E2-2009-80

N. P. Konopleva*

PHYSICS AND GEOMETRY

Submitted to the Proceedings of the International Seminar ISHEPP XIX,
29 September – 4 October 2008, JINR, Dubna, Russia

*Permanent address: All-Russian Scientific Research Institute
of Electromechanics, Moscow, Russia
E-mail: nelly@theor.jinr.ru; vniiem@orc.ru

Коноплева Н. П. E2-2009-80
Физика и геометрия

Обсуждаются базисные идеи методов описания физических полей и взаимодействий элементарных частиц. Одной из таких идей является концепция геометрии пространства-времени. В связи с этим анализируются методы экспериментальных измерений. Показано, что измерительные процедуры являются источником геометрических аксиом. Рассматривается связь между свойствами симметрии пространства и законами сохранения.

Работа выполнена в Лаборатории теоретической физики им. Н. Н. Боголюбова ОИЯИ.

Препринт Объединенного института ядерных исследований. Дубна, 2009

Konopleva N. P. E2-2009-80
Physics and Geometry

The basic ideas of description methods of physical fields and elementary particle interactions are discussed. One of such ideas is the conception of space-time geometry. In this connection experimental measurement methods are analyzed. It is shown that measure procedures are the origin of geometrical axioms. The connection between space symmetry properties and the conservation laws is considered.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

Preprint of the Joint Institute for Nuclear Research. Dubna, 2009

Talk dedicated to the 150th jubilee of M. Planck

1. INTRODUCTION

Where did physics and geometry go to at the end of the XX century? Is geometry the only simplest physics? Or, conversely, is physics the image of geometry? Where are geometrical axioms coming from?

These and other questions are the subject of this talk.

It is well known that in ancient Egypt geometry was experimental science. It consisted of the different prescriptions how length, area and volume must be measured. Axioms of Euclidean geometry came from experimental data. The ancient measurement procedures based on congruence of the object under consideration with standard [1].

For attainment congruence the object with standard it was necessary to move them in space. S. Lie and F. Klein noted that it is only possible in the special case of homogeneous space. The space of such kind is symmetrical one as a whole. Invariants of its motion group characterize the properties of figures in this space.

S. Lie and F. Klein meant equality as congruence. It was in 1872 [2].

But in 1854 (published in 1868) B. Riemann [3] proposed the quite different geometry conception. He remarked that really we cannot have the space as a whole. Any experiment is performed in the restricted domain of the space. Therefore in practice we must fulfil our measurements step by step. We can only introduce the infinitesimal element of length $dl^2 = g_{ik}dx^i dx^k$, where dx^i — differentials of coordinates, dl — element of length, g_{ik} — metric tensor. Measurement of the finite interval is fulfilling sequentially step by step. Symmetry properties of such space as a whole are unknown. Congruence of standard with the infinitesimal element of length is only approximately possible.

For a long time Lie–Klein's and Riemannian points of view were considered inconsistent with each other. Physicists of XX century inherit this problem.

The teacher of M. Planck, H. Helmholtz developed his own geometry close to Riemannian one [4]. Einstein used Riemannian geometry in General Relativity. Planck and Einstein were friends.

But Special Relativity, where Einstein worked also, is based on homogeneous 4D space-time having the global symmetry of Lie–Klein's type. In a broad fashion

H. Weyl [5] and P. A. M. Dirac [6] used Klein's geometry conception in quantum mechanics.

Are the above-mentioned points of view really inconsistent with each other? Both of them are experimentally confirmed!

In 1925, E. Cartan proposed to construct inhomogeneous Riemannian space from infinitesimal homogeneous Klein's spaces step by step [7]. In that way Lie–Klein space symmetry group became the local one. This point of view was extended to the spaces of any connections. These spaces were named the fibre bundle spaces. They were used by me in the geometrical interpretation of the gauge fields in 1967–1969 and later [8–10].

Now the problem is: How must the experiment be set up to confirm this point of view?

2. EXPERIMENTS IN SPACES WITH GLOBAL GEOMETRY

Geometry of spaces possessing a symmetry group as a whole is named global geometry. Axiomatics of global geometry was proposed by F. Klein in his Erlangen program in 1872 [2]. It is based on the Lie theory of transformation groups [11].

In a globally symmetric space properties of figures are independent on their positions in this space. They are characterized by a set of the space symmetry group invariants. Therefore the group of global space symmetry can be only simple or semisimple Lie group. The space-time symmetry groups in Special Relativity, classical and quantum mechanics are the groups of precisely this kind. Energy, momentum and spin are the invariants of corresponding space-time symmetry groups [12].

The law of energy conservation was established in 1847 by H. Helmholtz. More precisely, Helmholtz demonstrated physical sense of this law in different physical and chemical processes and proved its universality and great meaning. Later Helmholtz demonstrated universality and great physical sense of least action principle.

The problem of the meaning of energy was offered by Faculty of Philosophy of University of Göttingen for awarding of prize for 1887. The work represented by M. Planck was awarded the second prize (the first prize did not present to anybody). This success permitted Planck to become the extraordinary professor of theoretical physics in Kiel University. Planck was happy. At that time theoretical physics was not a separate subject [13].

In 1918, E. Noether proved the theorem (the first Noether theorem) [14], which established close connection of symmetry properties of action integral with conservation laws, in particular, the energy conservation law. According to this theorem the energy conservation law appears when the space-time symmetry group includes time translations as its invariant subgroup. Poincaré, Galilei and Lorentz groups include this invariant subgroup. Therefore in Special Relativity, classical and quantum mechanics energy conservation law is realized. Momemtum conservation law corresponds to space translations considered as invariant subgroup of the global space-time symmetry group. Poincaré, Galilei and Lorentz groups contain space translations as their invariant subgroup. Therefore in Special Relativity, classical and quantum mechanics momentum conservation law is also realized. When time and space translations united with each other in invariant subgroup of Lorentz group we obtained one conservation law of energy-momentum instead of four isolated conservation laws of energy and 3-momentum.

In a globally symmetric space-time an experimenter measures values of dynamical constants corresponding with the conservation laws generated by the symmetry group of space-time as a whole. Elementary particle classification is realized by invariants of the above-mentioned symmetry group.

But what is «as a whole»? The difference between Lobachevski and Euclidean geometries consists in the only one axiom. In Euclidean geometry each straight line has only one straight line parallel to it and passing through the given point outside it. In the Lobachevski geometry each straight line has an infinite number of such lines. Where can we really have the infinite Euclidean plane? Nowhere. But replacement of infinite plate by finite one leads to replacement of Euclidean geometry by Lobachevski one, because in finite domain of space there are many straight lines which do not intersect with given one (i. e., they are parallel to it in terms of Lobachevski geometry). Modern physics does not take into account this fact.

The straight lines of Euclidean geometry are the images of the trajectories, which describe the inertial motion of the centres of mass of the real objects (i. e. without any interaction). *They are inobservable lines because any observation implies some interaction!* In order that a light ray became visible it must pass through some dispersive medium, for example, dust. In vacuum a light ray is invisible. As well known in practice, a light ray can be used as realization of straight line.

In 1828, Hamilton found such representation of the corpuscular light theory according to that determination of a light ray path passing through any inhomogeneous (but isotropic) medium is a special case of usual mechanical problem of material point motion [15]. In 1891, Klein saw that in spaces of higher dimensions each mechanical problem can be reduced to determination of a light ray path passing through some relevant medium [16].

It is necessary to note that in spherical geometry there are not any parallel straight lines, which are in this case named geodesic one. All straight lines passing through two different points nonbelonging to the same straight line are mutually intersect (as meridians on a globe). The parallels on a globe are not straight lines in spherical geometry with the exception of meridian.

3. EXPERIMENTS IN SPACES WITH LOCAL GEOMETRY

The spaces possessing locally given geometry are named the spaces with local geometry. This geometry can be given by infinitesimal quadratic form: $ds^2 = g_{ik}dx^i dx^k$, where ds^2 — infinitesimal interval, dx^i, dx^k — differentials of coordinates x^i, x^k and g_{ik} — metric tensor.

The spaces with global geometry can be given by finite quadratic form: $s^2 = g_{ik}x^i x^k$.

The symmetry group of this space is the group preserving given quadratic form (i. e., the quadratic form is invariant of space symmetry group). Just so the geometry is given in Special Relativity.

As stated above, the space with local geometry as a whole can be asymmetric. Are the experiments possible in such space? What values can we measure in it?

Riemannian space of General Relativity is the space with local geometry. Let us consider experiment problem in General Relativity.

In 1915, in the paper «Foundation of Physics» D. Hilbert [17] proved the theorem according to that invariance of the integral depending on 14 potentials (tensor g_{ik} and vector q_i components) with respect to arbitrary continuous coordinate transformations in 4D Riemannian space-time leads to appearance of connection between left-hand sides of the Euler equations and their derivatives. Therefore four identities appear and four equations of the Euler system are superfluous, namely, four equations become the corollary of the other equations.

The Hilbert theory discribed two classical fields: gravity (by tensor g_{ik}) and electromagnetic field (by vector q_i). The Euler equation system consisted of ten gravitational equations and four electromagnetic ones. Appearance of four identities Hilbert decided to use for realization of Mie ideas of unified field theory. He proposed that four electromagnetic equations are the corollary of ten gravitational ones. Then the electrodynamic phenomena follow from gravity. Hence, electromagnetic energy just as a total one can be expressed in terms of curvature scalar R and its derivatives.

Hilbert derived the Maxwell equations from his gravitational one. His gravitational equation had the form

$$R_{ik} - \frac{1}{2}g_{ik} = T_{ik}^{\mathrm{em}}, \tag{1}$$

where R_{ik} — Ricci tensor, T_{ik}^{em} — tensor of density of electromagnetic energy-momentum (in modern terms).

It is evident that in absence of electromagnetic field the Hilbert gravitational equations coincide with the Einstein vacuum gravitational one.

As Hilbert hoped this unified theory will describe processes inside atoms. In his opinion by this way the possibility appears to turn physics into science similar to geometry which is excellent example of the axiomatic method.

Unfortunately, in this paper Hilbert introduced a vector of energy regardless of the conservation law of it and have used arbitrary out of theory values: vector p^j and tensor $p^{\mu\nu}$. But in reality the conservation laws of energy and 3-momentum disappeared! This is also the corollary of invariance of the Hilbert theory with regards to the arbitrary continuous coordinate transformations, which Einstein proposed.

Against the background of success of Special Relativity created in 1905 this fact was perceived by scientists as a catastrophe. General Relativity principle led to disappearance of the energy conservation law. Scientists tried to go around this difficulty introducing various pseudotensors for artificial construction of the lost conservation law. Unnecessary variables removed by the gauge condition choice. After this procedure the number of equations became equal to the number of the variables and equations became solvable one.

But in 1918, E. Noether generalized the result of her teacher Hilbert and proved the second theorem (second Noether theorem) [14]. This theorem states that the situation discovered by Hilbert is a total one. When an action integral depending on arbitrary number of variable is invariant with respect to the group which transformations depend on the coordinate functions, the identities between the left-hand sides of Euler equations and their derivatives appear. If the group transformations depend on r coordinate functions then r corresponding identities will arise. Naturally, r conservation laws predicted by the first Noether theorem will disappear.

In other words, when the symmetry group transformations become depending on the point of space (or space-time) and the space symmetry becomes a local one, instead of r conservation laws arising in the case of the global space symmetry we shall have r identities between left-hand sides of the Euler equations and their derivatives. These Noether identities reduce the number of independent equations by r.

So, General Relativity acquired its isolated place between other physical theories.

Gradually it became clear that any pseudotensors cannot help to solve the energy problem in GR. The way out of the situation went in the other direction.

In 1921, Einstein wrote «The Meaning of Relativity», where he analyzed the experimental situation described by his theory [18]. Equivalence principle was, in his opinion, so significant physical one that all difficulties which GR met in its development must be regarded as negligible.

In the above paper Einstein connected his famous equations with the density energy conservation law, which he postulated in covariant form. He noted that such a form did not permit to integrate it and derive from it the integral energy conservation law. But this problem he disregarded.

Moreover from his equation having in the right-hand side Maxwellian energy-momentum tensor of electromagnetic field Einstein derived both pairs of the

Maxwell equations under conditions that a current is null. In addition from his covariant energy conservation law Einstein obtained the equation of particle motion along the geodesical line.

The meaning of covariant conservation law of energy-momentum tensor $T^{\mu\nu}$ Einstein saw in the following. Instead of the Newtonian motion equation of material point

$$\frac{d}{dt}P^i = F^i, \tag{2}$$

where P^i — momentum, F^i — external force, in GR we can use the covariant energy-momentum tensor conservation law in the form:

$$\partial_\mu T^{\mu\nu} = f^\nu, \tag{3}$$

where f^μ — density of external forces in analogy with classical mechanics. As in Newtonian case the momentum P^i is 3D space integral of the components of energy-momentum tensor $T^{\alpha 0}$ $(\alpha = 1, 2, 3)$, the last equation can be regarded as similar to the Newtonian one for the densities of corresponding values. He hoped to apply new equations for explanation of cosmic processes but not atomic one. Supporters of Hilbert's point of view criticized Einstein's approach as improper.

Einstein regarded GR as the theory describing a set of experimenters disposed in each point of space. Each of them has the local rulers and watch for measurements of space and time segments. Due to equivalence principle such experimenter falling freely in gravitation field does not feel this field. In its infinitesimal domain the space-time seems to him the Minkowski one. Therefore instead of Riemannian space experimenters study infinite set of local Minkowski space-times, which connected with each other by affine connection. The affine connection coefficients are the images of forces of classical mechanics.

This approach was noticed by E. Cartan, which in 1925 proposed new formulation of Riemannian geometry known as Riemannian geometry in orthogonal frame [7]. By Cartan Riemannian space is constructed step by step from Euclidean spaces associated with each point of Riemannian space. New geometrical objects necessary for this procedure and absent in global geometry are the connection coefficients. It is evident that this formulation permits us to take into account approximate character of the local measurements fulfilled by freely falling experimenters of Einstein. With the help of new mathematics it became possible to describe real experimental situation.

Other important problem of GR is a test body problem. In correspondence with GR axiomatics a test body must be subjected to gravity field action but has not any back influence on this field. What real objects can be the objects of such a kind? At first sight, it seems that a test body must be a small one. But more careful analysis shows that the role of the test bodies can play more correctly the massive bodies of big size. Centres of mass of these bodies must move along the geodesic lines of Riemannian space-time of GR. These lines are similar to the straight lines of Euclidean space-time in some respect, namely, the motion of test bodies along the geodesic lines performs by inertia. In order to have mutual influence of massive test bodies small, these bodies must be located far enough from each other. The estimate shows that masses of test bodies and distances between them must be of cosmic size. Therefore Einstein's theory of gravity is really the theory of cosmic space [19, 20].

4. EXPERIMENTS IN THE FIBRE BUNDLE SPACE AND GAUGE FIELDS

The Cartan approach turns Riemannian space-time of GR into the fibre bundle space, where the fibre is each local Minkowski space-time associated with some point of original Riemannian space-time named in this case the base of given fibre bundle space.

Einstein considered the local Minkowski spaces as the images of the frames where the local experimenter fulfilled his measurements by local rules and watches. Three space and one time dimensions correspond with the instruments used by experimenter for his geometrical investigations.

But what will happen if experimenter intends to measure nongeometrical parameters or to investigate electromagnetic field instead of gravity? How can we geometrically describe this situation? For a long time this question has not any answer. In the '60s of XX century when the fibre bundle space geometry was created the above question got the answer [21].

Set of the local frame origins must belong to the same Riemannian space-time of GR. But the fibre will be differ from the local Minkowski space-time. It must be the image of other instruments for measurements. Since any measurement results form some manifold they can be represented in some space of the parameters. Just this parameter space will play a role of a fibre of a new fibre bundle space. Such a geometrical construction was proposed by me in 1965. It permitted to geometrize all fundamental interactions known in modern physics [9].

In proposed by me unified geometrical theory of all interactions the merits and problems of GR became total. Due to more general approach some of them were solved. For example, the problem of integration of the covariant conservation law of energy-momentum tensor $T^{\mu\nu}$. The answer consists in the new procedure of invariant integration in Riemannian space. As $T^{\mu\nu}$ is a symmetric tensor of rank two the Gauss theorem cannot be used for integration of its divergence all over 4D Riemannian space-time. It is necessary to input some vector ξ^a_ν, which satisfies conditions: $\xi^a_{(\nu;\mu)} = 0$. Then the value $p^{a\mu} = T^{\mu\nu}\xi^a_\nu$ is divergence-free vector in Riemannian space-time. Index a is related to the parameter space which is the fibre and does not take part in the base operations. After integration of relation $p^{a\mu};\ \mu = 0$ with application of the Gauss theorem we shall obtain the set of a integrals invariant with regards to arbitrary continuous tranformations of base coordinates [22].

In GR the physical sense of this procedure becomes clear when the motion groups of Riemannian space are regardered as its symmetry groups. Then vector $P^a = \int p^{a0} d^3x$ turns into the multiplet of a motion integrals, for example, the energy-momentum components. The number of these components depends on the number of motion group parameters. Minkowski, de Sitter and anti de Sitter spaces have 10-parametric motion group. But it is necessary to note that only in Minkowski space we can obtain 4-component invariant set of integrals which corresponds to 4-vector energy-momentum. In de Sitter and anti de Sitter cases energy, momentum and angular momentum form the single multiplet in correspondence with algebra structure of the motion group.

In the geometrical gauge field theory of interactions the vector-potential of each gauge field is the image of connection coefficients of proper fibre bundle space. The fibre of this space is the group space of the gauge group. As the gauge symmetry is considered a local one, each copy of the gauge group space is associated with each point of base. The base is the space of GR. This geometrical picture is being used in all my works, beginning with 1965.

In spite of the fact that metric tensor $g_{\mu\nu}$ is not a connection coefficients, its interpretation as the gauge field was also obtained by me in 1967 [23]. The corresponding equations of $g_{\mu\nu}$ found by me coincides with the Einstein equations. The equation system of the nongravitational gauge fields and gravity generalizes the system of equations of Maxwell and Einstein. The energy-momentum tensor of this field system is sum of the energy-momentum tensors of each gauge field. The covariant conservation law of the total energy-momentum tensor permits one to obtain the particle motion equations as a corollary of the field system equations, as it is in GR [24, 25].

The fundamental question of the role of the Einsteinian equations in the elementary particle theory is now connected with the question of vacuum structure in this theory [26].

5. CONCLUSION

So, it can be chosen two independent directions of geometry and physics linking in XX century. One of them begins from the Hilbert VI problem of physics axioms [27, 28]. Another one was created by Einstein [29]. Hilbert based on the ideas of the Mie electrodynamics which offered to use only fields without material particles for explanation of physical phenomena. Einstein analyzed the nature of gravity and inertia in accordance with Mach ideas.

When Hilbert discovered that locality of the coordinate transformations in 4D Riemannian space-time leads to reduction of independing equation number, and this fact is connected with general relativity principle, he joined the gravity equations for metric tensor $g_{\mu\nu}$ and Maxwell equations for vector potential A_μ in the single system. In this way he, in his opinion, carried the number of independent equations into correspondence with the number of independent variables. Hilbert regarded the obtained equation system as the equations of a single electromagnetic-gravitational field which govern atomic processes. Riemann also assumed a nature unity of above fields.

But Weyl in 1918 called his attention to the local gauge invariance in Maxwell electrodynamics. This is one-parameter transformation of A_μ dependent on coordinates of space-time. The second Noether theorem was known to him. It followed that in the system of the Hilbert equations one more equation is superfluous. Weyl offered new approach to the problem of electromagnetism and gravity unification. He decided to connect the local gauge invariance in electrodynamics and local space symmetry. To this end he input additional principle in GR, namely, the local gauge invariance of 4D interval: $ds^{2'} = \lambda(x)ds^2$.

Unexpectedly, new Weyl principle led to the theory incompatible with GR and Riemannian geometry. Weyl took first step in generalization of Riemannian geometry having applied new connection coefficients disconnected with metrics.

In contrast to Hilbert, Einstein was interested in explanation of nature unity of gravitational and inertial fields. He began with the equivalence principle. In GR the equations of geodesic motion of material particles are simultaneously used with the gravity equations. Einstein added four equations of the inertial motion to the gravity equations instead of four Maxwell equations in Hilbert theory. Here any hidden local invariance is absent. Due to the local coordinate invariance the geodesic lines equations follow from the gravity one. Einstein demonstrated this fact using the covariant conservation law of the energy-momentum tensor. He regarded GR as the theory of cosmic objects motion.

Extension of Weyl point of view taking account of Einstein and Klein positions permitted Cartan to formulate Riemannian geometry in a new fashion and create the base of the geometry of fibre bundle spaces. In last geometry unrestricted number of interacting fields can be described. New moment in modern situation consists in the fundamental role of GR in determination of vacuum structure in the elementary particle theory.

As the final result, we can see that the fibre bundle space geometry and the geometrical gauge field theory unite all fundamental interactions of elementary particles and, on the other hand, permit to describe the processes in the Universe. They include all points of view which were regarded as incompatible for a long time.

Acknowledgments. It is a pleasure to thank the Organizers of the International Seminar ISHEPP XIX for their support of this investigation.

REFERENCES

1. *Einstein A.* Geometrie und Erjahrung, Sitzungsber. preuss. Acad. Wiss. 1921. V. 1. P. 123–130.
2. *Klein F.* Vergleichende Betrachtungen, über neure geometrische Vorschungen, Programm zum Eintritt in die philosophische Facultät und den Senat der Universität zu Erlangen, Erlangen, 1872; Math. Ann., Leipzig, 1893. V. 43. P. 63–109.
3. *Riemann B.* Ueber die Hypothesen, welche der Geometrie zu Grunde liegen, Nach. von der Kön // Ges. von Wissenschaften, Göttingen. 1868. V. 13. P. 133–152.
4. *von Helmholtz H.* Ueber die Tatsachen, die der Geometrie zu Grunde liegen, Nach. von der Kön // Ges. von Wissenschaften, Göttingen. 1868. V. 14. P. 193–221.
5. *Weyl H.* The Theory of Groups and Quantum Mechanics. Dover Publications, Inc., 1931.
6. *Dirac P. A. M.* The Principles of Quantum Mechanics, 4th ed. Oxford: Clarendon Press, 1958.
7. *Cartan E.* La théorie des groupes et la géométrie // L'Enseignement mathématique. 1927. P. 200–225.
8. *Konopleva N. P.* Geometrical Description of Interactions. Ph. D. Thesis. M.: Lebedev Institute of Physics, 1969.
9. *Konopleva N. P.* Geometrical Description of the Gauge Fields // Proc. of the Intern. Sem. on Vector Mesons and Electromagn. Interactions / Ed. A. M. Baldin. Dubna: JINR, 1969.
10. *Konopleva N. P., Popov V. N.* Gauge Fields. Chur; London; New York: Harwood Academic Publishers, 1981.
11. *Lie S.* Theorie der Transformationsgruppen. Unter Mitwirkung v. Dr. F. Engel bearbeitet. I. Allgemeine Eigenschaften, Leipzig, 1888; II. Theorie der Berührungstransformationen, Leipzig, 1890; III. Spezielle Untersuchungen, Leipzig, 1893.
12. *Konopleva N. P., Sokolik H. A.* Symmetries and Physical Theories Types // Voprosy Philos. 1972. V. 1. P. 118–127 (in Russian).
13. *Planck M.* Wissenschaftliche Selbstbiographie. Leipzig, 1955.
14. *Nöther E.* Invariante Variation probleme, Nach. von der Kön // Ges. der Wissenschaften zu Göttingen, Math.-Phys. Kl., Heft, 1918. V. 2. P. 235–258.
15. *Hamilton W.* On a General Method of Expressing the Paths of Light and of the Planets. Dublin University Review, 1933.
16. *Klein F.* Über neuere englische Arbeiten zur Mechanik, Gesammelte mathematische Abhandlungen. Berlin, 1923.
17. *Hilbert D.* Die Grundlagen der Physik, Nach. von der Kön // Ges. der Wissenschaften zu Göttingen, Math.-Phys. Kl., Heft, 1915. V. 3. P. 395–407.
18. *Einstein A.* The Meaning of Relativity. N.Y.: Princeton Univ. Press., 1921.
19. *Fock V. A.* Theory of Space, Time and Gravitation. M.: GTTL, 1955 (English: London: Pergamon Press, 1959).
20. *Konopleva N. P.* Gravitational Experiments in Space // Sov. Phys. Usp. 1977. V. 20. P. 973.

21. *Konopleva N. P.* Coordinate Transformations and Compensating Fields // Vestnik MGU. 1965. ser. III, No. 3. P. 73–80.
22. *Konopleva N. P.* On the Integral Conservation Laws in General Relativity // Dokl. AN SSSR. 1970. No. 6. P. 1070–1073.
23. *Konopleva N. P.* Variational Formalism for Infinite Groups and Field Theory. Gravitation and the Theory of Relativity. Kazan State University, 1968. Nos. 4–5. P. 67.
24. *Konopleva N. P.* On the Motion of Matter in the Geometrical Gauge Field Theory // Proc. of the Intern. Sem. ISHEPP XVII, 27 September – 2 October 2004, Dubna, Russia.
25. *Konopleva N. P.* Relativistic Physics as Geometry // Proc. of the Intern. Sem. ISHEPP XV, 25–29 Sept. 2000, Dubna, Russia. Dubna, 2001. P. 77–86.
26. *Konopleva N. P.* Hyperbolical Instantons and Gauge Fields Vacuum Structure. JINR Commun. E2-98-388. Dubna, 1998. P. 1–11.
27. *Konopleva N. P.* VI Hilbert's Problem and S. Lie Infinite Groups. JINR Commun. E2-99-301. Dubna, 1999. P. 1–8.
28. *Hilbert D.* // Gesammelte Abhandlungen. 1935. V. III. P. 290–329.
29. *Einstein A.* // Sitz. Preuss. Akad. Wiss. 1916. V. 2. P .1111–1116.

Received on May 29, 2009.

Корректор *Т. Е. Попеко*

Подписано в печать 26.08.2009.
Формат 60 × 90/16. Бумага офсетная. Печать офсетная.
Усл. печ. л. 0,93. Уч.-изд. л. 1,29. Тираж 415 экз. Заказ № 56706.

Издательский отдел Объединенного института ядерных исследований
141980, г. Дубна, Московская обл., ул. Жолио-Кюри, 6.
E-mail: publish@jinr.ru
www.jinr.ru/publish/