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SELF-ORGANIZATION OF PHYSICAL FIELDS AND SPIN

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Самоорганизация физических полей и спин

Темой настоящего исследования являются законы внутренней самоорганизации фундаментальных физических полей. В рамках теории самоорганизации открыта геометрическая и физическая природа спиновых явлений. Спиновая симметрия (фундаментальная реализация концепции геометрической внутренней симметрии) и спиновое поле (пространство определяющего представления спиновой симметрии) являются ключевыми элементами. Показано, что сущность спина проявляется как биполярная структура спиновой симметрии, индуцированная гравитационными потенциалами. Биполярная структура обеспечивает естественное нарушение спиновой симметрии и ведет к спинстатике (теории спинового поля вне времени) и спиндинамике. Выведены уравнения спинстатики и спиндинамики. Показано, что формула Зоммерфельда может быть выведена из уравнений спиндинамики и, следовательно, принцип соответствия выполняется. Это означает, что теория самоорганизации обеспечивает новое понимание спиновых явлений.

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Self-Organization of Physical Fields and Spin

The subject of the present investigation is the laws of intrinsic self-organization of fundamental physical fields. In the framework of the Theory of Self-Organization the geometrical and physical nature of spin phenomena is uncovered. The key points are spin symmetry (the fundamental realization of the concept of geometrical internal symmetry) and the spinning field (space of defining representation of spin symmetry). It is shown that the essence of spin is the bipolar structure of spin symmetry induced by the gravitational potentials. The bipolar structure provides natural violation of spin symmetry and leads to spinstatics (theory of spinning field outside the time) and spindynamics. The equations of spinstatics and spindynamics are derived. It is shown that Sommerfeld's formula can be derived from the equations of spindynamics and hence the correspondence principle is valid. This means that the Theory of Self-Organization provides the new understanding of spin phenomena.

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INTRODUCTION

Einstein's Theory of Gravitation and Quantum Mechanics show clearly that the physical laws have a more objective character than presupposed earlier. This is easy seen since coordinates have no physical meaning in both General Theory of Relativity and Quantum Mechanics (notion of trajectory disappears in quantum theory of particles, uncertainty principle [1], principle of general invariance [2]). Thus, the General Theory of Relativity and Quantum Mechanics are very important evidences of the existence of a unique geometric field theory that expresses the laws of nature in the absolutely objective form. The «field theory without observers» is evidently based on the principle that laws of the real world are absolutely independent of any outer and a priori conditions. The existence of these laws is not evident since, by its nature, the field theory based on the principle of absolute objectivity establishes the intrinsic laws of Self-Organization of the physical fields and raises the questions such as: What is the status of geometry in this case? What is physical space of the self-organizing field system? What is time? What is evolution in time? What is causality? Is it possible to formulate discrete symmetries in a geometrical coordinate-independent form? What is internal symmetry from the geometrical point of view? What is spin and charge? All these questions have a very general character but this seems to be unavoidable in order to understand the deeper levels of living reality. It is no doubt that the concept of physical space is basic and especially important since we have very strong evidences that the essential properties of physical space are tightly connected with the most fundamental laws governing the behavior of matter and, hence, these properties predetermine the objective physical regularities.

We express all details of this intimate relationship in the exact mathematical form and establish the basic principles and laws of the Theory of Self-Organization as an integral and logically coherent system. The observer himself can use Theory of Self-Organization as a unique instrument to learn more objective information about the physical phenomena itself, and to find new creative approaches and innovative possibilities for his own goals (the calculations and any activity can be fruitful only on the reliable basis of understanding that Theory of Self-Organization provides in full measure).

The development of Theory of Self-Organization was originated by the creation of the natural, argued, and novel concept of time [3]. Time by itself exists in the form of the scalar temporal field and is the cornerstone of any dynamical theory and, hence, the dynamical theory of spin as well. The coming-into-being of Theory of Self-Organization has been completed here with formulation of the theory of spin as manifestation of the geometrical structure of physical space in the form of spin symmetry with its bipolar structure. From the new concept of time it follows directly that there are phenomena outside the time (in this case a temporal field is simply absent) [3]. In Theory of Self-Organization this takes the form of two divisions: the static (timeless) Theory of Self-Organization and the dynamical Theory of Self-Organization. Timeless Theory of Self-Organization of the physical fields represents entirely a new division of the field theory and involves the following chapters: spinstatics and gefstatics (the theory of the general electromagnetic field outside the time). Gravistatics as an independent chapter of Theory of Self-Organization is not considered since its equations with the absence of other fields have only a trivial solution. In the dynamical Theory of Self-Organization we correspondingly distinguish gravidynamics, spindynamics, the dynamical theory of general electromagnetic field (gefdynamics). Gefdynamics involves the Maxwell theory as the theory of a singlet state of the general electromagnetic field. The equations of gefdynamics in the geometrical form were established in [4]. The connection between statics and dynamics is very simple. We consider the 3-dimensional physical space in the static Theory of Self-Organization as an initial space cross section of 4-dimensional physical space in the dynamical Theory of Self-Organization. Due to this the problem of initial singularity disappears. The Big Bang can be considered as a transition of a physical system from a timeless state to a dynamical state (as a release of the internal potential energy of a timeless system).

The paper is organized as follows. In Sec. 1, we begin a carefull consideration of the conceptual structure of Theory of Self-Organization. The concept of physical space is introduced and the fundamental role of the positive-definite Riemann metric is recognized. In Theory of Self-Organization the physical space is a strict realization of the abstract notion of manifold. In Sec. 2, the concepts of a really geometrical quantity and geometrical internal symmetry are introduced. Geometry has always been at the center of science and it is natural that the really geometrical quantities are the parts of nature. With geometrical internal symmetry we have the real understanding of the mysterious internal symmetries of modern physics (without the introduction of the artificial «charged spaces»). Self-organization presupposes that the physical space and internal symmetry are tightly connected and kept inseparable. In Theory of Self-Organization only three realizations of the general concept of geometrical internal symmetry define actually all intrinsic laws of self-organization. The set of the really geometrical quantities includes the fundamental physical fields. It is very restricted and the fundamental fields are listed. Section 3 represents the novel concept of time and the discrete symmetries in the geometric coordinate-independent form with bilateral (left-right) symmetry as the main topic. The bilateral symmetry is the fundamental realization of the general concept of geometrical internal symmetry and has especially important meaning in spindynamics providing the nontrivial introduction of the temporal structure and sufficient reason of complexification. In Sec. 4, we introduce the concepts of the spinning field and spin symmetry which is the fundamental realization of the general concept of geometrical internal symmetry. Spin symmetry actually involves the concept of the isotopic spin of current physics and in Theory of Self-Organization it is a unique image and a characteristic to describe spin and express its essence. The spinning field is the space of representation of the spin symmetry (general linear group $GL(2^n, \mathbf{R})$. Section 5 is devoted to spinstatics where the spinning field is at absolute rest (outside the time). It is shown that the equations of spinstatics represent the trace of broken spin symmetry but this violation is not complete, and unbroken spin symmetry appears in the form of chiral and orient symmetry. In Sec. 6, we derive the equations of spindynamics in geometrical form. In spindynamics the temporal field enters into the Lagrangian of the spinning field as a symplectic structure due to the nontrivial representation of bilateral symmetry. It is found that the symplectic (skew-symmetric) scalar product is invariant with respect to the peculiar transformations. This observation leads directly to the informal (nonartificial) complexification of the theory and ensures the existence of the probability measure in space of solutions of the spindynamics equations. The importance of this result can be understood as follows. An experimentalist cannot observe a quantum system without producing a serious disturbance and hence he cannot expect to find any causal relation between the results of his observations. However, causality will still be assumed to apply to undisturbed systems and the equations which will be set up to describe an undisturbed system will be differential equations expressing a causal connection between the conditions at one time and the conditions at later time. The deterministic equations of this kind should guarantee the existence of the probabilistic measure in the space of solutions, which is invariant with respect to time translation. This measure has the foundational meaning since it opens up the possibilities of probabilistic description when signals from the disturbed system may be nonpredictable; however, their statistical properties are reproduced with time. For example, a similar situation occurs in the theory of turbulence [5] where the methods of the probability theory have the strict and quite practical meaning. It is evident that the same situation holds for the so-called quantum systems. In spindynamics unbroken spin symmetry plays a more fundamental role than in spinstatics and appears in the form of quaternion structure that can be realized dynamically as the Yang–Mills field. In Sec. 7, the equations of spindynamics are represented in the most symmetrical and obviously dynamical form, using the formalism of vector algebra and vector analysis in the 4-dimensional and general covariant form (developed in [3] in investigating the role of temporal field in the theory of electromagnetic field). Through this, we recognize the operator of evolution in spindynamics as the covariant derivative along the stream of time. It differs from the operator of evolution for the (general) electromagnetic field and the gravitational field, where it is the Lie derivative along the stream of time. In Sec. 8, the concepts of internal spin and spin current are introduced and the problem of central field is considered, which leads exactly to the Sommerfeld formula. Through this, we identify one of the states of the spinning field with an electron in a hydrogen atom. Thus, the correspondence principle holds valid. Summary is given in Sec. 9. Self-organization of the physical fields is an internal field theory in which the physical space and fields are tightly connected and kept inseparable and thus the fundamental problem of geometrization gets an adequate solution. Spin phenomena emerge in a quite definite system of really geometrical quantities. Spinstatics and spindynamics describe these phenomena and give a natural and unique way to connect spin with geometry of physical space. The concept of spin involves the whole complex of foundational notions and the theory of spin phenomena is the fundamental chapter of Theory of Self-Organization which helps one to understand the basic structures and the intrinsic connections of physics and geometry. By and large the theory of spin phenomena gets in the framework of Theory of Self-Organization the status that can be compared with the status of the law of energy conservation.

1. CONCEPT OF PHYSICAL SPACE

The concept of abstract differential manifold is basic in modern differential geometry. In both Geometry and General Relativity a manifold is a priori an element and on the given manifold one considers the different metrics and other structures. This is not convenient for Theory of Self-Organization since it is free from any external and a priori conditions. The abstract theory of manifolds learns a given manifold itself. In this case, a process of coming-into-being cannot be considered. Thus, this abstract structure from the physical point of view is not the most interesting aspect of the theory of manifolds. It is more important to know where and how a manifold of a physical system emerges. After Whitney, we know that the class of abstract differential manifolds is not wider than the class of submanifolds of Euclidean spaces and we use this result to show that the realization of abstract manifold as a surface in Euclidean space of a fairly large number of dimensions permits one to consider a manifold as physical space, i.e., as the inner element of the absolute self-organization.

The algebraic model of the familiar Euclidean space provides a natural generalization. The n-dimensional Euclidean space R^n is the linear structure in the set of n-tuples (vectors)

$$\mathbf{x} = (x^1, \cdots, x^n),$$

which is defined by the natural rules

$$\lambda\mathbf{x} = (\lambda x^1,\ \lambda x^2, \cdots, \lambda x^n),$$

$$\mathbf{x} + \mathbf{y} = (x^1 + y^1, \cdots, x^n + y^n),$$

where λ is a real number and x^a are independent real variables (the coordinates of points in R^n). Every n-tuple $\mathbf{x} = (x^1, \cdots, x^n)$ corresponds a definite position in R^n with the known visualization of this position in the dimensions $n = 1, 2, 3$. The distance function is given by

$$d(\mathbf{x}, \mathbf{y}) = \sqrt{(x^1 - y^1)^2 + \cdots + (x^n - y^n)^2}.$$

From the known formula $d^2(\mathbf{y}, \mathbf{z}) = d^2(\mathbf{x}, \mathbf{y}) + d^2(\mathbf{x}, \mathbf{z}) - 2d(\mathbf{x}, \mathbf{y})d(\mathbf{x}, \mathbf{z}) \cos\varphi$ one can derive the algebraic representation for the $\cos\varphi$ and scalar product

$$(\mathbf{x} \cdot \mathbf{y}) = x^1 y^1 + \cdots + x^n y^n.$$

We give the following definition of physical space which is only possible in Theory of Self-Organization based on the principle of absolute objectivity. The physical space is the 3-dimensional manifold (in the timeless theory of self-organization) and the 4-dimensional manifold (in the dynamical theory of self-organization) that is realized as a surface in the imbedding Euclidean space of a fairly large number of dimensions as follows. The region of 4-dimensional physical space is analytically defined by the equations

$$x^a = F^a(u^1, u^2, u^3, u^4),$$

where the functions $F^a(u^1, u^2, u^3, u^4)$ of four independent variables u^1, u^2, u^3, u^4 (the Gauss coordinates) are the solution of the characteristic system of nonlinear equations in partial derivatives

$$\delta_{ab} \frac{\partial F^a}{\partial u^i} \frac{\partial F^b}{\partial u^j} = g_{ij}(u^1, u^2, u^3, u^4), \quad a,\, b = 1, \cdots, 4 + k,\ k \geqslant 0. \tag{1}$$

The known functions $g_{ij}(u^1, u^2, u^3, u^4)$ in the right-hand side of equations (1) represent the positive-definite Riemann metric

$$ds^2 = g_{ij} du^i du^j, \tag{2}$$

which we put in correspondence to the gravitational field [3]. The functions $g_{ij}(u)$ are the solution of the controlling system of equations which connects the gravitational field $g_{ij}(u)$ (positive-definite Riemann metric) with other fundamental physical fields. In Theory of Self-Organization all physical fields should be a manifestation of the geometrical structure of physical space. The full set of these fields will be outlined in the following section. The controlling system of equations is the essence of the theory and involves the equations of spinstatics and spindynamics, and the equations of gefstatics and gefdynamics. For the gravitational field and the electromagnetic field the controlling system of equations was found in [3] (the case of the general electromagnetic field was considered in [4]). Solving the controlling system of equations we find the functions $g_{ij}(u^1, u^2, u^3, u^4)$ and with this result we can design local physical space and find a minimal dimension of imbedding Euclidean space as a result of solution of the characteristic system of equations (1). It should be noted that one cannot exclude the Riemann metric g_{ij} as an independent element and works with functions $F^a(u^1, \cdots u^4)$ only, since in this case the dimension of the imbedding Euclidean space should be fixed a priori but this is in contradiction with the foundational principle of self-organization that theory should be absolutely independent of any outer conditions. The 3-dimensional physical space of the static Theory of Self-Organization is defined quite analogously.

In Theory of Self-Organization of the physical fields the physical space and a minimal dimension of the imbedding Euclidean space arises as a result of solution of a characteristic system of differential equations (1). The general conclusion lies in the fact that the physical space of a self-organizing system is generated by the system itself and, hence, they are interdependent. The so-defined physical space is the basic notion. This means that all other definitions, notions and laws should be connected with the geometrical structure of physical space to be geometrical and physical. Galileo and Etvesh experiments demonstrate that the «gravitational forces» are inertial in nature and hence they are originated by constraints. The equations $x^a = F^a(u^1, \cdots u^4)$ realize these constraints. The field equations provide no means to rule out either multiply-connected physical spaces, or physical spaces which are nonorientable.

2. CONCEPT OF REALLY GEOMETRICAL QUANTITY

The guide principle of geometrization and self-organization reads: the geometrical structure of physical space (points, curves, congruences of curves, families of curves) determines a very restricted set of really geometrical quantities and along with that geometrical internal symmetry that makes these quantities variable.

There is a set of tensor fields [6] connected with the coordinate covering of a manifold. However, only some types of the tensor fields are connected with the geometrical structure of a manifold and, hence, can be considered as really geometrical quantities. In what follows, we shall extract fundamental physical fields out of a set of really geometrical quantities and this is evidently the geometrization of physics in the strict sense. The points, curves and families of curves (submanifolds) put together the geometrical structure of a manifold and hence a scalar field is the simplest really geometrical quantity which can be considered as a map that puts into correspondence any point of a manifold of a definite number. An important class of really geometrical quantities is defined by the functionals on the set of curves and submanifolds. Any curve on an n-dimensional manifold is a geometrical locus that is defined by the equations

$$\gamma : u^i = \varphi^i(t).$$

The families of curves (submanifolds S_p) are defined by the equations

$$S_p : u^i = \varphi^i(t_1, t_2, \cdots, t_p) \quad (p = 2, 3, ...n).$$

After Gauss and Riemann and the experiments with the calculations and measurements of the lengths in the Euclidean space and on the sphere, the functional

$$l(\gamma, g) = \int\limits_{t_0}^{t_1} \sqrt{g_{ij} \frac{du^i}{dt} \frac{du^j}{dt}} dt$$

on a manifold is called the length of the curve. Thus, the really geometrical quantity g_{ij} is a field introduced at first by Riemann as the positive-definite metric on a manifold and, as was explained above, this field plays the fundamental role in Theory of Self-Organization.

The important classes of the functionals on a manifold give the line integral

$$\int a_i du^i = \int a_i \frac{du^i}{dt} dt$$

and its generalization on the families of curves in the form of the iterated integrals

$$\int a_{i_1 \cdots i_p} \frac{\partial u^{i_1}}{\partial t^1} \cdots \frac{\partial u^{i_p}}{\partial t^p} dt^1 \cdots dt^p,$$

where a_i are components of a covector field and $a_{i_1 \cdots i_p}$ are components of an antisymmetrical tensor field of the rank p. Thus, we see (with Stoke's theorem as an additional argument) that the covariant antisymmetric tensor fields (including a scalar field and a covariant vector field) are the quantities connected with the geometrical structure of a manifold and, hence, they all are the really geometrical quantities.

A congruence of curves (a stream on a manifold) is given by a system of regular differential equations

$$\frac{du^i}{dt} = v^i(u^1(t), u^2(t), \cdots, u^n(t)).$$

The right-hand side of this system of equations is called a vector field on a manifold and, hence, the vector field belongs to the set of really geometrical quantities.

The equations of a parallel displacement for a vector field along the given curve γ

$$\frac{dv^i}{dt} + P^i_{jk} v^k \frac{du^j}{dt} = 0$$

give one more really geometrical quantity with the components P^i_{jk}, known as affine (linear) connection. If the curve γ belongs to the stream of the vector field v^i and hence $du^i/dt = v^i$, then the linear connection defines a geodesic stream on a manifold by the equations

$$\frac{d^2u^i}{dt^2} + P^i_{jk} \frac{du^j}{dt} \frac{du^k}{dt} = 0.$$

Thus, the set of quantities connected with the geometrical structure of a manifold is very restricted. Let us enumerate the really geometrical quantities:

1) the positive-definite Reimann metric g_{ij};

2) the vector field v^i;

3) the affine (linear) connection P^i_{jk};

4) the scalar and covariant vector fields, antisymmetric covariant tensor fields.

For our goals it is useful to show the last set of the fields as the 2^n-tuple

$$(a, a_i, a_{ij}, \cdots a_{ijk\cdots l}).$$

This very small zoo of the really geometrical quantities should be sufficient for understanding of everything and, hence, for understanding of the essence of spin phenomena. To this end, we first of all introduce the general notion of geometrical internal symmetry.

The symmetry principles made their appearance in the twentieth century physics with identification of the invariance group of the Maxwell equations. With this as a precedent, symmetries took on a character in physicists minds as a priori principles of universal validity. The natural generalization appears as the fundamental principle that basic laws should be defined by the widest possible groups of transformations. So the principle of general invariance (covariance) resulted above all in the Einstein theory of the gravitational field. This principle proved insufficient to reach the goal at which field physics is aimed: a unified field theory deriving all laws from one common structure of the world. Theory of Self-Organization gives this structure in the form of geometrical internal symmetry.

By definition, geometrical internal transformations come in after the introduction of the really geometrical quantities to make these really geometrical quantities variable.

An example of geometrical internal symmetry was discovered at first in Weyl's work [7], where he investigated the process of recalibration in which a metric g_{ij} is replaced by $\overline{g}_{ij} = \lambda g_{ij}$ in which λ is an arbitrary positive function of position. To formulate our guide principle connected with geometrical internal symmetry, we compare the group of local diffeomorfisms of a manifold and the Weyl recalibrations. The local diffeomorphisms change the really geometrical quantities on a manifold as follows. Let U be a domain of definition of coordinates on a manifold. In this domain one can consider functions $\varphi^i(u^1 \cdots u^n)$ for which a domain of definition and a range of values coincide. Hence, a local diffeomorphism $\widetilde{\varphi}$ is defined

$$
\begin{aligned}
&\widetilde{\varphi} : u^i \Rightarrow \varphi^i(u^1, \cdots u^n), && \widetilde{\varphi}^{-1} : u^i \Rightarrow f^i(u^1, \cdots u^n),\\
&\varphi^i(f^1(u), \cdots f^n(u)) = u^i, && f^i(\varphi^1(u), \cdots \varphi^n(u)) = u^i.
\end{aligned}
$$

For $\widetilde{g} = \widetilde{\varphi} g$, we have

$$\widetilde{g}_{ij}(u) = g_{kl}(f(u)) f^k_i(u) f^l_j(u),$$

where $f^l_j(u) = \partial_j f^l(u)$. As can be verified by simple calculations, under a change of variables $\bar{u}^i = \bar{u}^i(u^1, \cdots, u^n), \quad u^i = u^i(\bar{u}^1, \cdots, \bar{u}^n)$ the components of $\widetilde{g}_{ij}(u)$ are transformed as components of a symmetrical tensor.

The calculations of the lengths of curves is a process on a manifold that is invariant with respect to the local diffeomorphisms in the following sense. One can calculate the lengths of the set of curves on a manifold with respect to the metrics $g_{ij}(u)$ and $\widetilde{g}_{ij}(u)$ and get the sets of values $l(\gamma, g)$ and $l(\gamma, \widetilde{g})$. The sets $l(\gamma, g)$ and $l(\gamma, \widetilde{g})$ are equivalent since $l(\gamma, g) = l(\tilde{\gamma}, \widetilde{g})$, where $\tilde{\gamma} = \widetilde{\varphi}\gamma$. We see that the Weyl recalibrations $\overline{g}_{ij} = \lambda g_{ij}$ do not induce transformations in the set of curves and the set $l(\gamma, \bar{g})$ in general case will differ from the set $l(\gamma, g)$. It is not the case if λ is defined from the equations

$$\widetilde{g}_{ij}(u) = g_{kl}(f(u)) f^k_i(u) f^l_j(u) = \lambda(u) g_{ij}(u),$$

but these equations have in general only a trivial solution. It is also clear that g_{ij} and $\overline{g}_{ij} = \lambda g_{ij}$ will design, generally speaking, the different physical spaces (see equations (1)). This means that if one considers a theory that is invariant with respect to the recalibrations $\overline{g}_{ij} = \lambda g_{ij}$, then the metric g_{ij} loses its fundamental physical and geometrical meaning and the Weyl geometrical internal symmetry should be broken, for example, by the equations for the really geometrical quantities in his theory. We consider this situation in more detail to give the exact realization of our last statement. Let

$$W^i_{jk} = \Gamma^i_{jk} + T^i_{jk}, \quad T^i_{jk} = \delta^i_j \phi_k + \delta^i_k \phi_j - g_{jk}\phi^i$$

be the Weyl connection in which Γ^i_{jk} are the Christoffel symbols of the metric g_{ij}. For the Riemann tensor ${B_{ijk}}^l$ of the Weyl connection we get

$${B_{ijk}}^l = {R_{ijk}}^l + \nabla_i T^l_{jk} - \nabla_j T^l_{ik} + T^l_{im}T^m_{jk} - T^l_{jm}T^m_{ik},$$

where ${R_{ijk}}^l$ is the curvature tensor and ∇_i is the covariant derivative with respect to the connection Γ^i_{jk}. Since the Weyl connection W^i_{jk} is invariant with respect to the recalibration, in which ϕ_i, g_{ij} are replaced by

$$\phi_i - \frac{1}{\lambda}\partial_i \lambda, \quad \lambda g_{ij},$$

respectively, the tensor ${B_{ijk}}^l$ is also invariant and, hence, the same holds for the tensors

$$B_{jk} = {B_{ljk}}^l = R_{jk} + \nabla_l T^l_{jk} - \nabla_j T^l_{lk} + T^l_{lm}T^m_{jk} - T^l_{jm}T^m_{lk}$$

and

$$H_{ij} = {B_{ijl}}^l = n(\nabla_i \phi_j - \nabla_j \phi_i),$$

where R_{jk} is the Ricci tensor. Since, it is desirable to have for ϕ_i, g_{ij} the equations of the second order, a violation of the Weyl geometrical internal symmetry becomes very natural. Setting

$$B = g^{jk}B_{jk} = R - (n-1)(n-2)\phi_l\phi^l + \nabla_l S^l, \quad S^l = T^l_{jk}g^{jk} - T^m_{mk}g^{kl},$$

we see that the recalibration being broken has the trace in the form of the Einstein equations for the gravitational field and the Maxwell equations for the electromagnetic field with mass. These equations can be derived from the geometrical Lagrangian $L = aB + bH_{ij}H^{ij}$ with a and b being constant. We conclude that the really geometrical quantities define the processes in the physical space that are not invariant with respect to the geometrical internal symmetry. We formulate the general principle that outlines the foundational role of the geometrical internal symmetry.

The geometrical internal symmetry makes the really geometrical quantities variable changing their geometrical status and being broken it leaves the trace in the form of the differential equations for these quantities that describe all phenomena connected with the geometrical internal symmetry.

Our main goal here is to define and investigate a realization of the concept of geometrical internal symmetry that describes such a phenomenon as spin (spin symmetry). Being broken, spin symmetry (as the widest possible group of the geometrical internal transformations connected with spin phenomena) designs the equations of spinstatics and spindynamics.

3. CONCEPT OF TEMPORAL FIELD AND DISCRETE SYMMETRIES

We start with the intrinsic (and coordinate-independent) representation of dynamics. From the quantum-mechanical causality discussed above it follows that all dynamical laws of the undisturbed systems have the following general form: **the rate of change with time of certain quantity equals to the result of action of some operator on this quantity**. The rate of change with time is the operator of evolution which defines causality in the field theory. The geometric and coordinate-independent definition of this foundational notion is the key point of any dynamical theory and cannot be given without the creation of the new concept of time. Indeed, in Theory of Self-Organization the coordinates have no physical sense. Hence, it is quite obvious that time by itself can be represented only as a really geometrical quantity in physical space. The idea was put forward [3] that **time by itself is a scalar field** suggesting, by way of justification, a self-consistent dynamical theory of fields which does not depend on any outer conditions (the dynamical division of Theory of Self-Organization). The absolute form of the energy conservation was discovered, the fundamental physical meaning of the energy flows was recognized, and the real possibility appeared to seize the energy of the gravitational field. In Theory of Self-Organization we deal with a redistribution of the different forms of energy and this is the general and the most fundamental characteristic of living reality. Thus, dynamics is first of all the dynamical equations with the operator of evolution as the manifestation of the temporal structure defined by the temporal field.

In the theory of self-organizing fields properties of time and physical space are not defined by the properties of devices and by the methods of measurements which are the aspects of the human being. This is the intrinsic matter of the physical system by itself. The temporal field (together with other fields) designs physical space, as it was explained above, but it has also other very important functions in the dynamical theory of self-organizing physical fields and in the context of spin as well. It is our goal here to represent shortly these fundamental properties of the temporal field.

The temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 in the region U of physical space is denoted as

$$f(u) = f(u^1, u^2, u^3, u^4).$$

The space cross sections of physical space are defined by the temporal field. For the real number t, the space cross section is given by the equation

$$f(u^1, u^2, u^3, u^4) = t. \tag{3}$$

Since the temporal field is a scalar field, the partial derivatives define the covector field $t_i = \partial_i f$. The gradient of the temporal field (or the stream of time) is the vector field $\mathbf{t}$ with the components

$$t^i = (\nabla f)^i = g^{ij} \frac{\partial f}{\partial u^j} = g^{ij} \partial_j f = g^{ij} t_j,$$

where g^{ij} are the contravariant components of the positive-definite Riemann metric (2). The gradient of the field of time defines the direction of the most rapid increase (decrease) of the field of time. The rate of change with time of some quantity is the Lie derivative or the covariant derivative in the direction of the gradient of the field of time. The symbols $D_{\mathbf{t}}$ and $\nabla_{\mathbf{t}} = t^i \nabla_i$ denote these operations, where ∇_i is the covariant derivative with respect to the connection that belongs to the Riemann metric g_{ij}.

The rate of change with time of the temporal field itself is given by the formula $D_{\mathbf{t}} f = \nabla_{\mathbf{t}} f = t^i \partial_i f = g^{ij} \partial_i f \, \partial_j f$. Since $D_{\mathbf{t}} f$ is a generalization of $\dfrac{d}{dt} t$, the temporal field obeys the fundamental equation

$$(\nabla f)^2 = g^{ij} \frac{\partial f}{\partial u^j} \frac{\partial f}{\partial u^j} = 1. \tag{4}$$

The rate of change with time of the gravitational potentials g_{ij} is given by the expression

$$D_{\mathbf{t}} g_{ij} = t^k \frac{\partial g_{ij}}{\partial u^k} + g_{kj} \frac{\partial t^k}{\partial u^i} + g_{ik} \frac{\partial t^k}{\partial u^j}.$$

For the antisymmetric covariant tensor field $a_{ij\cdots k}$ of rank p we have

$$D_{\mathbf{t}} a_{ij\cdots l} = t^k \partial_k a_{ij\cdots l} + p a_{k[j\cdots l} \partial_{i]} t^k.$$

Similar formulae can be presented for any geometrical quantities. The operator $\nabla_{\mathbf{t}}$ has no sense for the cases of the gravitational field and the general electromagnetic field since $\nabla_{\mathbf{t}} g_{ij} = 0$ identically and the potentials of the general electromagnetic field are themselves a linear connection (from the geometrical point of view) [4]. Thus, it was very difficult to recognize that in spindynamics the operator of evolution takes the form $\nabla_{\mathbf{t}}$. Now we shall introduce in the geometrical (coordinate-independent) form the following discrete symmetries: time reversal and bilateral symmetry. These symmetries play the foundational role in Theory of Self-Organization in general and especially as regards the theory of spin phenomena. First of all, let us consider the very important notion of time reversal and invariance with respect to this symmetry. It is almost evident that in the geometrical form the time reversal invariance means that a theory is invariant with respect to the transformations

$$T: \quad t^i \to -t^i.$$

It is clear that a theory will be time reversal invariant if the gradient of the temporal field appears in all formulae only as an even number of times, like $t^i t^j$. This definition should be explained in more detail. The definition and properties of the intrinsic system of coordinates t, x^1, x^2, x^3 can be found in [3]. In this system of coordinates for the temporal field we have $f(t, x^1, x^2, x^3) = t$ and thus, in the intrinsic system of coordinates the rate of change with time of any field is simply equal to the partial derivative with respect to t, i.e., $D_t = \partial/\partial t$ (see, for example, the formula for the rate of change with time of the symmetric tensor field given above). If we reverse time setting $\tilde{t}^i = -t^i$, then the lines of time are parametrized by the new variable $\tilde{t}$. It is not difficult to show that there is a one-to-one and mutually continuous correspondence between the parameters t and $\tilde{t}$ given by the relation $\widetilde{t} = -t$. From here it is clear that in the system of coordinates defined by the time reversal, the variable $-t$ will be the coordinate of time. Thus, the geometric (coordinate-independent) definition of the time reversal is adjusted to the familiar definition that is connected with the transformation of coordinates.

Now let us consider how to introduce the foundational notion of right and left in the framework of Theory of Self-Organization. It is very important to recognize that the symmetry of the right and left (bilateral symmetry) is the realization of the concept of geometrical internal symmetry and is tightly connected with the stream of time. A pair of the vector fields $\mathbf{v}$ and $\overline{\mathbf{v}}$ has the bilateral symmetry with respect to the stream of time if the sum of these fields is collinear to the gradient of the temporal field and their difference is orthogonal to it,

$$\overline{\mathbf{v}} + \mathbf{v} = \lambda \mathbf{t}, \quad (\overline{\mathbf{v}}, \mathbf{t}) = (\mathbf{v}, \mathbf{t}),$$

where $(\mathbf{v}, \mathbf{w}) = g_{ij} v^i w^j = v^i w_i$ is the scalar product. In components, we have

$$\overline{v}^i + v^i = \lambda t^i, \quad (\overline{v}^i - v^i) t_i = 0$$

and, hence,

$$\overline{v}^i = 2(\mathbf{v}, \mathbf{t})\, t^i - v^i = (2t^i t_j - \delta^i_j) v^j, \quad v^i = (2t^i t_j - \delta^i_j) \overline{v}^j.$$

If the vector field v^i is right-sided (lies to the right from the stream of time), then the vector field $\overline{v}^i$ will be left-sided (lies to the left from the stream of time).

The choice as to which is right or left is determined arbitrarily. We see that if $(\mathbf{v}, \mathbf{t}) = 0$, then $\overline{v}^i = -v^i$. Symmetry of right and left $\overline{v}^i = 2(\mathbf{v}, \mathbf{t})\, t^i - v^i$ may be presented in the form of the linear transformation $\overline{v}^i = R^i_j v^j$, where $R^i_j = 2t^i t_j - \delta^i_j$, $\mathrm{Det}(R^i_j) = -1$. This transformation is natural to call the reflection. Thus, for the other really geometrical quantities the bilateral symmetry may be represented as the reflection. Since $\overline{v}^i = R^i_j v^j$, for the metric tensor we have $\overline{g}_{ij} = g_{kl} R^k_i R^l_j = g_{ij}$ and, hence, g_{ij} is invariant with respect to the reflection. The same holds for the stream of time. This is the natural result. In the case of antisymmetric tensor fields we have

$$\overline{F}_{i_1 \cdots i_p} = R^{j_1}_{i_1} \cdots R^{j_p}_{i_p} F_{j_1 \cdots j_p} = (-1)^p (F_{i_1 \cdots i_p} - 2pt^k F_{k[i_2 \cdots i_p} t_{i_1]}), \qquad (5)$$

where the square bracket [...] denotes the process of alternation over p indices. For $p = n$ we obtain

$$\overline{F}_{i_1 \cdots i_n} = (-1)^{n+1} F_{i_1 \cdots i_n}. \qquad (6)$$

Thus, with the temporal field not only evolution is tightly connected but the notion of right and left as well. The equivalence of the right and left can be considered as an important principle. The scalar product is invariant with respect to the reflection. Indeed, $(\overline{\mathbf{v}}, \overline{\mathbf{w}}) = (\mathbf{v}, \mathbf{w})$ since g_{ij} is invariant with respect to the reflection.

Here it is convenient to make a general remark about the so-called pseudo-Riemann geometry. This title is connected with the quadratic form

$$\overline{\varphi} = \overline{g}_{ij} v^i v^j$$

that is not positive definite. Let

$$\varphi = g_{ij} v^i v^j$$

be the positive definite quadratic form. Then a tensor $\overline{g}_{ij}$ can be represented as follows: $\overline{g}_{ij} = g_{ik} S^k_j$. We get

$$\overline{\varphi} = \overline{g}_{ij} v^i v^j = g_{ik} v^i v^j S^k_j = g_{ik} v^i w^k,$$

where $w^k = v^j S^k_j$. It is evident that the pseudo-Reimann quadratic form has the geometric meaning of the angle between two directions. Thus, one can avoid using the pseudo-geometrical terminology and concentrate on recognizing the geometrical meaning of the operator S^i_j in the framework of the genuine Riemann geometry defined by the positive-definite Riemann metric. We should like to remind that the Euclidean geometry underlies geometry. Euclid put us on the truth track by taking space as the primary concept of science.

The associated scalar product defined by the bilateral symmetry has the form

$$\langle \mathbf{v}, \mathbf{w} \rangle = (\overline{\mathbf{v}}, \mathbf{w}).$$

It is not difficult to show that the associated scalar product is also invariant with respect to the reflection: $\langle \overline{\mathbf{v}}, \overline{\mathbf{w}} \rangle = \langle \mathbf{v}, \mathbf{w} \rangle$. Let us consider the physical meaning of the associated scalar product $\langle \mathbf{v}, \mathbf{w} \rangle$. Since

$$\langle \mathbf{v}, \mathbf{v} \rangle = (\overline{\mathbf{v}}, \mathbf{v}) = 2(\mathbf{v}, \mathbf{t})^2 - (\mathbf{v}, \mathbf{v}) = |\mathbf{v}|^2 (2\cos^2\varphi - 1) = |\mathbf{v}|^2 \cos 2\varphi,$$

where φ is the angle between the vectors v^i and t^i, the associated scalar product is indefinite and can be positive, negative or equal to zero, according to the value of the angle φ. In particular, $(\overline{\mathbf{v}}, \mathbf{v}) = 0$, if $\varphi = \pi/4$. Thus, the associated scalar product $\langle \mathbf{v}, \mathbf{w} \rangle$ is time reversal invariant and permits one to classify all vectors depending on what angle they form with the stream of time. As we see from the formula $(\overline{\mathbf{v}}, \mathbf{v}) = \overline{g}_{ij} v^i v^j$, where

$$\overline{g}_{ij} = 2t_i t_j - g_{ij}, \tag{7}$$

the associated scalar product may be formalized with the help of the auxiliary metric $\overline{g}_{ij}$ as the metric of the normal hyperbolic type, which is defined by the temporal field and the bilateral symmetry. The contravariant components of the tensor field $\overline{g}_{ij}$ are $\overline{g}^{ij} = g^{ij} 2t^i t^j - g^{ij}$. Hence, the bilateral symmetry defines the natural causal structure on the physical space and can be identified with it. The conclusion lies in the fact that with the temporal field we get full understanding of the Lorentzian metric on the basis of the positive-definite metric and the bilateral symmetry. From different points of view, it is very important to recognize that in GR time was eliminated by the pseudometric but this leads up to the insurmountable conceptual difficulties [8]. Since the bilateral symmetry is the realization of the concept of the geometrical internal symmetry, we conclude that equations of gravidynamics are defined by the internal symmetry because the simplest passage from statics to dynamics can be realized as the increasing of the dimension of physical space and the exchange of the positive definite metric by the auxiliary metric (7). Let us pay a bit of attention to this point. The bilateral symmetry gives an evident method of introduction of the temporal field into Lagrangians of the physical fields. The cases of the gravitational field and the electromagnetic field were considered in [3]. The geometrical representation of the dynamical laws of the gravitational field is defined by the Lagrangian $L = \overline{R}$, where $\overline{R}$ is the scalar that is constructed from the auxiliary metric (7) following the formulae of the Riemann geometry. Since the auxiliary metric (7) is defined by the two fields connected by equation (4), it is necessary to pay special attention when deriving the equations of field. A standard method is to incorporate the constraint (4) via a Lagrange multiplier $\varepsilon = \varepsilon(u)$, rewrite the action density for gravity field in the form $L_g = \overline{R} + \varepsilon(g^{ij} t_i t_j - 1)$ and treat the components of the fields g_{ij} and f as independent variables. Thus, one needs to vary the action

$$A = \frac{1}{2} \int \overline{R} \sqrt{g}\, d^4u + \int L_m(\overline{g}, F) \sqrt{g}\, d^4u + \frac{1}{2} \int \varepsilon\, (g^{ij} t_i t_j - 1) \sqrt{g}\, d^4u,$$

where $g = \mathrm{Det}(g_{ij}) > 0$ and $L_m(\overline{g}, F)$ is the Lagrangian density of the system of other fields F which incorporates time through the auxiliary metric (7) in the conventional form (Lagrangian $L_m(\overline{g}, F)$ depends only on the values of $\overline{g}_{ij}$ in point). It is a very important condition since the minimal interaction of the fields with gravity is supposed. The concept of potential field [4] is the realization of this principle. The method of introduction of causality with the help of the auxiliary metric (7) does not require special explanation but in the case of spinning field it demands a more fine consideration. In general, it is not evident also how to derive the dynamical laws (the form of which was defined earlier) from their geometrical representation. This problem was solved in the two important cases of the electromagnetic field and gravidynamics in [3]. Here we use the results obtained in this work to derive equations of spindynamics in evidently dynamical (Hamiltonian) form.

The bilateral symmetry can be considered also as the starting point in the theory of the general electromagnetic field. Indeed, let us consider a pair of vector fields $\overline{\mathbf{v}}$ and $\mathbf{v}$ with bilateral symmetry $\overline{v}^i = R^i_j v^j$. It is clear that the notion of the parallel displacement is not applied to this system of vector fields. From the equations of parallel displacement it follows that the parallel displacement of the system in question is defined by a pair of the connections $\overline{P}^i_{jk}$ and P^i_{jk} with bilateral symmetry, i.e.,

$$\overline{P}^i_{jk} = R^i_m P^m_{jn} R^n_k + R^i_m \partial_j R^m_k.$$

Be extended in the evident manner, the bilateral symmetry gets status of the geometrical internal symmetry $GL(n, \mathbf{R})$ with the following law of transformation of the potentials of the general electromagnetic field (linear connection)

$$\overline{P}^i_{jk} = S^i_m P^m_{jn} T^n_k + S^i_m \partial_j T^m_k,$$

where T^n_k are components of the field S^{-1} inverse to S, $S^i_k T^k_j = \delta^i_j$. In this framework the affine connection gets the status of the physical field with internal properties (the general electromagnetic field) and the problems of geometrization of the electromagnetic field and dark matter get their natural solution [4] (see also Ref. [10]). Here we introduce the electric ${E_{il}}^k$ and magnetic ${M_{il}}^k$constituents of the general electromagnetic field strength setting

$${E_{il}}^k = t^j {H_{jil}}^k, \quad {M_{il}}^k = t^j {\widetilde{H}_{jil}}{}^k, \quad {\widetilde{H}_{jil}}{}^k = \frac{1}{2} e_{jimn} H^{mnk}{}_l,$$

where ${H_{jil}}^k$ is the strength tensor of the general electromagnetic field.

It is interesting that the concept of the potential field (introduced in [4]) and the concept of the really geometrical quantity actually outline the same set of fundamental fields.

It should be noted that for any S^i_j we have the decomposition $S^i_j = P^i_j + Q^i_j$, where

$$P^i_j = \frac{1}{2}(S^i_j + \widetilde{S}^i_j), \qquad Q^i_j = \frac{1}{2}(S^i_j - \widetilde{S}^i_j), \qquad \widetilde{S}^i_j = g^{ik} S^l_k g_{lj}.$$

We have from the definition that $\widetilde{P}^i_j = P^i_j$, $\quad \widetilde{Q}^i_j = -Q^i_j$, or $P_{ij} = P_{ji}$, $\quad Q_{ij} = -Q_{ji}$, where $P_{ij} = P^l_i g_{lj}$, $\quad Q_{ij} = Q^l_i g_{lj}$. Thus, on the group $GL(n, \mathbf{R})$ there is a bipolar structure which is defined by the representatives of the set of really geometrical quantities.

Orientation is the antisymmetrical tensor $e_{ij\cdots kl}$ («element of volume») normalized as $e_{12\cdots n} = \sqrt{g}$, where $g = \mathrm{Det}(g_{ij})$. In what follows, it will be shown that the orientation $e_{ij\cdots kl}$ defines a generator of one parameter group of transformations, which will be called the group of orient symmetry. The group of orient symmetry plays the important role in spinstatics and especially in spindynamics.

4. SPIN AND BIPOLAR STRUCTURE OF SPIN SYMMETRY

With the general definition of linear space we have nothing to do with preferred dimension of this space. However, from the geometrical point of view, the dimension 2^n is singled out. Indeed, the binomial formula shows that $2^n = \sum_{p=0}^{n} C^n_p$, where C^n_p denotes the number of distinct combinations of p objects chosen from n. The covariant totally antisymmetrical tensor field $a_{i_1\cdots i_p}$ has C^n_p components. Thus, on the n-dimensional manifold there is natural linear space of the dimension $N = 2^n$ that can be constructed from the really geometrical quantities. The general element $\mathbf{A}$ of this space can be represented as 2^n-tuple

$$\mathbf{A} = (a,\, a_{i_1},\, a_{i_1 i_2},\, \cdots a_{i_1\cdots i_n}). \tag{8}$$

However, from the initial geometrical interpretation it follows that this object consists of independent blocks and cannot be considered as a unit. The concept of the geometrical internal symmetry makes it possible to remove this discrepancy. As a starting point, let us consider the linear space of the covector fields a_i. To make this really geometrical quantity variable, we can consider only two kinds of the general covariant internal transformations: $\overline{a}_i = L^k_i a_k$ and $\overline{a}_i = a_i + \partial_i \phi$, where L^i_j is a tensor field of type (1,1) (linear operator). Reversible transformations of the first type form the group $GL(n; \mathbf{R})$ of local transformations and the transformations of the second kind are known as the gauge transformations. It is easy to see that the group of geometrical internal transformations $GL(n; \mathbf{R})$ is not the group of external automorphisms of the gauge group and vice versa. Since the group $GL(n; \mathbf{R})$ violates the equivalence relation defined by the gauge group it is natural to separate the set of covector fields into the class of really covector fields and the class of differential forms. In the first case, the covector fields are a basis of the representation of the $GL(n;\mathbf{R})$ group and in the second one, it is a basis of the gauge group representation. It is clear that the gauge transformations

$$\overline{a}_{i_1\cdots i_p} = a_{i_1\cdots i_p} + p\partial_{[i_1}\phi_{i_2\cdots i_p]}$$

map p-form onto p-form and hence we cannot consider 2^n-tuple (8) as a set of p-form. The same situation occurs if we consider only transformations of the following form:

$$\overline{a}_{i_1\cdots i_p} = L^{j_1}_{i_1}\cdots L^{j_p}_{i_p} a_{j_1\cdots j_p}.$$

Hence, the gauge transformations and the transformations of the $GL(n;\mathbf{R})$ group are not suitable for solution of our problem and we shall consider the group of geometrical internal transformations $GL(2^n;\mathbf{R})$ which are defined as follows. Let us consider the set of totally antisymmetrical tensor fields of the type (p,q),

$$L^{i_1\cdots i_p}_{j_1\cdots j_q}, \quad (p,q = 0,1,\cdots,n).$$

A geometrical internal transformation (linear operator) in the space of 2^n-tuples (8) is defined by the equations $\overline{\mathbf{A}} = L\mathbf{A}$, where $(L\mathbf{A})_{i_1\cdots i_p} = \overline{a}_{i_1\cdots i_p},\ p = 0,1,\cdots n$ and

$$\overline{a}_{i_1\cdots i_p} = \sum\nolimits_{q=0}^{n} \frac{1}{q!} L^{j_1\cdots j_q}_{i_1\cdots i_p} a_{j_1\cdots j_q}.$$

Here and in what follows, it will be convenient to show only a general element $a_{i_1\cdots i_p}$ of 2^n-tuple (8) assuming that p runs from 0 to n. In accordance with the definition, we have for the multiplication of the two transformations in question $N = LM$

$$N^{i_1\cdots i_p}_{j_1\cdots j_q} = \sum\nolimits_{r=0}^{n} \frac{1}{r!} L^{k_1\cdots k_r}_{j_1\cdots j_q} M^{i_1\cdots i_p}_{k_1\cdots k_r}.$$

The identical transformation E is defined by the conditions

$$E^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad E^{i_1\cdots i_p}_{j_1\cdots j_p} = \delta^{i_1\cdots i_p}_{j_1\cdots j_p},$$

where the generalized Kronecker delta is used

$$a_{i_1\cdots i_p} = \frac{1}{p!}\delta^{j_1\cdots j_p}_{i_1\cdots i_p} a_{j_1\cdots j_p}.$$

The so-defined local internal transformations form the group of the spin symmetry $GL(2^n,\mathbf{R})$ which allows one to consider the 2^n-tuple (8) as a unit. Thus, the status of the group $GL(2^n,\mathbf{R})$ is established. With respect to the group of the spin symmetry the 2^n-tuple (8) will be known in what follows as the spinning field. We started from the set of the really geometrical quantities and introduced the concept of the spin symmetry as the realization of the concept of geometrical internal symmetry.

Let us define the positive definite fundamental bilinear quadratic form in the linear space in question as follows:

$$(\mathbf{A}|\mathbf{B}) = \sum\nolimits_{p=0}^{n} \frac{1}{p!} a_{i_1\cdots i_p} b_{j_1\cdots j_p} g^{i_1 j_1} \cdots g^{i_p j_p}. \tag{9}$$

We have $(\mathbf{A}|\mathbf{B}) = (\mathbf{B}|\mathbf{A})$, $\quad (\mathbf{A}|\mathbf{A}) \geqslant 0$. The scalar product (9) is invariant with respect to the transformations $\overline{\mathbf{A}} = L\mathbf{A}$ if $\widetilde{L}L = E$, where $\widetilde{L}$ is the transposed operator with respect to the metric tensor g_{ij}

$$\widetilde{L}^{i_1\cdots i_p}_{j_1\cdots j_q} = g^{i_1 k_1} \cdots g^{i_p k_p} L^{l_1\cdots l_q}_{k_1\cdots k_p} g_{j_1 l_1} \cdots g_{j_q l_q}.$$

It is easy to show that $\widetilde{LM} = \widetilde{M}\widetilde{L}$.

According to the principle of sufficient cause, nothing happens without there being a reason why it should be thus rather than otherwise. We consider real spin fields since in accordance with the principle of sufficient cause there is no essential informal reason to introduce the complex spinning field. Only the existence of these reasons has the foundational significance and represents real interest and motivation for the complexification.

To define the group of the spin symmetry, we have introduced a set of tensor fields that is not directly connected with the geometrical structure of the physical space. We shall give a representation of transformations of the group $GL(2^n, \mathbf{R})$ in the framework of the really geometrical quantities. We construct a natural general covariant basis in the Lee algebra $gl(2^n, \mathbf{R})$ of $GL(2^n, \mathbf{R})$ and uncover spin as the bipolar structure on the group of the spin symmetry $GL(2^n, \mathbf{R})$. After that the spin symmetry will be broken and visualized as equations of spinstatics and spindynamics. With this, we recognize the real significance and manifestation of the spin symmetry.

Let us consider in the space of the spinning fields (8) the natural algebraic operators $\overline{\mathbf{A}} = E_{\mathbf{v}}\mathbf{A}$ and $\overline{\mathbf{A}} = I_{\mathbf{v}}\mathbf{A}$ defined by the vector field v^i as follows:

$$E_{\mathbf{v}} : \bar{a}_{i_1\cdots i_p} = p v_{[i_1} a_{i_2\cdots i_p]}, \quad I_{\mathbf{v}} : \bar{a}_{i_1\cdots i_p} = v^k a_{k i_1\cdots i_p}, \quad (p = 0, 1, \cdots, n),$$

where the square brackets $[\cdots]$ denote the process of alternation and $v_i = g_{ij} v^j$. For any vector fields v^i and w^i we have

$$I_{\mathbf{v}} E_{\mathbf{w}} + E_{\mathbf{w}} I_{\mathbf{v}} = (\mathbf{v}, \mathbf{w}) \cdot E, \tag{10}$$

where $(\mathbf{v}, \mathbf{w}) = g_{ij} v^i w^j$. To prove formula (10) a very useful relation

$$(p+1) v_{[k} a_{i_1\cdots i_p]} = v_k a_{i_1\cdots i_p} - p a_{k[i_2\cdots i_p} v_{i_1]}$$

should be taken into account here and in what follows. We mention also the evident relations

$$E_{\mathbf{v}} E_{\mathbf{w}} + E_{\mathbf{w}} E_{\mathbf{v}} = 0, \quad I_{\mathbf{v}} I_{\mathbf{w}} + I_{\mathbf{w}} I_{\mathbf{v}} = 0.$$

To complete the preliminary, let us introduce the numerical diagonal operator Z that is defined by the conditions

$$Z^{i_1\cdots i_p}_{j_1\cdots j_q} = 0, \quad \text{if} \quad p \neq q, \quad Z^{i_1\cdots i_p}_{j_1\cdots j_p} = (-1)^p \delta^{i_1\cdots i_p}_{j_1\cdots j_p}.$$

From the definition of Z, we immediately have the following relations:

$$E_{\mathbf{v}} Z + Z E_{\mathbf{v}} = 0, \quad I_{\mathbf{v}} Z + Z I_{\mathbf{v}} = 0, \quad Z^2 = E.$$

Now we introduce the fundamental operators:

$$Q_{\mathbf{v}} = E_{\mathbf{v}} - I_{\mathbf{v}}, \quad \widetilde{Q}_{\mathbf{v}} = (E_{\mathbf{v}} + I_{\mathbf{v}}) Z,$$

which define a bipolar structure (spin) on the group of the spin symmetry $GL(2^n, \mathbf{R})$.

From the definition and (10), it follows that:

$$Q_{\mathbf{v}} Q_{\mathbf{w}} + Q_{\mathbf{w}} Q_{\mathbf{v}} = -2(\mathbf{v}, \mathbf{w}) \cdot E, \quad \widetilde{Q}_{\mathbf{v}} \widetilde{Q}_{\mathbf{w}} + \widetilde{Q}_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}} = -2(\mathbf{v}, \mathbf{w}) \cdot E, \tag{11}$$

and hence

$$Q^2_{\mathbf{v}} = \widetilde{Q}^2_{\mathbf{v}} = -(\mathbf{v}, \mathbf{v})\, E = -v^2\, E.$$

We also get the important relation

$$\widetilde{Q}_{\mathbf{v}} Q_{\mathbf{w}} = Q_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}} \tag{12}$$

that fulfills at any $\mathbf{v}$ and $\mathbf{w}$.

Further, let us introduce the operator

$$Q_{\mathbf{v}\wedge\mathbf{w}} = \frac{1}{2}(Q_{\mathbf{v}} Q_{\mathbf{w}} - Q_{\mathbf{w}} Q_{\mathbf{v}}).$$

It is easy to see that

$$Q^2_{\mathbf{v}\wedge\mathbf{w}} = -\triangle \cdot E, \tag{13}$$

where $\triangle$ is Gram's determinant

$$\triangle = \begin{vmatrix} (\mathbf{v}, \mathbf{v}) & (\mathbf{v}, \mathbf{w}) \\ (\mathbf{v}, \mathbf{w}) & (\mathbf{w}, \mathbf{w}) \end{vmatrix}.$$

Of course, the same relation holds for the operator

$$\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}} = \frac{1}{2}(\widetilde{Q}_{\mathbf{v}} \widetilde{Q}_{\mathbf{w}} - \widetilde{Q}_{\mathbf{w}} \widetilde{Q}_{\mathbf{v}}).$$

The last formula can be generalized as follows. We take $m (m = 2, 3, \cdots, n)$ linear-independent vector fields $\mathbf{v}, \mathbf{w}, \cdots, \mathbf{z}$ and introduce the operator that is defined as an alternated product of the operators $Q_{\mathbf{v}}, Q_{\mathbf{w}}, \cdots, Q_{\mathbf{z}}$ and $\widetilde{Q}_{\mathbf{v}}$, $\widetilde{Q}_{\mathbf{w}}, \cdots, \widetilde{Q}_{\mathbf{z}}$

$$Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}} = \frac{1}{m!}(Q_{\mathbf{v}}Q_{\mathbf{w}}\cdots Q_{\mathbf{z}} - Q_{\mathbf{w}}Q_{\mathbf{v}}\cdots Q_{\mathbf{z}} + \cdots),$$
$$\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}} = \frac{1}{m!}(\widetilde{Q}_{\mathbf{v}}\widetilde{Q}_{\mathbf{w}}\cdots \widetilde{Q}_{\mathbf{z}} - \widetilde{Q}_{\mathbf{w}}\widetilde{Q}_{\mathbf{v}}\cdots \widetilde{Q}_{\mathbf{z}} + \cdots).$$

We see that the operators $Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ and $\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ commute with each other and all their possible products put together the general covariant basis in the Lie algebra $gl(2^n, \mathbf{R})$ of the spin-symmetry group $GL(2^n, \mathbf{R})$). The total number of these operators is equal to $2^n \cdot 2^n$ (including the identical operator E). Thus, in the space in question transformations of the group $GL(2^n, \mathbf{R})$ can be represented in terms of the really geometrical quantities and spin symmetry is the fundamental realization of the concept of geometrical internal symmetry. We see that the Riemann metric looks like a «prism» and the spin symmetry (like light) demonstrates its bipolar structure passing through this prism and becomes apparent in the form of the operators $Q_{\mathbf{v}}, \cdots, \quad Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ on one pole and the operators $\widetilde{Q}_{\mathbf{v}}, \cdots \quad \widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ on the other pole.

To represent this bipolar structure more visually, let us consider the n linear-independent vector fields $\mathbf{v}_\mu \quad (\mu = 1, 2, \cdots, n)$. It is always possible to construct from this set of vector fields a system of the orthogonal and unit vector fields $\mathbf{E}_\mu$ with respect to the given metric g_{ij},

$$(\mathbf{E}_\mu, \mathbf{E}_\nu,) = g_{ij}E^i_\mu E^j_\nu = \delta_{\mu\nu},$$

where $\delta_{\mu\nu} = 0,\ \mu \neq \nu, \quad \delta_{\mu\mu} = 1,\ \mu = 1, 2, \cdots n$ and E^i_μ are functions of v^i_μ and g_{ij}. Let us denote the operators $Q_{\mathbf{E}_\mu}$ and $\widetilde{Q}_{\mathbf{E}_\mu}$ as Q_μ and $\widetilde{Q}_\mu$, respectively. With this, we get from (11) and (12)

$$Q_\mu Q_\nu + Q_\nu Q_\mu = -2\delta_{\mu\nu}\cdot E, \quad \widetilde{Q}_\mu\widetilde{Q}_\nu + \widetilde{Q}_\nu\widetilde{Q}_\mu = -2\delta_{\mu\nu}\cdot E, \quad \widetilde{Q}_\mu Q_\nu = Q_\nu\widetilde{Q}_\mu.$$

At last we introduce the operators

$$Q_{\mu\nu} = \frac{1}{2}(Q_\mu Q_\nu - Q_\nu Q_\mu), \quad \widetilde{Q}_{\mu\nu} = \frac{1}{2}(\widetilde{Q}_\mu\widetilde{Q}_\nu - \widetilde{Q}_\nu\widetilde{Q}_\mu).$$

The spin structure is quite clear from the consideration of the Lie algebras of the $GL(2^n, \mathbf{R})$ subgroups that are defined as follows. We set

$$K_\mu = \frac{1}{2}Q_\mu, \quad S_{\mu\nu} = \frac{1}{2}Q_{\mu\nu}, \quad \widetilde{K}_\mu = \frac{1}{2}\widetilde{Q}_\mu, \quad \widetilde{S}_{\mu\nu} = \frac{1}{2}\widetilde{Q}_{\mu\nu}$$

and get the following commutation relations:

$$[K_\mu, K_\nu] = S_{\mu\nu}, \quad [S_{\mu\nu}, S_{\sigma\tau}] = \delta_{\mu\tau}S_{\sigma\nu} - \delta_{\sigma\nu}S_{\mu\tau} + \delta_{\mu\sigma}S_{\nu\tau} - \delta_{\nu\tau}S_{\sigma\mu},$$
$$[\widetilde{K}_\mu, \widetilde{K}_\nu] = \widetilde{S}_{\mu\nu}, \quad [\widetilde{S}_{\mu\nu}, \widetilde{S}_{\sigma\tau}] = \delta_{\mu\tau}\widetilde{S}_{\sigma\nu} - \delta_{\sigma\nu}\widetilde{S}_{\mu\tau} + \delta_{\mu\sigma}\widetilde{S}_{\nu\tau} - \delta_{\nu\tau}\widetilde{S}_{\sigma\mu},$$
$$[K_\mu, \widetilde{K}_\nu] = [K_\mu, \widetilde{S}_{\sigma\nu}] = 0, \quad [S_{\mu\nu}, \widetilde{K}_\sigma] = [S_{\mu\nu}, \widetilde{S}_{\sigma\tau}] = 0.$$

We see that these commutation relations give two general covariant spinor representations of the group of conformable transformations of the sphere S^{n-1}. Since

$$(\mathbf{A}|Q_\mu\mathbf{B}) = -(Q_\mu\mathbf{A}|\mathbf{B}), \quad (\mathbf{A})|\widetilde{Q}_\mu\mathbf{B}) = -(\widetilde{Q}_\mu\mathbf{A}|\mathbf{B}),$$
$$(\mathbf{A}||Q_{\mu\nu}\mathbf{B}) = -(Q_{\mu\nu}\mathbf{A}|\mathbf{B}), \quad (\mathbf{A}|\widetilde{Q}_{\mu\nu}\mathbf{B})) = -(\widetilde{Q}_{\mu\nu}\mathbf{A}|\mathbf{B}),$$

the scalar product (9) is invariant with respect to the transformations $\bar{\mathbf{A}} = L\mathbf{A}$, $\bar{\mathbf{A}} = \widetilde{L}\mathbf{A}$, where

$$L = \exp\,(K_\mu\omega^\mu + \frac{1}{2}S_{\mu\nu}\omega^{\mu\nu}), \quad \widetilde{L} = \exp\,(\widetilde{K}_\mu\widetilde{\omega}^\mu + \frac{1}{2}\widetilde{S}_{\mu\nu}\widetilde{\omega}^{\mu\nu}),$$

and $\omega^\mu, \quad \omega^{\mu\nu}, \quad \widetilde{\omega}^\mu, \quad \widetilde{\omega}^{\mu\nu}$ is the set of real scalar fields.

Thus, spin emerges as the bipolar structure of the spin symmetry and two sets of the commuting operators $Q_{\mathbf{v}}, \cdots, Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ and $\widetilde{Q}_{\mathbf{v}}, \cdots, \widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}$ realize this structure. The bipolar structure of the spin symmetry gives a natural way of violation of the spin symmetry and derivation of equations of the spinning field.

5. SPINSTATICS

Here we consider natural general covariant differential operators for the spinning field that are defined by the bipolar structure. These differential operators give a natural method to violate the spin symmetry and develop the most objective theory of spin phenomena. In this section, we formulate the theory of the spinning field at absolute rest or outside the time (spinstatics). In this case, we consider the physical space without temporal structure. The physical space with temporal structure and the dynamical theory of the spinning field (spindynamics) will be formulated in what follows. By its nature Aether is at absolute rest (we may say now that outside the time). Thus, the timeless self-organized systems of fields actually represent Aether. This section may be considered as both an independent investigation and a basis for spindynamics.

Let ∇_i be the covariant derivative with respect to the connection belonging to the Riemann metric g_{ij}. The Christoffel symbols of this connection are

$$\Gamma^i_{jk} = \frac{1}{2}g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk}).$$

The evident correspondence between v_i and ∇_i (v^i and $\nabla^i = g^{ij}\nabla_j$) gives the opportunity to introduce two natural general covariant differential operators for the spinning field as manifestation of the bipolar structure

$$D = Q_\nabla, \quad \widetilde{D} = \widetilde{Q}_\nabla.$$

These operators inherit the property (12)

$$D\widetilde{D} = \widetilde{D}D.$$

The trace from the bipolar structure of the spin symmetry takes form of two general covariant equations for the real spinning field spatially extended, yet timeless. The spin symmetry will be broken if we establish the equation

$$D\mathbf{A} = m\mathbf{A} \tag{14}$$

or the dual equation

$$\widetilde{D}\mathbf{A} = m\mathbf{A}. \tag{15}$$

Equations (14) and (15) represent the bipolar structure of the spin symmetry and in view of this we formulate the duality principle: there are two universes dual to each other. The equation $D\mathbf{A} = m\mathbf{A}$ characterizes one world and the equation $\widetilde{D}\mathbf{A} = m\mathbf{A}$ characterizes the other world. In accordance with this, we give the full representation of the bipolar structure.

We have two identities

$$(\mathbf{A}|D\mathbf{B}) - (\mathbf{B}|D\mathbf{A}) = \nabla_i T^i, \quad (\mathbf{A}|\widetilde{D}\mathbf{B}) - (\mathbf{B}|\widetilde{D}\mathbf{A}) = \nabla_i \widetilde{T}^i. \tag{16}$$

The components of the vector fields T^i and $\widetilde{T}^i$ can be found from the relations

$$v_i T^i = (Q_{\mathbf{v}}\mathbf{A}|\mathbf{B}) = -(\mathbf{A}|Q_{\mathbf{v}}\mathbf{B}), \quad v_i\widetilde{T}^i = (\widetilde{Q}_{\mathbf{v}}\mathbf{A}|\mathbf{B}) = -(\mathbf{A}|\widetilde{Q}_{\mathbf{v}})\mathbf{B}).$$

We see that the operators D and $\widetilde{D}$ are self-adjoint and $\nabla_i T^i = 0$ if $\mathbf{A}$ and $\mathbf{B}$ are solutions to the spin equation (and the same for $\nabla_i\widetilde{T}^i$). From (16) it follows that equations (14) and (15) can be derived from the Lagrangians:

$$\pounds = \frac{1}{2}(\mathbf{A}|D\mathbf{A}) - \frac{m}{2}(\mathbf{A}|\mathbf{A}), \quad \widetilde{\pounds} = \frac{1}{2}(\mathbf{A}|\widetilde{D}\mathbf{A}) - \frac{m}{2}(\mathbf{A}|\mathbf{A}).$$

It is clear that $\pounds = 0$ if $\mathbf{A}$ is a solution of equation (14) and the same for $\widetilde{\pounds}$.

The spin symmetry is broken by equations (14) and (15). Let us consider in to what extent. Generally speaking, the operators $\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\wedge\cdots\mathbf{z}}$ can commute with the operator D and the same holds for the pair $\widetilde{D}, \quad Q_{\mathbf{v}\wedge\mathbf{w}\wedge\cdots\mathbf{z}}$. We remind the characteristic relation $\widetilde{Q}_{\mathbf{v}}Q_{\mathbf{w}} = Q_{\mathbf{w}}\widetilde{Q}_{\mathbf{v}}$. It is evident that

$$D\widetilde{Q}_{\mathbf{v}} = \widetilde{Q}_{\mathbf{v}}D, \quad \widetilde{D}Q_{\mathbf{v}} = Q_{\mathbf{v}}\widetilde{D},$$

if $\nabla_i v_j = 0$. This condition is a very strong restriction since the class of the Riemann spaces so defined is very poor. For example, the equations $\nabla_i v_j = 0$ have only trivial solution in the Riemann spaces of a constant curvature. Now we consider the operators of the form $Q_{\mathbf{v}\wedge\mathbf{w}}$ and $\widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}}$. The operators of this class are generated by the totally antisymmetric tensor field $v_i w_j - v_j w_i$ which is called simple. We consider the general antisymmetric tensor field S_{ij} and the dual spin operators generated by this field are denoted by $Q_{\mathbf{S}}$ and $\widetilde{Q}_{\mathbf{S}}$. The component representation takes the form

$$(Q_{\mathbf{S}}\mathbf{A})_{i_1\cdots i_p} = \frac{p(p-1)}{2} S_{[i_1 i_2} a_{i_3\cdots i_p]} - \frac{1}{2} S^{kl} a_{kl i_1\cdots i_p} + p S_{k[i_1} a^k_{\cdot i_2\cdots i_p]},$$
$$(\widetilde{Q}_{\mathbf{S}}\mathbf{A})_{i_1\cdots i_p} = -\frac{p(p-1)}{2} S_{[i_1 i_2} a_{i_3\cdots i_p]} + \frac{1}{2} S^{kl} a_{kl i_1\cdots i_p} + p S_{k[i_1} a^k_{\cdot i_2\cdots i_p]}.$$

We have $D\widetilde{Q}_{\mathbf{S}} = \widetilde{Q}_{\mathbf{S}} D$, and $\widetilde{D} Q_{\mathbf{S}} = Q_{\mathbf{S}} \widetilde{D}$, if

$$\nabla_i S_{jk} = 0.$$

On a manifold of an even dimension one can impose (without loss of generality) one more condition on S_{ij}

$$S_{ik} S_{jl} g^{kl} = g_{ij}.$$

Among the all conceivable Riemann manifolds of an even dimension these equations single out a very interesting class of manifolds which admit a complex analytical structure [11]. We have in this case $\frac{1}{4} Q^2_{\mathbf{S}} = -E, \quad \frac{1}{4}\widetilde{Q}^2_{\mathbf{S}} = -E$. One can put forward an idea that all gravitational phenomena are described by the set of the Kahler metrics, if we consider physical phenomena outside the time. However, it is natural to connect the complex analytical structure with the electric charge. However, in spinstatics the spinning field does not carry charge since charge is a dynamical effect due to the existence of the temporal structure (this will be clear under the formulation of spindynamics). That is why it is natural to suppose that in spinstatics dimension n of the physical space is odd (equal to three, $n = 3$.) Another essential argument is that equations (14) and (15) being written in the component form are very unsymmetrical and only in the dimension $n = 3$ we can present them in a simple and symmetrical form. We set in this case $a = \alpha, \quad a_{ijk} = -e_{ijk}\beta, \quad a_{ij} = e_{ijk} b^k$, and using the formalism of the usual vector analysis in the general covariant form, we get the following system of equations of spinstatics:

$$\begin{aligned} -\mathrm{div}\,\mathbf{a} &= m\,\alpha, \\ -\mathrm{div}\,\mathbf{b} &= m\,\beta, \\ \mathrm{rot}\,\mathbf{a} + \mathrm{grad}\,\beta &= m\,\mathbf{b}, \\ \mathrm{rot}\,\mathbf{b} + \mathrm{grad}\,\alpha &= m\,\mathbf{a}. \end{aligned} \tag{17}$$

The dual equation (15) takes the form

$$\begin{aligned} -\mathrm{div}\,\mathbf{a} &= m\,\alpha, \\ -\mathrm{div}\,\mathbf{b} &= m\,\beta, \\ -\mathrm{rot}\,\mathbf{a} + \mathrm{grad}\,\beta &= m\,\mathbf{b}, \\ -\mathrm{rot}\,\mathbf{b} + \mathrm{grad}\,\alpha &= m\,\mathbf{a}. \end{aligned}$$

Thus, the main equations of spinstatics are derived. In what follows, it will be useful to compare these equations with analogous equations in spindynamics for better understanding of the dynamical theory itself and the role of temporal field.

We see that in general the spin symmetry is broken by the equations of spinstatics itself or by taking into account the gravitational effects. However, there is a very interesting exclusion here. When the number of the vector fields is equal to the dimension of a manifold we can set $\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}=\mathbf{e}$, where $\mathbf{e}$ is antisymmetrical tensor $e_{i_1\cdots i_n}$. We set in this case

$$Q_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}=H,\quad \widetilde{Q}_{\mathbf{v}\wedge\mathbf{w}\cdots\wedge\mathbf{z}}=\widetilde{H}.$$

As above, for the operators H and $\widetilde{H}$ we have

$$D\widetilde{H}=\widetilde{H}D,\quad \widetilde{D}H=H\widetilde{D},$$

if $\nabla_k e_{i_1\cdots i_n}=0$. This is the case if $e_{i_1\cdots i_n}$ is orientation of manifold («element of volume») with normalization $e_{1\dots n}=\sqrt{g}$. We write these dual operators (operators of orient symmetry) in component form

$$\begin{aligned}(H\mathbf{A})_{i_1\cdots i_p}&=\frac{(-1)^n}{(n-p)!}(-1)^{\frac{(n-p)(n-p+1)}{2}}e_{i_1\cdots i_p j_1\dots j_{n-p}}a^{j_1\cdots j_{n-p}},\\ (\widetilde{H}\mathbf{A})_{i_1\cdots i_p}&=\frac{(-1)^n}{(n-p)!}(-1)^{\frac{p(p+1)}{2}}e_{i_1\cdots i_p j_1\dots j_{n-p}}a^{j_1\cdots j_{n-p}},\end{aligned}$$

and derive the following relations:

$$\begin{gathered}H^2=\widetilde{H}^2=(-1)^{\frac{n(n+1)}{2}}E,\quad HZ=(-1)^nZH,\quad \widetilde{H}Z=(-1)^nZ\widetilde{H},\\ HQ_{\mathbf{v}}+(-1)^nQ_{\mathbf{v}}H=0,\quad \widetilde{H}\widetilde{Q}_{\mathbf{v}}+(-1)^n\widetilde{Q}_{\mathbf{v}}\widetilde{H}=0.\end{gathered}$$

It is evident that

$$HD+(-1)^nDH=0,\quad \widetilde{H}\widetilde{D}+(-1)^n\widetilde{D}\widetilde{H}=0,$$

and with respect to the equation $D\mathbf{A}=m\mathbf{A}$ operator H will be called the operator of chirality and the same for the dual pair $\widetilde{D},\widetilde{H}$. We also have$(\widetilde{H}H\mathbf{A})_{i_1\cdots i_p}=(-1)^{p(n+1)}a_{i_1\cdots i_p}$, and hence

$$\widetilde{H}H=H\widetilde{H}=Z$$

for the even dimension, and

$$\widetilde{H}H=H\widetilde{H}=E$$

for the odd dimension. In the case $n=3$, we have $H^2=E,\quad \widetilde{H}^2=E,\quad H=\widetilde{H}$, and hence there are states with orient parity equal to ±1. For $\widetilde{H}A=A$, we have $\alpha = \beta, \quad \mathbf{a} = \mathbf{b}$, and in the other case $\alpha = -\beta, \quad \mathbf{a} = -\mathbf{b}$. We see that all phenomena of spinstatics are described by the equations

$$-\mathrm{div}\,\mathbf{a} = m\,\alpha,$$
$$\mathrm{rot}\,\mathbf{a} + \mathrm{grad}\,\alpha = m\,\mathbf{a},$$

or dual equations

$$-\mathrm{div}\,\mathbf{a} = m\,\alpha,$$
$$-\mathrm{rot}\,\mathbf{a} + \mathrm{grad}\,\alpha = m\,\mathbf{a}.$$

We supplement the equations of spinstatics by the equations of gravistatics

$$G_{ij} = T_{ij},$$

where $G_{ij} = R_{ij} - \frac{1}{2}Rg_{ij}$ is Einstein's tensor and T_{ij} can be found from the relation $\delta\pounds = \frac{1}{2}\,T_{ij}\delta g^{ij}$ or from the dual relation $\delta\widetilde{\pounds} = \frac{1}{2}T_{ij}\delta g^{ij}$. The equations of gravistatics have nontrivial solutions, if $T_{ij} \neq 0$, since in the dimension $n = 3$ the curvature tensor $R^k_{ijl} = 0$, if $G_{ij} = 0$. From equations (14) and (15) it follows that $\mathbf{A}$ is an eugenvector of the self-adjoint operators with eigenvalue m. On a given 3-dimensional Riemann manifold one can find eugenevectors and eigenvalues of the operators rot, D and $\widetilde{D}$. We conclude that if the full classification of the 3-dimensional Riemann manifolds is known, all solutions of the equations of spinstatics and gravistatics may be found.

6. SYMPLECTIC STRUCTURE AND SPINDYNAMICS

We start the consideration of spindynamics from the bilateral symmetry to introduce a bilinear quadratic form associated with (9) (see, for comparison, Sec. 3). In other words, we want to find an adequate realization of the bilateral symmetry for the case of the spinning field. In accordance with (5), it is natural to start from the consideration of the operator

$$(R\mathbf{A})_{i_1\cdots i_p} = R^{j_1}_{i_1}\cdots R^{j_p}_{i_p}a_{j_1\cdots j_p} = (-1)^p(a_{i_1\cdots i_p} - 2pt^k a_{k[i_2\cdots i_p}t_{i_1]})$$

that gives the straightforward representation of the bilateral symmetry. We consider this representation only as the starting point for a more fine consideration. Taking into account the definition of the operators $E_{\mathbf{v}}$, $I_{\mathbf{v}}$ and $Q_{\mathbf{v}}$, $\widetilde{Q}_{\mathbf{v}}$, we get sequentially

$$R = Z(E - 2E_{\mathbf{t}}I_{\mathbf{t}}) = -Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}.$$

This representation opens a new possibility to treatise the bilateral symmetry in the case of the spinning field. We define that a system of two spinning fields $\mathbf{A}$ and $\overline{\mathbf{A}}$ possesses the bilateral symmetry if

$$\overline{\mathbf{A}} = Q_{\mathbf{t}}\mathbf{A}.$$

On the other pole we have the following representation of the bilateral symmetry:

$$\overline{\mathbf{A}} = \widetilde{Q}_{\mathbf{t}}\mathbf{A}.$$

In view of this, let us consider two skew-symmetrical bilinear forms

$$[\mathbf{A},\mathbf{B}] = (Q_{\mathbf{t}}\mathbf{A}|\mathbf{B}), \quad \widetilde{[\mathbf{A},\mathbf{B}]} = (\widetilde{Q}_{\mathbf{t}}\mathbf{A}|\mathbf{B}),$$
$$[\mathbf{B},\mathbf{A}] = (Q_{\mathbf{t}}\mathbf{B}|\mathbf{A}) = -(\mathbf{B}|Q_{\mathbf{t}}\mathbf{A}) = -[\mathbf{A},\mathbf{B}].$$

We discovered that the symplectic scalar product is invariant with respect to the following peculiar transformations:

$$\begin{pmatrix}\mathbf{A}'\\ \mathbf{B}'\end{pmatrix} = \begin{pmatrix}\cos\varphi & -\sin\varphi\\ \sin\varphi & \cos\varphi\end{pmatrix}\begin{pmatrix}\mathbf{A}\\ \mathbf{B}\end{pmatrix}, \quad \begin{pmatrix}\mathbf{A}'\\ \mathbf{B}'\end{pmatrix} = \begin{pmatrix}\cosh\varphi & \sinh\varphi\\ \sinh\varphi & \cosh\varphi\end{pmatrix}\begin{pmatrix}\mathbf{A}\\ \mathbf{B}\end{pmatrix}.$$

The rotations in the space $(\mathbf{A},\mathbf{B})$ do not commute with the hyperbolic turns. If we consider the infinitesimal transformations we get the following two by two matrices:

$$i = \begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix}, \quad m = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$$

for which the following relations hold valid $i^2 = -1, m^2 = 1, im + mi = 0$. If we consider $\mathbf{A}\pm i\mathbf{B}$, $\mathbf{A}\pm m\mathbf{B}$, then $\mathbf{A}'\pm i\mathbf{B}' = \exp(\pm i\phi)(\mathbf{A}\pm i\mathbf{B})$, $\mathbf{A}'\pm m\mathbf{B}' = \exp(\pm m\phi)(\mathbf{A}\pm m\mathbf{B})$. We see that the space $(\mathbf{A},\mathbf{B})$ can be oriented by two different ways. We consider the first possibility since it leads to the probability measure in the space of solutions of spindynamics equations. This does not mean that the second possibility has no sense, but nevertheless we restrict here our consideration to the first case only.

Now the complexification has the fundamental geometrical and physical reasons (connected with the temporal structure and the bilateral symmetry) and we shall consider in what follows the complex spinning fields (oriented with respect to the peculiar transformations in question). We again have the bipolar structure but in the following form:

$$\mathbf{\Psi} = \frac{\sqrt{2}}{2}(\mathbf{A} + i\mathbf{B}), \quad \overset{*}{\mathbf{\Psi}} = \frac{\sqrt{2}}{2}(\mathbf{A} - i\mathbf{B}).$$

After the substitution

$$\mathbf{A} = \frac{\sqrt{2}}{2}(\overset{*}{\mathbf{\Psi}} + \mathbf{\Psi}), \quad \mathbf{B} = \frac{i\sqrt{2}}{2}(\overset{*}{\mathbf{\Psi}} - \mathbf{\Psi}),$$

we get

$$[\mathbf{A},\mathbf{B}] = i(Q_{\mathbf{t}}\overset{*}{\mathbf{\Psi}}|\mathbf{\Psi}), \quad \widetilde{[\mathbf{A},\mathbf{B}]} = i(\widetilde{Q}_{\mathbf{t}}\overset{*}{\mathbf{\Psi}}|\mathbf{\Psi}).$$

Our goal is to derive equations of spindynamics following the described above general method (see Sec. 3). This presupposes that at the first stage we derive equations of spindynamics in the geometrical form (using the formalism of the auxiliary metric (7)). After that we represent these equations of spindynamics in the dynamical form which reads: the rate of change with time of the spinning field is equal to the result of action of some operator on this field. To this end, let us consider the bilinear quadratic form in the spin space that is defined by the auxiliary metric $\overline{g}_{ij} = 2t_i t_j - g_{ij}$, $\quad \overline{g}^{ij} = 2t^i t^j - g^{ij}$,

$$\langle \mathbf{A} | \mathbf{B} \rangle = \sum\nolimits_{p=0}^{n} \frac{1}{p!} a_{i_1 \cdots i_p} b_{j_1 \cdots j_p} \overline{g}^{i_1 j_1} \cdots \overline{g}^{i_p j_p}. \tag{18}$$

Since $\overline{g}^{ij} = R^j_k g^{ik}$, we have the following relation between two quadratic forms:

$$\langle \mathbf{A} | \mathbf{B} \rangle = (\mathbf{A} | R\mathbf{B}) = -(\mathbf{A} | Q_{\mathbf{t}} \widetilde{Q}_{\mathbf{t}} \mathbf{B}) = (Q_{\mathbf{t}} \mathbf{A} | \widetilde{Q}_{\mathbf{t}} \mathbf{B}) = (\widetilde{Q}_{\mathbf{t}} \mathbf{A} | Q_{\mathbf{t}} \mathbf{B}). \tag{19}$$

We introduce also the operators $L_{\mathbf{v}}$ and $\widetilde{L}_{\mathbf{v}}$ that correspond to $Q_{\mathbf{v}}$ and $\widetilde{Q}_{\mathbf{v}}$ but are defined by the auxiliary metric (7). We have the following relations between these operators:

$$L_{\mathbf{v}} = \widetilde{Q}_{\mathbf{v}} Z - (\mathbf{t}, \mathbf{v})(\widetilde{Q}_{\mathbf{t}} Z - Q_{\mathbf{t}}), \quad \widetilde{L}_{\mathbf{v}} = Q_{\mathbf{v}} Z + (\mathbf{t}, \mathbf{v})(\widetilde{Q}_{\mathbf{t}} - Q_{\mathbf{t}} Z).$$

It is easy to see that

$$L_{\mathbf{t}} = Q_{\mathbf{t}}, \quad \widetilde{L}_{\mathbf{t}} = \widetilde{Q}_{\mathbf{t}} \quad \text{and} \quad L_{\mathbf{v}} = \widetilde{Q}_{\mathbf{v}} Z, \quad \widetilde{L}_{\mathbf{v}} = Q_{\mathbf{v}} Z,$$

when a vector $\mathbf{v}$ is orthogonal to the stream of time, $(\mathbf{t}, \mathbf{v}) = 0$.

Let $\overline{\Gamma}^i_{jk}$ be the Christoffel symbols belonging to the auxiliary metric $\overline{g}_{ij} = 2t_i t_j - g_{ij}$ and $\overline{\nabla}_i$ be the covariant derivative with respect to this auxiliary connection. We introduce the dual operators

$$\Pi = L_{\overline{\nabla}}, \quad \widetilde{\Pi} = \widetilde{L}_{\overline{\nabla}}.$$

The fundamental Lagrangians of spindynamics take the form

$$\mathcal{L}_{\mathbf{t}} = -\frac{i}{2} \langle \widetilde{Q}_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \Pi \mathbf{\Psi} \rangle + \frac{i}{2} \langle \widetilde{Q}_{\mathbf{t}} \mathbf{\Psi} | \Pi \overset{*}{\mathbf{\Psi}} \rangle + im \langle \widetilde{Q}_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi} \rangle,$$

$$\widetilde{\mathcal{L}}_{\mathbf{t}} = -\frac{i}{2} \langle Q_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \widetilde{\Pi} \mathbf{\Psi} \rangle + \frac{i}{2} \langle Q_{\mathbf{t}} \mathbf{\Psi} | \widetilde{\Pi} \overset{*}{\mathbf{\Psi}} \rangle + im \langle Q_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi} \rangle.$$

Thus, the temporal structure is incorporated into spindynamics in accordance with the new representation of the bilateral symmetry since

$$\langle \widetilde{Q}_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi} \rangle = (Q_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi}), \quad \langle Q_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi} \rangle = (\widetilde{Q}_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \mathbf{\Psi}).$$

We see that the Lagrangians of spindynamics are not invariant with respect to the time reversal T: $t_i \to -t_i$ (they change sign, $\mathcal{L}_{-\mathbf{t}} = -\mathcal{L}_{\mathbf{t}}$, $\widetilde{\mathcal{L}}_{-\mathbf{t}} = -\widetilde{\mathcal{L}}_{\mathbf{t}}$). We conclude that the energy-momentum tensor (that can be derived from the equations $\delta\mathcal{L}_{\mathbf{t}} = \frac{1}{2}T_{ij}\delta\overline{g}^{ij}$, $\delta\widetilde{\mathcal{L}}_{\mathbf{t}} = \frac{1}{2}T_{ij}\delta\overline{g}^{ij}$) changes sign under the time reversal. Thus, the equations of gravidynamics [3]

$$G_{ij} + T_{ij} = \varepsilon t_i t_j$$

are not invariant under the time reversal (Einstein's tensor G_{ij} of the auxiliary metric $\overline{g}_{ij}$ is invariant under time reversal). This means that the gravitational interactions of the spinning fields break invariance with respect to the time reversal and the arrow of time appears. However, this consideration is not full. Indeed, the Lagrangians $\mathcal{L}_{\mathbf{t}}$ and $\widetilde{\mathcal{L}}_{\mathbf{t}}$ change sign under the transformation

$$C: \; \mathbf{\Psi} \to \overset{*}{\mathbf{\Psi}}, \quad \overset{*}{\mathbf{\Psi}} \to \mathbf{\Psi}.$$

We conclude that if a pair of $\mathbf{\Psi}$ and $\overset{*}{\mathbf{\Psi}}$ represents the spinning matter and moves forward in time, then a pair of $\overset{*}{\mathbf{\Psi}}$ and $\mathbf{\Psi}$ represents the spinning antimatter and moves backward in time (see, for comparison, Ref. [7]). Thus, the theory is invariant with respect to the transformation TC and arrow of time disappears. We conclude that an universe is invariant with respect to the time reversal if the galaxies exist that consist of antimatter and time flows in the opposite (with respect to the «usual») direction. The net result is the connection between spindynamics (with its arrow of time) and the visible asymmetry between matter and antimatter. It is evident from our consideration that the time reversal gives the natural explanation of the parity nonconservation in the so-called weak interactions.

From the Lagrangians of spindynamics we derive the following relations for the variations:

$$\delta\mathcal{L}_{\mathbf{t}} = \frac{i}{2}\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|\Pi\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}\rangle + \frac{i}{2}\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|\widetilde{Q}_{\mathbf{t}}\Pi\mathbf{\Psi}\rangle - im\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}\rangle + \frac{i}{2}\nabla_i Y^i + c.c.$$

and

$$\widetilde{\mathcal{L}}_{\mathbf{t}} = \frac{i}{2}\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|\widetilde{\Pi}Q_{\mathbf{t}}\mathbf{\Psi}\rangle + \frac{i}{2}\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|Q_{\mathbf{t}}\widetilde{\Pi}\mathbf{\Psi}\rangle - im\langle\delta\,\overset{*}{\mathbf{\Psi}}\,|Q_{\mathbf{t}}\mathbf{\Psi}\rangle + \frac{i}{2}\nabla_i\widetilde{Y}^i + c.c.$$

The components of the dual vector fields Y^i and $\widetilde{Y}^i$ can be found from the dual relations

$$v_i Y^i = \langle Q_{\mathbf{v}}\delta\,\overset{*}{\mathbf{\Psi}}\,|\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}\rangle, \quad v_i\widetilde{Y}^i = \langle\widetilde{Q}_{\mathbf{v}}\delta\,\overset{*}{\mathbf{\Psi}}\,|Q_{\mathbf{t}}\mathbf{\Psi}\rangle.$$

Our result is the equations of spindynamics in the following geometrical (not explicitly dynamical) form:

$$\frac{1}{2}(\widetilde{Q}_{\mathbf{t}}\Pi + \Pi\widetilde{Q}_{\mathbf{t}})\mathbf{\Psi} = m\widetilde{Q}_{\mathbf{t}}\mathbf{\Psi}, \quad \frac{1}{2}(\widetilde{Q}_{\mathbf{t}}\Pi + \Pi\widetilde{Q}_{\mathbf{t}})\,\overset{*}{\mathbf{\Psi}} = m\widetilde{Q}_{\mathbf{t}}\,\overset{*}{\mathbf{\Psi}}, \tag{20}$$

$$\frac{1}{2}(Q_{\mathbf{t}}\widetilde{\Pi}+\widetilde{\Pi}Q_{\mathbf{t}})\boldsymbol{\Psi}=mQ_{\mathbf{t}}\boldsymbol{\Psi},\quad \frac{1}{2}(Q_{\mathbf{t}}\widetilde{\Pi}+\widetilde{\Pi}Q_{\mathbf{t}})\overset{*}{\boldsymbol{\Psi}}=mQ_{\mathbf{t}}\overset{*}{\boldsymbol{\Psi}}. \tag{21}$$

We should like to remind that equations (20) and (21) are considered as dual to each other and characterize the dual «universes», that is why the full consideration is needed.

We use the equations of spindynamics (20) and (21) to prove a very important result about the existence of the probability measure in the space of solutions and to recognize the unbroken spin symmetry. Our Lagrangians are invariant with respect to the phase transformations and we can put $\delta\boldsymbol{\Psi}=i\phi\boldsymbol{\Psi},\quad \delta\overset{*}{\boldsymbol{\Psi}}=-i\phi\overset{*}{\boldsymbol{\Psi}}$, and the result is the equations

$$\nabla_k C^k=0,\quad \nabla_k\widetilde{C}^k=0.$$

The components C^k and $\widetilde{C}^k$ of the dual 4-currents $\mathbf{C}$ and $\widetilde{\mathbf{C}}$ can be found from the relations

$$v_iC^i=\langle Q_{\mathbf{v}}\overset{*}{\boldsymbol{\Psi}}|\widetilde{Q}_{\mathbf{t}}\boldsymbol{\Psi}\rangle,\quad v_i\widetilde{C}^i=\langle\widetilde{Q}_{\mathbf{v}}\overset{*}{\boldsymbol{\Psi}}|Q_{\mathbf{t}}\boldsymbol{\Psi}\rangle. \tag{22}$$

To find the dual charge densities and the dual physical currents we set

$$\begin{aligned} C^k&=t^k(\mathbf{t},\mathbf{C})+C^k-t^k(\mathbf{t},\mathbf{C})=\rho t^k+J^k,\\ \widetilde{C}^k&=t^k(\mathbf{t},\widetilde{\mathbf{C}})+\widetilde{C}^k-t^k(\mathbf{t},\widetilde{\mathbf{C}})=\widetilde{\rho}t^k+\widetilde{J}^k.\end{aligned}$$

Since $\rho=t_kC^k,\quad \widetilde{\rho}=t_k\widetilde{C}^k$, we have from (22)

$$\rho=\langle Q_{\mathbf{t}}\overset{*}{\boldsymbol{\Psi}}|\widetilde{Q}_{\mathbf{t}}\boldsymbol{\Psi}\rangle=(Q_{\mathbf{t}}Q_{\mathbf{t}}\overset{*}{\boldsymbol{\Psi}}|\widetilde{Q}_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}}\boldsymbol{\Psi})=(\overset{*}{\boldsymbol{\Psi}}|\boldsymbol{\Psi})=\widetilde{\rho}.$$

We have arrived at the needed fundamental result because $(\overset{*}{\boldsymbol{\Psi}}|\boldsymbol{\Psi})\geqslant 0$ and $(\overset{*}{\boldsymbol{\Psi}}|\boldsymbol{\Psi})=0$ provide that $\boldsymbol{\Psi}=0$.

Let A_i be components of the potential $\mathbf{a}$ of the electromagnetic field. In accordance with (22), we write the dual Lagrangians of interactions of the electromagnetic field and the spinning field:

$$\pounds_{\text{int}}=q\,A_iC^i=q\langle Q_{\mathbf{a}}\overset{*}{\boldsymbol{\Psi}}|\widetilde{Q}_{\mathbf{t}}\boldsymbol{\Psi}\rangle,\quad \widetilde{\pounds}_{\text{int}}=q\,A_i\widetilde{C}^i=\langle\widetilde{Q}_{\mathbf{a}}\overset{*}{\boldsymbol{\Psi}}|Q_{\mathbf{t}}\boldsymbol{\Psi}\rangle,$$

where q is a constant of interaction. Taking into account the electromagnetic interactions, we change equations (20) and (21) that can be realized as the substitution $\nabla_i\to\nabla_i-iq\,A_i$.

The unbroken spin symmetry is very interesting since it represents the quaternion structure in spindynamics. It is evident that the operator of temporal parity

$\widetilde{Q}_{\mathbf{t}}$ commutes with the operator $\frac{1}{2}(\widetilde{Q}_{\mathbf{t}}\Pi + \Pi\widetilde{Q}_{\mathbf{t}})$ and hence it acts in the space of solutions of equations (20) (the same for the operators $Q_{\mathbf{t}}$ and $\frac{1}{2}(Q_{\mathbf{t}}\widetilde{\Pi} + \widetilde{\Pi}Q_{\mathbf{t}})$. Further, in the definition of the dual operators H and $\widetilde{H}$ we substitute instead of the Riemann metric g_{ij} the auxiliary metric $\overline{g}_{ij}$, and hence we get the dual operators

$$J = -HQ_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = Q_{\mathbf{t}}H\widetilde{Q}_{\mathbf{t}}, \quad \widetilde{J} = -\widetilde{H}Q_{\mathbf{t}}\widetilde{Q}_{\mathbf{t}} = \widetilde{Q}_{\mathbf{t}}\widetilde{H}Q_{\mathbf{t}},$$

which together with the operators $Q_{\mathbf{t}}$ and $\widetilde{Q}_{\mathbf{t}}$ act in the space of solutions of the corresponding dual equations. The following relations:

$$J^2 = \widetilde{J}^2 = (-1)^{\frac{(n+1)(n+2)}{2}}E$$

give for the case $n = 4$

$$J^2 = -E, \quad \widetilde{J}^2 = -E, \quad JQ_{\mathbf{t}} + Q_{\mathbf{t}}J = 0, \quad \widetilde{J}\widetilde{Q}_{\mathbf{t}} + \widetilde{Q}_{\mathbf{t}}\widetilde{J} = 0.$$

We set

$$I = Q_{\mathbf{t}}, \quad IJ = -JI = K,$$
$$\widetilde{I} = \widetilde{Q}_{\mathbf{t}}, \quad \widetilde{I}\widetilde{J} = -\widetilde{J}\widetilde{I} = \widetilde{K},$$

and the quaternion structure of the unbroken spin symmetry becomes evident. We introduce the phases

$$\varphi = \frac{1}{2}\alpha I + \frac{1}{2}\beta J + \frac{1}{2}\gamma K, \quad \widetilde{\varphi} = \frac{1}{2}\alpha\widetilde{I} + \frac{1}{2}\beta\widetilde{J} + \frac{1}{2}\gamma\widetilde{K},$$

which generate the general elements of the dual quaternion groups G_q and $\widetilde{G}_q$:

$$S = \exp(\varphi), \quad \widetilde{S} = \exp(\widetilde{\varphi}).$$

The quaternion symmetry of spindynamics is the representation of $SU(2)$ group. The Yang–Mills field is the dynamical manifestation of the quaternion symmetry of spindynamics.

7. EQUATIONS OF SPINDYNAMICS IN HAMILTONIAN FORM

From this moment we shall consider equation (20) only and shall not reproduce dual results (here we leave this exercise for a dual observer). Our goal is to represent the equations of spindynamics in the most symmetrical and obviously dynamical (Hamiltonian) form. We have the following relation between the Christoffel symbols of the auxiliary metric $\overline{g}_{ij}$ and the Riemann metric g_{ij}:

$$\overline{\Gamma}^i_{jk} = \Gamma^i_{jk} + 2t^i\nabla_j t_k,$$

and on this ground we shall find relations between the differential operators D and Π ($\widetilde{D}$ and $\widetilde{\Pi}$) to derive dynamical equations of spindynamics from their geometrical representation (20) and (21). As the first step in this direction, we mention the formulae

$$\overline{\nabla}_i S^i = \nabla_i S^i + 2S^k \nabla_{\mathbf{t}} t_k = \nabla_i S^i, \quad \sqrt{-\overline{g}} = \sqrt{g}.$$

Uncovering the expression $\overline{\nabla}^k \psi_{ki_1\cdots i_p} = \overline{g}^{lk} \overline{\nabla}_l \psi_{ki_1\cdots i_p}$, we get

$$-\overline{\nabla}^k \psi_{ki_1\cdots i_p} = \nabla^k \psi_{ki_1\cdots i_p} - 2(\nabla_k t^k) t^l \psi_{li_1\cdots i_p} - \\ - 2t^k \nabla_k t^l \psi_{li_1\cdots i_p} + 2pt^l \psi_{lk[i_2\cdots i_p} \nabla_{i_1]} t^k.$$

Since the operator $D_{\mathbf{t}}$ can be presented as

$$(D_{\mathbf{t}} \Psi)_{i_1\cdots i_p} = \nabla_{\mathbf{t}} \psi_{i_1\cdots i_p} + p\psi_{k[i_2\cdots i_p} \nabla_{i_1]} t^k,$$

we obtain the following representation for Π:

$$\Pi = \widetilde{D}\, Z + 2\Omega\, I_{\mathbf{t}} - 2(\nabla_{\mathbf{t}} + \varphi E) I_{\mathbf{t}},$$

where $\Omega = D_{\mathbf{t}} - \nabla_{\mathbf{t}}, \quad \varphi = \nabla_k t^k$. Further, let us introduce the operators

$$\triangle_i = \nabla_i - t_i \nabla_{\mathbf{t}}, \quad P = Q_{\triangle}, \quad \widetilde{P} = \widetilde{Q}_{\triangle}.$$

The operator Π can be written as follows:

$$\Pi = Q_{\mathbf{t}} \nabla_{\mathbf{t}} + \widetilde{P}\, Z + 2(\Omega - \varphi E) I_{\mathbf{t}}.$$

Since $\overline{\nabla}_i t^k = \nabla_i t^k$,

$$\Pi \widetilde{Q}_{\mathbf{t}} - \widetilde{Q}_{\mathbf{t}} \Pi = -\varphi Z + 2\Omega Z.$$

We write equation (20)

$$\frac{1}{2}(\Pi \widetilde{Q}_{\mathbf{t}} + \widetilde{Q}_{\mathbf{t}} \Pi)\mathbf{\Psi} = m\widetilde{Q}_{\mathbf{t}} \mathbf{\Psi},$$

and after substitutions and multiplication on the operator $\widetilde{Q}_{\mathbf{t}} Q_{\mathbf{t}}$ it takes the following form:

$$(\nabla_{\mathbf{t}} + \frac{1}{2}\varphi - Q_{\mathbf{t}} \widetilde{P} Z - \Omega + E_{\mathbf{t}} I_{\mathbf{t}})\mathbf{\Psi} = -mQ_{\mathbf{t}} \mathbf{\Psi}. \tag{23}$$

The operator evolution $T = \nabla_{\mathbf{t}} + \frac{1}{2}\varphi$ has the property

$$(\mathbf{A}|T\mathbf{B}|) + (\mathbf{B}|T\mathbf{A}) = \nabla_k v^k,$$

where $v^k = (\mathbf{A}|\mathbf{B})\, t^k$ and it is an analog of the evolution operator $D_{\mathbf{t}} + \varphi$ in the theory of electromagnetic and gravitational fields [3]. We find the operator $\nabla_{\mathbf{t}} + \frac{1}{2}\varphi$ as the operator of evolution in spindynamics. Equation (23) being written in the component form looks like a very complicated and asymmetric. However, from this equation we shall derive equations of spindynamics in evidently Hamiltonian and symmetrical form using the results obtained under the derivation of the really Maxwell equations in the evidently dynamical and general covariant form [3]. First of all, remind some definitions of the vector algebra and vector analysis in the four-dimensional and general covariant form [3]. The operator rot is defined for the vector fields as follows:

$$(\mathrm{rot}\mathbf{M})^i = e^{ijkl} t_j \partial_k \mathrm{M}_l = \frac{1}{2} e^{ijkl} t_j (\partial_k \mathrm{M}_l - \partial_l \mathrm{M}_k).$$

It is easy to see that

$$(\mathbf{M},\, \mathrm{rot}\mathbf{N}) + \mathrm{div}[\mathbf{MN}] = (\mathrm{rot}\mathbf{M},\, \mathbf{N}),$$

where

$$[\mathbf{MN}]^i = e^{ijkl} t_j \mathrm{M}_k \mathrm{N}_l$$

is the vector product of two vector fields $\mathbf{M}$ and $\mathbf{N}$, $\mathrm{div}\,\mathbf{M} = \nabla_i \mathrm{M}^i$. Thus, the operator rot is self-adjoint. To find eigenvectors and eigenvalues of this operator, one should consider the equation $\mathrm{rot}\,\mathbf{M} = \sigma \mathbf{M}$. We also mention that $(\mathrm{grad}\varphi)_i = \triangle_i\, \varphi$ and

$$\mathrm{rot}\,\mathrm{grad} = 0, \quad \mathrm{div}\,\mathrm{rot} = 0.$$

To apply formalisms of the vector analysis to equation (23), we consider the mapping of the spinning field

$$\boldsymbol{\Psi} = (\psi, \psi_i, \psi_{ij}, \psi_{ijk}, \psi_{ijkl})$$

onto two scalars κ and μ, two pseudoscalars λ and ν, two vectors $\mathbf{K}$ and $\mathbf{M}$, two pseudovectors $\mathbf{L}$ and $\mathbf{N}$ all orthogonal to the stream of time,

$$(\mathbf{t}, \mathbf{K}) = (\mathbf{t}, \mathbf{L}) = (\mathbf{t}, \mathbf{M}) = (\mathbf{t}, \mathbf{N}) = 0.$$

This mapping is defined as follows:

$$\kappa = t^i \psi_i, \quad \mu = \psi, \quad \lambda = \frac{1}{3!} t_l e^{ijkl} \psi_{ijk}, \quad \nu = \frac{1}{4!} e^{ijkl} \psi_{ijkl},$$
$$\mathrm{K}^i = h^i_m \psi^m, \quad \mathrm{M}^i = t_k \psi^{ki}, \quad \mathrm{L}^i = \frac{1}{3!} h^i_m e^{mjkl} \psi_{jkl}, \quad \mathrm{N}^i = t_k \widetilde{\psi}^{ki},$$

where $h^i_j = \delta^i_j - t^i t_j$, $\quad \widetilde{\psi}^{ki} = \frac{1}{2} e^{kijl} \psi_{jl}$. The inverse mapping has the form

$$\psi = \mu, \quad \psi_i = \mathrm{K}_i + \kappa t_i, \quad \psi_{ij} = t_i \mathrm{M}_j - t_j \mathrm{M}_i + e_{ijkl} t^k \mathrm{N}^l,$$
$$\psi_{ijk} = e_{mijk} \mathrm{L}^m + t^m e_{ijkm} \lambda, \quad \psi_{ijkl} = e_{ijkl}\, \nu.$$

Further, we write the equation of spindynamics (23) in component form setting $p = 0, 1, 2, 3, 4$ and after some transformations arrive at the following symmetric system of spindynamics equations which involves four scalar and four vector equations:

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa &= \nabla_i \mathrm{K}^i - m\,\mu,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda &= \nabla_i \mathrm{L}^i - m\,\nu,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu &= \nabla_i \mathrm{M}^i + m\,\kappa,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu &= \nabla_i \mathrm{N}^i + m\,\lambda,
\end{aligned}
\tag{24}
$$

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{K}_i &= -(\mathrm{rot}\,\mathbf{L})_i + \triangle_i\,\kappa + m\,\mathrm{M}_i,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{L}_i &= (\mathrm{rot}\,\mathbf{K})_i + \triangle_i\,\lambda + m\,\mathrm{N}_i,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{M}_i &= (\mathrm{rot}\,\mathbf{N})_i + \triangle_i\,\mu - m\,\mathrm{K}_i,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathrm{N}_i &= -(\mathrm{rot}\,\mathbf{M})_i + \triangle_i\,\nu - m\,\mathrm{L}_i.
\end{aligned}
\tag{25}
$$

We also write these fundamental equations in the invariant form

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa &= \mathrm{div}\,\mathbf{K} - m\,\mu,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\lambda &= \mathrm{div}\,\mathbf{L} - m\,\nu,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu &= \mathrm{div}\,\mathbf{M} + m\,\kappa,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\nu &= \mathrm{div}\,\mathbf{N} + m\,\lambda,
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{K} &= -\mathrm{rot}\,\mathbf{L} + \mathrm{grad}\,\kappa + m\,\mathbf{M},\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{L} &= \mathrm{rot}\,\mathbf{K} + \mathrm{grad}\,\lambda + m\,\mathbf{N},\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{M} &= \mathrm{rot}\,\mathbf{N} + \mathrm{grad}\,\mu - m\,\mathbf{K},\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{N} &= -\mathrm{rot}\,\mathbf{M} + \mathrm{grad}\,\nu - m\,\mathbf{L}.
\end{aligned}
\tag{27}
$$

The dual operators J and $\widetilde{J}$ represent orient symmetry in spindynamics. Since $J^2 = \widetilde{J}^2 = -E$, we can define the orient and antiorient states by the algebraic equations

$$J\mathbf{\Psi} = \pm i\,\mathbf{\Psi}, \quad \widetilde{J}\mathbf{\Psi} = \pm i\,\mathbf{\Psi}.$$

For the orient state $\widetilde{J}\mathbf{\Psi} = i\mathbf{\Psi}$, we get

$$\kappa = i\,\lambda, \quad \mu = i\,\nu, \quad \mathrm{K}_i = i\,\mathrm{L}_i, \quad \mathrm{M}_i = i\,\mathrm{N}_i,$$

and the equations of spindynamics take the form

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\kappa &= \mathrm{div}\,\mathbf{K} - m\,\mu,\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mu &= \mathrm{div}\,\mathbf{M} + m\,\kappa,
\end{aligned}
\tag{28}
$$

$$
\begin{aligned}
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{K} &= i\,\mathrm{rot}\,\mathbf{K} + \mathrm{grad}\,\kappa + m\,\mathbf{M},\\
(\nabla_{\mathbf{t}} + \tfrac{1}{2}\varphi)\mathbf{M} &= -i\,\mathrm{rot}\,\mathbf{M} + \mathrm{grad}\,\mu - m\,\mathbf{K}.
\end{aligned}
\tag{29}
$$

Equations (28) and (29) (and similar them) correspond to the spin phenomena with given orient parity. From the first principles, it follows that the derived equations of spindynamics describe all phenomena connected with the spin symmetry and spin. We do not need to introduce the artificial concept of isotopic spin.

8. CONCEPT OF INTERNAL SPIN

In this section, we consider a concept of internal spin which is visible in experiments like Stern–Gerlach and after that consider the solution of equations (28) and (29) that corresponds to the artificial physical system known as a hydrogen atom. It has long been possible to dissociate diatomic hydrogen molecules at the laboratory, but the resulting hydrogen atoms recombine in less than a thousandth of second to form new hydrogen molecules. It should be emphasized that all constructions connected with internal spin have sense only in the dimension $n = 4$. We start from the remark that the spin operator introduced earlier has an interesting representation in dimension $n = 4$. Let t^i be components of a given unit vector field, then for any antisymmetric tensor field S_{ij}, we have the following representation:

$$S_{ij} = t_i v_j - t_j v_i + e_{ijkl} t^k w^l,$$

which we used under the derivation of the equations of spindynamics. Here v^i are components of a vector field and w^i are components of a pseudovector field orthogonal to the given vector field t^i, $(\mathbf{t}, \mathbf{v}) = 0$, $(\mathbf{t}, \mathbf{w}) = 0$. The inverse mapping is defined as follows:

$$v_i = t^k S_{ki}, \quad w_i = t^k \widetilde{S}_{ki}, \quad \widetilde{S}_{ij} = \frac{1}{2} e_{ijkl} S^{kl}.$$

Setting $N_{ij} = e_{ijkl} t^k w^l$, we get for the spin operator $Q_{\mathbf{S}} = Q_{\mathbf{t}\wedge\mathbf{v}} + Q_{\mathbf{N}}$. It can be shown that $Q_{\mathbf{N}} = -Q_{\mathbf{t}\wedge\mathbf{w}} H$ and hence

$$Q_{\mathbf{S}} = Q_{\mathbf{v}*\mathbf{w}} = Q_{\mathbf{t}\wedge\mathbf{v}} - Q_{\mathbf{t}\wedge\mathbf{w}} H = Q_{\mathbf{t}}(Q_{\mathbf{v}} - Q_{\mathbf{w}} H).$$

For the dual operator we have

$$\widetilde{Q}_{\mathbf{S}} = \widetilde{Q}_{\mathbf{v}*\mathbf{w}} = \widetilde{Q}_{\mathbf{t}}(\widetilde{Q}_{\mathbf{v}} - \widetilde{Q}_{\mathbf{w}} \widetilde{H}).$$

It is easy to derive the following formulae:

$$Q^2_{\mathbf{v}*\mathbf{w}} = -(v^2 + w^2)E + 2(\mathbf{v}, \mathbf{w})H, \quad \widetilde{Q}^2_{\mathbf{v}*\mathbf{w}} = -(v^2 + w^2)E + 2(\mathbf{v}, \mathbf{w})\widetilde{H}.$$

Now we carry over these results into spindynamics. Let us consider operators analogous to $Q_{\mathbf{v}*\mathbf{w}}$ and $\widetilde{Q}_{\mathbf{v}*\mathbf{w}}$:

$$\begin{aligned}
S_{\mathbf{v}*\mathbf{w}} &= \frac{1}{2} Q_{\mathbf{t}}(L_{\mathbf{v}} - L_{\mathbf{w}} J) = \frac{1}{2}(\widetilde{Q}_{\mathbf{v}} Q_{\mathbf{t}} + \widetilde{Q}_{\mathbf{w}} \widetilde{Q}_{\mathbf{t}} H) Z, \\
\widetilde{S}_{\mathbf{v}*\mathbf{w}} &= \frac{1}{2} \widetilde{Q}_{\mathbf{t}}(\widetilde{L}_{\mathbf{v}} - \widetilde{L}_{\mathbf{w}} \widetilde{J}) = \frac{1}{2}(Q_{\mathbf{v}} \widetilde{Q}_{\mathbf{t}} + Q_{\mathbf{w}} Q_{\mathbf{t}} \widetilde{H}) Z,
\end{aligned}$$

where $\mathbf{t}$ represents in the given case a stream of time. We have

$$S^2_{\mathbf{v}*\mathbf{w}} = \frac{1}{4}[(v^2 - w^2)E - 2(\mathbf{v}, \mathbf{w})\widetilde{Q}_{\mathbf{t}}Q_{\mathbf{t}}H],$$
$$\widetilde{S}^2_{\mathbf{v}*\mathbf{w}} = \frac{1}{4}[(v^2 - w^2)E - 2(\mathbf{v}, \mathbf{w})\widetilde{Q}_{\mathbf{t}}Q_{\mathbf{t}}\widetilde{H}].$$

By definition, $S_{\mathbf{v}*\mathbf{w}}$ and $\widetilde{S}_{\mathbf{v}*\mathbf{w}}$ are the dual operators of internal spin, the first operator with respect to equation (21) and the second operator with respect to equation (20). From the formula for the square of the operators of internal spin we see that it is interesting to consider the anti-Hermitian operator

$$\widetilde{S}_{\mathbf{o}*\mathbf{w}} = \frac{1}{2}Q_{\mathbf{w}}Q_{\mathbf{t}}\widetilde{H}Z = \frac{1}{2}Q_{\mathbf{w}}Q_{\mathbf{t}}H, \quad \mathbf{v} = \mathbf{o}.$$

Setting in this case

$$\widetilde{S}_1 = \frac{1}{2}Q_{\mathbf{u}}Q_{\mathbf{t}}H, \quad \widetilde{S}_2 = \frac{1}{2}Q_{\mathbf{v}}Q_{\mathbf{t}}H, \quad \widetilde{S}_3 = \frac{1}{2}Q_{\mathbf{w}}Q_{\mathbf{t}}H,$$

where $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ are three unit orthogonal vectors

$$[\mathbf{uv}] = \mathbf{w}, \quad [\mathbf{vw}] = \mathbf{u}, \quad [\mathbf{wu}] = \mathbf{v},$$

and taking into account that $\widetilde{S}_1\widetilde{S}_2 = \frac{1}{4}Q_{\mathbf{u}}Q_{\mathbf{v}}$ and $u_iv_j - u_jv_i = -e_{ijkl}t^kw^l$, we get

$$\widetilde{S}_1\widetilde{S}_2 - \widetilde{S}_2\widetilde{S}_1 = \frac{1}{2}Q_{\mathbf{u}\wedge Q_{\mathbf{v}}} = \frac{1}{2}Q_{\mathbf{t}}Q_{\mathbf{w}}H = -\widetilde{S}_3$$

and similar relations for other commutators. Thus, for the Hermitian operators $\widetilde{I}_\mu = -i\widetilde{S}_\mu, \quad \mu = 1, 2, 3$, the following relations hold valid:

$$\widetilde{I}_\mu\widetilde{I}_\nu - \widetilde{I}_\nu\widetilde{I}_\mu = ie_{\mu\nu\lambda}\widetilde{I}_\lambda,$$
$$\widetilde{I}_\mu\widetilde{I}_\nu + \widetilde{I}_\nu\widetilde{I}_\mu = \frac{1}{2}\delta_{\mu\nu}E.$$

The operators $\widetilde{I}_\mu$ of internal spin commute with the operator $\widetilde{J}$ and, hence, there is a state which is an eigenvector of the third component of internal spin and the operator of orient parity

$$\widetilde{J}\mathbf{\Psi} = \pm i\mathbf{\Psi}, \quad \widetilde{I}_3\mathbf{\Psi} = \pm\frac{1}{2}\mathbf{\Psi}.$$

Let $F_{ij} = \partial_iA_j - \partial_jA_i$ be tensor of the electromagnetic field, then

$$F_{ij} = t_i\mathrm{e}_j - t_j\mathrm{e}_i + e_{ijkl}t^k\mathrm{h}^l,$$

where $\mathrm{e}_i = t^k F_{ki}$ are covariant components of electric field strength $\mathbf{e}$ and $\mathrm{h}_i = t^k \widetilde{F}_{ki}$ are covariant components of magnetic field strength $\mathbf{h}$. The operators

$$S_{\mathbf{e}*\mathbf{h}} = \frac{1}{2}(\widetilde{Q}_{\mathbf{e}} Q_{\mathbf{t}} + i\widetilde{Q}_{\mathbf{h}} \widetilde{Q}_{\mathbf{t}} H)Z,$$
$$\widetilde{S}_{\mathbf{e}*\mathbf{h}} = \frac{1}{2}(Q_{\mathbf{e}} \widetilde{Q}_{\mathbf{t}} + iQ_{\mathbf{h}} Q_{\mathbf{t}} \widetilde{H})Z$$

represent additional potential energy due to the internal spin. We represent the density of this energy in the following dual forms:

$$\pounds_{is} = \frac{iq}{m}\langle \widetilde{Q}_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | \widetilde{S}_{\mathbf{e}*\mathbf{h}} \mathbf{\Psi} \rangle, \quad \widetilde{\pounds}_{is} = \frac{iq}{m}\langle Q_{\mathbf{t}} \overset{*}{\mathbf{\Psi}} | S_{\mathbf{e}*\mathbf{h}} \mathbf{\Psi} \rangle.$$

Further, we set

$$\pounds_{is} = -\frac{1}{2} S^{ij} F_{ij}, \quad \widetilde{\pounds}_{is} = -\frac{1}{2} \widetilde{S}^{ij} F_{ij},$$

and varying the vector potential A_i, we get

$$\delta \pounds_{is} = -\nabla_i (S^{ij} \delta A_j) + \nabla_i S^{ij} \delta A_j.$$

The dual vectors

$$S^k = \nabla_i S^{ik}, \quad \widetilde{S}^k = \nabla_i \widetilde{S}^{ik}$$

are the dual spin currents. The spin currents satisfy the equation $\nabla_i S^i = 0$ identically and being the manifestation of the bipolar structure they outline the region of spin phenomena which should be investigated and understood. For the energy density of the internal spin we also introduce the following representation:

$$\pounds_{is} = -\frac{1}{2} S^{ij} F_{ij} = -\frac{1}{2} S^{ij} (t_i \mathrm{e}_j - t_j \mathrm{e}_i + e_{ijkl} t^k \mathrm{h}^l) = \mathrm{e}_k d^k + \mathrm{h}_k \overline{d}^k.$$

The vector

$$d^k = t_i S^{ki}$$

represents the electrical dipole moment of spinning field, and the axial vector

$$\overline{d}^k = \frac{1}{2} e^{kijl} t_i S_{jl}$$

represents the magnetic dipole moment of spinning field. The dual case is considered quite analogously.

We can consider the physical situation when the energy of internal spin and the gravitational effects are very small. In this case, the operator of internal spin may be realized as a quantum number. Let us consider this situation in more detail. We prepare the state of spinning field $\mathbf{\Psi}_{++}$ which is characterized as follows: it is a solution of the equations

$$\widetilde{J}\mathbf{\Psi}_{++} = i\mathbf{\Psi}_{++}, \quad \widetilde{I}_3\mathbf{\Psi}_{++} = \frac{1}{2}\mathbf{\Psi}_{++}$$

and the equations of spindynamics (28) and (29) in 4-dimensional Euclidean space and in a central field of electric charge q. In physical space R^4 equation (4)

$$\left(\frac{\partial f}{\partial u^1}\right)^2 + \left(\frac{\partial f}{\partial u^2}\right)^2 + \left(\frac{\partial f}{\partial u^3}\right)^2 + \left(\frac{\partial f}{\partial u^4}\right)^2 = 1$$

has two fundamental solutions

$$f(u^1, u^2, u^3, u^4) = a^1u^1 + a^2u^2 + a^3u^3 + a^4u^4,$$

where $\mathbf{a} = (a^1, a^2, a^3, a^4)$ is a unit constant vector $(\mathbf{a}\cdot\mathbf{a}) = 1$, and

$$f(u^1, u^2, u^3, u^4) = \sqrt{(u^1)^2 + (u^2)^2 + (u^3)^2 + (u^4)^2}.$$

These solutions define two causal structure in physical space in question. It is easy to see that the first causal structure corresponds to special relativity. Analytically, the lines of time are defined as a solution of the autonomous system of differential equations

$$\frac{du^i}{dt} = g^{ij}\frac{\partial f}{\partial u^j} = t^i = a^i.$$

The general solution is a straight line that goes through the fixed point $\vec{u_0}$:

$$\vec{u}\,(t) = \vec{a}\,(t - t_0) + \vec{u_0}\,. \tag{30}$$

The causal structure defines the interval as follows. Let

$$\vec{u}_r = 2\,\vec{a}\,(\vec{a}\cdot\vec{u}) - \vec{u}$$

be the vector obtained by the reflection of the $\vec{u}$ with respect to the vector $\vec{a}$. Then in the coordinates u^1, u^2, u^3, u^4 the interval can be presented as follows:

$$s^2 = \vec{u}\cdot\vec{u}_r = 2(\vec{a}\cdot\vec{u})^2 - \vec{u}\cdot\vec{u} = |\,\vec{u}\,|^2\cos 2\varphi,$$

where φ is an angle between $\vec{a}$ and $\vec{u}$. To be sure that s is really well-known interval, let us introduce the system of coordinates compatible with causal structure [3]. To this end, suppose that all initial data in (30) belong to the space cross section

$$\vec{a}\cdot\vec{u}_0 = t_0. \tag{31}$$

Let us consider the natural system of four orthogonal unit vectors $\overrightarrow{E}_0=\overrightarrow{a}=(a^1, a^2, a^3, a^4)$, $\overrightarrow{E}_1=(-a^4, -a^3, a^2, a^1)$, $\overrightarrow{E}_2=(a^3, -a^4, -a^1, a^2)$, $\overrightarrow{E}_3=(-a^2, a^1, -a^4, a^3)$. Now the general solution to equation (31) has the form

$$\overrightarrow{u}_0= t_0 \overrightarrow{E}_0 +x \overrightarrow{E}_1 +y \overrightarrow{E}_2 +z \overrightarrow{E}_3 .$$

Substituting this representation into formula (30), we get $\overrightarrow{u}= t \overrightarrow{E}_0 +x \overrightarrow{E}_1 +y \overrightarrow{E}_2 +z \overrightarrow{E}_3$. It is easy to see that in the coordinates t, x, y, z,

$$s^2 = t^2 - x^2 - y^2 - z^2.$$

The second causal structure requires separate investigation.

We use the vector spherical harmonics $\mathbf{Y}^L_{JM}(\theta,\varphi)$ [12]. For the contravariant cyclic components of $\mathbf{Y}^L_{JM}(\theta,\varphi)$ we found the following relations:

$$\begin{aligned}
&\sqrt{\frac{(J-M+1)(2J+3)}{J+1}}\mathbf{Y}^{J+1}_{JM}(\theta,\varphi)-\\
&\qquad -\sqrt{\frac{(J+M+1)(J+2)}{J+1}}\mathbf{Y}^{J+1}_{J+1M}(\theta,\varphi)=\\
&=[\sqrt{2(J-M+2)}Y_{J+1M-1}(\theta,\varphi)]\mathbf{e}_{+1}-[\sqrt{J+M+1}Y_{J+1M}(\theta,\varphi)]\mathbf{e}_0,
\end{aligned}$$

$$\begin{aligned}
&\sqrt{\frac{(J+M)(2J-1)}{J}}\mathbf{Y}^{J-1}_{JM}(\theta,\varphi)-\sqrt{\frac{(J-M)(J-1)}{J}}\mathbf{Y}^{J-1}_{J-1M}(\theta,\varphi)=\\
&\qquad [\sqrt{2(J+M-1)}Y_{J-1M-1}(\theta,\varphi)]\mathbf{e}_{+1}+[\sqrt{J-M}Y_{J-1M}(\theta,\varphi)]\mathbf{e}_0.
\end{aligned}$$

With these formulae and known relations for the vector spherical harmonics [12], the algebraic equations for $\mathbf{\Psi}_{++}$ and equations of spindynamics (28) and (29) can be solved exactly and we arrive at Sommerfeld's formula for the hydrogen level at $q=e$.

We can conclude from this consideration that these configurations of the spinning field represent an electron in a hydrogen atom and they are artificial. Indeed, for the preparation of this state we need to accomplish work over the system equal to the potential energy of internal spin and hence, hydrogen atoms should «annihilate» (and sufficiently quickly). In other words, decomposing molecular hydrogen we should observe a rapid recombination of hydrogen atoms. Really, the gas combines explosively in familiar environments to form molecular hydrogen. Thus, it is natural to suppose that molecular hydrogen is a form of manifestation of the internal spin which appears here and in all other cases as the «electron pairing».

9. SUMMARY

Creation of the new concept of time [3] and the theory of potential fields [4] has resulted here in Theory of Self-Organization of the physical fields based on the principle of absolute objectivity and the existence of the absolutely objective truth. Theory of Self-Organization is the new, self-consistent, and integral structure in which geometry, symmetries, and fields are tightly connected and kept inseparable providing the adequate solution of the most difficult conceptual problems. For example, gefdynamics provides the natural solution of the problem of dark matter and gravidynamics gives the solution of the problem of dark energy identifying the dark energy with the energy of the gravitational field. In the dynamical Theory of Self-Organization all manifolds of phenomena are projected on the set of the four fundamental fields (the gravitational field, the temporal field, the general electromagnetic field, the spinning field) and, hence, these fields can be considered as the order parameters. In the static Theory of Self-Organization the temporal field is absent. The first principles and laws of self-organization of this defining system of fields are established and on this basis the novel theory of the spin phenomena is developed. It is shown, that spin is the diverse phenomenon which involves the concepts of the spinning field, spin symmetry with its bipolar structure, the equations of spinstatics and spindynamics as the natural and fundamental manifestation of bipolar structure, new understanding of the time reversal, unbroken spin symmetry with the concepts of internal spin and the spin current. Since the Theory of Self-Organization is integral structure, spindynamics involves all phenomena connected with spin. Hence, having in our disposal the bipolar structure of spin symmetry, we can maintain that observer does not need to consider the artificial concept of weak or strong isotopic spin. In this case, the geometrical and physical nature of the strong interactions can be understood only in the framework of the nontrivial causal structure defined by the temporal field. We can consider in the first approximation that physical space is 4-dimensional Euclidean space. It is easy to find that in R^4 the basic equation (4) of the temporal field has two fundamental solutions. One of these solutions defines the causal structure that corresponds to the special relativity. The second solution is considered as the starting point for understanding the strong interactions on the basis of the new causal structure tightly connected with rotations. This work is now in progress.

Spindynamics is mainly defined by the new field concept of time and as a consequence, the channel of the electromagnetic interactions, the Yang–Mills field and the internal spin are the manifestation of the temporal structure (with temporal field as a basic element). It is discovered that charge is the dynamical effect, a manifestation of the temporal structure in the framework of spindynamics. We establish that all spin phenomena are connected with spin symmetry which unifies a quite definite subset of really geometrical quantities and as a result the new quality appears in the form of the spinning field. In this sense, spin is the collective effect. There is no special geometrical quantity to describe the manifestations of spin. The spinning field is the entity that can be in the different states defined by the unbroken spin symmetry. We demonstrated this for the case that is known as a hydrogen atom. The example of hydrogen atom shows clearly that in this case we cannot see the manifestations of internal spin, orient symmetry and the Yang–Mills field. We shall find the physical manifestations of internal spin in interplay with the Yang–Mills fields, orient and chiral symmetry, in the physics of atoms and molecules, atomic nucleus, in superconductivity and solid physics. This is evident since all these natural (non artificial) physical systems by their essence are the objects of Theory of Self-Organization and, hence, spindynamics as well. Spinstatics and spindynamics will permit experimentalist to avoid straightforward attempts to change a natural course of physical processes conjugated with enormous expenditure of energy and time. The probabilistic measure of spindynamics has a sufficient reason from different points of view and hence spindynamics is the integral structure and the adequate theory of the spin phenomena.

For comparison, let us look at the historical development of Quantum Mechanics. From the geometrical point of view, in the Schrödinger theory the two real scalar fields are introduced and internal symmetry appears at first in the form of the complex scalar field. Here the principle of sufficient cause is substituted by the experiment but the question remains open. In the Dirac theory already the four complex scalar fields are introduced and, hence, it can be considered as the theory of the Higgs fields with nontrivial internal symmetry defined by the Dirac spin matrices [4]. In the electroweak theory and quantum chromodynamics the number of the scalar fields increases again and again and thus, the artificial internal symmetry is extended. This way of development of the theory looks like artificial and oriented on the explanation of the artificial phenomena since it is impossible to derive the theory of elementary particles from the first principles without understanding the essence of time. But nevertheless the final judgment should be leaved to the future development of the theory and experiment.

The general conclusion is rather evident and lies in the fact that Theory of Self-Organization of Physical Fields helps to understand and unify basic structures, the beauty of physics and geometry, and opens quite new possibilities for investigations in both physics and mathematics. In the last case we mention the informal unification of all mathematics for which we have a ready approach in the form of self-organization.

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