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GENERALIZED z -SCALING IN PROTON-PROTON
COLLISIONS AT HIGH ENERGIES

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Предложено новое обобщение z -скейлинга для инклюзивного рождения частиц. Скейлинговая переменная z представляет собой фрактальную меру, зависящую от кинематических характеристик элементарного подпроцесса, выраженных через доли импульсов x_1 и x_2 сталкивающихся протонов. В обобщенном подходе x_1 и x_2 являются функциями долей импульсов y_a и y_b рассеянного конститuenta и конститuenta отдачи, уносимых инклюзивной частицей и объектом отдачи соответственно. Построена скейлинговая функция $\psi(z)$ для заряженных и идентифицированных адронов, рожденных в протон-протонных взаимодействиях. Определены фрактальные размерности и удельные теплоемкости образовавшейся среды, которые входят в определение переменной z и обеспечивают энергетический и угловой скейлинг функции $\psi(z)$. Эти параметры обеспечивают также независимость $\psi(z)$ от плотности множественности процесса. Предложенная схема позволяет получить единое описание экспериментальных данных по инклюзивным сечениям рождения заряженных адронов, пионов, каонов, антипротонов и лямбда-гиперонов при высоких энергиях в рамках z -представления. Полученные результаты представляют интерес для использования z -скейлинга в качестве инструмента для поиска новых физических явлений в процессах с рождением частиц с большими поперечными импульсами и в процессах с большими множественностями на протон-протонных коллайдерах RHIC и LHC.

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New generalization of z -scaling in inclusive particle production is proposed. The scaling variable z is fractal measure which depends on kinematical characteristics of the underlying subprocess expressed in terms of the momentum fractions x_1 and x_2 of the incoming protons. In the generalized approach, the x_1 and x_2 are functions of the momentum fractions y_a and y_b of the scattered and recoil constituents carried out by the inclusive particle and recoil object, respectively. The scaling function $\psi(z)$ for charged and identified hadrons produced in proton-proton collisions is constructed. The fractal dimensions and heat capacity of the produced medium entering definition of the z are established to obtain energy, angular and multiplicity independence of the $\psi(z)$. The scheme allows unique description of data on inclusive cross sections of charged particles, pions, kaons, antiprotons, and lambdas at high energies. The obtained results are of interest to use z -scaling as a tool for searching for new physics phenomena of particle production in high transverse momentum and high multiplicity region at the proton-proton colliders RHIC and LHC.

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1. INTRODUCTION

Production of particles with high transverse momenta from collision of hadrons and nuclei at sufficiently high energies has relevance to constituent interactions at small scales. This regime is of interest to search for new physical phenomena in elementary processes such as quark compositeness [1], extra dimensions [2], black holes [3], fractal space-time [4], etc. Other aspects of high energy interactions are connected with small momenta of secondary particles and high multiplicities. This regime has relevance to collective phenomena of particle production. Search for new physics in both regions is one of the main goals of investigations at the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN. Experimental data of particle production can give constraints for different theoretical models. Processes with high transverse momenta of produced particles are most suitable for precise test of perturbative QCD. The soft regime is of interest for verification of nonperturbative QCD and investigation of phase transitions in non-Abelian theories.

Nucleus–nucleus interactions are very complicated. In order to understand their nature one often exploits phenomenology and comparison with more simple proton–nucleus and proton–proton collisions. Many approaches to description of particle production are used to search for regularities reflecting general principles in these systems at high energies [5–14]. One of the most basic principles is the self-similarity of hadron production valid both in soft and hard physics. Other general principles are locality and fractality which can be applied to hard processes at small scales. The locality of hadronic interactions follows from numerous experimental and theoretical studies. These investigations have shown that interactions of hadrons and nuclei can be described in terms of the interactions of their constituents. Fractality in hard processes is specific feature connected with substructure of the constituents. This includes the self-similarity over wide scale range.

Fractality of soft processes concerning the multi-particle production was investigated by many authors. Fractality in inclusive reactions with high- p_T particles was considered for the first time in the framework of z -scaling [15]. The approach is based on principles of locality, self-similarity, and fractality. It takes into account fractal structure of the colliding objects, interaction of their constituents and particle formation. The scaling function $\psi(z)$ and the variable z are constructed via the experimentally measured inclusive cross section $E d^3\sigma/dp^3$

and the multiplicity density $dN/d\eta$. In the original version, the construction was based on the assumption that gross features of the inclusive particle distribution for the inclusive reaction

$$M_1 + M_2 \rightarrow m_1 + X \quad (1)$$

at high energies can be described in terms of the corresponding exclusive sub-process

$$(x_1 M_1) + (x_2 M_2) \rightarrow m_1 + (x_1 M_1 + x_2 M_2 + m_2). \quad (2)$$

The M_1 and M_2 are masses of the colliding hadrons (or nuclei) and m_1 is the mass of the inclusive particle. The mass parameter m_2 is introduced in connection with internal conservation laws (for isospin, baryon number, strangeness. . .). The x_1 and x_2 are fractions of the incoming four-momenta P_1 and P_2 of the colliding objects. The scaling variable z was constructed as fractal measure in terms of the fractions x_1 and x_2 . It depends on the nucleon anomalous dimension δ and the average multiplicity density $dN/d\eta|_0$ of charged particles produced in the central region of the interaction ($\eta = 0$). The scaling for non-biased collisions was established for single constant value of δ as independence of the scaling function $\psi(z)$ on the collision energy \sqrt{s} and angle θ of the inclusive particle.

The concept of z -scaling was generalized for various multiplicities of produced particles [16]. Relation of the z -scaling to entropy S and heat capacity c of the colliding system was established. The generalization was connected with introduction of momentum fraction y in the final state written in the symbolic way

$$(x_1 M_1) + (x_2 M_2) \rightarrow (m_1/y) + (x_1 M_1 + x_2 M_2 + m_2/y). \quad (3)$$

It was shown that generalized scaling represents regularity in both soft and hard regimes in proton-(anti)proton collisions over a wide range of initial energies and multiplicities of the produced particles. However, the generalization of the scaling for various multiplicities was at the expense of the angular independence of the scaling function observed for $y = 1$.

In this paper, we show that independence of the scaling function $\psi(z)$ on the collision energy \sqrt{s} , multiplicity density $dN/d\eta$ and angle θ can be restored simultaneously, if two fractions y_a and y_b for the scattered constituent and its recoil are introduced, respectively.

2. NEW GENERALIZATION OF z -SCALING

We consider collision of extended objects (hadrons and nuclei) at sufficiently high energies as an ensemble of individual interactions of their constituents. The constituents are partons in the parton model or quarks and gluons in the theory of

QCD. Single interaction of the constituents is illustrated in Fig. 1. Structures of the colliding objects are characterized by the anomalous dimensions δ_1 and δ_2 in the space of momentum fractions. Interacting constituents carry the fractions x_1 and

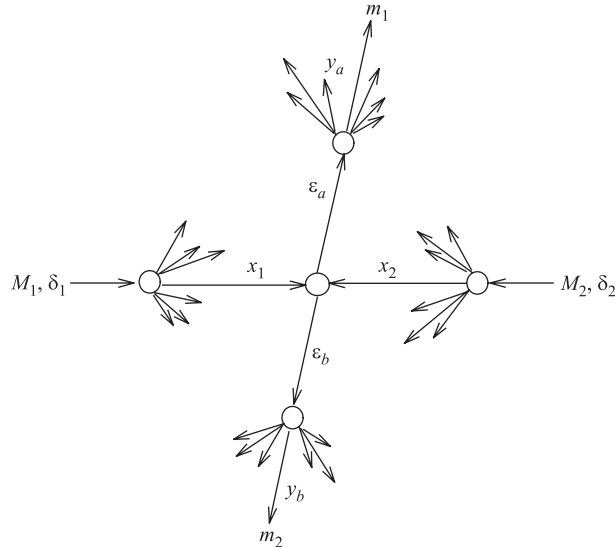


Fig. 1. Diagram of the constituent subprocess

x_2 of the incoming objects. The subprocess is considered as a binary collision with production of the scattered and recoil constituents, respectively. The inclusive particle carries the momentum fraction y_a of the scattered constituent which fragmentation is characterized by the anomalous dimension ϵ_a . Fragmentation of the recoil constituent is described by the corresponding fractal dimension ϵ_b and momentum fraction y_b .

Multiple interactions are considered to be similar. The property is manifestation of the self-similarity of hadronic interaction at the constituent level. The interactions are governed by local energy-momentum conservation law. The self-similarity at small scales reflects fractality of the interaction objects and their constituents characterized by the corresponding fractal dimensions. The fractality concerns parton content of the composite structures involved.

2.1. Locality, Self-Similarity and Fractality. The idea of z -scaling is based on the assumption [10] that gross features of inclusive particle distribution of reaction (1) can be described at high energies in terms of the kinematic characteristics of the corresponding constituent subprocesses. We consider the subprocess as binary collision

$$(x_1 M_1) + (x_2 M_2) \rightarrow (m_1/y_a) + (x_1 M_1 + x_2 M_2 + m_2/y_b) \quad (4)$$

of the constituents (x_1M_1) and (x_2M_2) resulting in the scattered (m_1/y_a) and recoil $(x_1M_1 + x_2M_2 + m_2/y_b)$ constituents in the final state. The inclusive particle with the mass m_1 and the 4-momentum p carries out the fraction y_a of the 4-momentum of the scattered constituent. Its counterpart (m_2) , moving in the away side direction, carries out the 4-momentum fraction y_b of the produced recoil. The binary subprocess satisfies the energy-momentum conservation written in the form

$$(x_1P_1 + x_2P_2 - p/y_a)^2 = (x_1M_1 + x_2M_2 + m_2/y_b)^2. \quad (5)$$

The equation is expression of the locality of hadron interaction at constituent level. It represents kinematical constraint on the fractions x_1 , x_2 , y_a , and y_b .

The self-similarity of hadron interactions reflects property that hadron constituents and their interactions are similar. This is connected with dropping of certain dimensional quantities out of description of physical phenomena. The self-similar solutions are constructed in terms of self-similarity parameters. We search for the solution

$$\psi(z) = \frac{1}{N\sigma_{\text{in}}} \frac{d\sigma}{dz} \quad (6)$$

depending on single self-similarity variable z . Here σ_{in} is the inelastic cross section of reaction (1) and N is the average particle multiplicity. The variable z is specific dimensionless combination of quantities which characterizes particle production in high energy inclusive reactions. It depends on momenta and masses of the colliding and inclusive particles, structural parameters of the interacting objects and dynamical characteristics of the produced system. We define the self-similarity variable z in the form

$$z = z_0\Omega^{-1}, \quad (7)$$

where

$$\Omega(x_1, x_2, y_a, y_b) = (1 - x_1)^{\delta_1} (1 - x_2)^{\delta_2} (1 - y_a)^{\epsilon_a} (1 - y_b)^{\epsilon_b}. \quad (8)$$

The variable z has character of a fractal measure. For a given reaction (1), its finite part z_0 is proportional to the transverse kinetic energy of the constituent subprocess consumed on the production of the inclusive particle (m_1) and its counterpart (m_2) . The divergent factor Ω^{-1} describes resolution at which the subprocess can be singled out of this reaction. The $\Omega(x_1, x_2, y_a, y_b)$ is relative number of parton configurations containing the incoming constituents which carry the fractions x_1 and x_2 of the momenta P_1 and P_2 and the outgoing constituents which fractions y_a and y_b are carried out by the inclusive particle (m_1) and its counterpart (m_2) , respectively. The δ_1 (δ_2) and ϵ_a (ϵ_b) are anomalous fractal dimensions of the incoming and fragmenting objects, respectively. For inelastic

proton–proton collisions we have $\delta_1 = \delta_2 \equiv \delta$. We also assume that fragmentation of the scattered and recoil constituents is governed by the same anomalous dimensions $\epsilon_a = \epsilon_b \equiv \epsilon$.

Common property of fractal measures is their divergence with the increasing resolution

$$z(\Omega) \rightarrow \infty, \quad \text{if} \quad \Omega^{-1} \rightarrow \infty. \quad (9)$$

For the infinite resolution, all momentum fractions become unity ($x_1 = x_2 = y_a = y_b = 1$) and $\Omega = 0$. The kinematical limit corresponds to the fractal limit $z = \infty$.

2.2. Principle of Minimal Resolution. The momentum fractions x_1, x_2, y_a , and y_b are determined from principle of minimal resolution of the fractal measure z . The principle states that resolution Ω^{-1} should be minimal with respect to all binary subprocesses (4) in which the inclusive particle m_1 with the momentum p can be produced. This singles out the underlying interaction of the constituents. The momentum fractions x_1, x_2, y_a , and y_b are found from minimization of $\Omega^{-1}(x_1, x_2, y_a, y_b)$,

$$\frac{\partial \Omega(x_i, y_j)}{\partial x_1} = 0, \quad \frac{\partial \Omega(x_i, y_j)}{\partial x_2} = 0, \quad \frac{\partial \Omega(x_i, y_j)}{\partial y_a} = 0, \quad \frac{\partial \Omega(x_i, y_j)}{\partial y_b} = 0, \quad (10)$$

taking into account the energy-momentum conservation (5). The momentum fractions x_1 and x_2 can be decomposed as follows:

$$x_1 = \lambda_1 + \chi_1(\alpha), \quad x_2 = \lambda_2 + \chi_2(\alpha). \quad (11)$$

The parameter $\alpha = \delta_2/\delta_1$ is ratio of the anomalous fractal dimensions of the colliding objects. Using the decomposition, expression (4) can be rewritten to the symbolic form

$$x_1 + x_2 \rightarrow (\lambda_1 + \lambda_2) + (\chi_1 + \chi_2). \quad (12)$$

This relation means that λ parts of the interacting constituents contribute to the production of the inclusive particle, while the χ parts are responsible for the creation of its recoil. The λ 's are functions of y_a and y_b

$$\lambda_1 = \kappa_1/y_a + \nu_1/y_b, \quad \lambda_2 = \kappa_2/y_a + \nu_2/y_b, \quad \lambda_0 = \bar{\nu}_0/y_b^2 - \nu_0/y_a^2, \quad (13)$$

where

$$\kappa_1 = \frac{(P_2 p)}{(P_1 P_2) - M_1 M_2}, \quad \kappa_2 = \frac{(P_1 p)}{(P_1 P_2) - M_1 M_2}, \quad (14)$$

$$\nu_1 = \frac{M_2 m_2}{(P_1 P_2) - M_1 M_2}, \quad \nu_2 = \frac{M_1 m_2}{(P_1 P_2) - M_1 M_2}, \quad (15)$$

$$\nu_0 = \frac{0.5 m_1^2}{(P_1 P_2) - M_1 M_2}, \quad \bar{\nu}_0 = \frac{0.5 m_2^2}{(P_1 P_2) - M_1 M_2}. \quad (16)$$

The χ' 's are expressed via λ' 's as follows:

$$\chi_1 = \sqrt{\mu_1^2 + \omega_1^2} - \omega_1, \quad \chi_2 = \sqrt{\mu_2^2 + \omega_2^2} + \omega_2, \quad (17)$$

where

$$\mu_1^2 = (\lambda_1 \lambda_2 + \lambda_0) \alpha \frac{1 - \lambda_1}{1 - \lambda_2}, \quad \mu_2^2 = (\lambda_1 \lambda_2 + \lambda_0) \alpha^{-1} \frac{1 - \lambda_2}{1 - \lambda_1}, \quad (18)$$

and $\omega_i = \mu_i U$ ($i = 1, 2$).

The quantity

$$U = \frac{\alpha - 1}{2\sqrt{\alpha}} \xi \quad (19)$$

has physical meaning of longitudinal component of the «structural 4-velocity» [17]. It is function of α and the kinematical factor

$$\xi = \sqrt{\frac{\lambda_1 \lambda_2 + \lambda_0}{(1 - \lambda_1)(1 - \lambda_2)}} \quad (20)$$

($0 \leq \xi \leq 1$). The ξ characterizes kinematical scale of the underlying constituent interaction.

Solution of the system (10) with the condition (5) can be obtained by searching for the unbounded maximum of the function

$$F(y_a, y_b) \equiv \Omega(x_1(y_a, y_b), x_2(y_a, y_b), y_a, y_b) \quad (21)$$

of two independent variables y_a and y_b . Here $x_i(y_a, y_b)$ are given explicitly by expressions (11). There exists single maximum of the function $F(y_a, y_b)$ in the allowed kinematical region, and we determined it numerically.

2.3. Scaling Variable z . Search for an adequate, physically meaningful but still sufficiently simple form of the self-similarity parameter z plays a crucial role in our approach. We define the scaling variable z in the form

$$z = \frac{s_{\perp}^{1/2}}{(dN/d\eta|_0)^c \cdot m} \cdot \Omega^{-1}. \quad (22)$$

Here m is a mass constant which we fix at the nucleon mass. The transverse kinetic energy of the constituent subprocess consumed on the production of the inclusive particle (m_1) and its counterpart (m_2) is determined by the formula

$$s_{\perp}^{1/2} = T_a + T_b, \quad (23)$$

where

$$T_a = y_a (s_{\lambda}^{1/2} - M_1 \lambda_1 - M_2 \lambda_2) - m_1, \quad (24)$$

$$T_b = y_b(s_\chi^{1/2} - M_1\chi_1 - M_2\chi_2) - m_2. \quad (25)$$

For more details see Appendix. The terms

$$s_\lambda^{1/2} = \sqrt{(\lambda_1 P_1 + \lambda_2 P_2)^2}, \quad s_\chi^{1/2} = \sqrt{(\chi_1 P_1 + \chi_2 P_2)^2} \quad (26)$$

represent energy for production of the scattered constituent and its recoil, respectively. The boundaries of the range of the variable z are 0 and ∞ . They are accessible at any collision energy.

The $dN/d\eta|_0$ is average multiplicity density of charged particles produced in the central region of reaction (1) at pseudorapidity $\eta = 0$. It depends on state of the produced medium in the colliding system. The parameter c characterizes properties of this medium. The quantity

$$W = (dN/d\eta|_0)^c \cdot \Omega \quad (27)$$

is proportional to all parton and hadron configurations of the colliding system which can contribute to production of the inclusive particle with the momentum p . The scaling variable (22) is proportional to the ratio

$$z \sim \frac{s_\perp^{1/2}}{W} \quad (28)$$

of the transverse kinetic energy $s_\perp^{1/2}$ and maximal number of the configurations W .

2.4. Scaling Variable z and Entropy S . According to statistical physics, entropy of a system is given by number of all statistical states W of the system as follows:

$$S = \ln W. \quad (29)$$

In thermodynamics, entropy for ideal gas is determined by the formula

$$S = c_V \ln T + R \ln V + \text{const}. \quad (30)$$

The c_V is heat capacity and R is universal constant. The temperature T and the volume V characterize state of the system. Using (27) and (29), we can write

$$S = c \ln [dN/d\eta|_0] + \ln [(1-x_1)^{\delta_1} (1-x_2)^{\delta_2} (1-y_a)^{\epsilon_a} (1-y_b)^{\epsilon_b}]. \quad (31)$$

Exploiting analogy between Eqs. (30) and (31), we interpret the parameter c as «heat capacity» of the produced medium. The multiplicity density $dN/d\eta|_0$ has physical meaning of «temperature» of the colliding system. The second term in Eq. (31) depends on volume in space of the momentum fractions $\{x_1, x_2, y_a, y_b\}$. This analogy emphasizes once more interpretation of the parameters $\delta_1, \delta_2, \epsilon_a$

and ϵ_b as fractal dimensions. As seen from Eq. (31), entropy of the colliding system increases with the multiplicity density and decreases with increasing resolution Ω^{-1} .

Let us note that entropy (29) of a system is determined up to an arbitrary constant $\ln W_0$. Dimensional units entering definition of the entropy can be included within this constant. Therefore, it allows us to make analogy between dimensionless $dN/d\eta|_0$ and the temperature. This degree of freedom is connected with the transformation

$$z \rightarrow W_0 \cdot z, \quad \psi \rightarrow W_0^{-1} \cdot \psi. \quad (32)$$

In such a way the scaling variable and the scaling function are determined up to an arbitrary multiplicative constant.

2.5. Scaling Function $\psi(z)$. The scaling function $\psi(z)$ is expressed in terms of the experimentally measured inclusive invariant cross section $E d^3\sigma/dp^3$, multiplicity density $dN/d\eta$ and the total inelastic cross section σ_{in} . Exploiting the definition (6) one can obtain the expression

$$\psi(z) = -\frac{\pi s}{(dN/d\eta)\sigma_{\text{in}}} J^{-1} E \frac{d^3\sigma}{dp^3}. \quad (33)$$

Here, s is the centre-of-mass collision energy squared and

$$J = \frac{\partial\eta}{\partial\kappa_1} \frac{\partial z}{\partial\kappa_2} - \frac{\partial\eta}{\partial\kappa_2} \frac{\partial z}{\partial\kappa_1} \quad (34)$$

is the corresponding Jacobian. Angular properties of the $\psi(z)$ showed that the scaling is valid rather in pseudorapidity than in rapidity (see Appendix). The function $\psi(z)$ is normalized as follows:

$$\int_0^\infty \psi(z) dz = 1. \quad (35)$$

The relation allows us to interpret the $\psi(z)$ as a probability density to produce inclusive particle with the corresponding value of the variable z .

3. PROPERTIES OF THE SCALING FUNCTION $\psi(z)$

Let us investigate properties of z presentation of experimental data obtained in proton-proton collisions at high energies.

3.1. Energy Independence of $\psi(z)$. We analyze experimental data [18–25] on inclusive hadron (h^\pm , π^- , K^- , and \bar{p}) production in minimum-biased proton–proton collisions. The data on inclusive cross sections were measured in central rapidity region at FNAL, ISR and RHIC energies $\sqrt{s} = 19 - 200$ GeV.

The energy dependence of the charged hadron spectra on the transverse momentum is shown in Fig. 2, *a*. The distributions cover the range up to $p_T \simeq 10$ GeV/c. The cross sections change within the range of 12 orders of magnitudes. Strong dependence of the spectra on the collision energy \sqrt{s} increases with transverse momentum. Figure 2, *b* shows z -presentation of the same data. The scaling variable z depends on the average multiplicity density

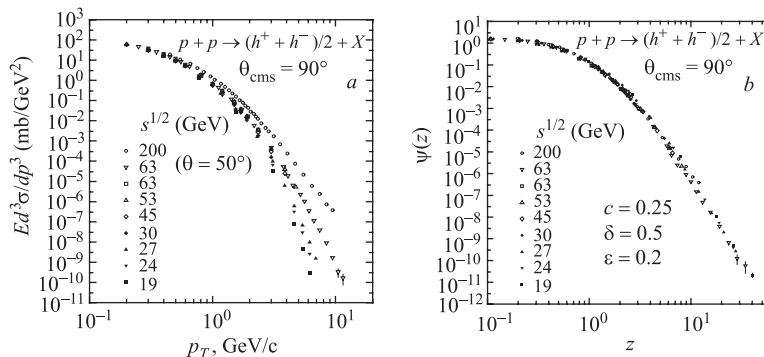


Fig. 2. *a*) Transverse momentum spectra of charged hadrons produced in pp collisions at $\sqrt{s} = 19 - 200$ GeV. Experimental data are taken from Refs. [18, 19, 20, 22]. *b*) The corresponding scaling function $\psi(z)$

of charged particles produced in the central pseudorapidity region of the collision. We have used experimentally measured values of $dN/d\eta|_{\eta=0}$ [26] for minimum-biased collisions in the analysis of energy and angular properties of $\psi(z)$. Independence of the scaling function $\psi(z)$ on collision energy \sqrt{s} is found for the constant values of the parameters $c = 0.25$, $\delta = 0.5$ and $\epsilon = 0.2$. The form of $\psi(z)$ manifests two regimes of particle production. The hard regime is characterized by the power law $\psi(z) \sim z^{-\beta}$ at large z . Soft processes correspond to behavior of the $\psi(z)$ at small z . Slope of the scaling curve decreases with z in this region.

Invariant cross sections for π^- -meson production as function of the collision energy and transverse momentum are plotted in Fig. 3, *a*. The spectra were measured over a wide transverse momentum range $p_T = 0.1 - 10$ GeV/c. The cross sections change from 10^2 to 10^{-10} mb/GeV 2 . Strong dependence of the pion spectra on \sqrt{s} as for charged hadrons is observed. The z -presentation of the same data is shown in Fig. 3, *b*. For pions (as well as for all other types of the particles — kaons, antiprotons, . . .), the dependence of z on the charged particle

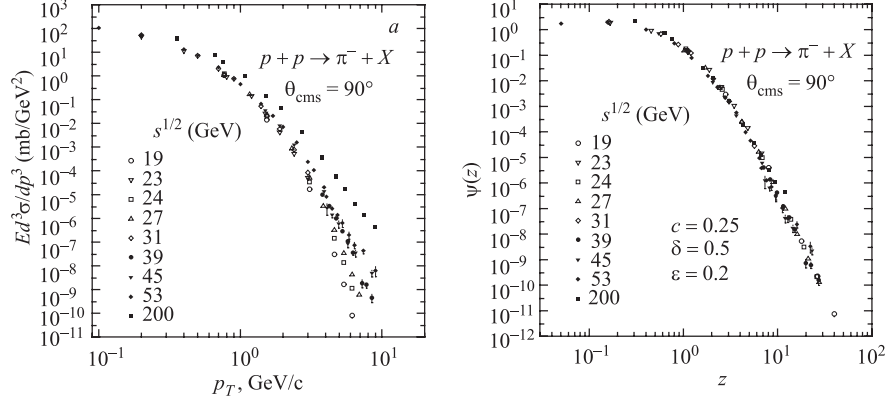


Fig. 3. *a*) Transverse momentum spectra of π^- mesons produced in pp collisions at $\sqrt{s} = 19 - 200$ GeV. Experimental data are taken from Refs. [18, 19, 21, 23]. *b*) The corresponding scaling function $\psi(z)$

multiplicity density $dN/d\eta|_{\eta=0}$ have been used in formula (22). Let us stress that, unlike this, the scaling function (33) is normalized to the multiplicity density of pions. Independence of the scaling function for pions on \sqrt{s} was obtained at $c = 0.25, \delta = 0.5$ and $\epsilon = 0.2$ as for charged hadrons. Shape of the $\psi(z)$ is similar in both cases, as well.

Transverse momentum spectra for K^- -meson production are shown in Fig. 4, *a*. The cross sections were measured in the range $p_T = 0.1 - 8$ GeV/c. Data for K_S^0 mesons obtained by the STAR Collaboration at RHIC are presented

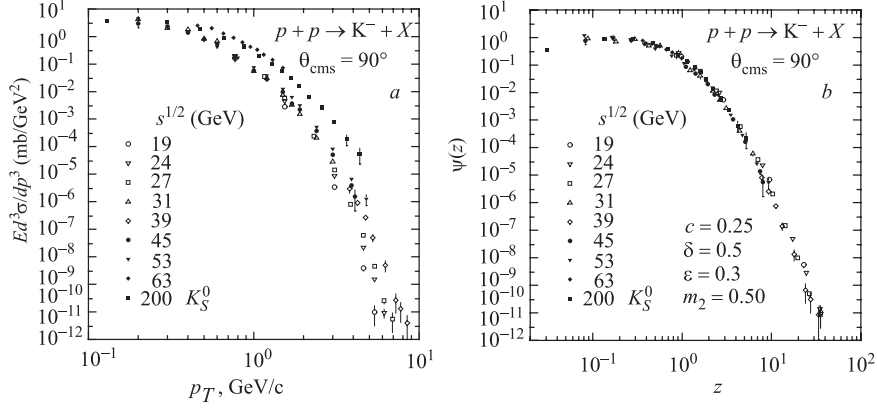


Fig. 4. *a*) Transverse momentum spectra of K^- mesons produced in pp collisions at $\sqrt{s} = 19 - 200$ GeV. The spectrum of K_S^0 mesons is shown by full squares. Experimental data are taken from Refs. [18, 19, 21, 24, 25]. *b*) The corresponding scaling function $\psi(z)$

in Fig. 4, *a* as well. The K -meson spectra demonstrate strong dependence on the collision energy \sqrt{s} . The corresponding scaling function $\psi(z)$ is depicted in Fig. 4, *b*. Independence of the $\psi(z)$ on \sqrt{s} is restored at $c = 0.25, \delta = 0.5$ and $\epsilon = 0.3$. Similar features of p_T and z presentations of experimental data on antiproton production are presented in Fig. 5, *a, b*. The energy independence of $\psi(z)$ for antiprotons is established at $c = 0.25, \delta = 0.5$ and $\epsilon = 0.35$.

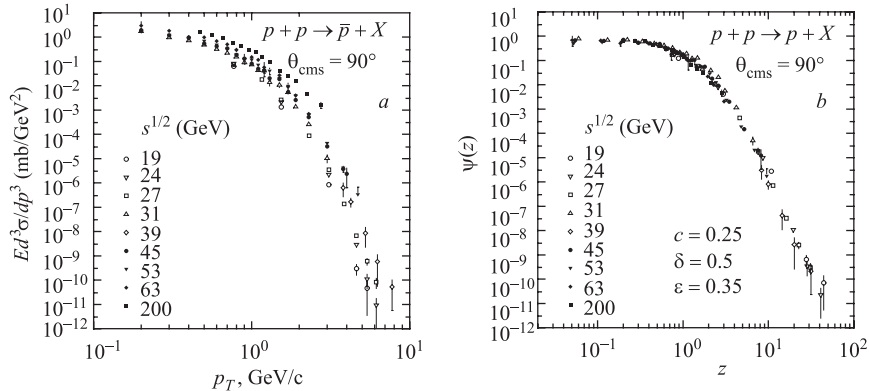


Fig. 5. *a*) Transverse momentum spectra of antiprotons produced in pp collisions at $\theta_{\text{cms}} = 90^\circ$. Experimental data are taken from Refs. [18, 19, 21, 24]. *b*) The corresponding scaling function $\psi(z)$

As a result, we can conclude that energy independence of the scaling function $\psi(z)$ is valid for different types of hadrons in a wide range of centre-of-mass energy \sqrt{s} and transverse momentum p_T .

3.2. Angular Independence of $\psi(z)$. We analyze experimental data [19, 27] on angular dependence of negative hadrons (pions, kaons and antiprotons) measured at ISR energies. The data were measured both in the central and fragmentation regions. Results of our analysis are demonstrated at the energy $\sqrt{s} = 53$ GeV.

Invariant cross sections for π^- -meson production as function of the centre-of-mass angle θ and transverse momentum are shown in Fig. 6, *a*. The angles cover the range $\theta = 3 - 90^\circ$. The central and fragmentation regions are distinguished by different behavior of differential cross sections. The z -presentation of the same data is demonstrated in Fig. 6, *b*. The charged hadron multiplicity density $dN/d\eta|_{\eta=0}$ represents angular independent factor in the definition of the variable z . Contrary to this, the scaling function (33) is normalized to the multiplicity density $dN/d\eta$ of pions depending on the angle θ . Angular and energy independence of the scaling function for pions was obtained at the same values of $c = 0.25, \delta = 0.5$ and $\epsilon = 0.2$. The function $\psi(z)$ for small θ is sensitive to the value of m_2 . This parameter is determined from the corresponding exclusive

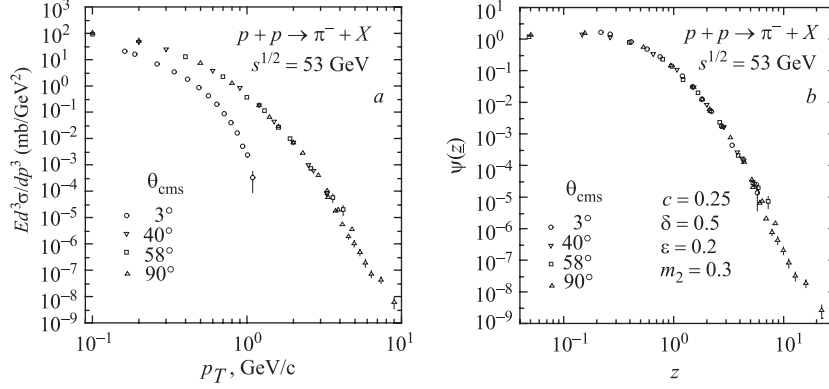


Fig. 6. *a*) Transverse momentum spectra of π^- mesons produced in pp collisions for different angles at $\sqrt{s} = 53$ GeV. Experimental data are taken from Refs. [19, 27]. *b*) The corresponding scaling function $\psi(z)$

reaction

$$p + p \rightarrow \pi^- + p + \Delta^{++}. \quad (36)$$

The reaction is limiting case of the subprocess (4) for $x_1 = x_2 = y_a = y_b = 1$. According to Eqs. (5) and (36), we obtain $m_2 = m(\Delta^{++}) - m(p) = 0.3$ GeV. This value was used in our analysis for inclusive π^- -meson production.

Transverse momentum spectra for K mesons and antiprotons produced in pp collisions at different angles are shown in Figs. 7, *a* and 8, *a*. In addition to ISR data at $\sqrt{s} = 53$ GeV, the data from RHIC at $\sqrt{s} = 200$ GeV are shown

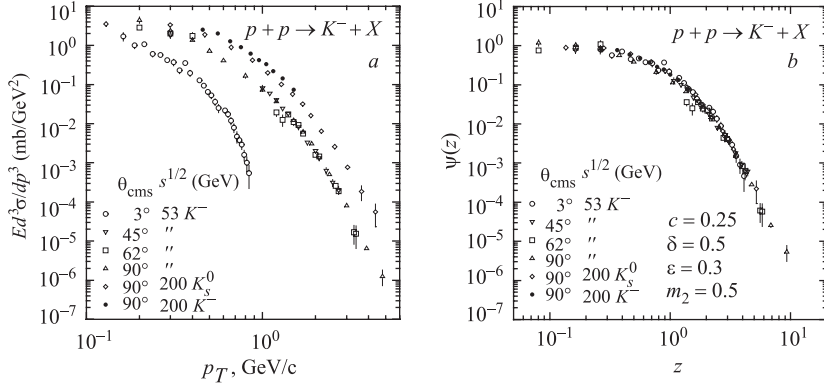


Fig. 7. *a*) Transverse momentum spectra of K^- mesons produced in pp collisions for different angles at $\sqrt{s} = 53$ GeV. The K^- - and K_s^0 -meson spectra for $\theta_{\text{cms}} \simeq 90$ at $\sqrt{s} = 200$ GeV are shown by \bullet and \diamond , respectively. Experimental data are taken from Refs. [19, 24, 25, 27]. *b*) The corresponding scaling function $\psi(z)$

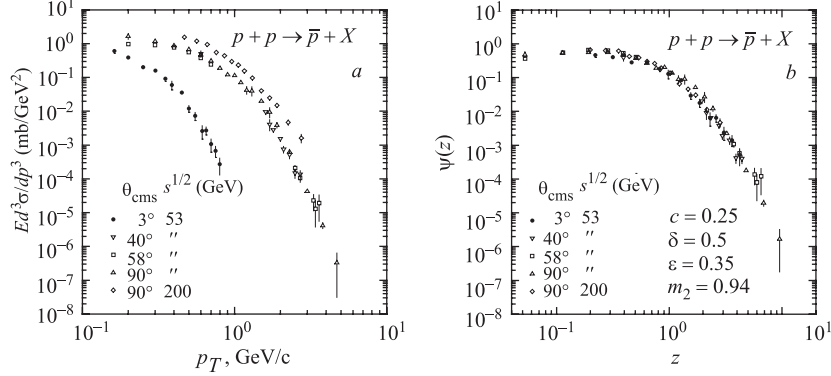


Fig. 8. *a*) Transverse momentum spectra of antiprotons produced in pp collisions for different angles at $\sqrt{s} = 53$ GeV. The antiproton spectrum for $\theta_{\text{cms}} \simeq 90$ at $\sqrt{s} = 200$ GeV is shown by \diamond . Experimental data are taken from Refs. [19, 24, 27]. *b*) The corresponding scaling function $\psi(z)$

as well. The angular dependence of the spectra demonstrates strong difference between central and fragmentation regions. The corresponding function $\psi(z)$ for kaons and antiprotons is plotted in Figs. 7, *b* and 8, *b*, respectively. For both particles, the charged hadron multiplicity density $dN/d\eta|_{\eta=0}$ represents angular independent factor in the definition of the variable z . The scaling function (33) is normalized to the angular-dependent multiplicity density $dN/d\eta$ of kaons and antiprotons, respectively.

Independence of the $\psi(z)$ on the angle θ is obtained at $c = 0.25, \delta = 0.5$ and $\epsilon = 0.3$ for kaons and $c = 0.25, \delta = 0.5$ and $\epsilon = 0.35$ for antiprotons. Values of these parameters allow us to obtain simultaneously angular and energy independence of the scaling function. Like in the case of π^- mesons, the function $\psi(z)$ for kaons and antiprotons is sensitive to the value of m_2 at small angles θ . The corresponding exclusive reactions

$$p + p \rightarrow K^- + p + p + K^+, \quad (37)$$

$$p + p \rightarrow \bar{p} + p + p + p \quad (38)$$

were used to determine the parameter m_2 . Exploiting Eq. (5) for $x_1 = x_2 = y_a = y_b = 1$ and Eqs. (37) and (38), we obtain $m_2 = m(K^+) = 0.5$ GeV and $m_2 = m(p) = 0.94$ GeV for the inclusive production of K^- mesons and antiprotons, respectively. These values were used in our analysis.

3.3. Multiplicity Independence of $\psi(z)$. The STAR Collaboration obtained the new data [28] on multiplicity dependence of the inclusive spectra of charged hadrons produced in pp collisions in the central rapidity range $|\eta| < 0.5$ at the energy $\sqrt{s} = 200$ GeV. The transverse momentum distributions were measured up to

9.5 GeV/c using different multiplicity selection criteria. Figure 9, *a* demonstrates strong dependence of the spectra on multiplicity density at $dN/d\eta = 2.5, 6.0$ and 8.0 . The same data are presented in Fig. 9, *b* in the scaling form. The scaling

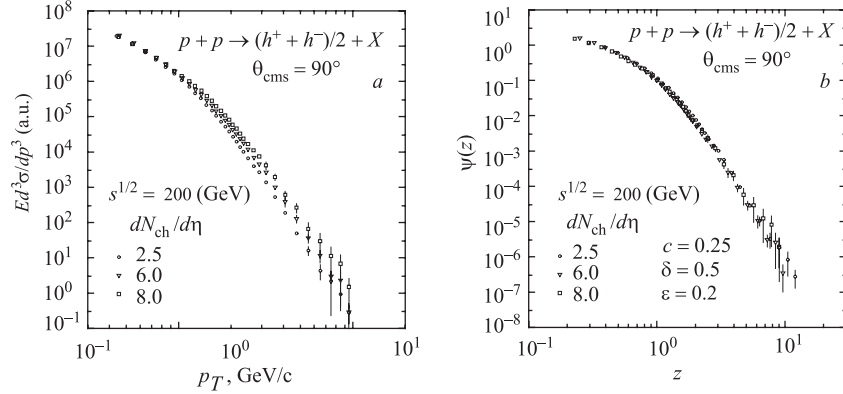


Fig. 9. *a*) Transverse momentum spectra of charged hadrons produced in pp collisions for different multiplicity densities at $\sqrt{s} = 200$ GeV. The spectra are normalized at $p_T = 0.4$ GeV/c. Experimental data are taken from Ref. [28]. *b*) The corresponding scaling function $\psi(z)$

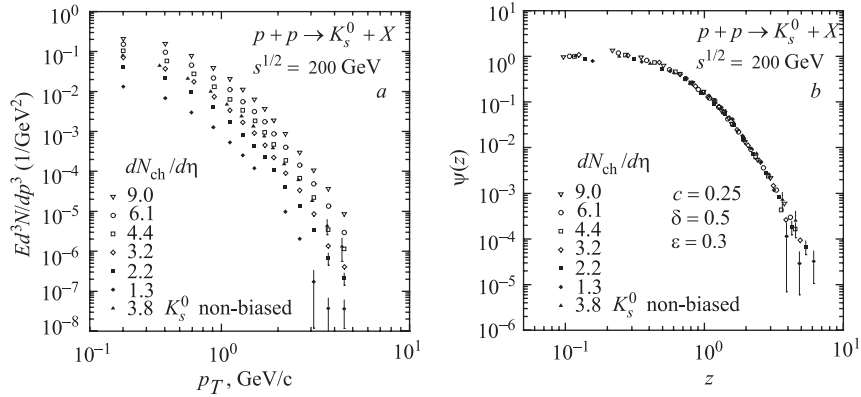


Fig. 10. *a*) Transverse momentum spectra of K_s^0 mesons produced in pp collisions for different multiplicity densities at $\sqrt{s} = 200$ GeV. The K_s^0 -meson spectrum for non-biased pp collisions at $\sqrt{s} = 200$ GeV is shown by black triangles. Experimental data are taken from Ref. [29]. *b*) The corresponding scaling function $\psi(z)$

function $\psi(z)$ changes over 6 orders of magnitude in the range of $z = 0.2 - 10$. The independence of $\psi(z)$ on multiplicity density $dN/d\eta$ is obtained. The result gives strong restriction on the parameter c . It was found to be $c = 0.25$.

The STAR Collaboration measured the multiplicity dependence of the K_s^0 -meson and Λ_s -baryon spectra [29] in the central rapidity range $|\eta| < 0.5$ at the energy $\sqrt{s} = 200$ GeV. The spectra are presented in Figs. 10, *a* and 11, *a*. The multiplicity density was varied in the range $dN/d\eta = 1.3 - 9.0$. The transverse momentum distributions were measured up to 4.5 GeV/c. The corresponding

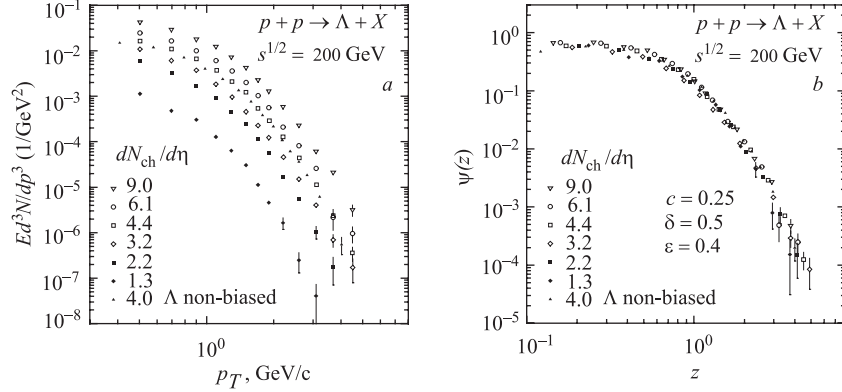


Fig. 11. *a*) Transverse momentum spectra of Λ baryons produced in pp collisions for different multiplicity densities at $\sqrt{s} = 200$ GeV. The Λ -baryon spectrum for non-biased pp collisions at $\sqrt{s} = 200$ GeV is shown by black triangles. Experimental data are taken from Ref. [29]. *b*) The corresponding scaling function $\psi(z)$

function $\psi(z)$ is shown in Figs. 10, *b* and 11, *b*, respectively. The scaling behavior gives strong restriction on the value of c . Data prefer $c = 0.25$ in both cases.

Thus we conclude that the available experimental data on the multiplicity dependence of spectra of charged hadrons, K_s^0 mesons and Λ_s baryons produced in pp collisions at the RHIC confirm generalized z -scaling for the same value of the parameter c .

4. CONCLUSIONS

Generalized z -scaling for the inclusive particle production in proton-proton collisions was suggested. The scaling variable z is function of multiplicity density $dN/d\eta|_0$ of charged particles in the central region of collision. The variable z depends on the parameters c , δ and ϵ . They are interpreted as specific heat of the produced medium, anomalous fractal dimension of the proton and fractal dimension of the fragmentation process, respectively. Connection between the scaling variable z and entropy S of the interacting system was established.

We have analyzed experimental data on inclusive cross sections of hadrons (h^\pm , π^- , K^- , K_s^0 , \bar{p} , and Λ) measured in proton-proton collisions at FNAL, ISR

and RHIC. The data cover a wide range of collision energy, transverse momenta and angles of the produced particles. Spectra from minimum biased events and events with various multiplicity selection criteria have been studied. The energy, angular and multiplicity independence of the scaling function was established. It gives strong constraints on the values of the parameters c , δ , and ϵ . It was shown that the parameters are constants in the considered kinematical region. The parameters $c = 0.25$ and $\delta = 0.5$ were found to be the same for all types of the considered inclusive hadrons. The value of ϵ increases with mass of the produced hadron.

The variable z has property of a fractal measure connected with parton content of the composite structures involved. The fractal dimensions δ and ϵ determine fractal properties of z in space of the momentum fractions. The scaling function $\psi(z)$ manifests two regimes of particle production. The hard regime is characterized by the power law $\psi(z) \sim z^{-\beta}$ at large z . Soft processes correspond to behavior of the $\psi(z)$ at small z . Slope of the scaling curve decreases with z in this region. On basis of the performed analysis we conclude that z -scaling in proton–proton collisions is regularity which reflects self-similarity, locality and fractality of hadron interaction at constituent level. It concerns structure of the colliding objects, interactions of their constituents and fragmentation process.

We consider that obtained results are of interest for searching and study of new physics phenomena in particle production over a wide range of collision energies, high transverse momenta and large multiplicities in proton–proton and nucleus–nucleus interactions at the RHIC and LHC.

APPENDIX

The invariant differential cross section for production of the inclusive particle is normalized as follows:

$$\int E \frac{d^3\sigma}{dp^3} d\eta d^2p_{\perp} = \sigma_{\text{inel}} N. \quad (\text{A1})$$

The σ_{inel} is the inelastic cross section and N is the average multiplicity. The differential cross section of identified hadrons of certain type is normalized to the corresponding average multiplicity of this type. The inclusive cross section can be expressed in terms of κ_1 and κ_2 as follows:

$$E \frac{d^3\sigma}{dp^3} = -\frac{1}{2\pi} \frac{\sqrt{(P_1 P_2)^2 - M_1^2 M_2^2}}{[(P_1 P_2) - M_1 M_2]^2} \frac{d^2\sigma}{d\kappa_1 d\kappa_2}. \quad (\text{A2})$$

In the region of high energies, the formula can be written in the approximate form

$$E \frac{d^3\sigma}{dp^3} = -\frac{1}{\pi s} \frac{d^2\sigma}{d\kappa_1 d\kappa_2}, \quad (\text{A3})$$

where s is square of the centre-of-mass energy. We suppose that the inclusive cross section is given by solution (6) as function of a single variable $z = z(\kappa_1, \kappa_2)$. Another independent combination of κ_1 and κ_2 is rapidity

$$y = \frac{1}{2} \ln \frac{\kappa_2}{\kappa_1}. \quad (\text{A4})$$

Using the variables z and y , we get the normalization

$$\int \frac{d^2\sigma}{d\kappa_1 d\kappa_2} d\kappa_1 d\kappa_2 = \int \frac{d^2\sigma}{dy dz} dy dz = \sigma_{\text{inel}} \int \rho(y) \psi(z) dy dz = \sigma_{\text{inel}} N, \quad (\text{A5})$$

where $\rho(y) \equiv dN/dy$ is the rapidity distribution of particles of the considered type. Detailed analysis of experimental data on angular properties of the scaling function showed that the factorization

$$\frac{d^2\sigma}{d\eta dz} = \sigma_{\text{inel}} \rho(\eta) \psi(z) \quad (\text{A6})$$

is valid using rather pseudorapidity than rapidity. In the centre of mass of the symmetric systems (e.g., for NN collisions) we can exploit the relations

$$\eta = \frac{1}{2} \ln \frac{1 - \cos \theta_2}{1 + \cos \theta_1} \sim y = \frac{1}{2} \ln \frac{\kappa_2}{\kappa_1}, \quad (\text{A7})$$

$$\begin{aligned} \kappa_1 &= \frac{(P_2 p)}{(P_1 P_2) - M_1 M_2} \sim \frac{E_p + p \cos \theta_1}{\sqrt{s}}, \\ \kappa_2 &= \frac{(P_1 p)}{(P_1 P_2) - M_1 M_2} \sim \frac{E_p - p \cos \theta_2}{\sqrt{s}} \end{aligned} \quad (\text{A8})$$

and set at the end $\theta_1 = \theta_2 \equiv \theta$. Then we get

$$\frac{\partial \eta}{\partial \kappa_1} = \frac{\partial \eta}{\partial \theta_1} \frac{\partial \theta_1}{\partial \kappa_1} = -\frac{\sqrt{s}}{2p} \frac{1}{(1 + \cos \theta)}, \quad (\text{A9})$$

$$\frac{\partial \eta}{\partial \kappa_2} = \frac{\partial \eta}{\partial \theta_2} \frac{\partial \theta_2}{\partial \kappa_2} = +\frac{\sqrt{s}}{2p} \frac{1}{(1 - \cos \theta)}. \quad (\text{A10})$$

The derivatives $\partial z / \partial \kappa_i$, which enter the Jacobian (34), were calculated numerically.

Finally, we examine the transverse kinetic energy $s_{\perp}^{1/2}$ which enter the definition (22) of the scaling variable z . The transverse momentum balance in the constituent subprocess is guaranteed by the identity

$$\chi_1 \chi_2 = \mu_1 \mu_2 = \lambda_1 \lambda_2 + \lambda_0. \quad (\text{A11})$$

Neglecting masses $M_i\lambda_i$ and $M_i\chi_i$ of the interacting constituents in expressions (24) and (25), we get for the transverse kinetic energies T_a and T_b the following relations:

$$T_{\perp}^a \sim y_a \sqrt{\lambda_1 \lambda_2 2P_1 P_2} - m_1 \sim \sqrt{(p_{\perp}^a)^2 + m_1^2} - m_1 \quad (\text{A12})$$

and

$$\begin{aligned} T_{\perp}^b &\sim y_b \sqrt{\chi_1 \chi_2 2P_1 P_2} - m_2 \sim y_b \sqrt{(\lambda_1 \lambda_2 + \lambda_0) s} - m_2 \\ &\sim y_b \sqrt{\kappa_1 \kappa_2 s / y_a^2 + m_2^2 / y_b^2 - m_1^2 / y_a^2} - m_2 \\ &\sim y_b \sqrt{((p_{\perp}^a)^2 + m_1^2) / y_a^2 + m_2^2 / y_b^2 - m_1^2 / y_a^2} - m_2 = \sqrt{(p_{\perp}^b)^2 + m_2^2} - m_2. \end{aligned} \quad (\text{A13})$$

In the last equation we have used the transverse momentum balance

$$p_{\perp}^a / y_a = p_{\perp}^b / y_b \quad (\text{A14})$$

valid in the constituent subprocess. The p^a and p^b are momenta of the inclusive particle (m_1) and its counterpart (m_2), respectively.

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