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THE ISOSPIN SYMMETRY BREAKING EFFECTS IN K_{e4} DECAYS

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Эффекты нарушения изотопической инвариантности в K_{e4}-распадах

Теорема Ферми–Ватсона обобщена на случай двух каналов с равными массами и применена к проблеме взаимодействия пионов в конечном состоянии в распаде K_{e4}. Оценено влияние этого эффекта на величину фазы пионного рассеяния, и показано, что он может быть существенным при извлечении длин рассеяния из экспериментальных данных по распаду K_{e4}.

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The Isospin Symmetry Breaking Effects in K_{e4} Decays

The Fermi–Watson theorem is generalized to the case of two coupled channels with different masses and applied to final state interaction in K_{e4} decays. The impact of the considered effect on the phase of $\pi\pi$ scattering is estimated and it is shown that it can be crucial for the scattering lengths extraction from experimental data on K_{e4} decays.

The investigation has been performed at the Veksler and Baldin Laboratory of High Energy Physics, JINR.

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1. INTRODUCTION

The $\pi\pi$ scattering at low energies provides a testing ground for strong interaction study [1]. As the free pion targets cannot be created, the experimental evaluation of $\pi\pi$ scattering characteristics is restricted to the study of a dipion system in a final state of more complicated reactions. One of the most suitable processes for such a study is the K_{e4} decays:

$$K^{\pm} \to \pi^{+}\pi^{-}e^{\pm}\nu, \tag{1}$$

$$K^{\pm} \to \pi^{0}\pi^{0}e^{\pm}\nu. \tag{2}$$

For many years [2, 3], the decay (1) was considered as the cleanest method to determine the s-wave isospin zero scattering length a_0^0. At present its value is predicted by Chiral Perturbation Theory with high precision [4] $a_0^0 = 0.220 \pm 0.005$, thus the extraction of this quantity from experimental data with highest possible accuracy becomes an urgent problem. The appearance of new precise experimental data [5–7] requires the relevant theoretical approaches, taking into account the effects neglected up to now in comparison of experimental data with theoretical models predictions.

The usual method used in the scattering length extraction from decays (1) and (2) is based on the classical works [8,9]. The K_{e4} decay rates are determined by three axial form factors* F, G, R and vector form factor H. Making the partial-wave expansion of the hadronic current with respect to the angular momentum of the dipion system and restricted to s- and p-waves**, the hadronic form factors can be written in the following form:

$$F = f_s e^{i\delta_s(s)} + f_p e^{i\delta_p(s)} \cos\theta_\pi; \quad G = g_p e^{i\delta_p(s)}; \quad H = h_p e^{i\delta_p(s)}. \tag{3}$$

Here $s = M_{\pi\pi}^2$ is the square of the dipion invariant mass and θ_π is the polar angle of pion in the dipion rest frame measured with respect to the flight direction of dipion in the K-meson rest frame. The coefficients f_s, f_p, g_p, h_p can be

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*The form factor R is suppressed by a factor $\dfrac{m_e^2}{S_e}$ and can't be determined from K_{e4} decay.

**As was shown in [10], the contribution of higher waves are small and can be safely neglected.

parameterized as functions of pions momenta q in dipion rest system and invariant mass of lepton pair $s_{e\nu}$. It is widely accepted that the s- and p-wave phases δ_s and δ_p of dipion system due to Fermi–Watson theorem [11] coincide with the corresponding phase shifts in elastic $\pi\pi$ scattering. Nevertheless, this statement is true if the isospin symmetry takes place. On the other hand, in the real world the isospin symmetry breaking effects [12–14] would play an important part leading to corrections, which can be essential in scattering length extraction from K_{e4} decays*.

Recently, in experiment NA48/2 at CERN [16] in the $\pi^0\pi^0$ mass distribution from the decay $K^\pm \to \pi^\pm\pi^0\pi^0$ the effect of cusp was observed, which, as was pointed by N. Cabibbo [17], is the result of isospin symmetry breaking in final-state $\pi^0\pi^0$ interaction, provided by inelastic $\pi\pi$ reactions and difference in masses of neutral and charged pions**.

The same effects can take place in K_{e4} decays. Usually the final-state interaction of two pions in K_{e4} decay is considered using the Fermi–Watson theorem [11], which is valid only in the isospin symmetry limit, i.e., at $m_c = m_0$. The main result of the present work is the generalization of accepted approach to K_{e4} decays, taking into account the inelastic processes in the final state and different masses of neutral and charged pions.

2. FINAL-STATE INTERACTIONS AND ISOSPIN SYMMETRY BREAKING

The s-wave phase shift δ_s has impact only on hadronic form factor F, whereas the form factors G and H depend only on p-wave phase shift δ_p. If one confines oneself to s- and p-waves contributions, the charge exchange process $\pi^+\pi^- \to \pi^0\pi^0$ and vice versa are forbidden for $\pi\pi$ system in the state with $l = 1$ (p-wave) due to identity of neutral pions. Thus, the inelastic transitions can change only the first (s-wave) term in the hadronic form factor F.

Keeping this in mind, let us denote by T_c the decay amplitude corresponding to two charged s-wave pions in the final state, whereas the s-wave amplitude for K-meson decay to two neutral pions is T_n. In one loop approximation of nonperturbative effective field theory (see, e.g., [19]), these amplitudes take the form

$$\begin{aligned} T_n &= \tilde{T}_n(1 + ik_n a_n(s)) + ik_c a_x(s)\tilde{T}_c; \\ T_c &= \tilde{T}_c(1 + ik_c a_c(s)) + ik_n a_x(s)\tilde{T}_n. \end{aligned} \tag{4}$$

*The isospin breaking effects in photoproduction were considered in [15].

**The possibility of cusp in $\pi^0\pi^0$ scattering due to different pion masses in charge exchange reaction $\pi^+\pi^- \to \pi^0\pi^0$ was first predicted in [18].

Here, $\tilde{T}_c, \tilde{T}_n$ are so-called [17,21] «unperturbed» amplitudes of decays (1) and (2); $k_n = \dfrac{\sqrt{s-4m_0^2}}{2}$, $k_c = \dfrac{\sqrt{s-4m_c^2}}{2}$ are the neutral and charged pions momenta in $\pi^0\pi^0$ and $\pi^+\pi^-$ systems with the same invariant mass $s = M_{\pi\pi}^2$. The real functions [17,21] $a_n(s)$, $a_c(s)$, $a_x(s)$ in the isospin symmetry limit ($k_1 = k_2 = k$) can be expressed through the s-wave pion-pion «amplitudes» with definite isospin, which at the threshold coincide with relevant scattering lengths a_0^0, a_0^2 *:

$$a_n(s) = \frac{a_0(s) + 2a_2(s)}{3}; \quad a_c(s) = \frac{2a_0(s) + a_2(s)}{3};$$
$$a_x(s) = \frac{\sqrt{2}}{3}(a_0(s) - a_2(s)). \tag{5}$$

On the other hand, these functions are connected with s-wave phases with certain isospin:

$$a_0(s) = \frac{\tan\delta_0^0(s)}{k}; \quad a_2(s) = \frac{\tan\delta_2^0(s)}{k}. \tag{6}$$

In the isospin symmetry limit, from the rule $\Delta I = 1/2$ for semileptonic decays follows the simple relation between the «unperturbed» amplitudes $\tilde{T}_c = \sqrt{2}\tilde{T}_n$. Substituting these relations in (4), one gets

$$T_n = \tilde{T}_c(1 + ika_0(s)) = \tilde{T}_n\sqrt{1 + k^2a_0^2(s)}e^{i\delta_0^0(s)};$$
$$T_c = \tilde{T}_c(1 + ika_0(s)) = \tilde{T}_c\sqrt{1 + k^2a_0^2(s)}e^{i\delta_0^0(s)}. \tag{7}$$

These equations are nothing else than Fermi–Watson theorem for the pion–pion interaction in final states.

But in the real world, where $m_c \neq m_0$, the Fermi–Watson theorem in its original form is not valid and the two-channel problem in this case demands a special consideration.

The considered picture can be generalized [22] to all orders in $a_j(s)$:

$$T_n = \tilde{T}_n(1 + ik_nf_n) + ik_cf_x\tilde{T}_c;$$
$$T_c = \tilde{T}_c(1 + ik_cf_c) + ik_nf_x\tilde{T}_n. \tag{8}$$

Here f_x, f_c, f_n are the amplitudes of the processes $\pi^+\pi^- \to \pi^0\pi^0$; $\pi^+\pi^- \to \pi^+\pi^-$; $\pi^0\pi^0 \to \pi^0\pi^0$ accounting for different masses of charged and neutral pions. These amplitudes can be expressed through the real functions a_x, a_c, a_n

*Our definition of amplitudes coincides with the one adopted in [20] and differs from that accepted in [19, 21].

and relevant S-matrix elements:

$$\begin{aligned}
S_x &= 2i\sqrt{k_c k_n} f_x = 2i\sqrt{k_c k_n}\frac{a_x(s)}{D};\\
S_n &= 1 + 2ik_n f_n = \frac{(1+ik_n a_n(s))(1-ik_c a_c(s)) - k_n k_c a_x^2(s)}{D};\\
S_c &= 1 + 2ik_c f_c = \frac{(1-ik_n a_n(s))(1+ik_c a_c(s)) - k_n k_c a_x^2(s)}{D};\\
D &= (1-ik_n a_n(s))(1-ik_c a_c(s)) + k_n k_c a_x^2(s).
\end{aligned} \tag{9}$$

In the isospin symmetry limit ($k_n = k_c = k$) using the expressions (8), (9) and relations between S-matrix elements with certain isospin:

$$S_c = \frac{2}{3}S_0 + \frac{1}{3}S_2; \quad S_n = \frac{1}{3}S_0 + \frac{2}{3}S_2, \tag{10}$$

after a bit algebra we obtain

$$\begin{aligned}
S_0 &= \frac{1+ika_0}{1-ika_0} = e^{2i\delta_0}; \quad f_0 = \frac{a_0(s)}{1-ika_0(s)};\\
S_2 &= \frac{1+ika_2}{1-ika_2} = e^{2i\delta_2}; \quad f_2 = \frac{a_2(s)}{1-ika_2(s)}.
\end{aligned} \tag{11}$$

In the real world where the isospin symmetry breaking takes place, equations (8) can be rewritten in the following form:

$$\begin{aligned}
T_n &= \frac{\tilde{T}_n\sqrt{1+k_c^2(a_c(s)-\sqrt{2}a_x(s))^2}}{|D|}e^{i\delta_n};\\
T_c &= \frac{\tilde{T}_c\sqrt{1+k_n^2(a_n(s)-\frac{1}{\sqrt{2}}a_x(s))^2}}{|D|}e^{i\delta_c};\\
\delta_n &= \arctan\frac{k_n a_n(s) + k_c a_c(s)}{1+k_n k_n(a_x^2(s) - a_n(s)a_c(s))} - \\
&\quad - \arctan k_c(a_c(s) - \sqrt{2}a_x(s));\\
\delta_c &= \arctan\frac{k_n a_n(s) + k_c a_c(s)}{1+k_n k_c(a_x^2(s) - a_n(s)a_c(s))} - \\
&\quad - \arctan k_n(a_n(s) - \frac{1}{\sqrt{2}}a_x(s)).
\end{aligned} \tag{12}$$

In the case of exact isospin symmetry $(m_c = m_0)$

$$T_n = \frac{\tilde{T}_n}{\sqrt{1+k^2 a_0^2(s)}} e^{i\delta_0^0}; \qquad T_c = \frac{\tilde{T}_c}{\sqrt{1+k^2 a_0^2(s)}} e^{i\delta_0^0}, \tag{13}$$

which is the manifestation of the Fermi–Watson theorem.
Let us note that, unlike the common wisdom, the generalized phases (12) depend not only on $a_0(s)$, but also on $a_2(s)$.

The obtained relations are valid in the region above the charged pions production threshold $M_{\pi\pi} = 2m_c$. To go under charged pions threshold in reaction (2), one has to make the simple substitution $k_n \to i\kappa$ in the expression for T_n with the result

$$\begin{aligned} T_n &= \frac{\tilde{T}_n[1+\kappa(a_c - \sqrt{2}a_x)]}{D} e^{i\delta_n}; \\ \delta_n &= \arctan \kappa \left(a_n - \frac{\kappa a_x^2}{1+\kappa a_c} \right). \end{aligned} \tag{14}$$

Thus, as in the case of $K \to 3\pi$ [16], in the decay (2) the cusp phenomenon also takes place.

The above expressions completely solve the problem of generalization of the Fermi–Watson theorem to the case of two coupled channels with different masses in the final state. To estimate the numerical difference of the proposed approach from usually accepted, we use the fact that at the leading order in chiral perturbation theory [19, 23] the isospin breaking changes the relations (5) in the following way:

$$\begin{aligned} a_n(s) &= \frac{a_0(s) + 2a_2(s)}{3}(1-\eta); \\ a_c(s) &= \frac{2a_0(s) + a_2(s)}{3}(1+\eta); \\ a_x(s) &= \frac{\sqrt{2}}{3}(a_0(s) - a_2(s))\left(1 + \frac{\eta}{3}\right); \\ \eta &= \frac{m_c^2 - m_0^2}{m_c^2}. \end{aligned} \tag{15}$$

The relations (6) are valid also in the case of isospin symmetry breaking, with simple replacement [19] $k \to k_c$. Substituting these relations in the expression

(12) for phase shift relevant to decay (1), we obtain

$$\delta_c = \arctan\left(\frac{A\tan\delta_0^0 + B\tan\delta_2^0}{1+\lambda W}\right) + \arctan\left(C\tan\delta_0^0 + D\tan\delta_2^0\right);$$
$$A = \frac{2(1+\eta)+\lambda(1-\eta)}{3}; \quad B = \frac{(1+\eta)+2\lambda(1-\eta)}{3};$$
$$C = \frac{4\eta\lambda}{9}; \quad D = -\frac{(3-\eta)\lambda}{3}; \quad \lambda = \frac{k_n}{k_c}; \tag{16}$$
$$W = \frac{2}{9}\left(1+\frac{\eta}{3}\right)^2(\tan\delta_0^0 - \tan\delta_2^0)^2 -$$
$$-\frac{1}{9}(1-\eta^2)(2\tan\delta_0^0 + \tan\delta_2^0)(2\tan\delta_2^0 + \tan\delta_0^0).$$

In Fig. 1 we depicted the dependence of phase $\delta_c(s)$ obtained by expression (16) (solid line) and in the case of exact isospin symmetry (dotted curve). The phases δ_0^0, δ_2^0 were calculated according to the Appendix D of the review [24] for the values of scattering lengths $a_0^0 = 0.225{m_c}^{-1}$, $a_0^2 = -0.03706{m_c}^{-1}$. The isospin breaking effect provided by different masses of neutral and charged pions increase the s-wave phase and hence would has impact on the value of scattering lengths extracted from K_{e4} decays.

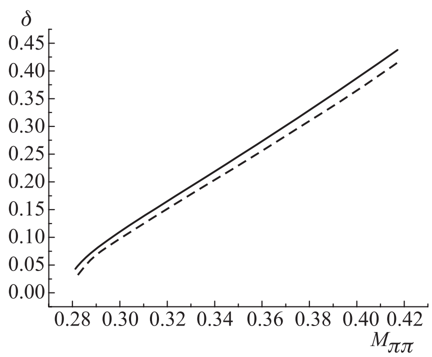

Fig. 1. The dependence of s-wave phase $\delta_c(s)$ on invariant mass of $\pi^+\pi^-$ pair in the case of isospin symmetry (dotted line) and isospin breaking case (solid line)

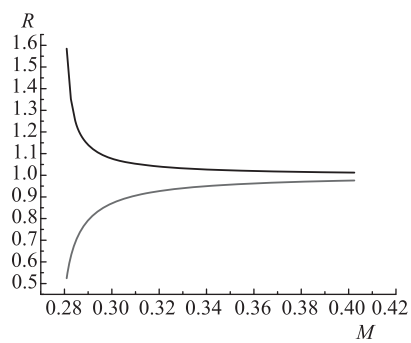

Fig. 2. The dependence of s-wave phase shifts ratios (17) for $\pi^+\pi^-$ (upper curve) and $\pi^0\pi^0$ (lower curve) on invariant mass of the pion pair

In Fig. 2 we show the invariant mass dependence of ratios

$$R_n = \frac{\tan\delta_n(s)}{k_n a_0(s)}; \qquad R_c = \frac{\tan\delta_c(s)}{k_c a_0(s)}. \tag{17}$$

The proposed approach allows one to extract from decays (1) and (2) besides the scattering length a_0^0 the scattering length a_0^2, the challenge which is absent in common approach. At present the high-quality data on K_{e4} from NA48/2 experiment at CERN are published [6] and their fitting by the expressions of the present work would be very useful and can shed light not only on the true values of scattering lengths a_0^0, a_0^2, but also help to understand the limits and validity of the proposed approach.

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