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ABOUT ONE CONTRADICTION IN CLASSICAL ELECTRODYNAMICS

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Об одном противоречии в классической электродинамике

Рассматривается стандартная задача классической электродинамики, излучение, поглощение и интерпретация сопутствующих эффектов. Рассматривается движение заряженной частицы с постоянной скоростью. Внешняя сила меняет скорость движения. Изменение энергии системы равняется работе внешней силы. Из этого очевидного равенства получается нефизический результат: энергия электромагнитного поля движущейся с постоянной скоростью заряженной частицы не зависит от скорости движения.

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About One Contradiction in Classical Electrodynamics

A standard problem is considered which arises in classical electrodynamics when the processes of energy absorption and its radiation, and accompanying effects are studied and interpreted. It primarily concerns the processes of energy propagation in electromagnetic fields of charged particles moving in various regimes.

We consider two identical isolated systems; in each one we have a charged particle moving at a constant velocity. One particle's velocity is changed by an external force. Thus the difference between the energies of the considered systems is equal to the external force's work. From this obvious equality we have obtained a nonphysical result: the energy of the electromagnetic field of the charged particle moving at a constant velocity does not depend on the velocity of movement.

The investigation has been performed at the Dzhelepov Laboratory of Nuclear Problems, JINR.

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INTRODUCTION

During the last years an interest to Maxwell electrodynamics principles, to results arising from them, and to their possible interpretation [1–3] has been renewed. First of all this interest is related with transfer processes of electromagnetic field energy under conditions of different regimes [4] of charged particles' movement. Of great interest are also the processes of energy absorption and its radiation in a strong electromagnetic field, the investigation and interpretation of accompanying effects [5], nonstationary interference of electromagnetic waves [6], etc. Main attention in the cited works is focused on the study of localization and propagation of electromagnetic field energy. In addition, in a part of the discussed problems the effects of so-called «tachyons» [7–9] take place, i. e. exchange of energy between sources located in points with space-like interval between them, dislocation of interference maximums' with velocity higher than the speed of light. Some authors noted the misbalance in an energy-momentum 4-vector during calculation of its variations caused by radiation reaction force, etc. A number of authors see the roots of these difficulties in the existence of singularities of point sources (or, generally, central-symmetric fields), which complicates correct analysis of obtained results and realistic conclusion-making process.

In the present article two new contradictions are found in the framework of classical electrodynamics. While deriving the first result we used the quantity of field energy (but not its explicit form) localized in the vicinity of the point charge, and that's why this contradiction may be explained by the causes mentioned above. The second result appears in the area of weak fields and therefore its explanation requires different ideas.

Two systems are discussed in the framework of classical electrodynamics. In the first system a charge moves at a constant velocity υ_1, the second system is identical to the first one until moment t'; then work A is done on it, causing change of the velocity of a charge from υ_1 to υ_2 ($\upsilon_1 || \upsilon_2$). Based on the energy conservation law it is derived that starting from moment $t = -\infty$ the energy of the electromagnetic field of a charge moving at a constant velocity does not depend on the velocity of movement.

1. ENERGY OF LAYER

Let us consider a linear but nonuniform movement of a charge (Fig. 1). Fields radiated by the charge during its movement along AB interval are being localized for moment t in an eccentric spherical layer between *AD* and *BC* spherical surfaces. Calculation of the energy localized in this layer gives the following (see Appendix A, formula (28)):

$$W_{t',t'+\Delta t';t} = \frac{2e^2}{3c} \int\limits_{t'}^{t'+\Delta t'} \frac{\dot{\beta}^2 dt_1}{(1-\beta^2)^3} + \frac{e^2}{6c} \left[\frac{3+\beta^2}{1-\beta^2} \frac{1}{t-t_1} \right]_{t_1=t'}^{t_1=t'+\Delta t'}, \quad (1)$$

where $\beta = \upsilon_x(t_1)/c$;

$$BC = c\,(t-t'-\Delta t'), AD = c\,(t-t').$$

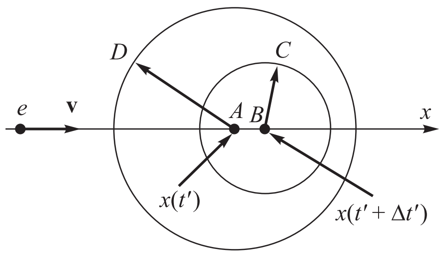

Fig. 1

From formula (1) for a particle moving at a constant velocity we obtain

$$W_{t',\,t'+\Delta t';t}\,(\upsilon) = \frac{e^2}{6c} \frac{3+\beta^2}{1-\beta^2} \left(\frac{1}{t-t'-\Delta t'} - \frac{1}{t-t'} \right). \quad (2)$$

2. WORK OF RADIATION REACTION FORCE

A charge moving with acceleration is impacted by radiation reaction force of its own field. The work of this force during period of time $(t',\ t'+\Delta t')$ is calculated by means of the following formula (see Appendix B, formula (37)):

$$A_{\mathrm{RRF}} = -\frac{2e^2}{3c} \int\limits_{t'}^{t'+\Delta t'} \frac{\dot{\beta}^2 dt_1}{(1-\beta^2)^3} + \left[\frac{\beta\dot{\beta}}{(1-\beta^2)^2} \right]_{t_1=t'}^{t_1=t'+\Delta t'} \frac{2e^2}{3c}. \quad (3)$$

Here again $\beta = \dfrac{\upsilon_x(t_1)}{c}$ (motion of the particle is linear).

3. LAW OF ENERGY CONSERVATION

1) Let us consider two systems:

I — a charge e moves at conserved velocity v_1 from $t = -\infty$ up to observation moment t;

II — a charge e moves at velocity v_1 from $t = -\infty$ up to t' moment, $t' < t$; then during $(t', t' + \Delta t')$ time interval, $t' + \Delta t' < t$, it undergoes work A as a result of which its velocity becomes v_2. From $t' + \Delta t'$ up to the moment of observation t the particle moves at constant velocity v_2. In accordance with the law of energy conservation,

$$E_{\mathrm{II}} - E_{\mathrm{I}} = A. \tag{4}$$

On the other hand, the work done on system II consists of the work done against the radiation reaction force and the work for the change of kinetic energy of the particle:

$$A = -A_{\mathrm{RRF}} + \frac{m_0 c^2}{\sqrt{1-\beta_2^2}} - \frac{m_0 c^2}{\sqrt{1-\beta_1^2}}. \tag{5}$$

Here $\beta_1 = v_{1x}/c$, $\beta_2 = v_{2x}/c$.

2) Let us calculate E_{II} and E_{I} energies in formula (4) (Fig. 2, a and 2, b). In these figures $AD = c(t - t')$, $BC = c(t - t' - \Delta t')$, $W_{t_1,t_2;t_3}(v)$ is the energy of fields radiated by the particle moving at a *constant velocity* v during the time interval $(t_1,\ t_2)$, the localization of which is being observed for the t_3 moment. $W_{t',\,t'+\Delta t';\,t}$ is the energy of fields radiated by the charge moving at a *variable velocity* during the time interval (t', $t' + \Delta t'$), whose localization is being observed for the moment t.

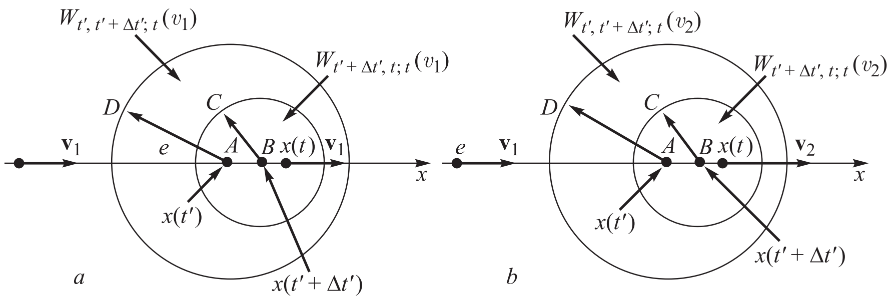

Fig. 2

From Fig. 2, a and 2, b we can obtain

$$E_{\rm II} - E_{\rm I} = W_{t',t'+\Delta t;t} + W_{t'+\Delta t',t;t}(\upsilon_2) + \frac{m_0c^2}{\sqrt{1-\beta_2^2}} - \\ - W_{t',t'+\Delta t';t}(\upsilon_1) - W_{t'+\Delta t',t;t}(\upsilon_1) - \frac{m_0c^2}{\sqrt{1-\beta_1^2}}. \quad (6)$$

3) Let us insert formulae (5) and (6) into formula (4):

$$W_{t',t'+\Delta t';t} + W_{t'+\Delta t',t;t}(\upsilon_2) - W_{t',t'+\Delta t';t}(\upsilon_1) - W_{t'+\Delta t',t't}(\upsilon_1) = -A_{\rm RRF}.$$

With account of formulae (1) and (3) the last equation turns to the following:

$$\frac{2c^2}{3c}\int\limits_{t'}^{t'+\Delta t'}\frac{\dot\beta^2 dt_1}{(1-\beta^2)^3} + \frac{e^2}{6c}\left[\frac{3+\beta^2}{1-\beta^2}\frac{1}{t-t_1}\right]_{t_1=t'}^{t_1=t'+\Delta t'} + W_{t'+\Delta t',t;t}(\upsilon_2) - \\ - W_{t',t'+\Delta t';t}(\upsilon_1) - W_{t'+\Delta t',t;t}(\upsilon_1) = \\ = \frac{2e^2}{3c}\int\limits_{t'}^{t'+\Delta t'}\frac{\dot\beta^2 dt_1}{(1-\beta^2)^3} - \frac{2e^2}{3c}\left[\frac{\beta\dot\beta}{(1-\beta^2)^2}\right]_{t_1=t'}^{t_1=t'+\Delta t'}. \quad (7)$$

The integrals in both sides of equation (7) are annihilated, and the second summand on the left side of the equation can be rewritten, with the use of formula (2), in the following way:

$$\frac{e^2}{6c}\left[\frac{3+\beta^2}{1-\beta^2}\frac{1}{t-t_1}\right]_{t_1=t'}^{t_1=t'+\Delta t'} = \frac{e^2}{6c}\frac{3+\beta_2^2}{1-\beta_2^2}\frac{1}{t-t'-\Delta t'} - \\ - \frac{e^2}{6c}\frac{3+\beta_1^2}{1-\beta_1^2}\frac{1}{t-t'} = W_{-\infty,t'+\Delta t',t}(\upsilon_2) - W_{-\infty,t';t}(\upsilon_1). \quad (8)$$

Inserting the obtained expression into formula (7) we get the following:

$$W_{-\infty,t;t}(\upsilon_2) - W_{-\infty,t;t}(\upsilon_1) = -\frac{2e^2}{3c}\left[\frac{\beta\dot\beta}{(1-\beta^2)^2}\right]_{t_1=t'}^{t_1=t'+\Delta t'}. \quad (9)$$

If the velocity of a particle is changing smoothly, then $\dot\beta(t') = \dot\beta(t'+\Delta t') = 0$ and it follows from equation (9) that for the time interval $(-\infty, t)$ the energy of the electromagnetic field induced by a charge moving at a constant velocity does not depend on the velocity of the charge's movement.

CONCLUSIONS

The two main results obtained in the paper are the following:

1) The energy of the electromagnetic field induced by a charge moving a constant velocity does not depend on the velocity of the charge's movement.

2) According to formula (2) the energy of the electromagnetic field induced by a charge moving along (A, B) section (Fig. 1) at a constant velocity and localized within the asymmetric layer formed by spheres with CB and AD radii depends on the observation moment t (it decreases as t^{-2}). It is unclear where this energy disappears, because the field and, therefore, its energy outside the layer is unambiguously determined by motion of the charge beyond the (A, B) section (by Lienar–Wiechert potentials).

The aforementioned results are based on the standard idea that the electromagnetic field's energy and momentum in a volume enclosed by some surface is calculated by integrating the densities of these quantities in the considered area. But it is known that Lorentz transformation for energy and momentum densities generally includes the densities of flows of energy and momentum [10]. If these flows are not equal to zero, the quantities obtained by integrating energy and momentum densities do not have proper transformational qualities. That is why these quantities cannot be considered as the energy and momentum of the field localized in some space area. It means that if we cut some area from space filled by electromagnetic field, we cannot consider it as an independent physical object because it does not satisfy Maxwell equations and its energy and momentum do not have proper transformational qualities.

We think that the reason for the appearance of nonphysical results when we consider complicated nonstationary processes and try to reach energy–momentum balance by traditional methods is that we just do not take into account the circumstances mentioned above.

APPENDIX A. ENERGY OF LAYER

1) The field of a moving charge is described by potentials:

$$\mathbf{E}(\mathbf{r}, t) = \frac{e(1-\beta^2)}{[R - (\mathbf{R}\boldsymbol{\beta})]^3}(\mathbf{R} - \boldsymbol{\beta}R) + \frac{e}{[R - (\mathbf{R}\boldsymbol{\beta})]^3}\left[\mathbf{R}, \left[(\mathbf{R} - \boldsymbol{\beta}R), \frac{\boldsymbol{\beta}}{c}\right]\right], \quad (10)$$

$$\mathbf{H}(\mathbf{r}, t) = \frac{1}{R}[\mathbf{R}, \mathbf{E}]. \quad (11)$$

In formulae (10) and (11) **[a, b]** is a vector product of **a** and **b** vectors, the values of **E** and **H** are $M(\mathbf{r})$ defined in some point (Fig. 3) at moment t, and the values of $\boldsymbol{\beta}$ and $\mathbf{R}$ in the right-hand parts of the formulae are given for the moment t' which is defined by the following relation:

$$t' + \frac{R(t')}{c} = t. \tag{12}$$

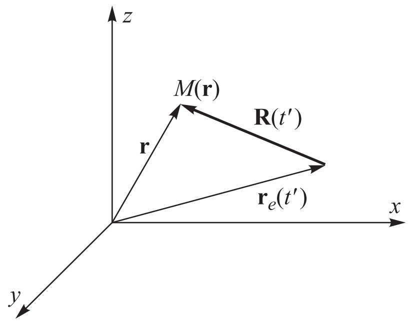

Fig. 3

2) Let us discuss the case of a charge moving along the *Ox* axis (Fig. 4). Formulae (10) and (11) are rewritten in the following way:

$$\mathbf{E}(\mathbf{r},t) = \frac{e}{(1-\beta\cos\alpha)^3}\left\{\frac{1-\beta^2}{R^2}\left(\hat{\mathbf{R}} - \hat{\mathbf{x}}\beta\right) + \frac{1}{R}\frac{\dot{\beta}}{c}\left(\hat{\mathbf{R}}\cos\alpha - \hat{\mathbf{x}}\right)\right\}, \tag{13}$$

$$\mathbf{H}(\mathbf{r},t) = \frac{e}{(1-\beta\cos\alpha)^3}[\hat{\mathbf{x}},\hat{\mathbf{R}}]\left[\frac{1-\beta^2}{R^2}\beta + \frac{1}{R}\frac{\dot{\beta}}{c}\right], \tag{14}$$

where $\beta = \dfrac{v_x}{c}$, $\hat{\mathbf{x}} = \dfrac{\mathbf{x}}{|\mathbf{x}|}$, $\hat{\mathbf{R}} = \dfrac{\mathbf{R}}{|\mathbf{R}|}$ (Fig. 4).

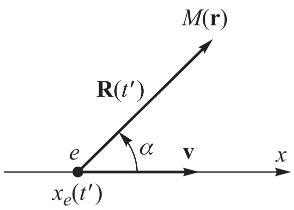

Fig. 4

Let us calculate the density of field energy:

$$w = \frac{E^2 + H^2}{8\pi} = \frac{e^2}{8\pi(1 - \beta\cos\alpha)^6}\times \\ \times\left\{\frac{(1-\beta^2)^2}{R^4}(1 + 2\beta^2 - 2\beta\cos\alpha - \beta^2\cos^2\alpha) + \right. \\ \left. + 2\sin^2\alpha\left(\frac{1}{R^2}\frac{\dot\beta^2}{c^2} + 2\frac{1-\beta^2}{R^3}\frac{\dot\beta}{c}\beta\right)\right\}. \quad (15)$$

3) Let us assume that the particle at the moment t' was in point A (Fig. 5) and at the moment $t' + \Delta t'$ was in point P. For the moment t $(t > t' + \Delta t' > t')$

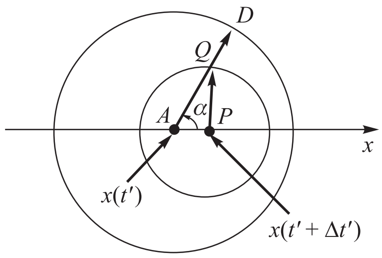

Fig. 5

during the motion on *AP* section the field radiated by a charge is localized in a layer formed by spherical surfaces with *AD* and *PQ* radii. Let us calculate the volume element of this layer. According to formula (12),

$$AD = c(t - t') = R(t'), \quad (16)$$

$$PQ = c(t - t' - dt') = R + dR, \quad (17)$$

$$\text{i.e.}\quad dR = -c dt', \quad AP = v dt'. \quad (18)$$

Let us consider the triangle ΔAPQ. According to cosine theorem,

$$PQ^2 = AQ^2 + AP^2 - 2AQ\,AP\cos\alpha. \quad (19)$$

Inserting formulae (16) and (18) into Eq. (19) gives the following:

$$c^2(t - t' - dt')^2 = AQ^2 + v^2 dt'^2 - 2\,AQ\,v dt'\,\cos\alpha.$$

Let us solve this equation relative to *AQ* (in the first approximation to *dt'*):

$$AQ = c(t - t') - (c - v\cos\alpha)dt'.$$

Therefore,

$$QD = (c - v\cos\alpha)dt'. \tag{20}$$

Let us transfer to spherical coordinates and measure radius R from point A (Fig. 5). Then we obtain

$$dV = (c - \cos\alpha)dt'\, R^2 \sin\alpha\, d\alpha\, d\varphi = -(1 - \beta\cos\alpha)dR\, R^2 \sin\alpha\, d\alpha\, d\varphi. \tag{21}$$

Formula (17) was used in deriving the last equality.

4) Let us use formula (15) for the energy density and calculate the amount of energy localized in an infinitely thin layer drawn on Fig. 5:

$$dW = -\int\limits_0^{\pi}\int\limits_0^{2\pi} w(\mathbf{r},t)(1-\beta\cos\alpha)R^2 dR \sin\alpha\, d\alpha\, d\varphi =$$

$$= -\frac{e^2}{4}dR\left\{\frac{(1-\beta^2)^2}{R^2}\int\limits_0^{\pi}\frac{(1+2\beta^2-2\beta\cos\alpha-\beta^2\cos^2\alpha)\sin\alpha\, d\alpha}{(1-\beta\cos\alpha)^5}+\right.$$

$$\left.+\left(2\frac{\dot\beta^2}{c^2}+4\frac{1-\beta^2}{R}\frac{\dot\beta}{c}\beta\right)\int\limits_0^{\pi}\frac{\sin^3\alpha\, d\alpha}{(1-\beta\cos\alpha)^5}\right\}. \tag{22}$$

The calculation of integrals in this expression gives the following:

$$I_1 = \int\limits_0^{\pi}\frac{\sin\alpha\, d\alpha}{(1-\beta\cos\alpha)^5} = \frac{2(1+\beta^2)}{(1-\beta^2)^4}, \tag{23}$$

$$I_2 = \int\limits_0^{\pi}\frac{\sin\alpha\, d\alpha}{(1-\beta\cos\alpha)^4} = \frac{2}{3}\frac{3+\beta^2}{(1-\beta^2)^3}, \tag{24}$$

$$I_3 = \int\limits_0^{\pi}\frac{\sin\alpha\, d\alpha}{(1-\beta\cos\alpha)^3} = \frac{2}{(1-\beta^2)^2}, \tag{25}$$

$$I_4 = \int\limits_0^{\pi}\frac{\sin^3\alpha\, d\alpha}{(1-\beta\cos\alpha)^5} = \frac{\beta^2-1}{\beta^2}I_1 + \frac{2}{\beta^2}I_2 - \frac{1}{\beta^2}I_3 = \frac{4}{3}\frac{1}{(1-\beta^2)^3}. \tag{26}$$

After inserting the obtained results into (22) we get the expression

$$dW = -\frac{e^2}{4}dR\left\{\frac{(1-\beta^2)^2}{R^2}(2\beta^2 I_2 + I_3) + \left(2\frac{\dot\beta^2}{c^2}+4\frac{1-\beta^2}{R}\frac{\dot\beta}{c}\beta\right) I_4\right\} =$$

$$= -\frac{e^2}{6}dR\left\{\frac{3+\beta^2}{R^2(1-\beta^2)} + \left(\frac{\dot\beta^2}{c^2}+2\frac{1-\beta^2}{R}\frac{\dot\beta}{c}\beta\right)\frac{4}{(1-\beta^2)^3}\right\}. \tag{27}$$

5) Let us calculate the energy ΔW (Fig. 1) localized in a layer of finite thickness. Let us remember that $dR = -cdt'$, $R = c(t-t')$, and β is taken for the moment t'. Integration of expression (27) from the t' to $t' + \Delta t'$ moment gives the following:

$$\Delta W = \frac{e^2}{6} \int\limits_{t'}^{t'+\Delta t'} \left[\frac{3+\beta^2}{1-\beta^2} \frac{1}{c^2(t-t')^2} + \frac{4\dot{\beta}^2}{c^2(1-\beta^2)^3} + \frac{8}{(1-\beta^2)^2} \frac{\dot{\beta}}{c} \times \right.$$

$$\left. \times \beta \frac{1}{c(t-t')} \right] cdt' = \frac{2e^2}{3c} \int\limits_{t'}^{t'+\Delta t'} \frac{\dot{\beta}^2}{(1-\beta^2)^3} dt_1 + \frac{e^2}{6c} \left[\frac{3+\beta^2}{1-\beta^2} \frac{1}{t-t_1} \right]_{t_1=t'}^{t_1=t'+\Delta t'} . \tag{28}$$

APPENDIX B. WORK OF RADIATION REACTION FORCE

1) Four-vector of radiation reaction force acting on a nonuniformly moving charge is given by the following expression (Landau, Lifshitz, vol.2, §76 (76.2)):

$$g^i = \frac{2e^2}{3c} \left(\frac{d^2u^i}{ds^2} - u^i u^k \frac{d^2 u_k}{ds^2} \right), \tag{29}$$

where $u^i = \left(\frac{1}{\sqrt{1-\beta^2}}, \frac{\boldsymbol{\beta}}{\sqrt{1-\beta^2}} \right)$, $ds = c\sqrt{1-\beta^2}dt$, $i,\ k = 0, 1, 2, 3$.

Component g^0 of this four-vector has the meaning of the work done by the force acting on the charge during a unit of time — a power of radiation reaction force. Let us write the equation of motion in covariant form:

$$g^i = mc \frac{du^i}{ds} = \frac{d}{ds} p^i. \tag{30}$$

Here $p^i = \left(\frac{\boldsymbol{\varepsilon}}{c}, \mathbf{p} \right) = \left(\frac{m_0 c}{\sqrt{1-\beta^2}}, \frac{m\mathbf{v}}{\sqrt{1-\beta^2}} \right)$ is the four-vector of momentum, ε and $\mathbf{p}$ are relativistic energy and three-dimensional momentum, respectively. Taking into account these notations, from equation (30) we can obtain that

$$\mathbf{g} = \frac{d}{ds}\mathbf{p} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{d}{dt} \frac{m_0 \mathbf{v}}{\sqrt{1-\beta^2}} = \frac{m\boldsymbol{\beta}(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})}{(1-\beta^2)^2} + \frac{m\dot{\boldsymbol{\beta}}}{1-\beta^2} = \frac{1}{\sqrt{1-\beta^2}} \frac{\mathbf{F}}{c},$$

$$g^0 = \frac{d}{ds} \frac{\boldsymbol{\varepsilon}}{c} = \frac{1}{\sqrt{1-\beta^2}} \frac{1}{c} \frac{d}{dt} \frac{m_0 c}{\sqrt{1-\beta^2}} = \frac{m(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})}{(1-\beta^2)^2} = (\mathbf{g}\boldsymbol{\beta}) = \frac{1}{\sqrt{1-\beta^2}} \frac{(\mathbf{F}\boldsymbol{\beta})}{c}, \tag{31}$$

where $\mathbf{F}$ is the vector of three-dimensional force acting on the particle.

2) It is easy to obtain that

$$\frac{du^i}{ds} = \frac{1}{c}\left(\frac{(\boldsymbol{\beta},\dot{\boldsymbol{\beta}})}{(1-\beta^2)^2}, \frac{\dot{\boldsymbol{\beta}}}{1-\beta^2} + \frac{\boldsymbol{\beta}(\boldsymbol{\beta},\dot{\boldsymbol{\beta}})}{(1-\beta^2)^2}\right), \tag{32}$$

$$\frac{d^2u^i}{ds^2} = \frac{1}{c^2}\left[\frac{\dot{\beta}^2 + (\boldsymbol{\beta}\ddot{\boldsymbol{\beta}})}{(1-\beta^2)^{5/2}} + \frac{4(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})^2}{(1-\beta^2)^{7/2}},\right.$$
$$\left.\frac{\ddot{\boldsymbol{\beta}}}{(1-\beta^2)^{3/2}} + \frac{3\dot{\boldsymbol{\beta}}(\boldsymbol{\beta},\dot{\boldsymbol{\beta}}) + \boldsymbol{\beta}\dot{\beta}^2 + \boldsymbol{\beta}(\boldsymbol{\beta}\ddot{\boldsymbol{\beta}})}{(1-\beta^2)^{5/2}} + \frac{4\boldsymbol{\beta}(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})^2}{(1-\beta^2)^{7/2}}\right], \tag{33}$$

$$u^i\frac{d^2u_i}{ds^2} = u_i\frac{d^2u^i}{ds^2} = \frac{\dot{\beta}^2}{c^2(1-\beta^2)^2} + \frac{(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})^2}{(1-\beta^2)^3}. \tag{34}$$

Let us insert expressions (33) and (34) into formula (29), and after some simplifications we obtain

$$g^0 = \frac{2e^2}{3c}\left[\frac{(\boldsymbol{\beta}\ddot{\boldsymbol{\beta}})}{(1-\beta^2)^{5/2}} + \frac{3(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})^2}{(1-\beta^2)^{7/2}}\right]. \tag{35}$$

3) From formulae (31) and (35) we can obtain the work done by the radiation reaction force (RRF):

$$A_{r.Z.} = \int\limits_{t'}^{t'+\Delta t'} (\mathbf{F},\mathbf{v})dt' = \frac{2e^2}{3c}\int\limits_{t'}^{t'+\Delta t'}\left[\frac{(\boldsymbol{\beta}\ddot{\boldsymbol{\beta}})}{(1-\beta^2)^2} + \frac{3(\boldsymbol{\beta}\dot{\boldsymbol{\beta}})^2}{(1-\beta^2)^3}\right]dt'. \tag{36}$$

If a charge moves along the Ox axis, then we can introduce the notation $\beta = v_x/c$ and expression (36) can be rewritten in the following way:

$$A_{r.Z.} = -\frac{2e^2}{3c}\int\limits_{t'}^{t'+\Delta t'}\frac{\dot{\beta}^2 dt_1}{(1-\beta^2)^3} + \frac{2e^2}{3c}\left[\frac{\beta\dot{\beta}}{(1-\beta^2)^2}\right]_{t_1=t'}^{t_1=t'+\Delta t'}. \tag{37}$$

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