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GEOMETRIZATION
OF THE ELECTROMAGNETIC FIELD
AND DARK MATTER

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Геометризация электромагнитного поля и темная материя

Введена общая концепция потенциального поля. Темной материи ставится в соответствие потенциальное поле, которое имеет фундаментальную геометрическую интерпретацию (параллельный перенос) и обладает внутренне присущей ему локальной симметрией. Выведены уравнения поля, инвариантные относительно локальных преобразований. Показано, как редуцировать эти уравнения к уравнениям Максвелла. Таким образом, поле, описывающее темную материю, может рассматриваться как обобщенное электромагнитное поле, что содержит также решение старой проблемы: как связать электромагнитное поле со свойствами самого физического многообразия. Показано, что при фиксировании калибровки кванты обобщенного электромагнитного поля становятся массивными, в то время как фотоны остаются безмассовыми. Для выяснения характера взаимодействия между спинорной и темной материей выведены общековариантные уравнения Дирака.

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Geometrization of the Electromagnetic Field and Dark Matter

A general concept of potential field is introduced. The potential field that one puts in correspondence with dark matter, has fundamental geometrical interpretation (parallel transport) and has intrinsically inherent local symmetry. The equations of dark matter field are derived that are invariant with respect to the local transformations. It is shown how to reduce these equations to the Maxwell equations. Thus, the dark matter field may be considered as generalized electromagnetic field and a simple solution of the old problem is given to connect electromagnetic field with geometrical properties of the physical manifold itself. It is shown that gauge fixing renders generalized electromagnetic field effectively massive while the Maxwell electromagnetic field remains massless. To learn more about interactions between matter and dark matter on the microscopical level (and to recognize the fundamental role of internal symmetry) the general covariant Dirac equation is derived in the Minkowski space-time which describes the interactions of spinor field with dark matter field.

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INTRODUCTION

The problem of invisible mass [1, 2] is acknowledged to be among the greatest puzzles of modern cosmology and field theory. The most direct evidence for the existence of large quantities of dark matter in the Universe comes from the astronomical observation of the motion of visible matter in galaxies [3]. One neither knows the identity of the dark matter nor whether there is one or more types of its structure elements. The most commonly discussed theoretical elementary particle candidates are a massive neutrino, a supersymmetric neutralino and the axion. So, at present time there is a good probability that the set of known fields is by no means limited to those fields. Moreover, we are free to look for deeper reasons for the existence of new entity unusual in many respects. Of course, such reasoning is grounded on the point of view that there is a general and easily visible mathematical structure that stands behind the all phenomena that we observe.

Here a field theory of the so-called dark matter is derived from the first principles. A general concept of potential field is introduced. We connect one of these fields with the problem of dark matter. The field that we put in correspondence with dark matter has fundamental geometrical interpretation (parallel transport) and has intrinsically inherent local symmetry. The equations of dark matter field that are invariant with respect to the local transformations are derived. It is shown how to reduce these equations to the Maxwell equations. Thus, the dark matter field may be considered as generalized electromagnetic field and at the same time we get a simple solution of the old problem raised by Weyl, Einstein and Eddington to connect electromagnetic field with geometrical properties of the physical manifold itself. The idea is that process of local symmetry breaking is an intrinsic property of the system itself which means that gauge fixing cannot be arbitrary. This approach is realized here in the framework of the concept of dark matter field vacuum.

It is interesting that the vacuum field belongs to the set of potential fields. It should be noted that gauge fixing renders generalized electromagnetic field effectively massive while the Maxwell electromagnetic field remains massless (particle of dark matter is a heavy photon). To learn more about interactions between matter and dark matter on the microscopical level the general covariant Dirac equation is derived in the Minkowski space-time and in course of this the fundamental role of internal symmetry is recognized. On this ground the Dirac equation is derived which describes the interactions of spinor field with dark matter field. From this it follows the general conclusion that interactions of generalized electromagnetic field with Dirac spinor field occur only via the

electromagnetic field and the above-introduced dark matter field vacuum. The general conclusion is that a dark matter gravitates but there is no actually direct interactions of this new form of matter with known physical fields that represent luminous matter. A rather simple and feasible experiment is proposed to verify this conclusion.

The paper is organized as follows. The first two sections are the basis for all considerations. Section 1 contains the geometrically motivated general definition of the concept of potential field. The conjecture is put forward that all potential fields have a geometrical interpretation. It is shown that in general case a parallel transport is an exact realization of the abstract concept of potential field. We consider this realization as a new physical field (dark matter field). In Sec.2 the equations of the dark matter field are derived. Section 3 deals with the vacuum of this field. The equations of vacuum field are considered in Secs.4 and 5. Section 6 treats the general covariant Dirac equation in the Minkowski space–time with careful consideration of internal and space–time symmetries and connection between them. In Sec.7 the theory of interactions of the mentioned above potential fields with matter (spinor field) is formulated. The source of the dark matter vacuum field is the circulation of the energy of the spinning matter, which is expressed in the direct connection between the potential of the dark matter vacuum field and the canonical tensor energy–momentum of spinning matter. Thus, it is shown that the canonical energy–momentum tensor plays fundamental role in the theory of the spinor fields. And finally, Sec.8 provides a proposal of rather simple experiment that can give answer the series of principal questions.

1. CONCEPT OF POTENTIAL FIELD

First of all, we shall consider the necessary elements of general mathematical structure. According to the modern viewpoint a fundamental physical theory is the one that possesses a mathematical representation whose elements are smooth manifold and geometrical objects defined on this manifold. Most physicists nowadays consider a theory to be fundamental only if it makes explicit use of this concept. It is thought that curvature of the manifold itself provides an explanation of gravity. Within the manifold, further structures are defined including vector fields, connections, particle path, and so forth, and these are taken into account for the behavior of physical world. This picture is generally accepted and it is based on such a long history of physical research, that there is no reason to question it. Another element is the concept of potential field.

If we take the components of symmetrical covariant tensor field g_{ij} and form its derivatives ($\partial_i g_{jk}$) then these derivatives are neither the components of a tensor nor of any geometrical object. However, from g_{ij} and these partial derivatives

one can form (with the help of algebraic operations only) a new geometrical object

$$\Gamma_{jk}^i = \frac{1}{2}g^{il}(\partial_j g_{kl} + \partial_j g_{kl} - \partial_l g_{jk}), \quad (1)$$

which is called Christoffel connection, where g^{il} are contravariant components of the g_{ij} . Now we can formalize this particular case and give general definition of the potential field.

If some geometrical object (or a geometrical quantity) is given and from the components of this object and its partial derivatives one can form (using the algebraic operations only) a new geometrical object (or geometrical quantity), then we deal with a new geometrical quantity that will be called a potential field. Potential field is characterized by the potential P and strength H and in what follows will be written in the form (P, H) . Connection between the potential and strength is then called a natural derivative and in symbolic form can be written as $H = \partial P$. If we go back to our starting point, then g_{ij} is a potential and Γ_{jk}^i is a strength of potential field (g, Γ) , known after Einstein as the gravitational field.

Now we introduce another very important and geometrically motivated potential field. The most important geometrical notions are the metric g_{ij} and parallel transport or linear (affine) connection P_{jk}^i . Tensor field g_{ij} is symmetric, $g_{ij} = g_{ji}$, but linear connection P_{jk}^i in general is nonsymmetric with respect to the covariant indices, $P_{jk}^i \neq P_{kj}^i$ and in any way does not link with the metric g_{ij} . In fact, these notions define, on a manifold M , different geometric operations. Namely, a metric on a manifold gives for each curve its length and linear connection defines the parallel transport of vectors along any path on M . Let us consider a vector field $E^i(x)$. Equation of local parallel transport from a point x^i to a point $x^i + dx^i$ has in general the form

$$dE^i(x) = -P_{jk}^i(x)E^k(x)dx^j, \quad (2)$$

where functions $P_{jk}^i(x)$ are components of a new geometrical object on the manifold, called a linear connection P . Under a parallel transport along the infinitesimal closed curve the change of the vector is equal to the quantity

$$\Delta E^k = -H_{ijl}{}^k E^l dx^i \delta x^j,$$

where

$$H_{ijl}{}^k = \partial_i P_{jl}^k - \partial_j P_{il}^k + P_{im}^k P_{jl}^m - P_{jm}^k P_{il}^m \quad (3)$$

is a tensor field of type (1, 3), called the Riemann tensor of the connection P_{jk}^i . Now we go back to the definition of potential field and see that parallel transport defines new potential field (P, H) . At first glance, this is in contradiction with fundamental principle, which means that only irreducible quantity should enter

into the theory. Indeed, from (2) it follows that under a coordinate mapping

$$\tilde{x}^i = \tilde{x}^i(x), \quad x^i = x^i(\tilde{x}),$$

the transformation law for a P_{jk}^i has the form

$$\tilde{P}_{jk}^i = \frac{\partial \tilde{x}^i}{\partial x^l} (P_{mn}^l \frac{\partial x^m}{\partial \tilde{x}^j} \frac{\partial x^n}{\partial \tilde{x}^k} + \frac{\partial^2 x^l}{\partial \tilde{x}^j \partial \tilde{x}^k}). \quad (4)$$

Recall that a geometrical quantity is reducible if it is possible to find linear combinations of its components which themselves constitute a new geometrical quantity. As for linear connection under the coordinate mappings, it is a reducible quantity which is easily seen from the expansion

$$P_{jk}^i = \frac{1}{2}(P_{jk}^i + P_{kj}^i) + \frac{1}{2}(P_{jk}^i - P_{kj}^i).$$

From (4) it follows that a symmetrical part of the connection $\frac{1}{2}(P_{jk}^i + P_{kj}^i)$, is again the linear connection and the antisymmetrical part, $\frac{1}{2}(P_{jk}^i - P_{kj}^i)$ transforms as a tensor field of type (1, 2). However, there is a very interesting structure which allows one to consider parallel transport as potential field.

Let S_j^i be components of a tensor field of type (1,1) (a field of linear operator), $\text{Det}(S_j^i) \neq 0$. Out of two tensor fields S_j^i and Q_j^i of type (1, 1) a tensor field $P_j^i = S_k^i Q_j^k$ of type (1, 1) may be constructed, called their product. With the operation of multiplication thus defined, the set of tensor fields of type (1, 1) with a nonzero determinant forms a group, denoted by G_i . This is a natural group of local symmetry on a manifold. At given vector field E^i , any element of the group G_i defines a bundle of vector fields, which is defined as follows:

$$\bar{E}^i = S_j^i E^j, \quad \tilde{E}^i = T_j^i E^j, \quad \text{etc.},$$

where T_j^i are components of the field S^{-1} inverse to S , $S_k^i T_j^k = \delta_j^i$. It is clear that notion of the parallel transport is not applied to the bundle of the vector fields. From (2) it follows that the parallel transport of the bundle of the vector fields is defined by the bundle of the linear connections, which is defined by the relation

$$\bar{P}_{jk}^i = S_m^i P_{jn}^m T_k^n + S_m^i \partial_j T_k^m.$$

It is easy to see that for the bundle of linear connections the expansion considered above has no sense, so the tensor $P_{jk}^i - P_{kj}^i$ is evidently not a geometrical quantity with respect to the transformations of the local group.

Thus, we shall expand the diffeomorphism group to include into the consideration the group of local symmetry G_i , defined above. It can be shown that the diffeomorphism group is the group of external automorphisms of the group of

local symmetry, i.e. the group G_i is invariant under the transformations of the group $\text{Diff}(M)$. Thus, we have a nontrivial unification of these symmetries and possibility to consider one more potential field.

We conclude that we really introduce geometrically motivated potential field (P, H) , but the theory of this field should be invariant not only with respect to the general transformations of the coordinates but with respect to the transformations of the local symmetry group G_i as well. We put in correspondence to this field the so-called dark matter and develop theory of the dark matter as the theory of this new potential field.

For brevity, we will use in what follows the matrix notation:

$$P_j = (P_{jk}^i), \quad E = (\delta_j^i), \quad H_{ij} = (H_{ijkl}^k), \quad S = (S_j^i), \quad \text{Tr}S = S_i^i,$$

in which the transformation law of the potential P_j is of the form

$$\bar{P}_j = SP_j S^{-1} + S \partial_j S^{-1} = P_i + SD_i S^{-1}, \quad (5)$$

where D_i stands for the important operator

$$D_i S = \partial_i S + P_i S - SP_i = \partial_i S + [P_i, S],$$

which is especially convenient when one deals with local symmetry in question. In what follows we shall meet many examples of this. Relation (5) is indeed the transformation of the connection, since $SD_i S^{-1}$ is a tensor field of type (1, 2) and on this reason \bar{P}_j is the connection with respect to the coordinate transformations. Since the connection between the potential and strength in matrix notation is given by the formula

$$H_{ij} = \partial_i P_j - \partial_j P_i + [P_i, P_j],$$

from (5) it follows that under the transformations of the group G_i the strength is transformed as follows:

$$\bar{H}_{ij} = SH_{ij} S^{-1}. \quad (6)$$

For H_{ij} we have

$$D_i H_{jk} = \partial_i H_{jk} + [P_i, H_{jk}],$$

and if \bar{D}_i is defined by potential \bar{P}_i , then from (5) and (6) it follows that:

$$\bar{D}_i \bar{H}_{jk} = S(D_i H_{jk}) S^{-1}.$$

In general case, the operator D_i is not general covariant, however, the commutator $[D_i, D_j]$ is always general covariant and we get the important relation for the strength tensor of dark matter

$$[D_i, D_j] H_{kl} = [H_{ij}, H_{kl}]. \quad (7)$$

Thus, in our approach the theory of the dark matter is tightly connected with the local symmetry, it is general covariant and has a profound geometrical interpretation.

2. FIELD EQUATIONS

The simplest general covariant and gauge invariant Lagrangian of the potential P_i is a direct consequence of (6)

$$L_P = -\frac{1}{4}\text{Tr}(H_{ij}H^{ij}), \quad (8)$$

where $H^{ij} = g^{ik}g^{jl}H_{kl}$. Varying the Lagrangian L_P with respect to P_i and using the relation $\delta H_{ij} = D_i\delta P_j - D_j\delta P_i$, we obtain

$$\delta L_P = \text{Tr} \left(\left(\frac{1}{\sqrt{g}} D_i(\sqrt{g}H^{ij}) \right) \delta P_j \right) - \frac{1}{\sqrt{g}} \partial_i \text{Tr}(\sqrt{g}H^{ij} \delta P_j),$$

and hence the following equations of the field hold valid:

$$\frac{1}{\sqrt{g}} D_i(\sqrt{g}H^{ij}) = 0, \quad (9)$$

where $g = -\text{Det}(g_{ij})$. From the properties of the operator D_i it is not difficult to see that Eq.(9) is invariant with respect to the local symmetry group. The tensor character of this equation can be seen from the identity

$$\frac{1}{\sqrt{g}} D_i(\sqrt{g}H^{ij}) = \overset{p}{\nabla}_i H^{ij} + \omega_i H^{ij} - \frac{1}{2}(P_{ik}^j - P_{ki}^j)H^{ik},$$

where $\overset{p}{\nabla}_i$ is the usual covariant derivative with respect to the connection P_i , and $\omega_i = \partial_i \ln \sqrt{g} - P_{ki}^k$ are the components of the covector field. Thus, it is shown that the group of diffeomorphisms is the group of covariance of Eq. (9). Equations (9) form the first group of the equations. The second one is presented by the identity

$$D_i H_{jk} + D_j H_{ki} + D_k H_{ij} = 0. \quad (10)$$

From definition of the operator D_i it follows that left-hand side of relation (10) is a tensor and hence it is general covariant.

Varying the Lagrangian L_P with respect to g^{ij} we obtain the so-called metric tensor of energy–momentum of the dark matter field

$$T_{ij} = \text{Tr}(H_{ik}H_j{}^k) + g_{ij}L_P, \quad (11)$$

where $H_j{}^k = H_{jl}g^{kl}$. One can establish the identity

$$\nabla^i T_{ij} = \text{Tr}(H_{jk} \frac{1}{\sqrt{g}} D_i(\sqrt{g}H^{ik})) + \frac{1}{2} \text{Tr}(H^{ik}(D_i H_{jk} + D_j H_{ki} + D_k H_{ij})).$$

With this equation and Eqs. (9) and (10) we see that the metric tensor of the energy–momentum satisfies the equation

$$\nabla_i T^{ij} = 0, \quad (12)$$

where ∇_i denotes as usual the covariant derivative with respect to the Christoffel connection (1) and $\nabla^i = g^{ik}\nabla_k$. It is evident that the metric tensor energy–momentum is invariant with respect to the group of local transformations in question. Now we can write down the full action for the fields g_{ij} and P_i

$$A = -\frac{c^3}{G} \int R\sqrt{g} d^4x - \frac{\beta^2 \hbar}{4} \int \text{Tr}(H_{ij}H^{ij})\sqrt{g} d^4x,$$

where R is the scalar curvature, G is the Newton gravitational constant, \hbar is the Planck constant, and β is dimensionless constant. From the geometrical interpretation of the field P it follows that it has the dimension of cm^{-1} . As all coordinates can be considered to have the dimension of cm , the action A has a correct dimension.

Varying the full action A with respect to g^{ij} we derive the Einstein equation

$$R_{ij} - \frac{1}{2}g_{ij}R = \beta^2 l^2 T_{ij}, \quad (13)$$

where $l = \sqrt{\hbar G/c^3}$ is the Planck length, and T_{ij} is the metric tensor of energy–momentum of dark matter field. Thus, it is shown that the interactions of the dark matter field with the gravitational field are characterized by some length $\lambda = \beta l$. Equations (9), (10) and (13) are compatible in view of (12).

Equations (9) and (10) constitute the full system of the generalized Maxwell equations in geometrical representation and new field (dark matter field) can be considered as the generalized electromagnetic field. The arguments are as follows. From the strength of the potential field in question it can be constructed very interesting quantity that is invariant with respect to the transformations of the group of local symmetry G_i , namely

$$F_{ij} = \text{Tr}H_{ij}. \quad (14)$$

It is evident that F_{ij} is an antisymmetrical tensor with respect to the transformations of coordinates. If H_{ij} satisfies Eqs. (9) and (10), then taking the trace we obtain that bivector F_{ij} satisfies the Maxwell equations

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}F^{ij}) = 0, \quad \partial_i F_{jk} + \partial_j F_{ki} + \partial_k F_{ij} = 0. \quad (15)$$

Now consider a question concerning the vector potential of the electromagnetic field. We put $A_i = \text{Tr}P_i = P_{ik}^k$. According to (5) and the differentiation rule for

determinants, the transformation law for A_i under the local transformations has the form

$$\bar{A}_i = A_i - \partial_i \ln |\Delta|,$$

where $\Delta = \text{Det}(S_j^i)$. Thus, the local transformations of potential P_{jk}^i reduced to the gauge transformations of the potential of the electromagnetic field A_i . For completeness of the picture we shall also consider the arbitrary coordinate transformations of A_i . From (4) one can derive that A_i transforms as follows:

$$\tilde{A}_i = (A_m - \partial_m \ln |J|) \frac{\partial x^m}{\partial \tilde{x}^i},$$

where $J = |\frac{\partial \tilde{x}}{\partial x}|$ is the Jacobian of the transformation. It is interesting to point out that any arbitrary coordinate transformation is accompanied by the gauge transformation. Since $F_{ij} = \partial_i A_j - \partial_j A_i$, the question on the nature of the gauge transformations is completely solved and geometrical origin of the electromagnetic field is recognized.

Now we have to solve two problems. If generalized electromagnetic field represents dark matter it should be massive (whereas electromagnetic field is massless), and the other problem is the general covariant gauge fixing that is provided by the Cauchy problem for the field in question. The distinctive feature of the generalized electromagnetic field is that it is self-interacting: it is non-linear even in the absence of other fields. Two potentials \bar{P}_i and P_i are physically equivalent if there is a local transformation which takes P_i into \bar{P}_i , and clearly \bar{P}_i satisfies the field equations if and only if P_i does. In order to obtain a definite member of the equivalence class of potentials one has to introduce general covariant gauge conditions. These conditions have to remove the sixteen degrees of freedom and lead to unique solution for the potential components. To solve these problems we suggest that gauge fixing is an internal property of the system in question and introduce very important notion of the vacuum of generalized electromagnetic field.

3. VACUUM

We have a vacuum if $H_{ij} = 0$ and so the energy density of the generalized electromagnetic field is equal to zero. On the other hand, $P_i \neq 0$ so the vacuum has a structure. Let four linear-independent covector fields be given E_i^μ , $p = \text{Det}(E_i^\mu) \neq 0$. Greece indices belong to the internal symmetry which we shall in what follows connect with internal symmetry inherent in the Dirac equation, whereas Latin indices are coordinate. Under a general transformation $x^i = \tilde{x}^i(x)$ of the coordinate system, each of these fields transforms as follows:

$$\tilde{E}_i^\mu = E_k^\mu \frac{\partial x^k}{\partial \tilde{x}^i}, \quad \mu = 0, 1, 2, 3.$$

From the E_i^μ one can purely algebraically construct components of the four vector field E_μ^i , so that

$$E_\mu^i E_j^\mu = \delta_j^i, \quad E_\mu^i E_i^\nu = \delta_\mu^\nu \quad (16)$$

hold valid. If we put $P_{jk}^i = V_{jk}^i$, where

$$V_{jk}^i = E_\mu^i \partial_j E_k^\mu, \quad (17)$$

then it is easy to show that this is the solution of the vacuum equation $H_{ij} = 0$ for any E_k^μ . If \bar{V}_i is another solution then it can be shown that $\bar{E}_\mu^i = S_k^i E_k^\mu$ and $\bar{V}_i = S V_i S^{-1} + S \partial_i S^{-1}$. Thus, the vacuum of generalized electromagnetic field is again a potential field (E, V) with E_i^μ being potential and V_i being strength.

Now we introduce the tensor field

$$Q_{jk}^i = P_{jk}^i - V_{jk}^i, \quad (18)$$

which can be called the deviation of the generalized electromagnetic field with respect to a vacuum. It is evident that under the local transformations the deviation tensor transforms as follows:

$$\bar{Q}_i = S Q_i S^{-1}.$$

The tensor Q is reducible and in what follows we shall consider the irreducible deviation tensor

$$T_{jk}^i = Q_{jk}^i - \frac{1}{4} Q_{jm}^m \delta_k^i, \quad T_j = Q_j - \frac{1}{4} (\text{Tr} Q_j) E.$$

With this we can consider the general covariant and gauge invariant Lagrangian of the generalized electromagnetic field in the following form:

$$L_P = -\frac{1}{4} \text{Tr}(H_{ij} H^{ij}) - \frac{\mu^2}{2} \text{Tr}(T_i T^i), \quad (19)$$

where μ is a constant, which has dimension of cm^{-1} . It is natural to identify this constant with the length that characterizes the interactions of the dark matter field with gravitational field, $\mu = 1/\lambda$. Varying (19) with respect to P_i we get the following equations:

$$\frac{1}{\sqrt{g}} D_i (\sqrt{g} H^{ij}) = \mu^2 T^j. \quad (20)$$

We see that in some sense one can treat μ as the effective mass of the heavy photon. Since trace of T^i equals zero, from (20) it follows that photon remains massless. From (20) it follows that T^i has to satisfy the equation

$$\frac{1}{\sqrt{g}} D_i (\sqrt{g} T^i) = 0, \quad (21)$$

since in accordance with (7) $D_i D_j (\sqrt{g} H^{ij}) = 0$. It is very important that the same equation appears under varying (19) with respect to E_μ^i . Equation (21) represents sixteen additional constraints on the potential P_i .

However Eqs. (20) and (21) are invariant with respect to the local transformations and hence we still have a problem of gauge fixing. To find its natural solution we can look for the geometrically motivated equations for the vacuum field, which are not invariant with respect to the transformations of the local symmetry group G_i . It is interesting that such a possibility really exists.

4. EQUATIONS OF THE VACUUM FIELD

The local symmetry will be broken if we introduce the quantity

$$U_{jk}^i = E_\mu^i (\partial_j E_k^\mu - \partial_k E_j^\mu). \quad (22)$$

From the definition it follows that U_{jk}^i is evidently a tensor field antisymmetric in covariant indexes. On the other hand, from (5) it follows that this tensor is not geometrical object with respect to the local symmetry group. The tensor U_{jk}^i defines no representation of the group G_i . Thus, it is convenient for our goal. Further we shall establish geometrically motivated Lagrangian that can be constructed for this vacuum tensor field. It leads us to the investigation of the geometry of affine space which is characterized by the connection

$$L_{jk}^i = \Gamma_{jk}^i + U_{jk}^i, \quad (23)$$

where the first summand is given by expression (1). Physical meaning of this connection is to investigate two quite independent potential fields in the uniform geometrical framework. Consider the most important geometrical quantity defined by connection (23). For the Riemann tensor as a function of the potentials of gravity and vacuum we have

$$B_{ijk}{}^l = R_{ijk}{}^l + \nabla_i U_{jk}^l - \nabla_j U_{ik}^l + U_{im}^l U_{jk}^m - U_{jm}^l U_{ik}^m, \quad (24)$$

where

$$R_{ijk}{}^l = \partial_i \Gamma_{jk}^l - \partial_j \Gamma_{ik}^l + \Gamma_{im}^l \Gamma_{jk}^m - \Gamma_{jm}^l \Gamma_{ik}^m \quad (25)$$

is the Riemann curvature tensor of metric g_{ij} and ∇_i as earlier stands for the covariant derivative with respect to the Christoffel connection (1)

$$\nabla_i U_{jk}^l = \partial_i U_{jk}^l + \Gamma_{im}^l U_{jk}^m - \Gamma_{ij}^m U_{mk}^l - \Gamma_{ik}^m U_{jm}^l.$$

By contraction we get from (24) the tensor

$$B_{jk} = B_{ijk}{}^i = R_{jk} + \nabla_i U_{jk}^i - \nabla_j U_{ik}^i + U_{im}^i U_{jk}^m - U_{jm}^i U_{ik}^m, \quad (26)$$

where R_{jk} is the Ricci tensor. From (26) one can find by contraction with metric the following expression for the scalar:

$$B = g^{jk} B_{jk} = R + g^{jk} U_{jm}^l U_{kl}^m - \nabla_j U^j,$$

where R is the Ricci scalar curvature and $U^j = g^{jk} U_k = g^{jk} U_{lk}^l$. Hence, connection (23) uniquely determines the geometrical Lagrangian of the potential fields of curvature and vacuum which is a natural generalization of the Einstein–Gilbert Lagrangian of the gravitational field. Thus, we shall derive equations describing the interactions of the gravitational and vacuum fields from the action

$$A = \frac{c^3}{2G} \int B \sqrt{g} d^4x. \quad (27)$$

From (27) it follows that connection (23) uniquely determines the Lagrangian L_v of the vacuum field itself

$$L_v = \frac{1}{2} g^{jk} U_{jm}^l U_{kl}^m. \quad (28)$$

It is natural that the Lagrangian of the vacuum field like the dark matter Lagrangian contains no derivatives of the components of the gravitational potential, since U_{jk}^i can be considered as a strength with respect to E_i^μ .

To conclude this section, we establish one more interesting connection between two potential fields in question. Standard Lagrangian of the gravitational field $L_g = R$ contains the second order derivatives of g_{ij} and this leads to the known difficulties [4]. Let us show, that this Lagrangian can be generally covariantly reduced to the Lagrangian without the second order derivatives of g_{ij} .

Introduce a binary tensor field

$$B_{jk}^i = E_\mu^i \nabla_j E_k^\mu = V_{jk}^i - \Gamma_{jk}^i. \quad (29)$$

Setting

$$V_{jk}^i = \Gamma_{jk}^i + V_{jk}^i - \Gamma_{jk}^i = \Gamma_{jk}^i + B_{jk}^i$$

and following closely the line defined by (24) and (26), we derive the relation

$$0 = R_{jk} + \nabla_i B_{jk}^i - \nabla_j B_{ik}^i + B_{im}^i B_{jk}^m - B_{jm}^i B_{ik}^m.$$

From the last formula it follows that:

$$R + \nabla_i (g^{jk} B_{jk}^i - g^{ik} B_{lk}^l) = g^{jk} (B_{jm}^i B_{ik}^m - B_{im}^i B_{jk}^m).$$

Thus, the Einstein–Hilbert Lagrangian is equivalent to the Lagrangian

$$L_{gv} = g^{jk} (B_{jm}^i B_{ik}^m - B_{im}^i B_{jk}^m),$$

which is defined by the vacuum field and may be more convenient in the quantum theory of the gravitational field.

5. CURVATURE AND VACUUM IN INTERACTION

Varying action (27) with respect to g_{ij} , we get the Einstein equations

$$G_{ij} = T_{ij},$$

where

$$T_{ij} = g_{ij}L_v - U_{il}^k U_{jk}^l \quad (30)$$

is the metric tensor energy–momentum of the vacuum field. From (28) and (30) it follows that $g^{ij}T_{ij} = 2L_v$, and hence equations of the vacuum field are not conformally invariant. It is yet another general property of gravity and vacuum fields.

Now we make small variations in our field quantities E_μ^l . It is convenient to introduce tensor

$$F_k^{ij} = g^{il}U_{lk}^j - g^{jl}U_{lk}^i = U_k^{ij} - U_k^{ji}$$

with inverse transformation

$$U_{jk}^i = \frac{1}{2}(g^{il}F_l^{mn}g_{jm}g_{kn} + g_{jl}F_k^{il} - g_{kl}F_j^{il}).$$

Since

$$U_{jk}^i = E_\mu^i(\partial_j E_k^\mu - \partial_k E_j^\mu) = E_\mu^i(\nabla_j E_k^\mu - \nabla_k E_j^\mu),$$

we get sequentially (28),

$$\delta B = \delta L_v = F_l^{jk}(\nabla_j E_k^\mu)\delta E_\mu^l + F_l^{jk}E_\mu^l \nabla_j \delta E_l^\mu. \quad (31)$$

With (16) we get

$$\delta E_k^\nu = -E_l^\nu E_k^\mu \delta E_\mu^l.$$

By this, the second term in the right-hand side of (31) can be presented in the following form:

$$\nabla_j(F_l^{jk}E_\mu^l \delta E_l^\mu) + E_k^\mu(\nabla_j F_l^{jk} + F_m^{jk}E_l^\nu \nabla_j E_\nu^m)\delta E_\mu^l.$$

Thus, the variational principle provides the following equation for the potential of the vacuum field:

$$E_k^\mu \nabla_j F_l^{jk} + F_l^{jk} \nabla_j E_k^\mu + F_m^{jk} E_k^\mu E_l^\nu \nabla_j E_\nu^m = 0.$$

It is possible to rewrite this equation in more symmetrical form (without covariant derivative of the potential). With (16) and (29) we have

$$E_l^\nu \nabla_j E_\nu^m = -E_\nu^m \nabla_j E_l^\nu = -B_{jl}^m, \quad \nabla_j E_k^\mu = B_{jk}^m E_m^\mu$$

and hence equation of the vacuum field has the following form:

$$\nabla_j F_l^{jk} + B_{jm}^k F_l^{jm} - B_{jl}^m F_m^{jk} = 0. \quad (32)$$

Like the equations of the gravitational field and dark matter field the equation of the vacuum field is essentially nonlinear. Let $\overset{v}{\nabla}_i$ be a covariant derivative with respect to the connection (17). Since

$$\nabla_j F_l^{jk} = \overset{v}{\nabla}_j F_l^{jk} - B_{ji}^j F_l^{ik} - B_{jm}^k F_l^{jm} + B_{jl}^m F_m^{jk},$$

equation of the vacuum field (32) can be presented in the following most simple form:

$$(\overset{v}{\nabla}_j - B_j) F_l^{jk} = 0, \quad (33)$$

where B_i is a contraction of the binary tensor field (29), $B_i = B_{ki}^k$.

In conclusion of this section we would like to point out on possible applications of the equations derived. It is of interest to find spherically symmetric solution of the system of equations (Einstein equations plus (33)) and then investigate the corresponding metric of the generalized Schwarzschild solution.

Now we shall consider the interactions of generalized electromagnetic field with matter in the framework of the Dirac theory that is very important since it is known nothing about the interactions of the dark matter field with luminous matter.

6. THE DIRAC EQUATION IN GENERAL COVARIANT FORM

The description of the interactions between the matter and dark matter we will provide in the framework of the Dirac equation, which is the basis for the description of matter. It is one of the fundamental principles of modern geometry and theoretical physics that laws of geometry and physics do not depend on the choice of coordinate systems. It is natural to write all equations in the coordinate basis since the problem to rewrite these equations in any other basis is formal and hence trivial task. In our days this statement is as canonical as the energy conservation. Let us show that original Dirac equation is in full agreement with this fundamental statement and that it is defined by the internal symmetry. As is known, internal symmetries play fundamental role in modern physical theories and hence it is very important to have clear understanding of the role of internal symmetries in the Dirac equation, which is the basis for all modern theories of elementary particles and their interactions, in particularly, Dirac's Hamiltonian defines entirely the space-time sector of the standard model.

Let \mathbf{C}^4 be a linear space of columns of four complex numbers $\psi_1, \psi_2, \psi_3, \psi_4$. Linear transformations in this space can be presented by the complex matrices

(4×4) . The set of all invertible (4×4) complex matrices forms a group denoted by $GL(4, \mathbf{C})$. Dirac's γ^μ matrices belong to $GL(4, \mathbf{C})$ and obey anticommutation relations

$$\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2\eta^{\mu\nu},$$

where $\eta^{\mu\nu}$ is digital matrix such as the inverse matrix $\eta_{\mu\nu}$ defines the commutation relations of the Poincaré group. In the case of the Poincaré group it is possible to write the structure relations with help of matrix $\eta_{\mu\nu}$ and signs plus and minus but for our consideration the explicit form of the matrix $\eta_{\mu\nu}$ is not important. One should only not confuse $\eta_{\mu\nu}$ with the Minkowski metric g_{ij} , which has quite another sense.

From γ^μ one can construct sixteen linear-independent matrices that form a basis of the Lie algebra of $GL(4, \mathbf{C})$. This basis is especially important since the matrices $S_{\mu\nu} = \frac{1}{4}(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)$ form the basis of the Lie algebra of the Lorentz group (subgroup of $GL(4, \mathbf{C})$.) Thus, we suppose that the Dirac spinor is an element of the space \mathbf{C}^4 where the group $GL(4, \mathbf{C})$ acts that is equipped with the matrix $\eta_{\mu\nu}$. For better understanding it should be noted that in the space \mathbf{C}^3 there are no matrices like γ^μ .

If one considers $\psi_1, \psi_2, \psi_3, \psi_4$ as a set of complex scalar fields on the space–time manifold then the Dirac spinor field emerges on the manifold which is a basis of irreducible representation of the group $GL(4, \mathbf{C})$. It is not difficult to understand that $GL(4, \mathbf{C})$ is a group of internal symmetry since its transformations involve only functions of the spinor field and do not affect the coordinates. In other words, spin symmetry is **an internal symmetry**.

Now, on this ground we consider general covariant formulation of the Dirac equation in the Minkowski space–time. We shall follow the fundamental physical principle that was mentioned above. With respect to an arbitrary curvilinear system of coordinates Minkowski space–time is characterized by the metric

$$ds^2 = g_{ij} dx^i dx^j$$

of the Lorentz signature, which satisfies the equation $R_{ijk}{}^l = 0$ and topology R^4 . At given g_{ij} , the generators of the group of space–time symmetry can be presented as a set of linear-independent solutions of general covariant system of equations (Killing's equations)

$$K^i \partial_i g_{jk} + g_{ik} \partial_j K^i + g_{ji} \partial_k K^i = 0$$

for a vector field K^i . In the case of the Minkowski metric we have ten linear-independent solutions of the Killing equations, which are denoted K_μ^i and $K_{\mu\nu}^i = -K_{\nu\mu}^i$ and hence the Greek indices enumerate vector fields and take the values 0, 1, 2, 3, like coordinate Latin indices.

It is well known that the generators of the Poincaré group

$$\mathbf{P}_\mu = K_\mu^i \frac{\partial}{\partial x^i}, \quad \mathbf{M}_{\mu\nu} = K_{\mu\nu}^i \frac{\partial}{\partial x^i}$$

satisfy the following commutation relations:

$$[\mathbf{P}_\mu, \mathbf{P}_\nu] = 0, \quad (34)$$

$$[\mathbf{P}_\mu, \mathbf{M}_{\nu\lambda}] = \eta_{\mu\nu} \mathbf{P}_\lambda - \eta_{\mu\lambda} \mathbf{P}_\nu. \quad (35)$$

It is evident that all these relations are general covariant and that the operators

$\mathbf{P}_\mu = K_\mu^i \frac{\partial}{\partial x^i}$ transform a scalar field into the scalar one.

Now we shall show that the general covariant Dirac equation has the form

$$i\gamma^\mu \mathbf{P}_\mu \psi = \frac{mc}{\hbar} \psi, \quad (36)$$

where ψ is a column of four complex scalar fields in question and \mathbf{P}_μ are the generators of space–time translations. To be exact in all details let us explain what does it mean that the Dirac equation is general covariant. Transformation φ of the local group of diffeomorphisms (group of general coordinate transformations) can be represented by the smooth functions

$$\varphi : x^i \Rightarrow \varphi^i(x), \quad \varphi^{-1} : x^i \Rightarrow f^i(x), \quad \varphi^i(f(x)) = x^i.$$

Induced transformation of the metric tensor is of the form

$$\tilde{g}_{ij}(x) = g_{kl}(f(x)) f_i^k(x) f_j^l(x),$$

where $f_i^k(x) = \partial_i f^k(x)$. For the scalar and vector fields we have

$$\tilde{\psi}(x) = \psi(f(x)), \quad \tilde{P}^i(x) = P^k(f(x)) \varphi_k^i(f(x)),$$

where $\varphi_k^i(x) = \partial_k \varphi^i(x)$. It is not difficult to verify that if $K^i(x)$ is a solution of the Killing equations for the metric $g_{ij}(x)$, then $\tilde{K}^i(x)$ is a solution of the Killing equations for the metric $\tilde{g}_{ij}(x)$. Further, if $\psi(x)$ is a solution of Dirac equation (36), then $\tilde{\psi}(x)$ will be a solution of Eq. (36) when $K_\mu^i(x)$ is substituted by the $\tilde{K}_\mu^i(x)$. Besides the transformations of the diffeomorphisms group conserve the form of the commutation relations of the Poincaré group. Dirac's equation is covariant with respect to the general coordinate transformations. It is known that in the Minkowski space–time there is preferred class of the coordinate systems. In the preferred system of coordinates Dirac equation (36) has a customary form.

It is also clear that Eq. (36) is equivalent to the equation

$$i\tilde{\gamma}^\mu \mathbf{P}_\mu \psi = \frac{mc}{\hbar} \psi,$$

if $\tilde{\gamma}^\mu = S\gamma^\mu S^{-1}$, where $S \in GL(4, \mathbf{C})$ (Dirac equation (36) is covariant with respect to the transformations of the group $GL(4, \mathbf{C})$).

Now we have found enough to provide some valuable insights into the connection between the space–time and internal transformations. Let us consider again the generators of the internal Lorentz group $S_{\mu\nu} = \frac{1}{4}(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$ and pay attention to the commutation relations:

$$[\gamma_\mu, S_{\nu\lambda}] = \eta_{\mu\nu}\gamma_\lambda - \eta_{\mu\lambda}\gamma_\nu. \quad (37)$$

Comparing (35) and (37) it is not difficult to verify that the operators

$$\mathbf{L}_{\mu\nu} = \mathbf{M}_{\mu\nu} + S_{\mu\nu}$$

commute with the Dirac operator $D = i\gamma^\mu \mathbf{P}_\mu$ and satisfy the commutation relations of the Poincaré group. Thus, in the Minkowski space–time there is a relation between the internal symmetry group and the space–time symmetry group. The consequence is that Dirac’s equation (36) is invariant with respect to the transformations of the Poincaré group. Thus, the geometrical and group-theoretical meaning of both spinor and original Dirac equation is quite clear. We see that structure of the Dirac equation is defined by the internal symmetry and the derivatives with respect to the given directions. In considered case these derivatives coincide with generators of the translation group. In this respect the Dirac equation differs radically from the Einstein equation, where internal symmetry has no role at all. The spinor enters into the world of tensors as four-component complex scalar field being a carrier of internal symmetry, which, thus, was discovered together with the Dirac equation.

Consider now the possible natural generalizations of the general covariant Dirac equation. We will strive to realize project when diffeomorphisms group is the group of invariance (not covariance) of generalized theory and internal symmetry remains without change. There is only one natural way to do this and it will be subject of our consideration in later sections.

7. GENERALIZATION OF THE DIRAC THEORY

In this section it is shown that a spinor field can be presented as a natural origin of the vacuum potential field, considered above.

We take that the canonical energy–momentum tensor plays fundamental role in the theory of the spinor fields and in accordance with this the generalized Dirac’s Lagrangian has the form

$$L_D = \frac{i}{2} E_\mu^i \left(\bar{\psi} \gamma^\mu D_i \psi - (D_i \bar{\psi}) \gamma^\mu \psi \right) - m \bar{\psi} \psi, \quad (38)$$

where E_μ^i are contravariant components of the potential of vacuum,

$$D_i\psi = (\partial_i - iqA_i)\psi, \quad D_i\bar{\psi} = (\partial_i + iqA_i)\bar{\psi}, \quad A_i = P_{ik}^k.$$

It is evident that varying (38) with respect to E_μ^i results in canonical energy-momentum tensor of the spinor field. Lagrangian (38) is invariant with respect to the substitutions

$$\psi \Rightarrow e^{i\varphi}\psi, \quad \bar{\psi} \Rightarrow e^{-i\varphi}\bar{\psi}, \quad A_i \Rightarrow A_i + \partial_i\varphi,$$

and hence it is general covariant and invariant with respect to the local transformation of the group G_i . Action has the form

$$A = \hbar \int L_D p d^4x,$$

where $p = \text{Det}(E_i^\mu)$. Since

$$E_\mu^i \partial_j E_i^\mu = \frac{1}{p} \partial_j p,$$

this action leads to the Dirac equations in the presence of external vacuum and electromagnetic fields

$$iE_\mu^i \gamma^\mu (D_i + \frac{1}{2}U_i)\psi = m\psi, \quad (39)$$

$$iE_\mu^i (D_i + \frac{1}{2}U_i)\bar{\psi}\gamma^\mu = -m\bar{\psi}, \quad (40)$$

where, as earlier, $U_i = U_{ik}^k$.

Setting

$$W_i^\mu = \frac{i}{2}(\bar{\psi}\gamma^\mu D_i\psi - (D_i\bar{\psi})\gamma^\mu\psi),$$

we have $L_D = P_\mu^i W_i^\mu - m\bar{\psi}\psi$. Hence, from the action

$$A = \hbar \int L_D p d^4x + \frac{c^3}{G} \int L_v \sqrt{g} d^4x, \quad g = -\text{Det}(g_{ij})$$

we verify (in accordance with (33)) the following equations for the potential of the vacuum field:

$$\nabla_j F_l^{jk} + B_{jm}^k F_l^{jm} - B_{jl}^m F_m^{jk} + l^2 W_l^k = 0, \quad (41)$$

where

$$W_l^k = \epsilon E_\mu^k W_l^\mu, \quad \epsilon = p/\sqrt{g}.$$

Equations (41) generalize Eq. (33) and together with Dirac equations (39) and (40) explain clearly how the vacuum field interacts with the spinor field. The potential of the generalized electromagnetic field enters into the Dirac Lagrangian only in the form of the trace of P_{jk}^i . The other possibility does not exist. From Eqs. (41) an interesting relation can be derived. By summing over the indices k and l we get that a trace of U_{jk}^i satisfies the following equation:

$$\nabla_i U^i = m\bar{\psi}\psi, \quad (42)$$

where $U^i = g^{ik}U_k$. We conclude that for $m = 0$ the interactions of the vacuum and spinor fields are characterized by a new conserved quantity. Indeed, this fact simply means that the action is invariant under the mapping

$$E_i^\mu \rightarrow aE_i^\mu, \quad \psi \rightarrow a^{-\frac{1}{2}}\psi,$$

where a is dimensionless constant. Thus, the introduction of the vacuum field into the framework of the standard model may shed new light on the mechanism of the lepton mass generation.

From the action

$$A = \beta^2 \hbar \int L_P \sqrt{g} d^4x + \hbar \int L_D p d^4x$$

we derive the equations of the generalized electromagnetic field (dark matter field) in interaction with the spinor field

$$\frac{1}{\sqrt{|g|}} D_i(\sqrt{|g|} H^{ik}) + \frac{q}{\beta^2} J^k E = \mu^2 T^j, \quad (43)$$

where

$$J^i = \epsilon E_\mu^i \bar{\psi} \gamma^\mu \psi$$

is the Dirac vector of current. Thus, the basic equations of the considered fields are derived.

Now it is important to exhibit the equations that follow from the equations derived. To this end we shall establish the identities for the Lagrangians of the fields in question. Identity for the Lagrangian of the vacuum field may be written as follows:

$$\partial_j L_v = \nabla_i S_j^i - B_{jk}^i S_i^k + \nabla^i (H_{il}^k H_{jk}^l), \quad (44)$$

where

$$S_j^i = \nabla_k F_j^{ki} + B_{kl}^i F_j^{kl} - B_{kj}^l F_l^{ki} = E_\mu^i \frac{\delta L_v}{\delta E_\mu^j}.$$

It is necessary to illuminate the important points under the derivation of identity (44). We have

$$\partial_j L_v = F_l^{ik} (\nabla_j E_\mu^l) \nabla_i E_k^\mu + F_l^{ik} E_\mu^l \nabla_j \nabla_i E_k^\mu.$$

With Ricci's identity

$$\nabla_j \nabla_i E_k^\mu = \nabla_i \nabla_j E_k^\mu - R_{jik}{}^l E_l^\mu$$

we can represent the second term in the right-hand side of the starting relation in the following form:

$$\nabla_i (F_l^{ik} E_\mu^l \nabla_j E_k^\mu) - (\nabla_i (F_l^{ik} E_\mu^l)) \nabla_j E_k^\mu - F_l^{ik} R_{jik}{}^l.$$

For the further transformations one needs to use identity

$$F_l^{ik} R_{jik}{}^l = \nabla_i \nabla_k F_j^{ik}.$$

For Dirac's Lagrangian one can derive the identity

$$\partial_j L_D = D_j \bar{\psi} \frac{\delta L_D}{\delta \bar{\psi}} - \frac{\delta L_D}{\delta \psi} D_j \bar{\psi} + \frac{1}{\epsilon} (\nabla_i W_j^i - B_{jk}^i W_i^k + e F_{ji} J^i).$$

From this identity it follows in accordance with (39) and (40) that the circulation of the energy of the spinning matter is defined by the equation

$$\nabla_i W_j^i - B_{jk}^i W_i^k + q F_{ji} J^i = 0, \quad (45)$$

when the electromagnetic and vacuum fields are present. The canonical energy-momentum tensor W_j^i of the spinning matter is not symmetric.

CONCLUSION

Here we suggest an experiment to test the formulated theory. It is suggested to measure the gravitational acceleration of electrons and positrons in the Earth gravitational field. The motivation is as follows.

In 1967 Witteborn and Fairbank measured the net vertical component of gravitational force on electrons in vacuum enclosed by a copper tube [5]. This force was shown to be less than $0.09 mg$, where m is the inertial mass of the electron and g is 980 cm/s^2 . They concluded that this result supports the contention that gravity induces an electric field outside a metal surface, of such magnitude and direction that the gravitational force on electrons is cancelled. If this is true, then the positrons will fall in this tube with the acceleration of $a = 2g$. The conclusion from the theory presented here is that electrons and positrons do not interact with the gravitational field directly but only through the vacuum field and electromagnetic channel. And the result presented by the measurements may be considered as an estimation for the energy of vacuum field generated by electron (and positron). Thus, the new measurements of the net vertical component of the force on positrons in vacuum enclosed by a copper tube will have the fundamental significance for understanding of the conceptual basis of contemporary theoretical physics and for the understanding of the nature of dark matter as well.

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