E2-2006-57

A. I. Ahmadov[1], Yu. M. Bystritskiy, E. A. Kuraev,
E. Zemljanaja, T. V. Shishkina[2]

MEASURING CHARGE-ODD CORRELATIONS AT LEPTON–PROTON AND PHOTON–PROTON COLLISIONS

Submitted to «Journal of Physics G»

[1] Institute of Physics of Azerbaijan National Academy of Sciences, Baku, Azerbaijan
[2] Belarus State University, Minsk, Belarus

Ахмедов А. И. и др. E2-2006-57

Измерение зарядово-нечетных корреляций в лептон-протонных и фотон-протонных столкновениях

Рассмотрены зарядово-нечетные корреляции в сечениях процессов с рождением заряженных частиц. В частности, рассмотрены случаи рождения мюонной и пионной пар, а также системы трех пионов $\pi^+\pi^-\pi^0$, в электрон-протонных и фотон-протонных столкновениях в области фрагментации протона. Зарядово-нечетные корреляции возникают как интерференция амплитуд различных механизмов рождения заряженных лептонов (пионов). Один из них соответствует рождению системы заряженных частиц в зарядово-нечетном состоянии (распад одного виртуального фотона или векторного мезона в эту систему частиц), а другой – в зарядово-четном состоянии (рождение системы частиц двумя фотонами). Зарядово-нечетный механизм рождения мюонной пары представляет собой чисто электродинамический процесс и может быть использован для калибровочных целей. Процессы с рождением пионов чувствительны к параметрам волновой функции пиона и, кроме того, могут быть использованы для измерения аномальной и простой частей эффективного пионного лагранжиана.

В процессе рождения лептонных пар в фотон-протонных столкновениях может быть измерен матричный элемент трех электромагнитных токов. Для этого зарядово-нечетное сечение $\gamma - p$-рассеяния представлено в виде свертки лептонного тензора третьего ранга с соответствующим адронным тензором. Этот эксперимент может рассматриваться как альтернатива к глубоконеупругому комптоновскому рассеянию.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2006

Ahmadov A. I. et al. E2-2006-57

Measuring Charge-Odd Correlations at Lepton–Proton and Photon–Proton Collisions

We consider the charge-odd correlations (COC) in cross sections of processes of charged particle production. The cases of muonic pair and pion systems $\pi^+\pi^-$, $\pi^+\pi^-\pi^0$ are considered in detail for electron–proton or photon–proton collisions in the proton fragmentation region kinematics. COC arise from interference of amplitudes which describe the different mechanisms of charged lepton (pion) creation. One of them corresponds to production of particles in the charge-odd state (one virtual photon or vector meson annihilation to this system of particles) and the other corresponds to the charge-even state of produced particles (creation by two photons). COC for muon–antimuon pair creation have a pure QED nature and can be considered as a normalization process. The processes with pion production are sensitive to some characteristics of proton wave functions and, besides, can be used for checking the anomalous and normal parts of the effective pionic Lagrangian.

Three electromagnetic currents operator matrix element can be measured in photon–proton interactions with lepton pair production. For this aim a charge-odd combination of cross sections can be constructed as a conversion of leptonic 3-rank tensor with hadronic ones. These experiments can be considered as an alternative to deep virtual Compton scattering.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2006

1. INTRODUCTION

The charge-odd contribution to cross section of processes

$$\begin{aligned} &e^-(p_1) + p(p) \to e^-(p_1') + p(p') + \mu^-(q_-) + \mu^+(q_+), \\ &e^-(p_1) + p(p) \to e^-(p_1') + p(p') + \pi^-(q_-) + \pi^+(q_+), \\ &e^-(p_1) + p(p) \to e^-(p_1') + p(p') + \pi^-(q_-) + \pi^+(q_+) + \pi^0(q_0) \end{aligned} \tag{1}$$

is caused by interference of amplitudes describing the two-photon mechanism and one-photon mechanism of meson set production (see Fig. 1, a, b). The similar

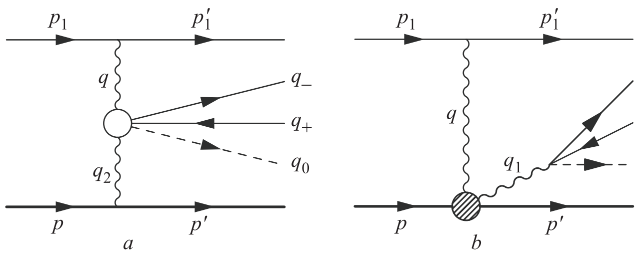

Fig. 1. The mechanisms of production of muons (pions) pair and three pions state

quantity can be constructed for inelastic collisions of photon with hadron by production of lepton pairs. The properties of effective meson Lagrangian as well as three-current correlator matrix elements, averaged on hadron states can be measured in relevant experiments, which is a motivation of our paper. We consider below the experimental set-up corresponding to the kinematical region of proton fragmentation, which means that the recoil proton and mesons created move in directions close to the initial proton motion in the center-of-mass system of initial particles, with invariant mass square of this jet s_1 much smaller than the center-of-mass square of the total energy of initial particles $s + M^2$ (M is a proton mass, we put below the electron mass as well as μ- and π-meson masses m to be small and neglect the terms of the order of m^2/s_1, s_1/s).

For laboratory system (initial proton in rest) the angles of emission of the produced particles can be of the order of unity (see discussion below).

This quantity can be measured using the combination of the double differential cross sections

$$\frac{d\sigma^{\text{odd}}}{d\Gamma} = \frac{1}{2}[F(q_+, q_-, X) - F(q_-, q_+, X)], \quad \frac{d\sigma}{d\Gamma} = F(q_+, q_-, X), \tag{2}$$

where X is the characteristics of other particles, $d\Gamma$ is the phase volume of final particles.

The paper is organized as follows. In Sec. 2 we calculate charge-odd cross section for processes (1). In Sec. 3 the charge-odd inelastic photon–hadron (proton) scattering is discussed. In Conclusion we estimate the order of charge-odd contribution, give spectral distribution and discuss the background effects.

2. MESON PRODUCTION

The remarkable feature of such a kinematics — the relevant contribution to the cross section does not depend on s in the high-energy limit. The corresponding matrix elements are proportional to s. This fact can be explicitly seen using the Gribov's representation of nominator of the virtual photon Green function in Feynman gauge:

$$g_{\mu\nu} = g_{\perp\mu\nu} + \frac{2}{s}[p_{1\mu}\tilde{p}_\nu + p_{1\nu}\tilde{p}_\mu], \tag{3}$$

with light-like vectors p_1, $\tilde{p} = p - \frac{M^2}{s}p_1$, $p_1^2 = \tilde{p}^2 = 0$.

Matrix element corresponding to one-photon mechanism of $\mu^+\mu^-$-meson pair production («bremsstrahlung» ones) has the form

$$M_1 = \frac{(4\pi\alpha)^2}{q_1^2 q_2^2}\frac{2}{s}sN_1 s\bar{u}(p')V_\mu^p u(p)J_\mu^m, \tag{4}$$

with $q = p_1 - p_1'$, $q_1 = q_+ + q_-$, $J_\mu^m = \bar{u}(q_-)\gamma_\mu v(q_+)$ — the conversion of virtual photon to muon pair current, and

$$\begin{aligned}
N_1 &= \frac{1}{s}\bar{u}(p_1')\tilde{p}_\nu\gamma^\nu u(p_1),\\
V_\mu^p &= \hat{p}_1\frac{\hat{p}' - \hat{q} + M}{d_1}\gamma_\mu + \gamma_\mu\frac{\hat{p} + \hat{q} + M}{d}\hat{p}_1,\\
d &= (p+q)^2 - M^2, \qquad d_1 = (p'-q)^2 - M^2.
\end{aligned}$$

When summing over spin states of electron (initial and scattered) we have $\sum |N_1|^2 = 2$. As well the averaged on waves of initial and recoil proton functions the quantity V_μ will be finite in the high-energy limit.

The two-photon mechanism matrix element has the form

$$M_2 = \frac{(4\pi\alpha)^2}{q^2 q_2^2}\frac{2}{s}sN_1\bar{u}(p')\gamma_\lambda u(p)sI_\lambda, \quad q_2 = p - p', \tag{5}$$

with $I_\lambda = \bar{u}(q_-)V_\lambda^m v(q_+)$ — two-photon conversion to muon pair current

$$V_\lambda^m = \hat{p}_1 \frac{\hat{q}_- - \hat{q}}{d_-}\gamma_\lambda + \gamma_\lambda \frac{-\hat{q}_+ + \hat{q}}{d_+}\hat{p}_1, \quad d_\pm = (q_\pm - q)^2 - m^2. \tag{6}$$

The similar expression is valid for creation of pion pair bremsstrahlung matrix element. It can be obtained from the muon pair by replacing J_μ^m by $J_\mu^\pi = (q_- - q_+)_\mu$. For pion pair creation by two-photon mechanism the replacement $I_\lambda^m \to I_\lambda^\pi$ must be done in the relevant matrix element for muons

$$I_\lambda^\pi = \frac{1}{s}p_1^\nu \left[\frac{(2q_- - q)_\nu(-2q_+ + q_2)_\lambda}{d_-} + \frac{(-2q_+ + q)_\nu(2q_- - q_2)_\lambda}{d_+} - 2g_{\nu\lambda} \right]. \tag{7}$$

For the case of three-pion production we must replace the one-photon conversion to two pions current J_μ by $J_\mu^{3\pi} = \frac{1}{4\pi^2 f_\pi^3}(\mu q_+ q_- q_0)$, with $(\mu q_+ q_- q_0) = \epsilon_{\mu\alpha\beta\gamma} q_+^\alpha q_-^\beta q_0^\gamma$.

For two-photon conversion to three pions we use (see [1] for details)

$$\begin{aligned} \Pi_\nu = \frac{p_1^\nu}{s} \Big[& \rho(\mu\nu q q_2) + (\mu\nu(q_2 - q)q_0) - \\ & - \frac{q_+^\nu}{q_+ q_2}(\mu q q_- q_0) - \frac{q_-^\nu}{q_- q_2}(\mu q q_+ q_0) - \frac{q_+^\mu}{q_+ q}(\nu q_2 q_- q_0) - \frac{q_-^\mu}{q_- q}(\nu q_2 q_+ q_0) \Big], \end{aligned} \tag{8}$$

with $\rho = \frac{5}{3} - 6(q_+q_-)/(q_+ + q_- + q_0)^2$. Here $f_\pi = 94$ MeV is the pion decay constant.

Due to gauge invariance the replacement $p_1 \to q$ in expressions V_μ^p, V_λ^m, I_λ^π, Π_ν turns them to zero. Keeping in mind the approximate kinematical relation $q = \frac{s_1}{s}p_1 + q_\perp$, $q_\perp p_1 = q_\perp p = 0$, with $q_\perp$ — transversal component of the transfer momentum q, one can be convinced that all these currents turn to zero at $q_\perp \to 0$ limit.

At this stage, we use the Sudakov's parametrization of 4-momenta of the problem (see Appendix A). Accepting it, we perform the phase volume of the process of type $2 \to 4$ and $2 \to 5$ defined as

$$\begin{aligned} d\Gamma_4 &= \frac{(2\pi)^4}{(2\pi)^{12}} \frac{d^3p_1'}{2E_1'} \frac{d^3q_+}{2E_+} \frac{d^3q_-}{2E_-} \frac{d^3p'}{2E'} \delta^4(p_1 + p - p_1' - p' - q_+ - q_-); \\ d\Gamma_5 &= \frac{(2\pi)^4}{(2\pi)^{15}} \frac{d^3p_1'}{2E_1'} \frac{d^3q_+}{2E_+} \frac{d^3q_-}{2E_-} \frac{d^3q_0}{2E_0} \frac{d^3p'}{2E'} \delta^4(p_1 + p - p_1' - p' - q_+ - q_- - q_0), \end{aligned}$$

to the form

$$d\Gamma_4 = \frac{1}{(2\pi)^8}\frac{d^2q^\perp\ d^2q_+^\perp\ d^2q_-^\perp\ dx_+\ dx_-}{8sx_+x_-(1-x_+-x_-)};$$
$$d\Gamma_5 = \frac{1}{(2\pi)^{11}}\frac{d^2q^\perp\ d^2q_+^\perp\ d^2q_-^\perp\ d^2q_0^\perp\ dx_+\ dx_-\ dx_0}{16sx_+x_-x_0(1-x_+-x_--x_0)}.$$

Deriving these expressions we had introduced an auxiliary integration $\int d^4q\delta^4(p_1 - p_1' - q) = 1$, using the relation $d^3q_i/(2E_i) = d^4q_i\delta(q_i^2 - m_i^2) = \frac{dx_i d^2q_{i\perp}}{2x_i}$ with x_i — the energy fraction of ith particle in the center of mass of colliding beams. Further we denote $q_i^\perp \equiv (0, \vec{q}_i)$, where $i = \pm, 0$ and $\vec{q}_i$ is the two-dimensional vector lying in the plane orthogonal to beam line.

The standard procedure leads to the charge-odd contribution to the cross sections:

$$d\sigma_{\mu\mu}^{\rm odd} = \frac{16\alpha^4(d^2\vec{q}/\pi)(d^2\vec{q}_+/(2\pi))(d^2\vec{q}_-/(2\pi))dx_+dx_-}{\pi(q^2)^2q_1^2q_2^2x_+x_-(1-x_+-x_-)}R^{(\mu)},$$
$$d\sigma_{\pi\pi}^{\rm odd} = \frac{4\alpha^4(d^2\vec{q}/\pi)(d^2\vec{q}_+/(2\pi))(d^2\vec{q}_-/(2\pi))dx_+dx_-}{\pi(q^2)^2q_1^2q_2^2x_+x_-(1-x_+-x_-)}R^{(\pi)},$$
$$d\sigma_{3\pi}^{\rm odd} = \frac{4\alpha^4(d^2\vec{q}/\pi)(d^2\vec{q}_+/(2\pi))(d^2\vec{q}_-/(2\pi))(d^2\vec{q}_0/(2\pi))dx_+dx_-dx_0}{\pi(q^2)^2q_1^2q_2^2x_+x_-x_0(1-x_+-x_--x_0)}\times$$
$$\times\left(\frac{M^3}{4\pi f_\pi^3}\right)^2 R^{(3\pi)},$$

with

$$R^{(\mu)} = \frac{1}{4}Tr(\hat{p}' + M)V_\mu^p(\hat{p} + M)\gamma_\lambda * \frac{1}{4}Tr\hat{q}_-V_\lambda^m\hat{q}_+\gamma_\mu,$$
$$R^{(\pi)} = \frac{1}{4}Tr(\hat{p}' + M)V_\mu^p(\hat{p} + M)\gamma_\lambda(q_- - q_+)_\mu I_\lambda^\pi,$$
$$R^{(3\pi)} = \frac{1}{M^4}\frac{1}{4}Tr(\hat{p}' + M)V_\mu^p(\hat{p} + M)\gamma_\lambda(\mu q_+ q_- q_0)\Pi_\lambda.$$

Explicit forms of V_μ^p, V_λ^m, I_λ^π, Π_λ in terms of the Sudakov's variables are given above; the ones for the case $\vec{q}^{\,2} < s_1$ are given in Appendix A. For general case, the expressions R_i are complicated. For the realistic case $-q^2 = \vec{q}^{\,2} < s_1$, they can be considerably simplified. Really, one can perform the angular averaging on transfer momentum $\vec{q}$ and can omit the terms of higher order of $\vec{q}^{\,2}$ in the nominators.

Performing the integration on transversal momenta of mesons we obtain

$$\frac{d\sigma_{\mu\mu}^{\rm odd}}{dx_+dx_-d\vec{q}^2} = \frac{\alpha^4}{\pi M^2\vec{q}^{\,2}}F^{(2\mu)}x_+, x_-, \tag{9}$$

$$F^{(2\mu)}(x_+, x_-) = \frac{(x_+ - x_-\Delta)(3 - 14\Delta + 16\Delta^2)}{6(1-\Delta)^3}. \tag{10}$$

Note that in the paper of one of us [2] the similar quantity was considered for process $e^+e^- \to e^+e^-\mu^+\mu^-$.

The similar manipulations for $\pi^+\pi^-$-pair production lead to

$$\frac{d\sigma^{\mathrm{odd}}_{\pi\pi}}{dx_+ dx_- d\vec{q}^{\,2}} = \frac{\alpha^4}{\pi M^2 \vec{q}^{\,2}} F^{(2\pi)} x_+, x_-, \tag{11}$$

$$F^{(2\pi)} x_+, x_- = \frac{(x_+ - x_-)\Delta(1+\Delta)}{3(1-\Delta)^4}. \tag{12}$$

For the case of three-pion production we have

$$\frac{d\sigma^{3\pi}_{\mathrm{odd}}}{dx_+ dx_- d\vec{q}^{\,2}} = \frac{\alpha^4 M^4}{16\pi^5 f_\pi^6 \vec{q}^{\,2}} F^{(3\pi)} x_+, x_-, \tag{13}$$

$$F^{(3\pi)} x_+, x_- = \frac{1}{x_+ x_- \vec{q}^{\,2}} \int \frac{dx_0 d^2q_+ d^2q_- d^2q_0}{\pi^3 x_0 (1 - x_+ - x_- - x_0) q_1^2 q_2^2} \, \bar{R}^{3\pi}, \tag{14}$$

where integration is performed with additional condition $0.2 < \Delta = 1 - x_+ - x_- - x_0 < 1$, and $\bar{R}^{3\pi}$ is averaged by azimuthal angle of transfer momentum $\vec{q}$

$$\bar{R}^{3\pi} = \int_0^{2\pi} \frac{d\phi}{2\pi} R^{3\pi}, \tag{15}$$

where ϕ is the azimuthal angle between vector $\vec{q}$ and the vector of problem. The results of numerical integration for $F^{(3\pi)}(x_+, x_-)$, as well as $F^{(2\mu)}$ and $F^{(2\pi)}$, are given in Tables 1, 2, 3.

Table 1. The results of integration for odd part of spectrum of 2μ production $100 \cdot F^{(2\mu)} x_+, x_-$ (see (10)) for the case $\vec{q}^{\,2} < s_1$

x_-/x_+	0.15	0.25	0.35	0.45	0.55	0.65	0.75
0.15	0.000	5.625	0.000	–0.370	1.399	2.734	2.414
0.25	–5.625	0.000	–0.123	0.700	1.641	1.610	
0.35	0.000	0.123	0.000	0.547	0.805		
0.45	0.370	–0.700	–0.547	0.000			
0.55	–1.399	–1.641	–0.805				
0.65	–2.734	–1.610					
0.75	–2.414						

Table 2. The results of integration for odd part of spectrum of 2π production $F^{(2\pi)}x_+, x_-$ (see (12)) for the case $\vec{q}^{\,2} < s_1$

x_-/x_+	0.15	0.25	0.35	0.45	0.55	0.65	0.75
0.15	0.000	1.250	0.800	0.432	0.217	0.098	0.034
0.25	–1.250	0.000	0.144	0.108	0.059	0.022	
0.35	–0.800	–0.144	0.000	0.020	0.011		
0.45	–0.432	–0.108	–0.020	0.000			
0.55	–0.217	–0.059	–0.011				
0.65	–0.098	–0.022					
0.75	–0.034						

Table 3. The results of numerical integration for odd part of spectrum of 3π production $100 \cdot F^{(3\pi)}x_+, x_-$ (see (14)) for the case $\vec{q}^{\,2} < s_1$

x_-/x_+	0.15	0.25	0.35	0.45	0.55
0.15	0.00	1.71	5.74	11.38	10.76
0.25	–1.72	0.00	3.52	4.86	
0.35	–5.74	–3.52	0.00		
0.45	–11.36	–4.88			
0.55	–10.74				

3. PROBING THREE-CURRENT CORRELATOR IN CHARGE-ODD EXPERIMENTAL SET-UP OF PHOTON–PROTON COLLISIONS

In photon–proton collisions with production of lepton pair

$$\gamma(k) + p(p) \to e^+(q_+) + e^-(q_-) + X(q_h) \tag{16}$$

with charge-odd experimental set-up

$$\frac{d\sigma_{\rm odd}}{d\Gamma} = \frac{1}{2}\Big[F(q_+, q_-, q_h) - F(q_-, q_+, q_h)\Big], \quad \frac{d\sigma}{d\Gamma} = F(q_+, q_-, q_h), \tag{17}$$

where $d\Gamma$ is the phase volume of final particles including leptons, we can to measure the three electromagnetic currents correlator

$$H_{\mu\nu\lambda} = < p|J_\mu(k)J_\nu(q)J_\lambda(q_1)|p > . \tag{18}$$

Really, in such a kind of experiment the interference of amplitudes of two mechanisms of lepton pair creation can be measured. One of them (two-photon mechanism) corresponds to charge-even state of lepton pair, another one (bremsstrahlung mechanism) describes the creation of a pair by the single virtual photon (see Fig. 2, a, b).

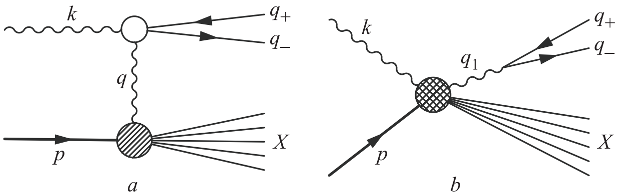

Fig. 2. The two mechanisms of lepton pair creation: a) production of lepton pair in charge-even state; b) the bremsstrahlung mechanism

Charge-odd cross section can be written in the form

$$\frac{d\sigma}{d\Gamma_+ d\Gamma_-} \sim \frac{\alpha^3}{q^2 q_1^2} L^{(e)}_{\mu\nu\lambda} H_{\mu\nu\lambda} d\Gamma_h, \tag{19}$$

with $d\Gamma_h$ — the phase volume of the final hadron system. Leptonic tensor $L^{(e)}_{\mu\nu\lambda}$ (we neglect lepton mass)

$$L^{(e)}_{\mu\nu\lambda} = \frac{1}{4} Tr \left[\hat{q}_- O_{\mu\nu} \hat{q}_+ \gamma_\lambda\right] \tag{20}$$

with

$$O_{\mu\nu} = \frac{1}{\kappa_-}\gamma_\mu(\hat{q}_- - \hat{k})\gamma_\nu + \frac{1}{\kappa_+}\gamma_\nu(-\hat{q}_+ + \hat{k})\gamma_\mu, \kappa_\pm = 2kq_\pm \tag{21}$$

obeys the gauge conditions $L^{(e)}_{\mu\nu\lambda} k^\mu = L^{(e)}_{\mu\nu\lambda} q^\nu = L^{(e)}_{\mu\nu\lambda} q_1^\lambda = 0$ with

$$k^2 = q_\pm^2 = 0, \quad k + q = q_1 = q_+ + q_-. \tag{22}$$

Leptonic tensor can be written in explicitly gauge-invariant form:

$$\begin{aligned}
L^{(e)}_{\mu\nu\lambda} &= Q_\lambda T^Q_{\mu\nu} + P_\nu T^P_{\mu\lambda} + R_\mu T^R_{\nu\lambda}, \\
Q_\lambda &= \frac{1}{2}\frac{1}{\kappa_+} - \frac{1}{\kappa_-}[\kappa_+ \tilde{q}_{-\lambda} + s_1 \tilde{k}_\lambda], \\
P_\nu &= \frac{s_1}{2}\frac{1}{\kappa_-} - \frac{1}{\kappa_+}\tilde{k}_\nu + \frac{\kappa_- + \kappa_+}{2\kappa_-}\tilde{q}_{-\nu}, \\
R_\mu &= \frac{s_1}{2}\frac{1}{\kappa_-} - \frac{1}{\kappa_+}\tilde{k}_\mu - \frac{2s_1 + \kappa_+ + \kappa_-}{2\kappa_-}\tilde{q}_{-\mu},
\end{aligned}$$

$$\tilde{q}_{-\mu} = q_{-\mu} - \frac{q_- k}{q_+ k} q_{+\mu}, \quad \tilde{k}_\mu = k_\mu,$$
$$\tilde{q}_{-\nu} = q_{-\nu} - \frac{q_- q}{q_+ q} q_{+\nu}, \quad \tilde{k}_\nu = k_\nu - \frac{kq}{q_+ q} q_{+\nu},$$
$$\tilde{q}_{-\lambda} = q_{-\lambda} - \frac{q_- q_1}{q_+ q_1} q_{+\lambda}, \quad \tilde{k}_\lambda = k_\lambda - \frac{kq_1}{q_+ q_1} q_{+\lambda},$$

besides,

$$T^Q_{\mu\nu} = \tilde{g}_{\mu\nu} + \frac{\tilde{k}_\nu \tilde{q}_{-\mu}}{kq}; \quad T^P_{\mu\lambda} = \tilde{g}_{\mu\lambda} + \frac{\tilde{k}_\lambda \tilde{q}_{-\mu}}{kq_1}; \quad T^R_{\nu\lambda} = \tilde{g}_{\nu\lambda} + \frac{\tilde{q}_{-\nu}(\tilde{q}_- - \tilde{k})_\lambda}{qq_1}; \tag{23}$$

and, finally,

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{k_\nu q_\mu}{kq}; \quad \tilde{g}_{\mu\lambda} = g_{\mu\lambda} - \frac{k_\lambda q_{1\mu}}{kq_1}; \quad \tilde{g}_{\lambda\nu} = g_{\lambda\nu} - \frac{q_\lambda q_{1\mu}}{qq_1}. \tag{24}$$

The form of hadronic 3-rank tensor depends on experimental conditions of detection of hadron jet particles. It won't be touched here.

4. CONCLUSION

The processes with meson production mentioned above can be studied at such facilities as HERA, HERMES and RHIC. Photon–hadron interaction processes (the analog of DIS experiments) can be realized at the facilities with the high-energy photon beams. The effective meson Lagrangian predictions can be examined for two- and three-pion productions for the experiments of the first class. In particular, the anomaly $2\gamma \to 3\pi$ can be measured.

For experiments with photon–hadron production the three electromagnetic current correlations can be studied. Unfortunately, these correlations are very poorly investigated in experiments as well, as theoretically [3].

Our results were obtained in the framework of QED with point-like mesons. In real applications we must include form factors of pion $J^{2\pi}_\mu \to F_\pi(q_1^2)(q_+ - q_-)_\mu$. Another modification is replacement of QED coupling constant used for proton–photon interaction by that ones for proton–ρ-meson interaction: $\alpha^4 \to \alpha^2(g^2_{\rho nn}/(4\pi))^2$. Besides, we must take into account the resonance character of vector meson propagators

$$\frac{1}{q_1^2} \to Re\frac{1}{q_1^2 - M_\rho^2 + iM_\rho\Gamma_\rho} = \frac{q_1^2 - M_\rho^2}{q_1^2 - M_\rho^{2\,2} + M_\rho^2\Gamma_\rho} \tag{25}$$

for $\gamma^* \to \rho \to 2\pi$ and the similar expression with replacement $M_\rho, \Gamma_\rho \to M_\omega, \Gamma_\omega$ for $\gamma^* \to \omega \to 3\pi$ case. All these factors were not included in calculations of spectra given above.

Charge-odd effects in two-pion production in proton fragmentation region provide besides the possibility to measure the deviation from point-pion approximation used above. Really, the subprocess of two charged pions production at two-photon collisions for the case when one of them is real and another is virtual

$$\gamma(q) + \gamma^*(q_2) \to \pi(q_-) + \pi(q_+) \tag{26}$$

can be described in terms of three kinematical singularities free amplitudes [4]:

$$\begin{aligned} T^{\pi_-\pi_+}_{\rho\sigma} &= a_1 L^{(1)}_{\rho\sigma} + a_2 L^{(2)}_{\rho\sigma} + a_3 L^{(3)}_{\rho\sigma}, \\ L^{(1)}_{\rho\sigma} &= (qq_2)g_{\rho\sigma} - q_{2\sigma}q_\rho, \\ L^{(2)}_{\rho\sigma} &= -(qq_2)Q_\rho Q_\sigma + (qQ)(q_{2\sigma}Q_\rho - q_\rho Q_\sigma) + (qQ)^2 g_{\rho\sigma}, \\ L^{(3)}_{\rho\sigma} &= (qQ)(q_2^2 g_{\rho\sigma} - q_{2\rho}q_{2\sigma}) + Q_\sigma((qq_2)q_{2\rho} - q_2^2 q_\rho), \end{aligned} \tag{27}$$

with $Q = (q_+ - q_-)/2$. All three tensor structures are gauge-invariant

$$L^{(i)}_{\rho\sigma}q^\sigma = L^{(i)}_{\rho\sigma}q_2^\rho = 0. \tag{28}$$

The case of point-like pions corresponds to the choice

$$a_0^{(1)} = \frac{-1}{\chi_+ + \chi_-}\frac{\chi_+}{\chi_-} + \frac{\chi_-}{\chi_+}, \quad a_0^{(2)} = \frac{4}{\chi_+\chi_-}, \quad a_0^{(3)} = 0, \tag{29}$$

where $2qQ = \chi_+ - \chi_-$, $qq_2 = \chi_+ + \chi_-$, $\chi_\pm = qq_\pm$. We note that in charge-odd experimental set-up differential cross section contains the linear combination of amplitudes.

Photon–proton deep inelastic interaction (see Sec. 3) can be considered as an alternative to deep inelastic Compton scattering (see Fig. 3, a, b) where as well three-current correlator $H_{\mu\nu\lambda}$ can be measured [6].

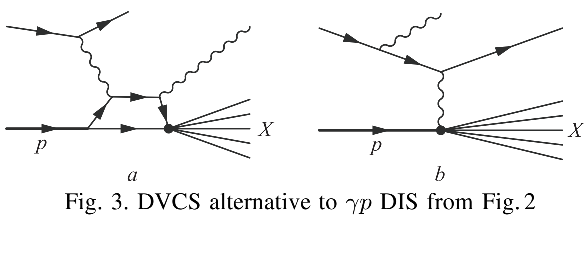

Fig. 3. DVCS alternative to γp DIS from Fig. 2

APPENDIX A

SUDAKOV'S PARAMETRIZATION

For light lepton–proton scattering $e(p_1) + p(p) \to e(p_1') + q(q_-) + \bar{q}(q_+) + \dots + p(p')$ in high-energy limit (keeping in mind the experimental requirement of detecting the final state particles, i. e., we must imply the polar angles between their 3-momenta and the beam axes to be sufficiently large) we can consider the leptons $(e^\pm, \mu^\pm)$ as well as pions $\pi^{+,-,0}$ to be massless. Errors caused by this assumptions is of the order

$$O\left(\left(\frac{m_\pi}{M}\right)^2, \frac{m_\pi^2}{s_1}, \frac{s_1}{s}\right), \tag{A1}$$

with m_π, M — masses of pion and proton; s_1 — invariant mass square of produced particles (excluding the scattered electron), $s = 2pp_1 \gg s_1 \sim M^2$.

Introducing the light-like 4-vector $\tilde{p} = p - p_1(M^2/s)$ we use the standard Sudakov parametrization of 4-momenta of problem

$$\begin{aligned}
&q_i = x_i\tilde{p} + \beta_i p_1 + q_{i\perp}, \qquad a_\perp p_1 = a_\perp p = 0, \qquad \tilde{p}^2 = p_1^2 = 0,\\
&p' = \Delta\tilde{p} + \beta_p p_1 + p'_\perp, \qquad q^2 = q_\perp^2 = -\vec{q}^{\,2}, \qquad p = \tilde{p} + \frac{M^2}{s}p_1,\\
&q = p_1 - p_1' = \alpha_q\tilde{p} + \beta_q p_1 + q_\perp.
\end{aligned} \tag{A2}$$

We imply $\vec{q}_i$ to be two-dimensional vectors situated in the plane transversal to the initial electron direction of motion (chosen as z-axis direction).

Putting on the mass-shell conditions $q_i^2 = 0$, $(p')^2 = M^2$, permits to exclude the «small» coefficients β_i:

$$\beta_i = \frac{\vec{q}_i^{\,2}}{sx_i}, \qquad \beta_p = \frac{(\vec{p}_i')^2 + M^2}{s\Delta}. \tag{A3}$$

Both light-cone components of transfer momentum q $(\alpha_q,\ \beta_q)$ are small (of the order of s_1/s), so we have $q^2 \approx -\vec{q}^{\,2}$. The conservation law reads as

$$\begin{aligned}
q + p &= q_+ + q_- + \ldots + p',\\
1 &= x_+ + x_- + \ldots + \Delta,\\
\vec{q} &= \vec{q}_+ + \vec{q}_- + \ldots + \vec{p}',\\
\beta_q + \frac{M^2}{s} &= \beta_+ + \beta_- + \ldots + \beta_p.
\end{aligned}$$

In the center of mass of initial particles the quantities x_i are the fractions of energy of the initial proton. Scattering angles of the set of particles, moving along initial proton direction of motion θ_i, are small quantities $\theta_i = \dfrac{2|\vec{q}_i|}{x_i\sqrt{s}}$.

Special attention must be paid for describing the processes in the laboratory frame with resting proton. In this frame the light-like 4-vectors are

$$\tilde{p} = \frac{M}{2}(1,-1,0,0), \quad p_1 = E(1,1,0,0), \quad s = 2ME. \tag{A4}$$

Energies of pions are $E_i = \frac{x_i M}{2} + \frac{\vec{q}_i^{\,2}}{2x_i M}$ and the energy of the scattered proton is $E' = \frac{\Delta M}{2} + \frac{(\vec{p}')^2 + M^2}{2M\Delta}$. The scattering angles of the pions and recoil proton are the quantities of the order of unity*:

$$\sin\theta_i = \frac{2x_i M|\vec{q}_i|}{M^2 x_i^2 + \vec{q}_i^{\,2}},$$
$$\tan\theta = \frac{2M\Delta|\vec{p}'|}{(\vec{p}')^2 + M^2(1-\Delta^2)}.$$

We put here the expressions of kinematical invariants entering R_i in terms of the Sudakov's variables:

$$\begin{aligned}
d_\pm &= q_\pm - q^2 - m^2 = -\vec{q}^{\,2} + 2\vec{q}_\pm\vec{q} - s_1 x_\pm, \\
2q_\pm q_2 &= 2\vec{q}_\pm\vec{p}' + \frac{\vec{q}_\pm^{\,2}\bar{\Delta}}{x_\pm} - \frac{x_\pm}{\Delta}[(\vec{p}')^2 + \bar{\Delta}M^2], \\
2q_\pm q &= -2\vec{q}\vec{q}_\pm + s_1 x_\pm, \\
2q_+ q_- &= \frac{(\vec{q}_- x_+ - \vec{q}_+ x_-)^2}{x_- x_+}, \\
2q_\pm q_0 &= \frac{(\vec{q}_\pm x_0 - \vec{q}_0 x_\pm)^2}{x_\pm x_0}, \\
q_2^2 &= -\frac{1}{\Delta}[(\vec{p}')^2 + \bar{\Delta}^2 M^2], \qquad \bar{\Delta} = 1 - \Delta, \\
q_1^2 &= (q + q_2)^2 = -\vec{q}^{\,2} + q_2^2 + 2\vec{q}\vec{p}' + s_1\bar{\Delta}.
\end{aligned} \tag{A5}$$

The quantity s_1 for $\pi_+, \pi_- p'$ jet has the form

$$s_1 = -M^2 + \frac{\vec{q}_+^{\,2}}{x_+} + \frac{\vec{q}_-^{\,2}}{x_-} + \frac{\vec{p}'^2 + M^2}{\Delta}, \vec{q} = \vec{q}_+ + \vec{q}_- + \vec{p}'; \tag{A6}$$

and for $\pi_+, \pi_-, \pi_0 p'$ jet is

$$s_1 = -M^2 + \frac{\vec{q}_+^{\,2}}{x_+} + \frac{\vec{q}_-^{\,2}}{x_-} + \frac{\vec{q}_0^{\,2}}{x_0} + \frac{\vec{p}'^2 + M^2}{\Delta}, \vec{q} = \vec{q}_+ + \vec{q}_- + \vec{q}_0 + \vec{p}'. \tag{A7}$$

*The relations of that type were first obtained by Benaksas and Morrison (see [5] and references therein).

The simplified expressions for vertex functions V_μ^p, V_λ^m, I_λ^π, Π_λ (the lowest order of $|\vec{q}|$ expression) are

$$\begin{aligned}
V_\mu^p &= \frac{2\vec{p}\vec{q}}{s_{10}^2\Delta}\gamma_\mu + \frac{\gamma_\mu\hat{q}_\perp\hat{p}_1}{ss_{10}} + \frac{\hat{p}_1\hat{q}_\perp\gamma_\mu}{ss_{10}\Delta}, \quad \vec{p} = \vec{q}_+ + \vec{q}_-,\\
V_\lambda^m &= \frac{2\vec{q}\vec{r}}{s_{10}^2 x_+ x_-}\gamma_\lambda + \frac{\hat{p}_1\hat{q}_\perp\gamma_\lambda}{s_1 x_-} - \frac{\gamma_\lambda\hat{q}_\perp\hat{p}_1}{s_1 x_+}, \quad \vec{r} = x_-\vec{q}_+ - x_+\vec{q}_-,\\
I_\nu^\pi &= \frac{1}{s_{10}}\left(\frac{2\vec{q}_-\vec{q}}{s_{10}x_-}2q_+ - q_{2\nu} + \frac{2\vec{q}_+\vec{q}}{s_{10}x_+}2q_- - q_{2\nu} + 2q_\nu^\perp\right),\\
\Pi^\nu &= \frac{1}{s}\left[\rho p_1 \nu q_\perp q_2 - p_1\nu q_\perp q_0 - \frac{q_+^\nu}{q_+q_2}p_1 q_\perp q_- q_0 - \frac{q_-^\nu}{q_-q_2}p_1 q_\perp q_+ q_0\right] -\\
&\quad - \frac{1}{s_{10}}\left[\frac{\vec{q}_+\vec{q}}{q_+q}\nu q_2 q_- q_0 + \frac{\vec{q}_-\vec{q}}{q_-q}\nu q_2 q_+ q_0\right],
\end{aligned}$$

with $s_{10} = s_1(\vec{q} = 0)$.

REFERENCES

1. *Witten E.* // Nucl. Phys. B. 1983. V. 223. P. 422;
   Wess J., Zumino B. // Phys. Lett. B. 1971. V. 37. P. 95;
   Kaymakcalan O., Rajeev S., Schechter J. // Phys. Rev. D. 1984. V. 30. P. 594.
2. *Kuraev E. A., Lipatov L. N., Merenkov N. P., Strikman M. I.* // Yad. Fiz. 1976. V. 23. P. 163.
3. *Mikhailov S.* Private communication;
   Ioffe B. L., Smilga A. V. // Nucl. Phys. B. 1983. V. 216. P. 373.
4. *Arbuzov A. B., Astakhov V. A., Fedorov A. V., Fedotovich G. V., Kuraev E. A., Merenkov N. P.* // JHEP. 1997. 9710. 006 [arXiv:hep-ph/9703456].
5. *Vinokurov E., Kuraev E.* // JETP. 1972. V. 63. P. 1142.
6. *Radyushkin A. V.* // Phys. Lett. B. 1996. V. 380. P. 417 [arXiv:hep-ph/9604317].

Received on April 24, 2006.

Корректор *Т. Е. Попеко*

Подписано в печать 12.09.2006.
Формат $60 \times 90/16$. Бумага офсетная. Печать офсетная.
Усл. печ. л. 0,93. Уч.-изд. л. 1,35. Тираж 415 экз. Заказ № 55458.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/