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MODEL OF INELASTIC COLLISIONS OF HIGH-ENERGY HADRONS WITH NUCLEUS

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Войцеховски А., Стругальска-Гола Э., Стругальски З. E1-2004-191
Модель неупругих столкновений
высокоэнергетических адронов с ядром

При прохождении высокоэнергетического адрона сквозь тяжелое ядро испускается (эмитируется) множество частиц, большинство из которых нуклоны и π^{+-0}-мезоны. Представлена математическая модель, которая является упрощенным объяснением основных процессов, протекающих внутри ядра во время прохождения сквозь него адрона. Вычислительный метод основан на модели неупругих столкновений, называемой файерболом. Результаты вычислений сравнивались с экспериментальными данными, полученными с помощью разных детекторов.

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Model of Inelastic Collisions of High-Energy
Hadrons with Nucleus

During passage of a high-energy hadron through a heavy nucleus, there are emitted nucleons, light nuclei and many other particles, from which the most numerous group is nucleons and π^{+-0} mesons. In this work, we present a mathematical model which is a simplified description of basic processes occurring in the interior of the nucleus during passage of the hadron through the nucleus. Method of calculation is based on inelastic collision model called a fireball. Results of calculations are compared with experimental data from different detectors.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energies, JINR, and in the Institute of Atomic Energy, Otwock-Swierk, Poland.

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INTRODUCTION

The hadron–hadron collisions can be elastic and inelastic. In this work, we analyze only inelastic collisions (we neglect elastic reactions). The work is based on the model of the hadron–hadron inelastic collisions called a fireball [1]. The basic assumption of this model is the assumption that the inelastic process of the collision proceeds through the intermediate state called the fireball. In literature, a well-described creation of the fireball is for the proton–antiproton reactions [1, 2]. This method quite well describes multiplicity and distribution of the energy and the momentum of produced particles. In our work, we will make the assumption that an analogous fireball is created during the collision of any hadrons and translocates inside the nuclear matter.

The hadron striking into the nucleus collides successively with the following nucleons located along the motion path and creates a group of nucleons of the mass m_g further called the fireball. The mass of the fireball grows along its displacement inside the nuclear matter. Particularly, this state can be the gluon–quark plasma. The process of the fireball increase occurs until the collisions with the successive nucleons are inelastic. The process is inelastic as long as the energy per nucleon (in barycentre of fireball system) is greater than $m_\pi c^2 \approx 140$ MeV. In other words, at small energies of the hadron, the inelastic process proceeds only in the first phase of the movement. If the energy of the hadron is large enough (for instance, about 4 MeV for ^{131}Xe nucleus), then the entire process of the passage of the hadron through the nucleus is inelastic. This causes the fireball creation from all nucleons which are along the path of the motion of the hadron and subsequent knock-out of the group of particles (fireball) outside nucleus. Figure 1 presents the scheme of such a fireball creation.

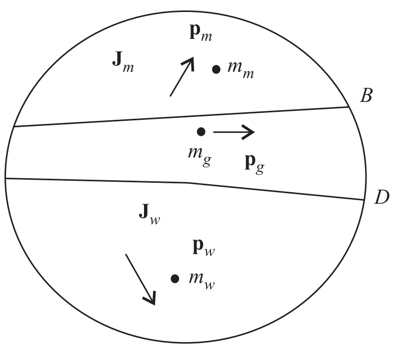

Fig. 1. The simplified scheme of interaction of the fireball of a mass m_g with fragments of the nucleus for the high-energy hadron in the laboratory system

The fireball consists of all nucleons situated along the motion path, has a mass m_g and moves with a momentum p_g (Fig. 1). The fireball interacts with the target-nucleus only by means of the emission of the particles, most often of pions produced inside the fireball. These particles transfer momenta p_m and p_w to the respective fragments of the nucleus with masses m_m and m_w. The nucleus-target as a product of the reaction breaks up into two nuclei of masses m_w and m_m and the corresponding angular momenta J_w and J_m (see Fig. 1).

Experimental data concerning fragmentation of nucleus-target during spallation process (for the high-energy hadron) are presented in many publications (e.g., [3–6]).

1. THE FIREBALL SIZE

In this chapter, we will present the method of calculation of the size of the fireball passing through the interior of the nucleus.

The following assumptions are made:

— The hadron moves in the nucleus along the straight line.

— The hadron interacts with all nucleons placed at a small distance (smaller than the range of the strong interaction). We can estimate this range D_0 on the basis of inelastic cross section σ_{in} of hadron–nucleon interaction $\sigma_{\text{in}} = \pi D_0^2$.

— The average kinetic energy of nucleons in the fireball have to be greater than the mass energy of pions $m_\pi c^2 \approx 140$ MeV.

— The density of the intranuclear matter distribution is continuous [7].

— During the reaction, the hadron–nucleon and nucleon–nucleon collisions are completely inelastic. It means that the process takes place through the intermediate state called the fireball. We can treat the fireball as a system of strongly interacting nuclear matter consisting of the hadron and group of nucleons of the nucleus and produced particles.

— The density of the matter of the fireball is equal to the density of the intranuclear matter in the center of the nucleus. In connection with this, the mass and the radius of the fireball increase during its moving in the intranuclear matter.

Below we will present the method of calculation of the number of emitted nucleons based on the above assumptions.

The number of nucleons being in the volume dV is equal to

$$dn = A\rho(r)dV, \tag{1}$$

$\rho(r)$ and A mean respectively the density of the nuclear matter normalized to unity and number of nucleons [1].

The number of nucleons n with which the moving hadron can interact and form a fireball is equal to a number of nucleons placed along the motion path of the hadron within the range of the strong interaction $D(l)$:

$$n = \int_{V'} A\rho(r)dV, \tag{2}$$

where V' is the volume of the cylinder along the motion path of the hadron and fireball of the radius $D(l)$ (7).

The volume V' is equal to

$$V' = \int_{-\infty}^{\infty} \int_{0}^{D(l)} \int_{0}^{2\pi} R dl dR d\theta, \tag{3}$$

where l, R, θ are cylindrical coordinates presented in Fig. 2. If the track of the hadron is at distance b from the center of the nucleus (b is the collision parameter), then any point of the space in the interior of the cylinder V' (l, R, θ) is connected with the distance from the center of the nucleus by the following formula:

$$r^2 = l^2 + (b + R\cos\theta)^2 + (R\sin\theta)^2. \tag{4}$$

Now by formulae (2), (3), (4) and using cylindrical coordinates (l, R, θ), we are able to calculate the number of nucleons which can interact with the hadron:

$$n(b) = \int_{-\infty}^{\infty} \int_{0}^{D(l)} \int_{0}^{2\pi} A\rho(l, R, \theta) R dl dR d\theta, \tag{5}$$

where $D(l)$ is a radius of the fireball in the point l described by formula (7).

The mass of the fireball increases at moving along the direction l. In the first phase of the movement, the mass is equal to the mass of the incident hadron, and the radius D_0 of the interaction is equal to $\pi D_0^2 = \sigma_{\rm in}$ ($\sigma_{\rm in}$ is inelastic hadron–nucleon cross section). In the further phase, the mass of the fireball is expressed in nucleons by formula (5).

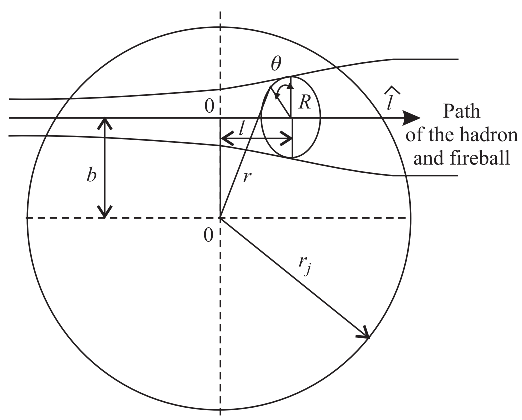

Fig. 2. The notation of coordinates

The mass of the fireball as the function of the parameters b and l is expressed by formula $m_g = m_h + n(b,l)m_n$, where m_h, m_n mean mass of the hadron and mass of nucleon respectively. The volume of the fireball is equal to $V_g = m_g/\rho$, where ρ is a density of the matter in the center of the nucleus (we assume that the nuclear matter is incompressible). On the other hand, the volume of the fireball is equal to $V_g = \frac{4}{3}\pi D^3$, where D is a radius of the fireball.

In respect to the foregoing we can express the radius of the fireball with the formula:

$$D(l) = \left(\frac{3V_g}{4\pi}\right)^{1/3} = \left(\frac{3(m_h + n(b,l)m_n)}{4\pi,\ \rho}\right)^{1/3}, \quad (6)$$

where $n(b,l)$ means the number of nucleons which create the fireball during his movement inside the nucleus to point l.

Taking into account the assumption that every hadron has the radius not less than D_0, we have the formula

$$D(l) = \left\{ \begin{array}{ll} \left(\dfrac{3(m_h + n(b,l)m_n)}{4\pi\ \rho}\right)^{1/3} & \text{when } D(l) > D_0 \\ D_0 & \text{when } D(l) \leqslant D_0 \end{array} \right\}. \quad (7)$$

In Fig. 3 $n(b)$ functions for a few reactions are presented.

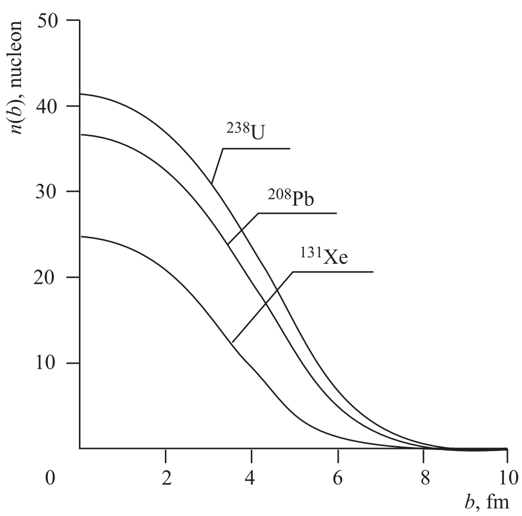

Fig. 3. The function $n(b)$ for reactions: $\pi^- + {}^{131}$Xe, $\pi^- + {}^{208}$Pb, and $\pi^- + {}^{238}$U

2. THE EMISSION OF NUCLEONS

The size of the fireball defined by the function $n(b)$ is equivalent to a number of nucleons emitted during the reaction with the collision parameter b.

The function $n(b)$ presents the maximum number of nucleons which create a fireball whose nucleons can be emitted from the nucleus if the hadron-projectile has enough energy. In the case when the hadron–projectile has relatively low energy such that the fireball is created only in the first phase of moving in the nucleus, the upper limit of the integration along the motion path (variable l) is equal to $l_{\max}(E, b)$ and it looks as follows:

$$n(b) = \int_{-\infty}^{l_{\max}(E,b)} \int_{0}^{D(l)} \int_{0}^{2\pi} A\rho(l, R, \theta) RdldRd\theta, \tag{8}$$

where $l_{\max}(E, b)$ is the point where the process of the fireball creation stops (kinetic energy in barycentric system of the fireball $E'_k(E, b)$ per nucleon is equal to $m_\pi c^2 \approx 140$ MeV).

We can estimate the maximum number of nucleons with which the fireball may be created:

$$n_{\max} = \frac{E'_k(E, b)}{\Delta E}, \quad \Delta E = m_\pi c^2, \tag{9}$$

where $E, E'_k(E, b)$, and m_π mean the initial kinetic energy of hadron, kinetic energy in barycentric system of fireball, and mass of π meson respectively.

Using formulae (8) and (9), we have the following function $n(E, b)$, by which one can determine the size of the fireball and the number of emitted nucleons:

$$n(E, b) = \left\{ \begin{array}{ll} \int_{-\infty}^{\infty} \int_{0}^{D(l)} \int_{0}^{2\pi} A\rho(l, R, \theta) RdldRd\theta & \text{when } n(b) \leqslant n_{\max} \\ \frac{E'_k(E, b)}{\Delta E} & \text{when } n(b) > n_{\max} \end{array} \right\}. \tag{10}$$

For a hadron of high enough initial energy, formula (10) transforms to formula (5) because $n(b) < n_{\max}$ for all collision parameters b.

The probability of the hadron–nucleus collision with impact parameter b is defined by the formula

$$P(b) = \frac{2\, b}{r_j^2} db, \tag{11}$$

where r_j is the radius of the nucleus.

On the other hand, the probability of the hadron–nucleus collision during the hadron passage through the intranuclear matter is defined by the formula

$$P_{\text{col}}(b) = 1 - \exp\left(-\sigma\ gr(b)\right), \tag{12}$$

where $gr(b) = \int\limits_{-\infty}^{\infty} \rho(l')dl'$.

Since both events are independent, the total probability of events $P_c(b)$ of the reaction with parameter b is a product of probabilities:

$$P_c(b)db = P_{\mathrm{col}}(b)P(b)db = (1 - \exp(-\sigma\ gr(b))\frac{2b}{r_j^2}db. \tag{13}$$

Using formulae (10) and (13), we can calculate the expected value $n_e(E, b)$ of emitted nucleons during the single reaction:

$$n_e(E, b) = P_c(b)n(E, b). \tag{14}$$

3. THE EMISSION OF PROTONS

Nucleons in the nucleus are a mixture of protons and neutrons. We will assume that temporary distribution of protons in the nucleus is random. One can calculate the probability $P'(n_p)$ that there are n_p protons among $n(E, b)$ of nucleons of the fireball using the following distribution:

$$P'(n_p) = \frac{\begin{pmatrix} Z \\ n_p \end{pmatrix}\begin{pmatrix} A - Z \\ n(E, b) - n_p \end{pmatrix}}{\begin{pmatrix} A \\ n(E, b) \end{pmatrix}}. \tag{15}$$

There is a problem related to the function $n(E, b)$ calculated by formula (10) as a real function. The distribution by formula (15) is defined for natural $n(E, b)$ function. The real function $n(E, b)$ can be approximated to the integer function by several methods. The simplest method is simply to transform the real values to integer values.

Formula (15) can be used for any nuclei because there are no limitations on the size of the nucleus A and the corresponding charge Z.

The $P'(n_p)$ is a function of the energy E and the collision parameter b; so in order to calculate the total probability of emitting n_p of protons, we must calculate the integral:

$$P_p(n_p) = \int\limits_0^{\infty} P'(n_p, E, b)P_c(b)db = \int\limits_0^{\infty} P'(n_p, E, b)\frac{2\pi}{r_j^2}(1 - \exp(\sigma gr(b)))db, \tag{16}$$

where $P'(n_p, E, b)$ is defined by formula (15), and $P_c(b)$, by formula (13).

4. THE PRODUCTION OF PIONS

In Sec. 2, we presented the method of calculation of the number of nucleons emitted from the nucleus during the passage of the hadron through the nucleus. We obtained this with the assumption that the process includes the intermediate state called the fireball. As we have already mentioned, inside the fireball, elementary particles are produced. By means of methods of the statistical physics, one can comparatively exactly describe the number and distribution energy of produced particles [1, 8].

The number and the type of produced particles depend mainly on masses of particles and energy of the incident hadron.

In what follows, we will calculate the production of pions only because their number in experiments is the greatest.

Obviously the particles are produced by using the kinetic energy of the initial hadron. In this respect, in formula (10) the energy E used for the creation of the fireball must be reduced to the energy used for emitted particle production.

Taking this fact into consideration, formula (9) looks as follows:

$$n_{\max} = \frac{E'_k(E,b) - \sum E_\pi}{\Delta E}. \quad (17)$$

Here ΣE_π means the sum of the total energy of emitted π^{+-0} mesons (and the average kinetic energy of the scattered pion when hadron-projectile is pion).

Taking into consideration the phenomenon of pion production, formula (10) looks as follows:

$$n(E,b,n_\pi) = \left\{ \begin{array}{ll} \int\limits_{-\infty}^{\infty} \int\limits_{0}^{D(l)} \int\limits_{0}^{2\pi} A\rho(l,R,\theta) dl dR d\theta & \text{when } \ n(b) \leqslant n_{\max} \\ \dfrac{E'_k(E,b) - \sum E_\pi}{\Delta E} & \text{when } \ n(b) > n_{\max} \end{array} \right\}. \quad (18)$$

5. THE ENERGY DISTRIBUTION OF NUCLEONS

We suppose that overall kinetic energy in the centre-of-mass system of hadrons colliding not elastically is exchanged for particle production. As a result of this assumption, nucleons inside the fireball take place at the lowest energy levels (in the moment when they are of the highest density). A second physical phenomenon having influence on the energy of nucleons is the emission of produced particles (we suppose, for the simplification, that there are only π mesons) and the recoil energy of nucleons.

The average kinetic energy $\overline{\varepsilon}$ of the nucleon inside the fireball can be approximately written in the form

$$\overline{\varepsilon} = \frac{3}{5}\varepsilon_f + \overline{\varepsilon}_{od}, \tag{19}$$

where ε_f and $\overline{\varepsilon}_{od}$ are the Fermi energy of nucleons and the average kinetic energy of the recoil energy of the nucleon during the emission of π meson respectively.

$$\varepsilon_f = \frac{\hbar^2}{2m_n}(3\pi^2\rho)^{2/3}, \quad \rho = n_n/V, \tag{20}$$

where m_n, ρ, n_n, and V are mass of nucleons, the density of nucleons, the number of nucleons inside the fireball, and volume of the fireball respectively.

We can estimate the average recoil energy per nucleon by the following formula:

$$\overline{\varepsilon_{od}} = \frac{n_\pi \varepsilon_{od}}{n(E,b)}, \tag{21}$$

where n_π, ε_{od}, and $n(E,b)$ are the average number of produced π mesons, the average recoil energy obtained by the nucleon during the emission of π mesons, and the number of nucleons of the fireball.

$$n_\pi = \frac{E'_k(b)}{\overline{E_\pi}}, \tag{22}$$

where $E'_k(b)$, and $\overline{E_\pi}$ are the kinetic energy in barycentric system of the fireball (the excitation energy of the fireball), and the average total energy of emitted π mesons respectively.

$$E'_k(b) = -(m(b)+m_h)c^2 + \sqrt{(m^2(b)+m_h^2)c^4 + 2m(b)c^2 E_{\text{lab}}}, \tag{23}$$

$$m(b) = n(b)m_n, \quad E_{\text{lab}} = m_h c^2 + E, \tag{24}$$

where m_h, m_n, and E mean mass of the hadron, mass of the nucleon, and the kinetic energy of the hadron in the laboratory system respectively.

$$\varepsilon_{od} = \sqrt{m_n^2 c^2 + p_n^2 c^2} - m_n c^2, \tag{25}$$

where the velocity of the nucleon p_n and the meson p_π satisfy conditions

$$\sqrt{m_\pi^2 c^4 + p_\pi^2 c^2} + \sqrt{m_n^2 c^2 + p_n^2 c^2} = \overline{E_\pi}, \quad p_n = p_\pi. \tag{26}$$

We can write the distribution of momentum of nucleons (as free independent particles) in the form of Fermi–Dirac statistics [9, 1]:

$$dn(p_n) = \frac{4\pi V g_s}{\hbar^3}\frac{p_n^2 dp_n}{\exp(\beta(E_k^n - \mu)) + 1}, \tag{27}$$

where μ, g_s, $\hbar$, and E_k^n mean the chemical potential, the degeneration of states of nucleons, Planck constant, and the kinetic energy of the nucleon respectively.

We can calculate energy distribution of nucleons using formula (27).

The distribution (27) is a function of the collision parameter b because the parameter β is a function of the collision parameter.

We can calculate the value of the parameter β using the dependence that the average energy of nucleons described by the Fermi–Dirac statistics is equal to the average energy of nucleons inside the fireball $\overline{\varepsilon}$ (19).

$$\overline{\varepsilon} = \frac{4\pi V g_s}{\hbar^3} \int\limits_0^\infty \frac{\varepsilon p_n^2 dp_n}{\exp\left(\beta(\varepsilon - \mu)\right) + 1}. \tag{28}$$

We can calculate the parameter β from Eq. (28) without reference to experimental data. The parameter β depends on the collision parameter b. The distribution of momentum (27) is a distribution in barycentric system of the fireball. To transform the momentum and energy distributions (27) to laboratory system, we used the Lorentz transformation.

6. RESULTS OF CALCULATIONS AND THE COMPARISON WITH EXPERIMENT

6.1. Multiplicity Distributions. We compared our calculation with experimental data from the bubble chamber with the liquid ^{131}Xe [10] (Figs. 4, 11), streamer chamber [11] (Fig. 6) and nuclear emulsion [12, 13] (Figs. 5, 7).

In Figs. 4, 11, presented are the experimental data from a 180 l xenon bubble chamber of the Institute of Theoretical and Experimental of Physics (ITEF, Moscow) irradiated with a beam of mesons of momentum 3.5 GeV/c.

Figure 4 presents calculated distribution of emitted protons. Graphs in turn for $n_\pi = 0, 1, 2, 3, 4, 5$ of pions, which will appear as the result of the reaction, are presented. It should be noticed that the hadron causing the reaction is a pion, so we have [the total number of pions] = [the number of created pions] $+1$

6.2. Energy Distributions. Experimental data and calculation results presented in this section are normalized. Experimental data are compared with calculation results done by a fireball method and Monte Carlo method. In calculations by Monte Carlo method we used the FRITIOF [14] code and Fluka code [15] (Figs. 8, 9, 10, 11). The FRITIOF code is a program of Monte Carlo simulation of inelastic hadron–hadron, hadron–nucleus and nucleus–nucleus interactions and takes into account:

— calculations of the Glauber cross sections,

— simulation of the string creation and fragmentation (Dual Parton Model),

— simulation of the residual nuclei relaxation and evaporation of nucleons.

According to the author (V. V. Uzhinski), one can use the FRITIOF code for the energy above 1–2 GeV/nucleon.

Hadronic interactions of FLUKA code [15] are simulated by different event generators, depending on kinetic energy E of the hadron:

— 4 GeV $< E <$ 20 TeV — Dual Parton Model (DPM);

— 3.5 GeV $< E <$ 4–5 GeV — Resonance Production and Decay Model;

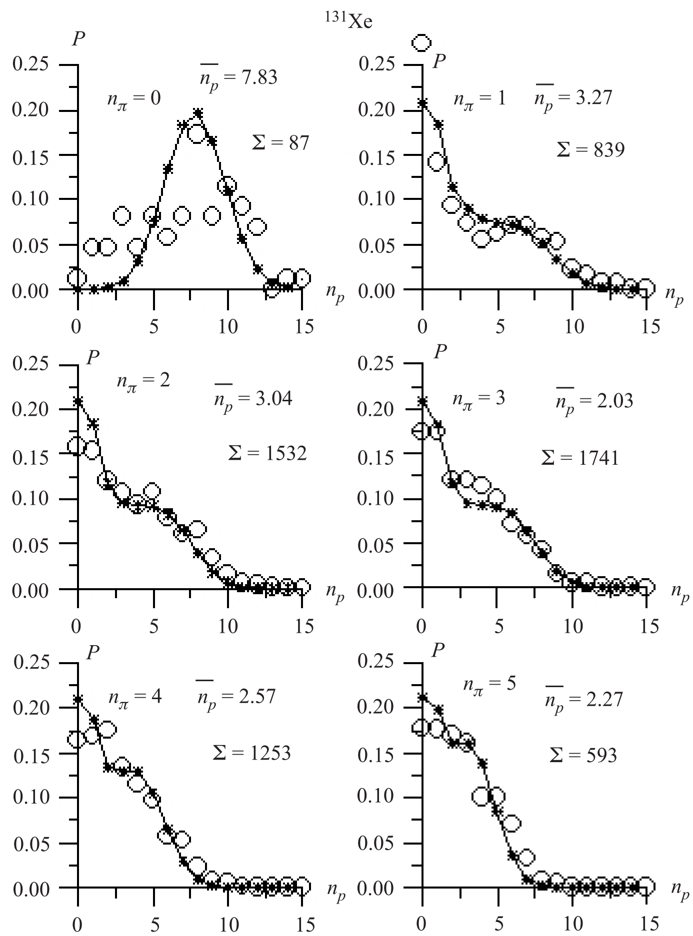

Fig. 4. The probability of the emission of protons in the reaction $\pi^- + \mathrm{Xe}$ with the energy of pions 3.3 GeV. n_p and P on axes mean the number of emitted protons and the probability respectively. $\circ$, $*$ are experimental data [10] and calculation results respectively

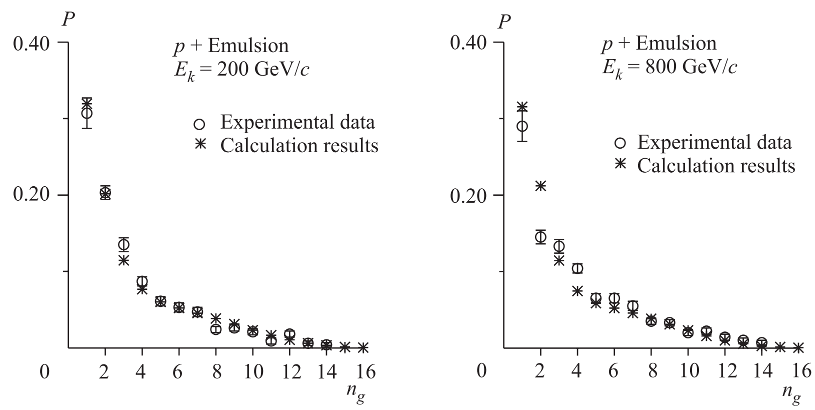

Fig. 5. Multiplicity of «grey» protons (40–400 MeV) in the reaction $p+\text{Emulsion}$ for energies 200 and 800 GeV. Experimental data are taken from [12]. n_p and P on axes mean the number of «grey» tracts and the probability, respectively

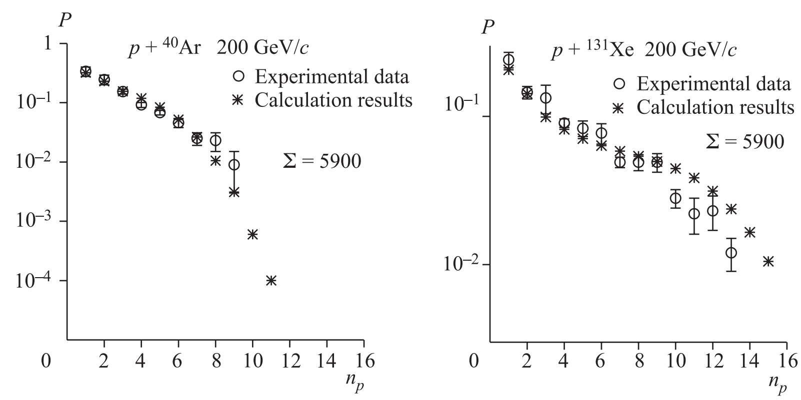

Fig. 6. Multiplicity of «grey» protons (100–600 MeV/c) in the reactions $p+\text{Ar}$ and $p+\text{Xe}$ for momentum 200 GeV/c. Experimental data are taken from streamer chamber [11]. n_g and P on axes mean the number of emitted protons and the probability, respectively

— 0.02 MeV $< E <$ 3.5 GeV — Preequilibrium-Cascade Model PEANUT.

Experimental data presented in Figs. 8, 9 are from the experiment described in [16]. To perform the experiment, a proton beam from accelerator SATURNE in Sacley was used. In the experiment, three types of detectors for measurement of

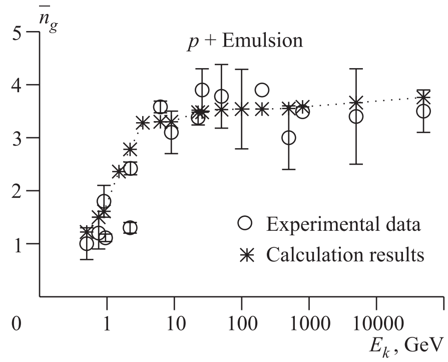

Fig. 7. Average multiplicity of «grey» tracts in the reaction $p+$Emulsion. Experimental data are taken from [12, 13]. E_k and $\overline{n}_g$ on axes mean the kinetic energy of the hadron and average number of «grey» tracts in emulsion, respectively

neutrons were used: DENSE (the energy range of neutrons 2–14 MeV), DEMON (the energy range 4–400 MeV), and the spectrometer (the energy range above 200 MeV).

Experimental data presented in Fig. 10 are from the experiment FNAL-E592 [17]. The experiment was performed using independent measurements of the time of flight in the telescope as well as dE/dx measurements in the scintillators traversed by the particles.

Experimental data presented in Fig. 11 are taken from the xenon bubble chamber [10].

7. SUMMARY

Advantages of the fireball method:

— It comparatively correctly foresees multiplicity (Fig. 5, 6) and energy distributions (Fig. 8, 9, 10, 11) of nucleons emitted in reactions with high-energy hadron.

— The accordance in a large range of the hadron energy (Fig. 7).

— The comparatively simple mathematical methods.

— Absence of any fit of parameters.

The comparatively good agreement of calculation results (approximated) with the experimental data means that basic assumptions of the model are valid.

Main disadvantages of calculation of the fireball method consists in the reduction elastic collisions and evaporation nucleons (for approximated calculation).

One should take into account that the graph 1 (for case $n_\pi = 0$) in Fig. 4 presents experimental results concerning $\Sigma = 87$ cases of the reactions. It is too little to check a statistical principle. The remaining graphs present a large enough number of experimental reactions; and one can compare them with the experiment.

Differences between calculations and experimental results do not exceed 5%.

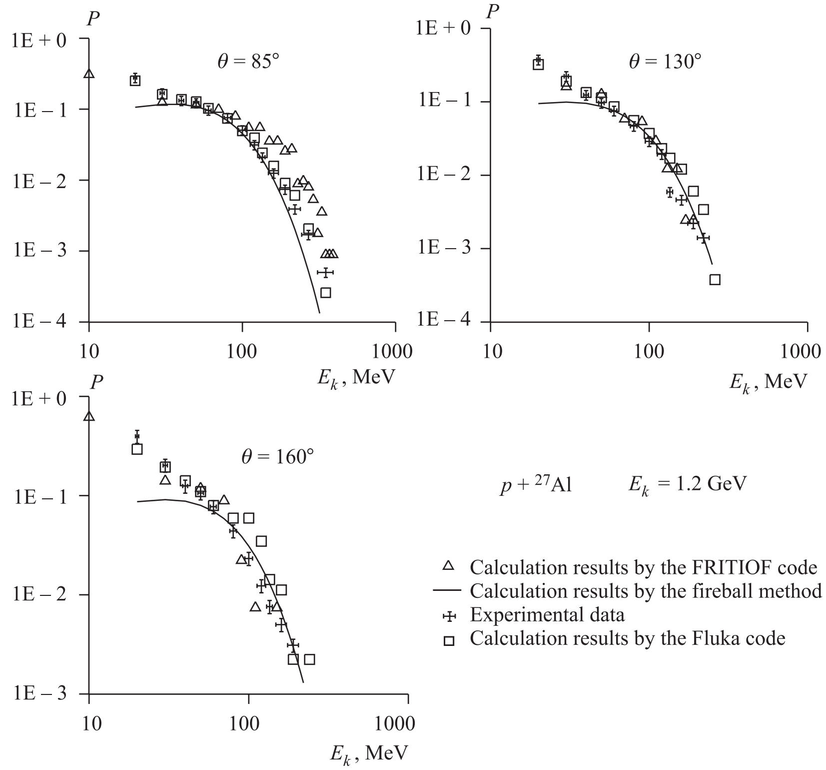

Fig. 8. Energy distributions of neutrons emitted in the reaction $p + {}^{27}$Al (1.2 GeV) into angles 85°, 130° and 160°. Experimental data are taken from work [16]. P, E_k are the probability and the kinetic energy, respectively. The number of calculated events by the FRITIOF code [14] and Fluka code [15] is equal to 7000 and 26000, respectively

Results of calculations agree with the experiment with the exactitude of experimental errors.

The exception is the point on the second graph in Fig. 4 concerning the number of pions $n_\pi = 1$ for $n_p = 0$. This means that the probability of the hadron–nucleus reaction (without the emission of nucleons and without the production of pions — the reaction of elastic scattering) is considerably greater in the experiment. The maximum difference of the probability (between experimental data and calculations) is equal to about 0.07, which means that the relative error equals about 30%.

Figure 7 has only the estimated value because it has big experimental and computational errors. Figures 8, 9, and 11 show considerably better agreement of

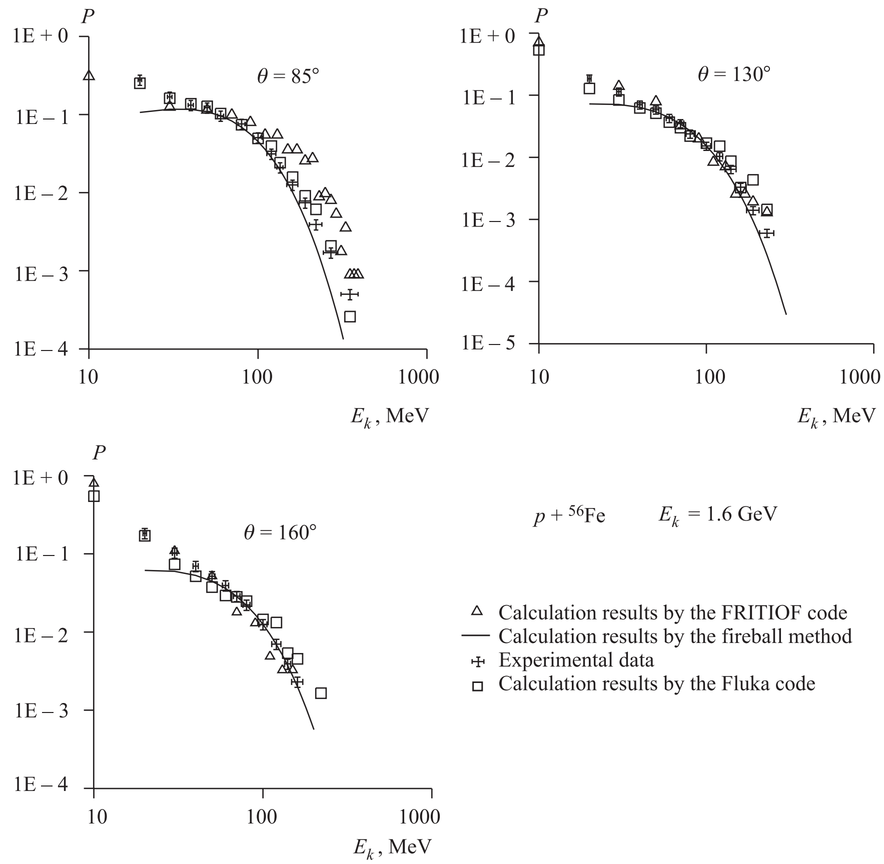

Fig. 9. Energy distributions of neutrons emitted in the reaction $p + {}^{56}$Fe (1.2 GeV) into angles 85°, 130° and 160°. Experimental data are taken from work [16]. P, E_k are the probability and the kinetic energy, respectively. The number of calculated events by the FRITIOF code [14] and Fluka code [15] is equal to 7000 and 65000, respectively

calculations of energy distributions with the experimental data for angles θ from the range $0 > \cos\theta > -1$ than from the range $1 > \cos\theta > 0$. This disagreement is due to the omission of elastic collisions (products of elastic collisions move at angles $1 > \cos\theta > 0$ in the laboratory system) in calculations by the fireball method (for the reduction of calculations).

Figures 8, 9 show disagreement of energy angular distributions of neutrons calculated by the fireball method with the experiment in the range of small energies (below 30 MeV). The omission (for the reduction of calculations) of the

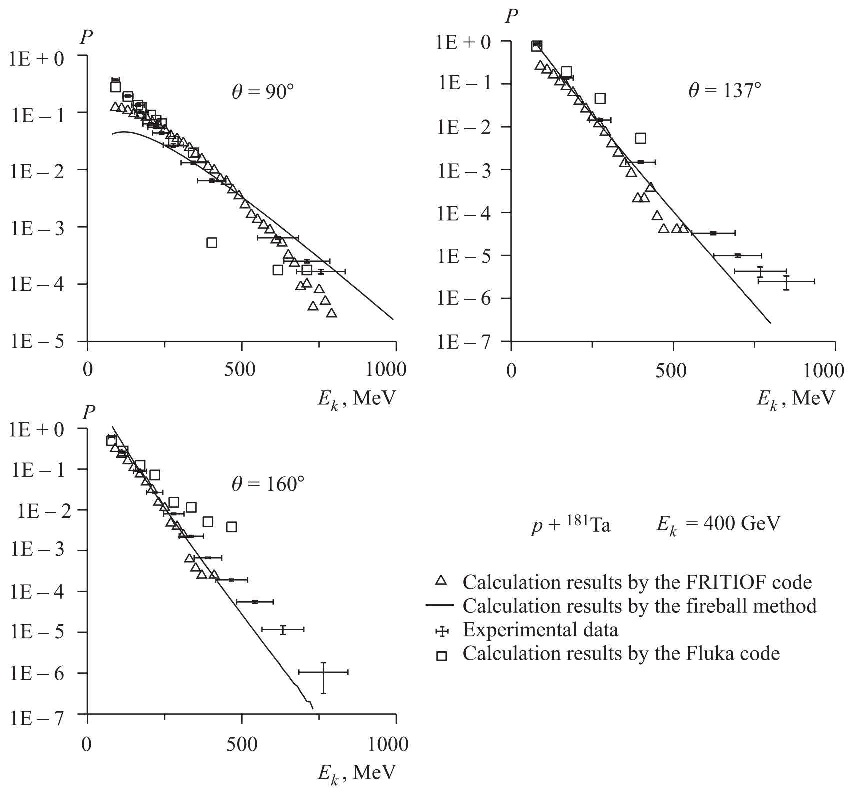

Fig. 10. Energy distributions of protons for the reaction $p\,+\,{}^{181}$Ta (400 GeV) into angles 90°, 137° and 160°. Experimental data are taken from work [17]. P, E_k are the probability and the kinetic energy, respectively. The number of calculated events by the FRITIOF code [14] and Fluka code [15] is equal to 15000 and $5.9 \cdot 10^5$, respectively

excitation of the nucleus and the emission of neutrons of the evaporation explain the disagreement.

Figure 10 shows disagreement of energy distribution of protons with high kinetic energies (above 500 MeV). The omission of producing of particles heavier than π mesons, i. e. mesons K, η and baryons, can be the reason of the disagreement.

Large accordance of calculation results with experimental data can mean that inelastic collision processes can proceed in intermediate state called the fireball or gluon–quark plasma.

To improve results of calculation for a wide energy range and all angles of the emission nucleons, one must additionally take into account elastic collisions (the

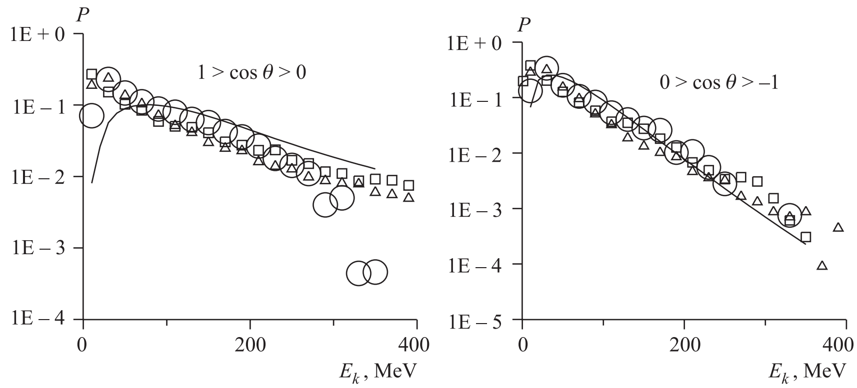

Fig. 11. The energy distribution of emitted protons in the reaction ^{131}Xe(π^-, p) emitted into the intervals of angles ($1 > \cos\,\theta > 0$) and ($0 > \cos\,\theta > -1$). The energy range of protons 10–350 MeV, the width of the channel 20 MeV. The solid line, ○, Δ, □ mean calculation results by the fireball method, experimental data [10], and calculation results by the FRITIOF code [14] and Fluka code [15], respectively. P, E_k mean the probability and the kinetic energy respectively. The number of calculated events by the FRITIOF code and Fluka code and the number of experimental data are equal to 6000, 6600 and 6000, respectively

intranuclear cascade of elastic processes) and the emission of nucleons during evaporation from the excited nucleus and the creation of heavy mesons and baryons. Results of calculations by the fireball method (for the reduction of calculations) presented in this paper take into account only inelastic processes during which only π mesons are created.

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