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EVOLUTION IN TIME OF MOVING UNSTABLE SYSTEMS

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Временна́я эволюция движущихся нестабильных систем

Релятивистская квантовая теория дает приближенное эйнштейновское замедление времени (ЭЗ) для закона распада нестабильной частицы с определенным импульсом. В этой работе используется другое определение движущейся частицы как состояния с определенной скоростью. Показано, что в этом случае распад не замедлен, а наоборот, ускорен по сравнению с распадом покоящейся частицы. ЭЗ не выполняется также для временно́й эволюции простой нестабильной системы типа осциллирующего нейтрино. Известны эксперименты, показывающие, что ЭЗ имеет место для движущихся мезонов. Используемая теория может объяснить этот факт в предположении, что измеряемые мезоны имеют определенный импульс.

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Evolution in Time of Moving Unstable Systems

Relativistic quantum theory shows that the known Einstein time dilation (ED) approximately holds for the decay law of the unstable particle having definite momentum p (DP). I use a different definition of the moving particle as the state with definite velocity v (DV). It is shown that in this case the decay law is not dilated. On the contrary, it is contracted as compared with the decay law of the particle at rest. It is demonstrated that ED fails in both DP and DV cases for time evolution of the simple unstable system of the kind of oscillating neutrino. Experiments are known which show that ED holds for mesons. The used theory may explain the fact by supposing that the measured mesons are in DP state.

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INTRODUCTION

Experimenters showed that the lifetime τ of a uniformly moving unstable particle is equal to $\tau_0\gamma$, where τ_0 is the lifetime of the particle at rest and γ is the Lorentz factor $\gamma = (1 - v^2/c^2)^{-1/2}$, e. g., see [1, 6]. In other words, if $F(t)$ is the decay law of the unstable particle moving in the laboratory frame and $F_0(t)$ is the decay law of the same particle at rest, then $F_0(t) = \exp(-t/\tau_0)$ and $F(t) = \exp(-t/\tau_0\gamma)$ or

$$F(t) = F_0(t/\gamma). \tag{1}$$

A usual explanation of the fact is based on the Einstein special theory of relativity and I call (1) the Einstein dilation (ED). For example, Møller [9] sets it forth as follows:

> «In view of the fact that an arbitrary physical system can be used as a clock, we see that any physical system which is moving relative to a system of inertia must have a slower course of development than the same system at rest. Consider for instance a radioactive process. The mean life τ of the radioactive substance, when moving with a velocity v, will thus be larger than the mean life τ_0 when the substance is at rest. From (2.36) we obtain immediately $\tau = (1 - v^2/c^2)^{-1/2}\tau_0$».

This argumentation may be complemented by the following possible definition of the unit of time provided by radioactive substance: this is the time interval during which the amount of the substance decreases twice.

However, the standard clocks of the relativity theory are used when obtaining Eq. (2.36) (from Ref. [9])

$$\Delta t = t_2 - t_1 = \gamma(t'_2 - t'_1) = (1 - v^2/c^2)^{-1/2}\Delta\tau$$

which Møller mentions. He begins the derivation of this equation with the phrase:

> «Consider a standard clock C' which is placed at rest in S' at a point on the x' axis with the coordinate $x' = x'_1$».

However, such a quantum clock as an unstable particle cannot be at rest (i. e., have zero velocity or zero momentum) and simultaneously be at a definite point (due to the quantum uncertainty relation). So the standard derivation of the moving clock dilation is inapplicable for the quantum clock.

Another way of theoretical derivation may be used: to find the relativistic quantum decay law $F(t)$ of the moving particle and to compare it with the decay law $F_0(t)$ of the particle at rest. Lorentz transformation of the space-time coordinates from one inertial frame to another is not needed as well as the space coordinates themselves. The approach was employed by Exner [5]; Stefanovich [12]; Khalfin [8]; Shirokov [11]. In these papers (below I shall refer to them as (ESKS)) the state of the moving unstable particle was described by the eigenvector Ψ_p of the momentum operator $\widehat{\vec{P}}$ (Exner [5] used a packet with almost exact momentum). One may state that the obtained decay law $F_p(t)$ is consistent with ED, Eq. (1), see Sec. 2 below. I use in Sec. 1 another definition of the moving particle: it is described by the vector Φ_v having a definite nonzero velocity $\vec{v}$. If the particle were stable, the vector Φ_v would coincide with Ψ_p at $\vec{p} = \vec{v}m_0\gamma$, m_0 being the particle mass. However, the unstable particle has no definite mass, it is described by a distribution over masses, see Sec. 1. Therefore, if $\vec{p}$ is definite, then $\vec{v}$ cannot be definite, see Eq. (12) below. The exclusion is the case $\vec{p} = 0$ when $\vec{v}$ is also zero.

In the case of unstable particles, whose decay laws can be measured one may expect that using either Φ_v or Ψ_p should give only slightly different results. Indeed, mass distributions of such particles are concentrated in small regions near average masses m, the dimension Γ of the regions being much less than m. However, the detailed calculation of $F_v(t)$ presented in Sec. 1 provides instead the unexpected result $F_v(t) = F_0(t\gamma)$, i. e., contraction instead of dilation: particles with exact nonzero velocity decay faster than the one at rest.

A simple unstable system is considered in Sec. 3. The oscillating neutrino may serve as an example. The usual formulae for the neutrino oscillation, e. g., see [2, 3], are valid when neutrino has definite momentum $\vec{p}$. The corresponding oscillation is dilated as compared to the oscillation of the neutrino with lesser momentum. However, the dilation is not Einsteinian, Eq. (1). In the case when neutrino has a definite velocity I obtain another formula for neutrino oscillation which gives the same contraction as in Sec. 1.

For summary and conclusion see Sec. 4.

1. DECAY LAW OF MOVING UNSTABLE PARTICLE WITH PRECISE VELOCITY

Let us consider a relativistic theory which describes unstable particles, products of their decay, and the corresponding interactions. A field theory may be an example. Such a theory must contain operators of total energy and momentum $\hat{H}$, $\widehat{\vec{P}}$ (the generators of time and space translations), total angular momentum, and generators of Lorentz boosts.

Suppose that at the initial moment $t = 0$ there is one unstable particle. Its state Φ_0 is defined as the eigenvector of the velocity operator $\widehat{\vec{V}}$ corresponding to eigenvalue being zero. The operator $\widehat{\vec{V}}$ is defined as $\widehat{\vec{V}} = \widehat{\vec{P}}/\hat{H}$. It commutes with $\hat{H}$ and, therefore, is conserved. Note that the total operators $\widehat{\vec{P}}$ and $\widehat{\vec{V}}$ are the unstable particle momentum and velocity as there are no other particles at the moment $t = 0$.

Let φ_μ be those eigenvectors of $\hat{H}$ which are simultaneously $\widehat{\vec{V}}$ eigenvectors with zero eigenvalue $\vec{v} = 0$. The corresponding $\hat{H}$ eigenvalues may be called masses and are denoted by μ: $\hat{H}\varphi_\mu = \mu\varphi_\mu$. Expand Φ_0 over φ_μ:

$$\Phi_0 = \int_\mu c(\mu)\varphi_\mu, \qquad c(\mu) = (\varphi_\mu, \Phi_0). \tag{2}$$

If the initial state of the unstable system under consideration is Φ_0, then later it is described by the vector $\Phi_0(t) = \exp(-iHt)\Phi_0$,

$$\Phi_0(t) = \int_\mu c(\mu)\mathrm{e}^{-i\mu t}\varphi_\mu. \tag{3}$$

Then the nondecay (survival) amplitude is

$$A_0(t) \equiv (\Phi_0, \Phi_0(t)) = \int_\mu |c(\mu)|^2 \mathrm{e}^{-i\mu t}. \tag{4}$$

The state Φ_0 is called unstable if $A_0(t) \to 0$ as $t \to \infty$. This property holds only if the convolution $\int_\mu$ in Eqs. (2)–(4) is integral over continual μ values. Besides, the spectrum of $\hat{H}$ must be bounded from below. So $\int_\mu$ may be understood as the integral $\int_0^\infty d\mu$.

The vectors φ_μ may be endowed with other indices (e. g., spin ones), upon which $\hat{H}$ eigenvalues do not depend. I do not write out these degeneration indices.

Let us define the initial state vector Φ_v of a moving unstable particle as $\Phi_v = L_v\Phi_0$, where L_v is the Lorentz transformation from the frame where the unstable particle is at rest to the frame where it has the velocity $\vec{v}$ (e. g., see [7]. Applying the operator L_v to both parts of Eq. (2) one obtains the expansion of Φ_v over the vectors $\varphi_{v\mu} \equiv L_v\varphi_\mu$:

$$\Phi_v = L_v\Phi_0 = \int d\mu\, c(\mu)\, \varphi_{v\mu}. \tag{5}$$

Let us show that $\varphi_{v\mu}$ is $\hat{H}$ eigenvector corresponding to the eigenvalue $\mu\gamma$, $\gamma = (1 - v^2)^{-1/2}$. Indeed, one has $L_v^{-1}HL_v = (\hat{H} - \vec{v}\cdot\widehat{\vec{P}})$ and, therefore,

$$HL_v\varphi_\mu = L_v(\hat{H} - \vec{v}\cdot\widehat{\vec{P}})\gamma\varphi_\mu = \gamma\mu L_v\varphi_\mu. \tag{6}$$

The equations $\hat{\vec{P}}\varphi_\mu = 0$ and $\hat{H}\varphi_\mu = \mu\varphi_\mu$ have been used.

Respectively, in place of Eqs. (3) and (4) one gets

$$\Phi_v(t) = \int d\mu\, c(\mu)\, \varphi_{v\mu}\, \exp(-i\mu\gamma t), \tag{7}$$

$$A_v(t) = \int d\mu\, |c(\mu)|^2\, \exp(-i\mu\gamma t). \tag{8}$$

Note. When calculating Eq. (7) the orthonormalization equation $(\varphi_{v\mu_1}, \varphi_{v\mu_2})$ = $\delta(\mu_1 - \mu_2)$ is used, implying unit normalization of $\hat{\vec{V}}$ eigenvectors. This is the case if $\hat{\vec{V}}$ has a discrete spectrum analogously to the spectrum the momentum has when the system is implied to be in a large space volume and usual periodicity conditions are imposed (or the volume opposite boundaries are identified).

Comparing Eq. (8) with Eq. (4) one obtains the following relation of survival amplitudes:

$$A_v(t) = A_0(\gamma t). \tag{9}$$

The same relation holds for the probabilities $F_v(t) = |A_v(t)|^2$ and $F_0(t) = |A_0(t)|^2$:

$$F_v(t) = F_0(\gamma t). \tag{10}$$

So one gets contraction instead of dilation, Eq. (1), if a moving unstable particle has a definite velocity. Note that Eq. (10) is the exact relation which does not depend upon the specific choice of $c(\mu)$.

In order to discuss this unexpected result I write out the corresponding survival amplitudes for particles with exact momentum.

2. DECAY LAW OF UNSTABLE PARTICLE WITH PRECISE MOMENTUM

Let us describe the moving unstable particle by an eigenvector Ψ_p of the momentum $\hat{\vec{P}}$. The corresponding decay law may be derived in the same way as in Sec. 1. I shall deal with a particular example of the eigenvector Ψ_p:

$$\Psi_p = \int d\mu\, c(\mu)\, \psi_{p\mu}, \qquad \psi_{p\mu} = L_{p\mu}\, \varphi_\mu \tag{11}$$

(other definitions are possible, e. g., when $c(\mu)$ depends upon $\vec{p}$). Here $L_{p\mu}$ denotes the operator of the Lorentz transformation of the rest state φ_μ into the frame where the velocity of the state is equal to $\vec{p}/\sqrt{p^2 + \mu^2}$, i. e., corresponds to the momentum $\vec{p}$. One may show that $\psi_{p\mu}$ is $\hat{\vec{P}}$ eigenvector with eigenvalue

$\vec{p}$. So is Ψ_p. Analogously, one may demonstrate that $\psi_{p\mu}$ is $\hat{H}$ eigenvector with eigenvalue $E_{p\mu} = \sqrt{p^2+\mu^2}$. The demonstrations are carried out in the same way as in Sec. 1 after Eq. (5). The value $\sqrt{p^2+\mu^2}$ for $E_{p\mu}$ has been obtained in another way in [11].

Let us stress that Ψ_p is not the eigenvector of the velocity operator $\hat{\vec{V}} = \hat{\vec{P}}/\hat{H}$. Indeed,

$$\hat{\vec{V}}\Psi_p = \int d\mu\, c(\mu)\, \hat{\vec{P}}/\hat{H}\psi_{p\mu} = \vec{p}\int d\mu\, c(\mu)\, (p^2+\mu^2)^{-1/2}\psi_{p\mu}. \qquad (12)$$

So the r.h.s. of (12) is not proportional to $\Psi_p = \int d\mu\, c(\mu)\, \psi_{p\mu}$.

Using Eq. (11) one obtains for survival amplitudes

$$A_p(t) \equiv \langle\Psi_p, \mathrm{e}^{-i\hat{H}t}\Psi_p\rangle = \int d\mu\, |c(\mu)|^2 \exp(-it\sqrt{p^2+\mu^2}), \qquad (13)$$

$$A_0(t) \equiv \langle\Psi_0, \mathrm{e}^{-i\hat{H}t}\Psi_0\rangle = \int d\mu\, |c(\mu)|^2 \exp(-i\mu t). \qquad (14)$$

Note that when $\vec{p} = 0$ the state $\Psi_p(t)$ coincides with $\Phi_p(t)$, see Eq. (3).

Now the survival law $A_p(t)$ is not connected with $A_0(t)$ by such a simple relation as $A_v(t)$ does, see Eq. (9). To compare $A_p(t)$ with $A_0(t)$, one has to calculate $A_p(t)$ and $A_0(t)$ separately. For this purpose one needs to know $|c(\mu)|^2$. The Breit–Wigner distribution

$$|c(\mu)|^2 = \frac{\Gamma}{2\pi}\left[(\mu-m)^2 + \Gamma^2/4\right]^{-1} \qquad (15)$$

has been used in (ESKS). Let us write out approximate expressions for $A_0(t)$ and $A_p(t)$ which are valid for time not too short and not too long when the decay laws are exponential [11]

$$A_0(t) \cong \exp(-imt - \frac{1}{2}\Gamma t), \qquad (16)$$

$$A_p(t) \cong \exp(-im\gamma_m t - \frac{1}{2}\Gamma t/\gamma_m), \qquad \gamma_m \equiv \sqrt{p^2+m^2}/m. \qquad (17)$$

Here m is the average (or the most probable) mass in distribution (15). It follows from Eqs. (16) and (17) that

$$|A_p(t)|^2 \cong |A_0(t/\gamma_m)|^2, \qquad (18)$$

i.e. ED holds for survival *probability* of unstable particle with precise momentum. Dilation (18) is to be juxtaposed to contraction (10). As was argued in Introduction, one may expect that the amplitudes $A_v(t)$ and $A_p(t)$ should not differ appreciably. Let us show that this expectation is realized in a sense. Note beforehand that Eqs. (16) and (9) result in the explicit approximate expression for $A_v(t)$ when $\vec{v} = \vec{p}/\sqrt{p^2+m^2}$

$$A_v(t) \cong \exp(-im\gamma t - \frac{1}{2}\Gamma t/\gamma). \tag{19}$$

Let us compare the exponents E_p and E_v of the corresponding exponentials in Eqs. (17) and (19)

$$E_p = -imt\gamma_m - \frac{1}{2}\Gamma t/\gamma_m, \qquad E_v = -imt\gamma - \frac{1}{2}\Gamma t\gamma, \tag{20}$$

assuming that $\gamma_m = \gamma$. As $\Gamma \ll m$ the exponents coincide in the zero approximation when the terms $\sim \Gamma$ are neglected in E_p and E_v. So in this approximation the corresponding *amplitudes* $A_p(t)$ and $A_v(t)$ coincide and they both satisfy the contraction property

$$A_p(t) \cong A_v(t) \cong A_0(t\gamma). \tag{21}$$

However, the main terms of E_p and E_v are purely imaginary and do not contribute to modules of $A_p(t)$ and $A_v(t)$. It is real parts $\frac{1}{2}\Gamma t/\gamma_m$ and $\frac{1}{2}\Gamma t\gamma$ which do contribute and determine the different dependencies of $|A_p|^2$ and $|A_v|^2$ upon t, see Eqs. (18) and (10).

In the next section I will consider a simple unstable system whose time evolution is determined by the interference of the main terms defined above (the terms $\sim \Gamma$ being absent). For this system one may expect the breakdown of ED (in view of Eq. (21)) even if the system has precise momentum.

3. TIME EVOLUTION OF MOVING TWO-MASS SYSTEM

Let us consider unstable system at rest whose state vector ϕ is a superposition of two $\hat{H}$ eigenvectors φ_1 and φ_2:

$$\phi = c_1\varphi_1 + c_2\varphi_2, \qquad |c_1|^2 + |c_2|^2 = 1, \tag{22}$$

$$\hat{H}\varphi_1 = m_1\varphi_1, \qquad \hat{H}\varphi_2 = m_2\varphi_2, \qquad m_1 \neq m_2. \tag{23}$$

The system time evolution is described by the survival amplitude

$$A_0(t) \equiv (\phi, \phi(t)) = |c_1|^2 \mathrm{e}^{-im_1 t} + |c_2|^2 \mathrm{e}^{-im_2 t}. \tag{24}$$

The survival amplitudes of the system with nonzero exact velocity $\vec{v}$ and exact momentum $\vec{p}$ are

$$A_v(t) = |c_1|^2 \exp(-im_1 t\gamma) + |c_2|^2 \exp(-im_2 t\gamma), \tag{25}$$

$$A_p(t) = |c_1|^2 \exp(-it\sqrt{p^2+m_1^2}) + |c_2|^2 \exp(-it\sqrt{p^2+m_2^2}), \tag{26}$$

respectively, cf. Eqs. (8) and (13).

As examples of such a system one may take electron neutrino, e. g., see [2, 3] and K_0 meson

$$|K_0\rangle = (|K_s\rangle + |K_l\rangle)/\sqrt{2},$$

provided that $\Gamma_s = \Gamma_l = 0$ (see [10]).

There exist different approaches to neutrino oscillations (see [4] and references therein). However, I am not aware of papers which consider moving neutrinos as eigenvectors of the velocity.

In what follows, I let $c_1 = c_2 = 1/\sqrt{2}$. Then

$$|A_0(t)|^2 = \cos^2\left[\frac{1}{2}(m_1 - m_2)t\right], \tag{27}$$

$$|A_v(t)|^2 = \cos^2\left[\frac{1}{2}(m_1 - m_2)t\gamma\right] = |A_0(t\gamma)|^2, \tag{28}$$

$$|A_p(t)|^2 = \cos^2\left[\frac{1}{2}\left(\sqrt{p^2+m_1^2} - \sqrt{p^2+m_2^2}\right)t\right] = |A_0(t/\tilde{\gamma})|^2,$$

$$\tilde{\gamma} = (\sqrt{p^2+m_1^2} + \sqrt{p^2+m_2^2})/(m_1+m_2). \tag{29}$$

The oscillatory behavior of these probabilities allows us to use the two-mass system as the quantum clock. Its unit of time may be defined as the period of oscillation (the oscillation frequency being equal to $m_1 - m_2$ in the case of $|A_0(t)|^2$), cf. with the definition of the unit of time provided by radioactive substance, see Introduction.

It follows from (27) and (29) that the time evolution $|A_p(t)|^2$ in the case of exact momentum is dilated as compared to $|A_0(t)|^2$, but the dilation is not Einsteinian if $m_1 \neq m_2$: $\tilde{\gamma}$ turns into the Lorentz factor only if $m_1 \cong m_2$. In the case of exact velocity we have the same contraction as for unstable particles, cf. Eqs. (27) and (28).

CONCLUSION

Relativistic quantum-mechanical derivation of the time evolution of moving unstable particles was considered in the papers (EKSS). There the state of the moving particle was defined as the eigenvector Ψ_p of the momentum operator $\widehat{\vec{P}}$ with eigenvalue $\vec{p}$. It was shown that then the nondecay law satisfied approximately ED, Eq. (1). Here in Sec. 1 the moving particle is described by eigenvector Φ_v of the velocity operator $\widehat{\vec{V}} = \widehat{\vec{P}}/\hat{H}$. If the particle were stable, then Φ_v would coincide with Ψ_p if $\vec{p} = \vec{v}m_0\gamma$. The vectors do not coincide in the case of unstable particle, but one may expect that they should give only slightly different results. It was shown in Sec. 2 that Φ_v and Ψ_p give indeed the same nondecay *amplitude* in the zero approximation. However, the approximation does not contribute to the corresponding *probability*, i. e., the nondecay law $F(t)$. As a result, the laws $F_v(t)$ and $F_p(t)$ turn out to be strongly different: $F_v(t)$ is contracted as compared to the nondecay law $F_0(t)$ of the particle at rest, see Eq. (10), meanwhile $F_p(t)$ is dilated, see Eq. (18).

Section 3 deals with unstable systems which are simpler than unstable particles. Oscillating neutrino may be the example. If it has exact momentum, then a dilation follows, but it is not ED, Eq. (1). In the case of exact velocity I obtain the formula which differs from the expressions for the neutrino oscillation known from the literature. It leads to the contraction, see Eq. (28).

I conclude that relativistic quantum theory of the time evolution of moving unstable systems does not ensure ED. The theory allows the possibility of moving unstable systems whose time evolution breaks ED.

Experiments are known which show that moving mesons have longer lifetimes than the immovable ones so that ED holds [1, 6]. The theory used here may explain this fact supposing that the experiments deal with mesons which are in states close to Ψ_p. In this case, the theory approximately gives ED.

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REFERENCES

1. *Bailey J. et al.* Measurement of relativistic time dilation for positive and negative muons in a circular orbit. // Nature 1977. V. 268. P. 301–304.

2. *Bilenky S. M., Pontecorvo B.* Lepton mixing and neutrino oscillations. // Phys. Rep. 1978. V. 41, No. 4. Sec. 3. P. 226–261.

3. *Bilenky S. M.* Neutrinos: a brief review. // Mod. Phys. Let. A. 2004. V. 19, No. 33. Sec. 3. P. 2451–2477.

4. *Dolgov A. D., Okun L. B., Rotaev M. V., Schepkin M. G.* Oscillation of neutrinos produced by a beam of electrons. arXiv: hep-ph/0407189, 2004.

5. *Exner P.* Representations of the Poincaré group associated with unstable particles. // Phys. Rev. D. 1983. V. 28, No. 10. P. 2621–2627.

6. *Farley F.* The CERN (g-2) measurements. // Zeitschr. Phys. C. 1992. V. 56. Sec. 5. P. S88.

7. *Gasiorowicz S.* Elementary particle physics. N. Y.: John Wiley, 1966. Ch. 4.

8. *Khalfin L. A.* Quantum theory of unstable particles and relativity. http://www.pdmi.ras.ru/preprint/1997/97-06.html.

9. *Møller C.* The Theory of Relativity. Oxford: Clarendon Press, 1972. Ch. 2.6.

10. *Perkins D.* Introduction to High Energy Physics. Addison-Wesley Co., Inc., 1987. Ch. 7.14.

11. *Shirokov M. I.* Decay law of moving unstable particle. // Int. J. Theor. Phys. 2004. V. 43, No. 6. P. 1541–1553.

12. *Stefanovich E.* Quantum effects in relativistic decays. // Int. J. Theor. Phys. 1996. V. 35, No. 12. P. 2539–2554.

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