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MEASUREMENT OF H^- BEAM EMITTANCE IN AXIAL INJECTION CHANNEL OF DC-72 CYCLOTRON

Казаринов Н. Ю. и др. E9-2004-146
Измерение эмиттанса пучка H^--ионов в канале
аксиальной инжекции циклотрона DC-72

В работе приводится методика измерения поперечного эмиттанса ионного пучка в канале аксиальной инжекции циклотрона DC-72. Она основана на методе градиентов с использованием стандартного сканера с вращающейся иглой для измерения поперечных размеров ионного пучка. Эта методика была разработана для ионных пучков с токами до 1000 мкА и позволяет восстанавливать эмиттанс с точностью около 30 %. Методика учитывает собственный заряд ионного пучка, что является существенным. В эксперименте не всегда удается получить аксиально-симметричный пучок. Поэтому предложен новый метод обработки экспериментальных данных при измерении поперечного эмиттанса аксиально-несимметричного ионного пучка. Приводятся формулы для определения дисперсий среднеквадратичных размеров пучка во вращающейся системе координат по сигналам с иглы сканера. Измерения среднеквадратичных эмиттансов $\varepsilon_{x,y}$ проводились на стенде канала инжекции циклотрона DC-72 при токе пучка H^--ионов 180 мкА и кинетической энергии ионов 16,82 кэВ. Приводятся также результаты обработки экспериментальных данных.

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Kazarinov N. Yu. et al. E9-2004-146
Measurement of H^- Beam Emittance in
Axial Injection Channel of DC-72 Cyclotron

A method of measuring the ion beam transversal emittance in the axial injection channel of DC-72 cyclotron is given. It is based on the gradient method using the standard rotating wire scanner for measurement of the transversal ion beam dimensions. This method was worked out for ion beam currents up to 1000 μA and allows one to reconstruct emittance with an accuracy about 30%. The method takes into account the ion beam self-charge, which is essential. It is not always a success to obtain an axial-symmetric ion beam in experiments. Therefore, a new experimental data processing method of measuring the transversal emittance for a non-axial-symmetric ion beam was suggested. The formulae for determination of the RMS dispersions of the ion beam dimensions in the rotating coordinate system by signals from the scanner wire are given. The measurements of the RMS emittances $\varepsilon_{x,y}$ were carried out in the test stand of the injection channel of DC-72 cyclotron with the H^- ion beam current of 180 μA and kinetic energy of ions of 16.82 keV. The results of the experimental data processing are adduced.

The investigation has been performed at the Flerov Laboratory of Nuclear Reactions, JINR.

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INTRODUCTION

A method of measuring the ion beam transversal emittance in the axial injection channel of the DC-72 cyclotron was given in [1]. It is based on the gradient method with a standard rotating wire scanner [2, 3] used to measure the transversal ion beam dimensions. This method was worked out for ion beam currents up to 1000 μA and allows one to reconstruct emittance with an accuracy of about 30%. The method takes into account the beam self-charge, which is essential. The main assumption in [1] is that the ion beam is axial-symmetric and has equal horizontal and vertical dimensions. But it is not always a success to obtain an axial-symmetric ion beam in experiments. Therefore, a new experimental data processing method was proposed to measure the transversal emittance of the non-axial-symmetric ion beams. The formulae to determine the RMS ion beam dimensions by signals from the scanner wire are given.

1. SCHEME OF THE EXPERIMENT

The scheme of the measuring section of the axial injection channel of H^- ions in the DC-72 cyclotron is shown in Fig. 1. The ion beam current was equal to 180 μA. The kinetic ion energy was equal to 16.82 keV. Ratio of the ion charge Z to its atomic weight $A(Z/A)$ was equal to 1.

Measurements were carried out by the gradient method using the focusing solenoid IS2. The beam RMS dimensions were measured by means of the scanner IC3. The axial dimension of the solenoid field was 450 mm. The distance between the solenoid center and the scanner was 750 mm. The method of the beam emittance reconstruction is given in detail in Appendix. Processing of the experimental data was carried out in the offline regime.

2. INITIAL PROCESSING OF SIGNALS FROM SCANNER

During measurements the signals were read out from the scanner and written into a file 196 times per one turn of the scanner wire. The first 98 signals correspond to measurement of the horizontal beam dimension (x) and the subsequent ones correspond to the measurement of the vertical beam dimension (y). An example of such signals is shown in Fig. 2.

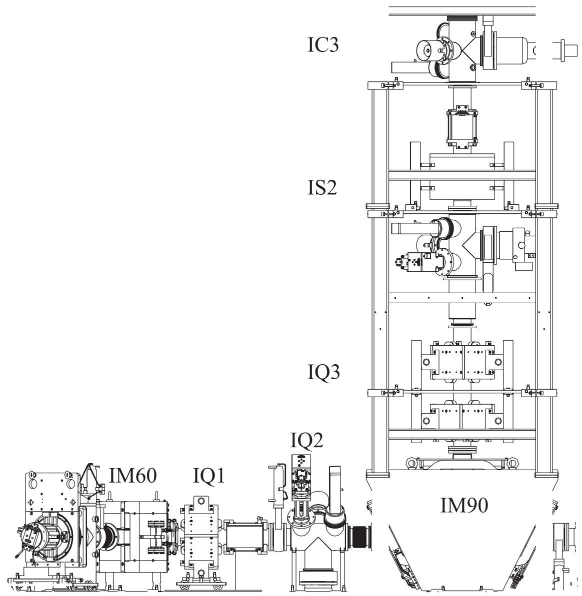

Fig. 1. Scheme of the measuring section of the channel. IM60 is a 60° horizontal bending magnet; IM90 is a 90° vertical bending magnet; IQ1, IQ2, IQ3 are quadrupoles; IS2 is a focusing solenoid, IC3 is a scanner

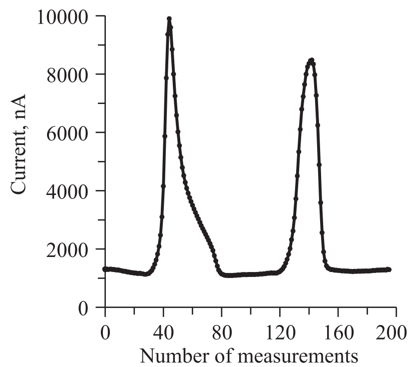

Fig. 2. Signals from the scanner before the background elimination

From Fig. 2 one can see that the background level can reach 15% from maximum of the useful signal. On the follow-up processing the background was subtracted from the useful signal as shown in Fig. 3.

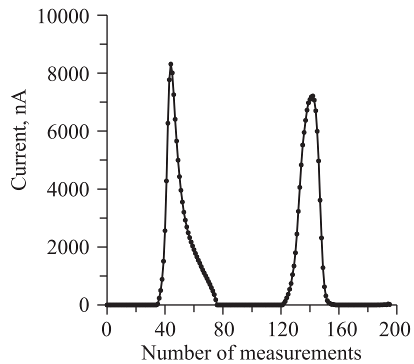

Fig. 3. Signals from the scanner after the background elimination

3. DISPERSIONS OF BEAM DENSITY DISTRIBUTION

Dispersions of the beam density distribution were determined by the following formulae:

$$\sigma_{x,y} = \frac{4L^2_{\max}}{N^2} \left[\, \overline{n^2_{x,y}} - (\overline{n_{x,y}})^2\right]$$

$$\overline{n_{x^2}} = \frac{\sum\limits_{n=0}^{97} n^2 I_n}{\sum\limits_{n=0}^{97} I_n}, \quad \overline{n_x} = \frac{\sum\limits_{n=0}^{97} n I_n}{\sum\limits_{n=0}^{97} I_n}, \tag{1}$$

$$\overline{n^2_y} = \frac{\sum\limits_{n=98}^{195} n^2 I_n}{\sum\limits_{n=98}^{195} I_n}, \quad \overline{n_y} = \frac{\sum\limits_{n=98}^{195} n I_n}{\sum\limits_{n=98}^{195} I_n}.$$

Here N is a number of divisions of the revolution period of the scanner wire ($N = 195$). $L_{\max}$ is the maximum linear dimension:

$$L_{\max} = \frac{\pi R}{\sqrt{2}}, \tag{2}$$

where R is the radius of the helical winding of the scanner wire. The angle of the spiral winding of the scanner wire was equal to 45° in this case. I_n is the current value from the scanner wire at one of the measurement moments.

Measured dependences of $\sigma_{x,y}$ versus the current in the solenoid IS2 winding are shown in Fig. 4 (dots).

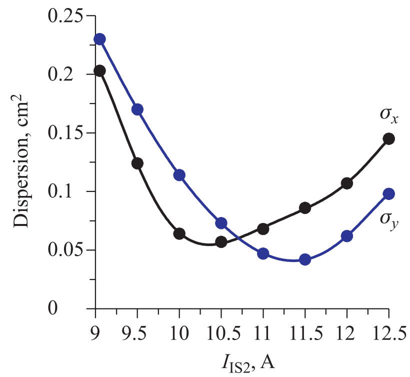

Fig. 4. Dependences of the dispersions $\sigma_{x,y}$ versus the current in the solenoid IS2

4. VIOLATION OF BEAM AXIAL SYMMETRY

As one can see from Fig. 4, the measured real ion beam was not axially symmetric at all. The new processing program has been proposed taking into account the violation of the beam axial symmetry. The emittances of the beam in the longitudinal magnetic field are invariants in the coordinate system rotating with Larmor frequency. Therefore, it is necessary to define the dispersions $\sigma_{1,2}$ in the coordinate system rotated through the angle φ, determined by the solenoid longitudinal magnetic field B_s:

$$\varphi = \frac{1}{2} \frac{\int_0^{L_s} B_s ds}{B\rho} \tag{3}$$

$$\begin{aligned} \sigma_1 &= \sigma_x \cos^2\varphi + \sigma_y \sin^2\varphi, \\ \sigma_2 &= \sigma_x \sin^2\varphi + \sigma_y \cos^2\varphi. \end{aligned} \tag{4}$$

Here L_s is the axial dimension of the solenoid field B_s. Recalculated dependences of the dispersions $\sigma_{1,2}$ on the current in the solenoid IS2 are shown in Fig. 5.

Values of the RMS ion beam emittances $\varepsilon_{x,y}$ found by means of the new processing program are equal to

$$\varepsilon_x = 19\pi \, \text{mm} \cdot \text{mrad}, \quad \varepsilon_y = 18.8\pi \, \text{mm} \cdot \text{mrad}. \tag{5}$$

Some difference of the emittances in planes $\{x, y\}$ can be explained by the ion losses while the beam passes through the bending magnet IM90. The

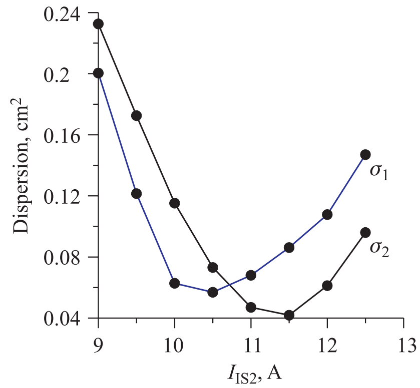

Fig. 5. Dependences of the dispersions $\sigma_{1,2}$ on the current in the solenoid IS2

obtained values of the ion beam transverse emittances (5) are two times greater than those measured earlier in the ion sources by using the multicusp magnetic plasma confinement [4, 5].

CONCLUSIONS

Measurement of the H^- ion beam emittance has been carried out by the gradient method.

The method has been worked out to reconstruct the transversal emittances of the beam of large space charge (the ion beam current up to several mA) by using the measured beam dimensions.

The software has been developed taking into account the violation of the beam axial symmetry.

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APPENDIX

For any value of the solenoid magnetic field B_s the magnitudes $\overline{x^2}$ and $\overline{y^2}$ are connected with momenta $\overline{x_0^2}$, $\overline{xx_0'}$, $\overline{x_0'^2}$ and $\overline{y_0^2}$, $\overline{yy_0'}$, $\overline{y_0'^2}$ at a solenoid entrance by the following equations:

$$\begin{aligned} \overline{x^2} &= Rx_{11}^2\overline{x_0^2} + 2Rx_{11}Rx_{12}\overline{xx_0'} + Rx_{12}^2\overline{x'2_0}, \\ \overline{y^2} &= Ry_{11}^2\overline{y_0^2} + 2Ry_{11}Ry_{12}\overline{yy_0'} + Ry_{12}^2\overline{y'2_0}. \end{aligned} \tag{1A}$$

For a small beam space charge, the elements of the transfer matrix, R_{11} and R_{12}, depend on parameters of the focusing system only. Initial RMS values $\overline{x_0^2}$, $\overline{xx_0'}$, $\overline{x_0'^2}$ and $\overline{y_0^2}$, $\overline{yy_0'}$, $\overline{y_0'^2}$ may be found by means of the least squares method with the measured (under several B_s magnitudes) dispersion values $\overline{x^2}$ and $\overline{y^2}$. These momenta are used to calculate RMS beam emittances [1]:

$$\begin{aligned} \varepsilon_x &= [\overline{x_0^2}\ \overline{x_0'^2} - (\overline{xx_0'})^2]^{1/2}, \\ \varepsilon_y &= [\overline{y_0^2}\ \overline{y_0'^2} - (\overline{yy_0'})^2]^{1/2}. \end{aligned} \tag{2A}$$

For a large space charge the elements of the transfer matrix, R_{11} and R_{12}, as it will be shown below, depend on the initial values $\overline{x_0^2}$, $\overline{xx_0'}$, $\overline{x_0'^2}$ and $\overline{y_0^2}$, $\overline{yy_0'}$, $\overline{y'2_0}$. However, equations (1A) remain valid. This allows one to use the equations (1A) as recursive formulae to determine the initial values of the momenta and the beam RMS emittances (2A).

Let us introduce matrix Λ related to matrix of the second-order momenta M in the following way:

$$M = \Lambda\Lambda^T. \tag{3A}$$

In non-axial-symmetric case, matrix Λ looks like

$$\Lambda = \begin{pmatrix} \Lambda^{(1)} & \Lambda^{(2)} \\ \Lambda^{(1)\prime} & \Lambda^{(2)\prime} \end{pmatrix}, \tag{4A}$$

where the prime denotes a derivative along the distance and $\Lambda^{(1,2)}$ are diagonal (2×2) matrices:

$$\Lambda^{(1)} = \begin{pmatrix} \Lambda_{11}^{(1)} & 0 \\ 0 & \Lambda_{22}^{(1)} \end{pmatrix} \quad \text{and} \quad \Lambda^{(2)} = \begin{pmatrix} \Lambda_{11}^{(2)} & 0 \\ 0 & \Lambda_{22}^{(2)} \end{pmatrix}. \tag{5A}$$

One seeks initial matrix Λ_0 in a lower triangular form:

$$\Lambda_0 = \begin{pmatrix} \Lambda_{x0} & 0 \\ \Lambda_{x0}' & \Lambda_{V0} \end{pmatrix}, \tag{6A}$$

where

$$\Lambda_{x0} = \begin{pmatrix} \sqrt{\overline{x_0^2}} & 0 \\ 0 & \sqrt{\overline{y_0^2}} \end{pmatrix}, \quad \Lambda_{x0}' = \begin{pmatrix} \dfrac{\overline{xx_0'}}{\sqrt{\overline{x_0^2}}} & 0 \\ 0 & \dfrac{\overline{yy_0'}}{\sqrt{\overline{y_0^2}}} \end{pmatrix},$$

$$\Lambda_{V0} = \begin{pmatrix} \sqrt{\overline{x'^2_0} - \dfrac{(\overline{xx'_0})^2}{\overline{x^2_0}}} & 0 \\ 0 & \sqrt{\overline{y'2_0} - \dfrac{(\overline{yy'_0})^2}{\overline{y^2_0}}} \end{pmatrix}. \tag{7A}$$

Then initial matrix M_0 looks like

$$M_0 = \begin{pmatrix} \Lambda_{x0}\Lambda^T_{x0} & \Lambda_{x0}\Lambda'^T_{x0} \\ \Lambda'_{x0}\Lambda^T_{x0} & (\Lambda'_{x0}\Lambda'^T_{V0} + \Lambda_{V0}\Lambda^T_{V0}) \end{pmatrix}. \tag{8A}$$

The variation of the values of the matrix Λ elements along the beam trajectory is given by the equation

$$\Lambda' = A\Lambda. \tag{9A}$$

Matrix is equal to

$$A = \begin{pmatrix} 0 & E \\ b & 0 \end{pmatrix}. \tag{10A}$$

Here and later matrix E is a unit (2×2) matrix, and symmetric matrix of the second-order b is defined by its own and external electromagnetic fields.

The initial conditions for the equation (9A) have the following form:

$$\Lambda^{(1)}_0 = \Lambda_{x0}, \quad \Lambda^{(1)'}_0 = \Lambda'_{x0}, \quad \Lambda^{(2)}_0 = 0, \quad \Lambda^{(2)'}_0 = \Lambda_{V0}. \tag{11A}$$

Solution of Eq. (9A) with the initial conditions (11A) allows one to define transfer matrix R:

$$R = \Lambda\Lambda^{-1}_0. \tag{12A}$$

When the ions move inside the solenoid, matrix b is equal to

$$b = \begin{pmatrix} -k^2 & 0 \\ 0 & -k^2 \end{pmatrix} = -k^2 E. \tag{13A}$$

Here $k = \sqrt{\frac{1}{2}\frac{B_s}{B\rho}}$, $B\rho$ is magnetic rigidity of the ions. In a drift space matrix b = 0. Taking into account the beam Coulomb field, one has $b \Rightarrow b + b_s$, where

$$
b_s = \frac{Z}{A}\frac{I}{I_0}\frac{1}{(\beta_z\gamma)^3}\frac{1}{\sqrt{\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2}}+\sqrt{\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2}}}\times
\times\begin{pmatrix} \dfrac{1}{\sqrt{\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2}}} & 0 \\ 0 & \dfrac{1}{\sqrt{\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2}}} \end{pmatrix}. \quad (14A)
$$

Here I is the beam current, $I_0 = 3.12\cdot 10^7$ A; β_z is the relative beam velocity; and γ is the relativistic factor.

From Eqs. (9A), (10A), (13A), (14A), we obtain the following system of nonlinear differential equations for defining the elements of matrix Λ:

$$
\begin{cases}
\Lambda_{11}^{(1)''} + k^2\Lambda_{11}^{(1)} - Q\dfrac{\Lambda_{11}^{(1)}}{\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2}+\sqrt{(\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2})(\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2})}} = 0 \\
\Lambda_{22}^{(1)''} + k^2\Lambda_{22}^{(1)} - Q\dfrac{\Lambda_{22}^{(1)}}{\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2}+\sqrt{(\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2})(\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2})}} = 0 \\
\Lambda_{11}^{(2)''} + k^2\Lambda_{11}^{(2)} - Q\dfrac{\Lambda_{11}^{(2)}}{\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2}+\sqrt{(\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2})(\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2})}} = 0 \\
\Lambda_{22}^{(2)''} + k^2\Lambda_{22}^{(2)} - Q\dfrac{\Lambda_{22}^{(2)}}{\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2}+\sqrt{(\Lambda_{11}^{(1)^2}+\Lambda_{11}^{(2)^2})(\Lambda_{22}^{(1)^2}+\Lambda_{22}^{(2)^2})}} = 0.
\end{cases} \quad (15A)
$$

Coulomb factor Q is equal to

$$
Q = \frac{Z}{A}\frac{I}{I_0(\beta_z\gamma)^3}. \quad (16A)
$$

According to Eqs. (4A)–(7A) and (12A), the values of elements of transfer matrix R are defined in the following way:

$$
R = \begin{pmatrix} (\Lambda^{(1)}\Lambda_{x0}^{-1} - \Lambda^{(2)}\Lambda_{V0}^{-1}\Lambda'_{x0}\Lambda_{x0}^{-1}) & \Lambda^{(2)}\Lambda_{V0}^{-1} \\ (\Lambda^{(1)'}\Lambda_{x0}^{-1} - \Lambda^{(2)'}\Lambda_{V0}^{-1}\Lambda'_{x0}\Lambda_{x0}^{-1}) & \Lambda^{(2)'}\Lambda_{V0}^{-1} \end{pmatrix}. \quad (17A)
$$

Then from (17A) we obtain

$$Rx_{11} = \frac{1}{\sqrt{\overline{x_0^2}}}\left(\Lambda_{11}^{(1)} - \frac{\Lambda_{11}^{(2)}}{E_x}\right), \quad Rx_{12} = \frac{\Lambda_{11}^{(2)}\sqrt{\overline{x_0^2}}}{E_x},$$

$$^2Ry_{11} = \frac{1}{\sqrt{\overline{y_0^2}}}\left(\Lambda_{22}^{(1)} - \frac{\Lambda_{22}^{(2)}}{E_y}\right), \quad Ry_{12} = \frac{\Lambda_{22}^{(2)}\sqrt{\overline{y_0^2}}}{E_y}. \qquad (18A)$$

Here $E_x = \sqrt{\overline{x_0^2 x_0'} - (\overline{xx_0'})^2}$ and $Ey = \sqrt{\overline{y_0^2 y_0'} - (\overline{yy_0'})^2}$.

Thus, we obtain the system of equations taking into account the dependence of elements of the transfer matrix on the beam Coulomb field. The elements of the transfer matrix depend on the initial momentum values (6A), (7A) due to nonlinearity of the system (15A).

The following algorithm is supposed to reconstruct the beam emittance:

1. The dependence of dispersions $\sigma_{x,y}$ on the solenoid magnetic field induction is defined.

2. Using the known elements of the transfer matrix for the zero beam current, the first approximation of the momentum values $\overline{x_1^2}$, $\overline{x_1'}$, $\overline{xx_1'}$, $\overline{y_1^2}$, $\overline{y_1'}$, $\overline{yy_1'}$ and RMS emittances (2A) are determined with the least squares method.

3. Using these values one determines the initial conditions (11A) to integrate the system (15A) when Coulomb parameter Q $\neq 0$ and finds the first approximation for the transfer matrix elements Rx_{11}, Rx_{12}, Ry_{11}, and Ry_{12} (18A) with the nonzero beam current.

4. Further the least squares method helps to find the second approximation for the initial momentum values $\overline{x_2^2}$, $\overline{x_2'}$, $\overline{xx'}$, $\overline{y_2^2}$, $\overline{y_2'}$, $\overline{yy'}$ and emittances ε_{x,y_2}.

5. This process of successive approximations continues until the necessary accuracy of the measured RMS values of the ion beam emittances is achieved.

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