calculation which has provided (11.12), (12.9). Consequently, in respect of all the aforesaid, the uncertainties in the result (12.9) prove to make up no more than

$$\Delta(\delta W) \approx 0.4\% , \qquad \Delta(\delta A) \approx 0.2\% . \tag{12.11}$$

Thus, our inferences are realized to hold true up to the accuracy about a few tenth of per cents, never worse.

If anything, let us behold the energy released in the $\beta-$decay of free neutrons is rather negligible as compared to the nucleon mass, $M_n - M_p \ll M_p$, whereas the energy released in manifold semileptonic decays is comparable to the masses of the hadrons involved in the process, or even greater than they. That is why accounting for compositeness of the hadron proves to play no decisive role in the neutron $\beta-$decay, but can be of significant value in other semileptonic decays (see, for example, [29,36]).

In the current treatment of the radiative corrections to the neutron $\beta-$decay, we have actually allowed for the effects of nucleon structure by introducing only one fit-parameter g_A to be specified, simultaneously with the fundamental quantity $|V_{ud}|$, by processing the experimental data on the lifetime [1] and electron momentum distribution [2]. Evidently, the ambiguities (12.11) put bounds on the accuracy which can be attained in obtaining the $|V_{ud}|$, g_A values thereby.

Thus, introducing only the usual parameters g_V, g_A, g_{WM}, g_{IP} to describe the weak nucleon transition current does not suffice to parameterize the whole effect of strong interactions in treating the neutron $\beta-$decay with allowance for the radiative corrections, in so far as the accuracy one per cent or better goes. Nowadays, no way is thought to get rid of the errors (12.11), but to parameterize ingeniously the effects of nucleon compositeness by expedient introducing some additional fit-parameters (besides g_A) to describe the radiative corrections to various characteristics of the neutron $\beta-$decay. These additional parameters are to be fixed by processing, simultaneously with

the results of measurements of τ [1] and A [2], the experimental data obtained in the additional experiments such as [3,4,6] and other in this line. For instance, these extra parameters might be conceived to render generically the "effective" mass in the intermediate state in (6.3), (6.4), (6.6), (9.2), (9.20) and the "effective" vertices (3.14), (3.10), (3.16)-(3.18). They are to be fixed, together with g_A, $|V_{ud}|$, M_S, from the simultaneous analysis of all the available experimental data, the kinematic corrections [11] respected as well.

So we are in need of the manifold tenable experiments to measure various characteristics of the neutron $\beta-$decay, besides τ and A, with an accuracy about 0.1%, and even better. Obtained such high-precision experimental data, the high accuracy, better than $\sim$0.1%, is believed to be attained within the unified self-contained analysis of the different experimental data amenably to the Standard Model.

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Радиационные поправки к β-распаду нейтрона
в стандартной модели

Согласно лагранжиану стандартной модели вычислены радиационные поправки к β-распаду нейтрона с последовательным учетом электрослабых взаимодействий в теории Вайнберга–Салама. Эффект сильных взаимодействий параметризуется введением электромагнитных формфакторов и слабого нуклонного тока перехода, который определяется слабыми формфакторами g_V, g_A, ... Радиационные поправки к полной вероятности β-распада W и к коэффициенту асимметрии импульсного распределения электронов A составляют $\delta W \approx 8{,}7$ %, $\delta A \approx -2$ %. Вклад в радиационные поправки от учета формфакторов и возбужденных состояний нуклона достигает нескольких процентов от полной величины этих поправок. Неопределенности в описании возбужденных состояний и формфакторов нуклона являются той причиной, что приводит к погрешности ~ 0,1 % в вычислении характеристик β-распада нейтрона. В настоящее время это определяет предел точности, достижимый при получении элемента V_{ud} матрицы CKM и значений величин g_V, g_A, ... из обработки экспериментальных данных.

Работа выполнена в Лаборатории нейтронной физики им. И. М. Франка ОИЯИ.

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Standard Model Treatment of the Radiative Corrections
to the Neutron β-Decay

Starting with the basic Lagrangian of the Standard Model, the radiative corrections to the neutron β-decay are acquired. The electroweak interactions are consistently taken into consideration amenably to the Weinberg–Salam theory. The effect of the strong quark–quark interactions on the neutron β-decay is parametrized by introducing the nucleon electromagnetic form factors and the weak nucleon transition current specified by the form factors g_V, g_A, ... The radiative corrections to the total decay probability W and to the asymmetry coefficient of the electron momentum distribution A are obtained to constitute $\delta W \approx 8.7$ %, $\delta A \approx -2$ %. The contribution to the radiative corrections due to allowance for the nucleon form factors and the nucleon excited states amounts up to a few per cent of the whole value of the radiative corrections. The ambiguity in description of the nucleon compositeness is surely what causes the uncertainties ~ 0.1 % in evaluation of the neutron β-decay characteristics. For now, this puts bounds to the precision attainable in obtaining the element V_{ud} of the CKM matrix and the g_V, g_A, ... values from experimental data processing.

The investigation has been performed at the Frank Laboratory of Neutron Physics, JINR.

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